

DOT-FAA-AFS-420-91

Flight Procedure Standards Branch
Flight Technologies and Procedures Division
Flight Standards Service

**Analysis of LPV Approach
Flight Test Data for
Categories A, B, and C Aircraft**

Dr. David N. Lankford
Gerry McCartor
Dr. David Stapleton, ISI
Donna Templeton, Editor

U.S. Department of Transportation
Federal Aviation Administration
Mike Monroney Aeronautical Center
Oklahoma City, OK 73125

May 2002

Technical Report

U.S. Department of Transportation
Federal Aviation Administration

NOTICE

This document is disseminated under the sponsorship of the U.S. Department of Transportation in the interest of information exchange. The United States Government assumes no liability for the contents or use thereof.

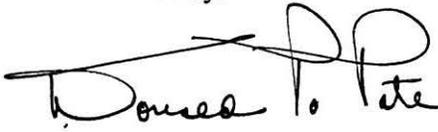
The United States Government does not endorse products or manufacturers. Trade or manufacturers' names appear herein solely because they are considered essential to the objective of this report.

DOT-FAA-AFS-420-91

Federal Aviation Administration
Flight Procedure Standards Branch
Mike Monroney Aeronautical Center
Oklahoma City, OK 73125

**ANALYSIS OF LPV APPROACH FLIGHT TEST DATA
FOR CATEGORIES A, B, AND C AIRCRAFT**

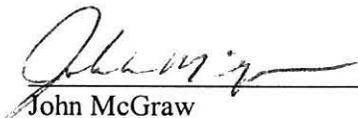
Reviewed by:

 9/18/02

Donald P. Pate
Manager, Flight Procedure
Standards Branch

Date

Released by:

 10/18/02

John McGraw
Manager, Flight Technologies and
Procedures Division

Date

May 2002

Technical Report

1. Report No. DOT-FAA-AFS-420-91	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Analysis of LPV Approach Flight Test Data for Categories A, B, and C Aircraft		5. Report Date May 2002
6. Author(s) Dr. David Lankford/Gerry McCartor Dr. David Stapleton, ISI		7. Performing Organization Code
8. Performing Organization Name and Address Federal Aviation Administration Standards Development Branch P.O. Box 25082, Oklahoma City, OK 73125		9. Type of Report and Period Covered Technical Report
10. Sponsoring Agency Name and Address Federal Aviation Administration Flight Procedure Standards Branch P.O. Box 25082, Oklahoma City, OK 73125		
11. Supplementary Notes		
12. Abstract The Total System Error (TSE), Flight Technical Error (FTE), and Navigation System Error (NSE) for category A, B, and C precision test approaches of aircraft by means of the Global Positioning System/Wide Area Augmentation System (GPS/WAAS) are analyzed. Statistical plots are presented. The total system error is shown to be dominated by flight technical error, as expected; and probability distributions for the components of the error near the glidepath are observed to be similar to those predicted by the Collision Risk Model (CRM) for Instrument Landing System approaches. (More flight data would be necessary in order to compare CRM predictions with flight statistics at greater deviations from the glidepath.) NSE is found to be independent of along-track position to within testing capabilities. The Atlantic City navigation system error is shown to contain short- and long-term components. Short-term statistics, computed from approaches performed within a single flight of approximately two and a half-hours, and long-term statistics, computed from the various averages of flights that contain five or more approaches, are calculated. It is observed that the root-sum-squares of short- and long-term navigation system error standard deviations match the total standard deviations. No correlation is observed between the horizontal and vertical components of any of the error types. The effect of system latency is not evaluated since latency values are not available for the test approaches. Tabulated TSE, NSE, and FTE from the tests are provided.		
13. Key Words WAAS Flight Test Approach Flight Technical Error Navigation System Error Total System Error LPV	14. Distribution Statement Controlled by AFS-420	
Unclassified	16. Security Classification of This Page Unclassified	

EXECUTIVE SUMMARY

The Total System Error (TSE), Flight Technical Error (FTE), and Navigation System Error (NSE) for category A, B, and C precision test approaches of aircraft by means of the Global Positioning System/Wide Area Augmentation System (GPS/WAAS) are analyzed. Category C calculations are based upon test flight data collected at the Federal Aviation Administration Technical Center (FAATC), in Atlantic City, New Jersey. Category A and B results are derived from data collected by the University of Oklahoma, at Westheimer Airport, in Norman, Oklahoma.

The TSE and FTE statistics (where $TSE = NSE + FTE$) are found to be nearly equal because NSE is relatively small, and the TSE probability distribution is observed to be graphically similar to that predicted by the Collision Risk Model (CRM) for Instrument Landing System (ILS) approaches within about three standard deviations of the glideslope. A greater number of test flights would be necessary; however, in order to validate or invalidate the CRM as a flight error model far from the glidepath. Tabulated TSE statistics include the following.

<i>Along-Track Position (m)</i>	<i>Category A Components (m)</i>		<i>Category B Components (m)</i>		<i>Category C Components (m)</i>	
	<i>Lateral</i>	<i>Vertical</i>	<i>Lateral</i>	<i>Vertical</i>	<i>Lateral</i>	<i>Vertical</i>
7800	19	-5	17	0	46	6
4200	8	0	23	-4	20	6
1200	6	9	17	5	4	5

**MEAN VALUES FOR LATERAL AND VERTICAL
COMPONENTS OF TOTAL SYSTEM ERROR**

<i>Along-Track Position (m)</i>	<i>Category A Components (m)</i>		<i>Category B Components (m)</i>		<i>Category C Components (m)</i>	
	<i>Lateral</i>	<i>Vertical</i>	<i>Lateral</i>	<i>Vertical</i>	<i>Lateral</i>	<i>Vertical</i>
7800	82	27	44	26	55	14
4200	53	10	35	15	36	9
1200	23	6	21	6	9	5

**STANDARD DEVIATIONS IN LATERAL AND VERTICAL
COMPONENTS OF TOTAL SYSTEM ERROR**

Navigation system errors appear to be independent of along-track position in each set of tests, and are summarized in the following table.

<i>Location</i>	<i>Mean Cross-Track NSE (m)</i>	<i>Std. Dev. Cross-Track NSE (m)</i>	<i>Mean Vertical NSE (m)</i>	<i>Std. Dev. Vertical NSE (m)</i>
Atlantic City, NJ	-0.7	1.3	.4	1.7
Norman, OK (Cat. B)	-0.1	.8	.6	1.0
Norman, OK (Cat. A)	-0.1	1.0	-0.7	1.9

NAVIGATION SYSTEM ERROR MEAN AND STANDARD DEVIATION VALUES

The Atlantic City NSE is shown to contain short- and long-term components. Short-term NSE statistics (computed from approaches performed within a single flight of approximately two and a half hours) and long-term NSE statistics (computed from the flight various averages of flights that contain five or more approaches) are presented in tables and graphs. It is observed that the root-sum-squares of short- and long-term NSE component standard deviations approximately match the overall Atlantic City component standard deviations. (University of Oklahoma data was not tested for these components).

<i>Short-Term Standard Deviation (m)</i>		<i>Long-Term Standard Deviation (m)</i>	
<i>Lateral</i>	<i>Vertical</i>	<i>Lateral</i>	<i>Vertical</i>
.6	1.1	1.0	1.2

**ATLANTIC CITY NAVIGATION SYSTEM ERROR
SHORT- AND LONG TERM STANDARD DEVIATIONS**

No correlation is evident between the vertical and lateral components, for any of the three error types.

TABLE OF CONTENTS

1.0	Test Flight Data	1
1.1	Test Flight Procedures	1
1.2	Test Flight Error Computations	4
2.0	GPS/WAAS Precision Approach Error Statistics	5
2.1	Category C Statistics	5
2.1.1	Correlation of Category C Error Components	15
2.1.2	Comparison with the Collision Risk Model	22
2.2	Category B Statistics	23
2.3	Category A Statistics	31
3.0	Summary	39
	Bibliography	41

LIST OF ILLUSTRATIONS

TABLES

Table 1.	Category C Total System Error Statistics for GPS/WAAS Test Approaches	13
Table 2.	Category C Navigation System Error Statistics for GPS/WAAS Test Approaches	14
Table 3.	Category C Flight Technical Error Statistics for GPS/WAAS Test Approaches	15
Table 4.	Lateral/Vertical Correlation Coefficients for Category C Test Approach Data	16
Table 5.	Category B Total System Error Statistics for GPS/WAAS Test Approaches	30
Table 6.	Category B Navigation System Error Statistics for GPS/WAAS Test Approaches	30
Table 7.	Category B Flight Technical Error Statistics for GPS/WAAS Test Approaches	31
Table 8.	Category A Total System Error Statistics for GPS/WAAS Test Approaches	38
Table 9.	Category A Navigation System Error Statistics for GPS/WAAS Test Approaches	38
Table 10.	Category A Flight Technical Error Statistics for GPS/WAAS Test Approaches	39

FIGURES

Figure 1.	Typical Vertical Full Scale Deflection and the Vertical TSE for February 5, 1999, PM Approach #5	3
Figure 2.	Typical Lateral Full Scale Deflection and the Lateral TSE for February 5, 1999, PM Approach #5	4
Figure 3.	Total System Error 2-Sigma and 6-Sigma Footprint for Category C Test Flights	6
Figure 4.	Total System Error 2-Sigma and 6-Sigma Profile for Category C Test Flights	7
Figure 5.	Navigation System Error 2-Sigma and 6-Sigma Flights Footprint for Category C Test	8
Figure 6.	Navigation System Error 2-Sigma and 6-Sigma Profile for Category C Test Flights	9
Figure 7.	Flight Technical Error 2-Sigma and 6-Sigma Footprint for Category C Test Flights	10
Figure 8.	Flight Technical Error 2-Sigma and 6-Sigma Profile for Category C Test Flights	11
Figure 9.	FAATC Flight Technical Error 2-Sigma Ellipses at 1 KM Along-Track Intervals, Viewed Down the Glidepath	12
Figure 10.	GPS/WAAS Category C Aircraft Pierce Points (TSE) at 5,000 Meters Along-Track with Two-Sigma Ellipse	17
Figure 11.	Category C GPS/WAAS Aircraft Pierce Points (TSE Points) at 2000 Meters Along-Track with Two-Sigma Ellipse	18
Figure 12.	Category C GPS/WAAS NSE at 5000 Meters Along-Track with Two-Sigma Ellipse	19
Figure 13.	Category C GPS/WAAS NSE at 5000 Meters Along-Track, for Flights Containing Five or More Admissible Approaches	21
Figure 14.	CRM and Category C Test Flight Vertical Cumulative Distributions (Showing Probabilities of being Above the Mean by at Least the Amount Shown)	22

Figure 15.	CRM and Category C Test Flight Lateral Cumulative Distributions (Showing Probabilities of being Right of the Mean by at Least the Amount Shown)	23
Figure 16.	Total System Error 2-Sigma and 6-Sigma Footprint for Category B Test Flights	24
Figure 17.	Total System Error 2-Sigma and 6-Sigma Profile for Category B Test Flights	25
Figure 18.	Navigation System Error 2-Sigma and 6-Sigma Footprint for Category B Test Flights	26
Figure 19.	Navigation System Error 2-Sigma and 6-Sigma Profile for Category B Test Flight	27
Figure 20.	Flight Technical Error 2-Sigma and 6-Sigma Footprint for Category B Test Flights	28
Figure 21.	Flight Technical Error 2-Sigma and 6-Sigma Profile for Category B Test Flights	29
Figure 22.	Total System Error 2-Sigma and 6-Sigma Footprint for Category A Test Flights	32
Figure 23.	Total System Error 2-Sigma and 6-Sigma Profile for Category A Test Flights	33
Figure 24.	Navigation System Error 2-Sigma and 6-Sigma Footprint for Category A Test Flights	34
Figure 25.	Navigation System Error 2-Sigma and 6-Sigma Profile for Category A Test Flight	35
Figure 26.	Flight Technical Error 2-Sigma and 6-Sigma Footprint for Category A Test Flights	36
Figure 27.	Flight Technical Error 2-Sigma and 6-Sigma Profile for Category A Test Flights	37

1.0 TEST FLIGHT DATA

Category A, B, and C precision test approaches were flown by means of GPS/WAAS guidance (1). The category C approaches were conducted at the FAATC in Atlantic City, New Jersey, from September 1998 through February 1999. The category A and B approach tests were made by the University of Oklahoma at Westheimer Airport in Norman, Oklahoma. Category B testing took place from January through July of 2001 and category A testing occurred in 1998.

This paper provides a statistical evaluation of the test flight data and briefly discusses its impact upon the proposed 27:1 Obstacle Clearance Surface (OCS) for LPV (2).

1.1 TEST FLIGHT PROCEDURES

In the FAATC tests, a prototype (emulation) of the GPS/WAAS system was provided by means of the National Satellite Test Bed (NSTB), and a Beechcraft BE-200, King Air aircraft performed a total of 116 usable category C approaches. The WAAS emulation used data from Wide Area Reference Stations (WARS) that were located within the United States and Southern Canada and were arranged in an approximate worst-case geometry. In this regard, data from the reference station located at Atlantic City was not used, and no stations were located substantially east of Atlantic City due to the presence of the Atlantic Ocean. The three closest reference stations were at Bangor, Maine, Anderson, South Carolina, and Dayton, Ohio, and were each approximately the maximum anticipated operational (U.S. mainland) distance of 450 to 500 nautical miles away. The master station which collected the reference station data and processed it to calculate the WAAS "differential corrections," was located at Atlantic City, New Jersey. This station sent the emulated WAAS signal to a terrestrial up-link station at Southbury, Connecticut, which transmitted it to an Inmarsat GEO stationary satellite (GEO) for broadcast. It is intended that the WAAS will eventually use its GEOs as additional ranging sources, but this was not the case for any tests discussed in this paper. Each test flight then obtained its emulated WAAS signal either from the GEO or from a VHF transmitter that provided the same signal with an analogous transmission delay. Selective availability (SA) was enabled during all test flights; and flights were generally conducted in pairs, with each pair of flights completed on a single day, by the same flight crew. Flights generally required a period of about two and a half hours and included approximately eight precision GPS/WAAS approaches. Some data reduction was necessary to omit portions of approaches when the NSTB WAAS became invalid or the pilot was beginning a (planned) missed approach. Details of the data reduction appear in (3). The final product consisted of 93 approach sets that were completed on runway 31 and 23 that were accomplished on runway 13.

In the University of Oklahoma test approaches the WAAS corrections were also calculated at the Atlantic City master station, but WAAS corrections were based upon data from reference stations located in the Oklahoma vicinity. The reference station at Oklahoma City was not used to compute corrections (nor was it available at the time). A total of 43 usable category B test approaches were performed in a Rockwell Commander AC 680W; and of these, 26 were performed on runway 17 and 17 were executed on runway 35. There were 53 usable category A approaches performed in a Piper Seneca; consisting of 32 approaches on runway 17 and 21 approaches on runway 35. SA was included during category A testing but not during category B

tests, and the true (developmental testing stage) WAAS signal was applied for category B corrections. Reduction of the University of Oklahoma category A data included deletion of some of the vertical TSE and FTE for approach 37 due to the presence of a blunder. The NSE values from 7,350 to 7,520 meters were omitted from approach 39 due to the presence of an 80+ meter spike that would have been filtered out by an operational WAAS. Approach 52 had invalid in-flight receiver data at distances less than 1,800 meters, and approach 54 had invalid data at distances less than 2,600 meters from threshold. Approaches 28 and 13 were cut at 2,000 meters, approach 35 was cut at 2,500 meters, and approaches 31, 36, and 42 were cut at 1,500 meters for technical reasons. Category B reduction consisted of cutting approach 23 at 1,500 meters.

For all (category A, B, and C) tests, two GPS/WAAS receivers were carried on board each flight (in which each flight typically included several GPS/WAAS precision approaches). The first of these was an "in-flight" receiver that was used to obtain and interpret the GPS/WAAS signal in real-time and whose data was presented through the instruments to the pilot. The second receiver was a truth position reference system, or "truth receiver," that provided no real-time data, but generated output suitable for extremely accurate position calculations by means of post-flight processing.

Aircraft were hand-flown in an instrument environment, using a 3.0-degree glidepath to a decision height of 200 feet above touchdown. All procedures were flown using raw data. (The category C aircraft had an Electronic Flight Information System (EFIS) but it was set to provide raw data to the pilots.)

Full Scale Deflection (FSD) geometry for GPS/WAAS testing was similar to the implementation for ILS approaches and is based upon angular vertical and lateral error from the glidepath during most of each descent. According to the MOPS, the vertical full-scale deflection value is the minimum of 500 feet or an angular error of .25 times the glidepath angle (from the glidepath) until a minimum deflection of 50 feet is reached, as shown in figure 1. The vertical error angle is measured from the point of intersection of the glidepath with a plane tangent to the earth at threshold in the 1984 World Geodetic System (WGS-84) earth model. Lateral full-scale deflection occurs at the minimum of one nautical mile or an angular deflection of two degrees from the glidepath. (The angle from the glidepath is measured from the point on the FAS that lies 10,000 feet behind threshold.) This FSD continues until a minimum value of 350 feet is achieved, as shown in figure 2. These nominal values were normalized (i.e., scaled) for use in the flight tests, according to runway requirements.

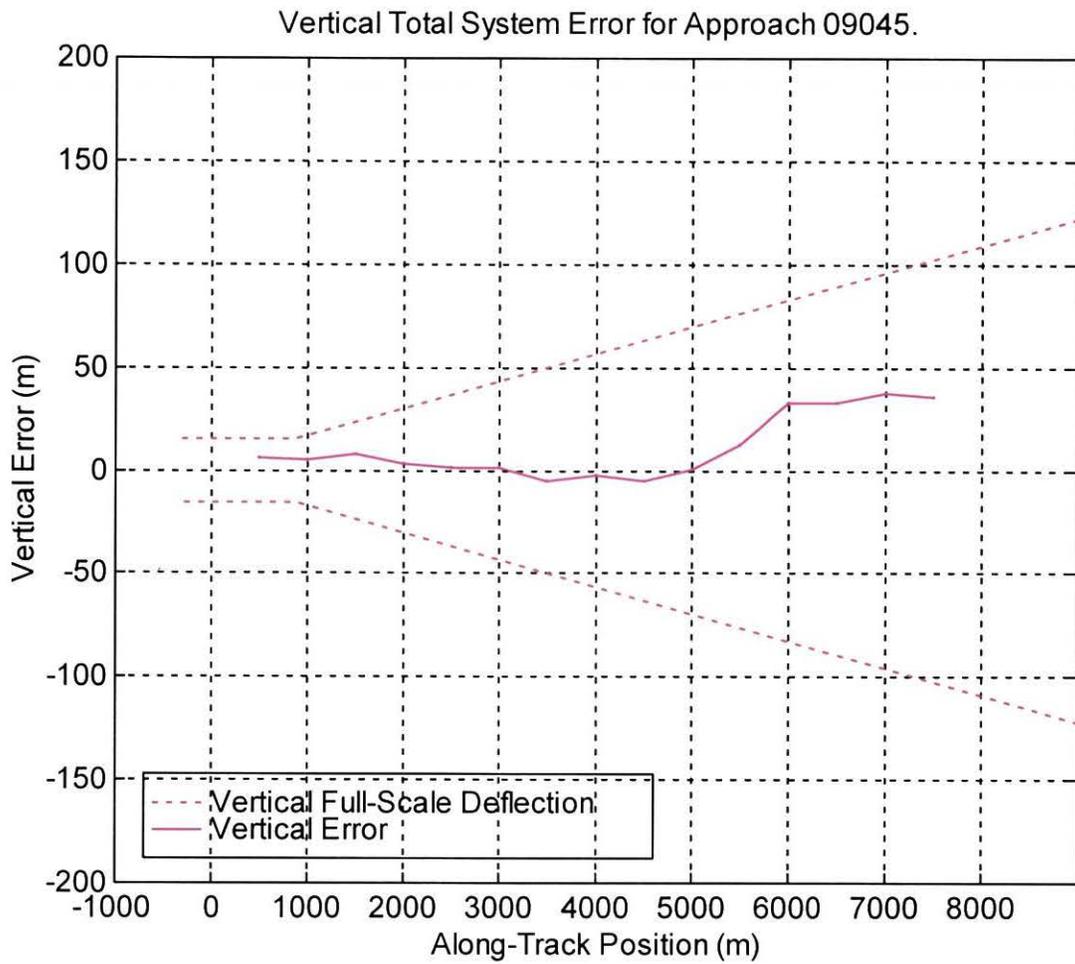


Figure 1: TYPICAL VERTICAL FULL-SCALE DEFLECTION AND THE VERTICAL TSE FOR FEBRUARY 5, 1999 PM APPROACH 5

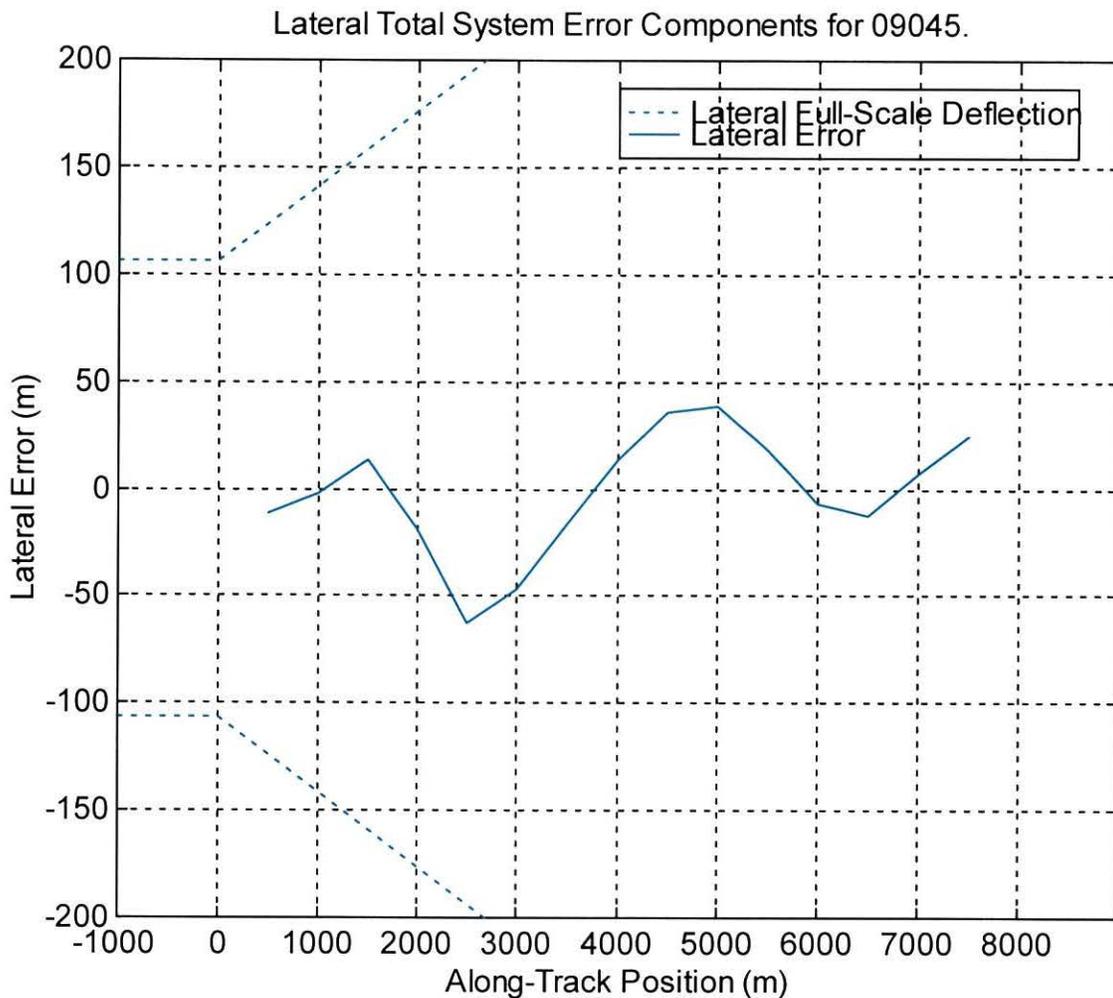


Figure 2: TYPICAL LATERAL FULL-SCALE DEFLECTION AND THE LATERAL TSE FOR FEBRUARY 5, 1999, PM APPROACH 5

1.2 TEST FLIGHT ERROR COMPUTATIONS

All errors were computed in an xyz-coordinate system in units of meters, with the x, y coordinates used for area navigation with the x-axis pointed along-track and the origin at the runway threshold. The z coordinate measures meters above threshold (height above the tangent plane at threshold to the WGS-84 ellipsoid). At any fixed along-track (i.e., x) position, the (y, z) position of the aircraft as given by the truth receiver is called the "aircraft pierce point." The vector difference between this point and the glidepath pierce point constitutes TSE, while the vector difference ($\Delta y, \Delta z$) between the aircraft pierce point and the in-flight receiver's estimated aircraft pierce point constitutes NSE. FTE is calculated either as the vector difference between the TSE and NSE or as the difference between the in-flight receiver's estimated aircraft pierce point and the glidepath pierce point.

2.0 GPS/WAAS PRECISION APPROACH ERROR STATISTICS

Tabulated values and plots of statistics are now considered. In all statistics, "sigma" refers to the value of one standard deviation from the mean position. An along-track range of (approximately) 872 meters appears in tables because it is the nominal range at which a 200-foot decision height occurs if the airplane is on the glidepath, and the ranges 1,200, 4,200, and 7,800 appear for purposes of comparison with particular CRM values that are tabulated at these ranges.

2.1 CATEGORY C STATISTICS

Figures 3 through 9 display the category C test approach errors (TSE, FTE, and NSE) in terms of various views and presentations of the statistical parameters are given in tables 1 through 4. The symbols + and * along the *edges* of the figures are the projections of the points marked with the same symbols along the glidepath and mean path. More parameters (such as skewness and kurtosis) have been computed and the Gaussian and non-Gaussian nature of error distributions presented, in reference (3). Distributions of all error types were found to be approximately Gaussian near the glidepath, as determined by Kolmogorov-Smirnov analysis and tests of skewness and kurtosis; but generally distributions had thicker-than-Gaussian tails. Consequently, the usual 95% rule for Gaussian probability of containment of a randomly sampled test flight point within a 2-sigma error range applies; but the probability of containment at the 6-sigma level does not appear to match that of a Gaussian prediction. In the TSE and FTE plots of vertical errors (figures 4 and 8), a side-view of the LPV obstacle clearance surface is also plotted. In the FTE plot the Height Above Touchdown (HAT) minus the (proposed 10^{-7} risk distance) quantity

$$T = ([5.33 \times FTE_z]^2 + 50^2)^{1/2}$$

is plotted (where FTE_z is the standard deviation in the z-component of FTE). The so-called "safety margin" is then the vertical difference between the HAT - T and the OCS height.

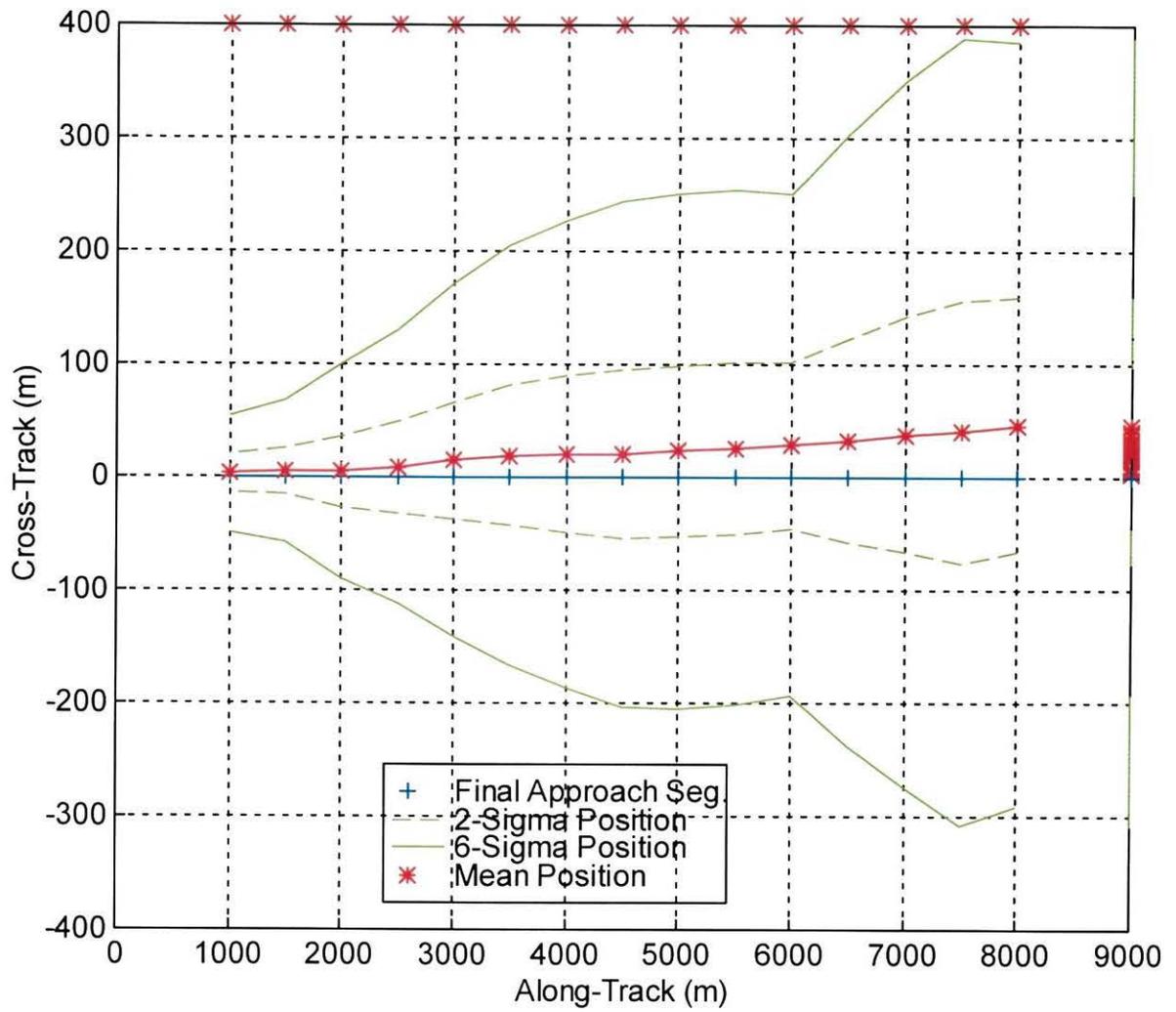


Figure 3: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY C TEST FLIGHTS

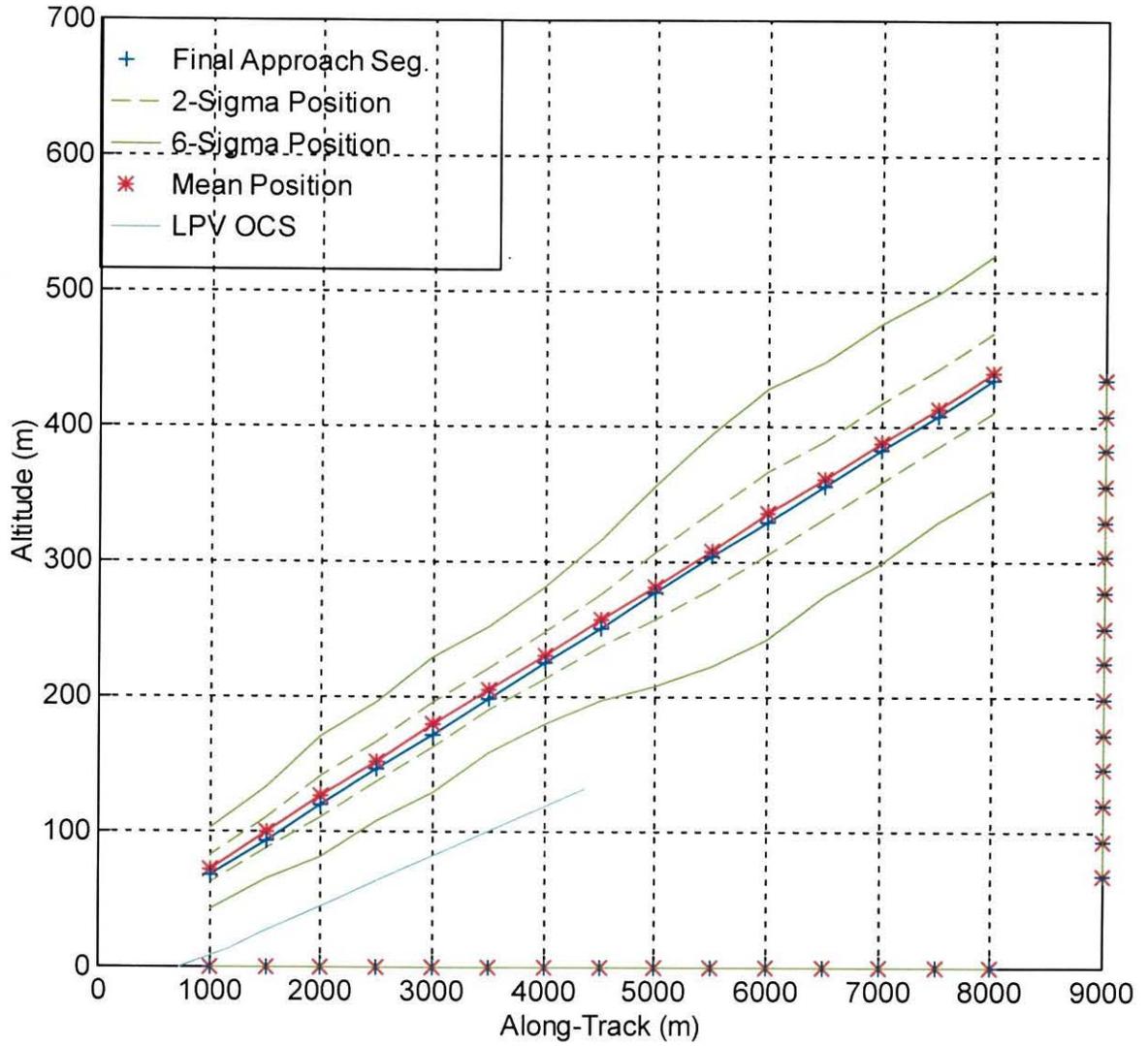


Figure 4: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY C TEST FLIGHTS

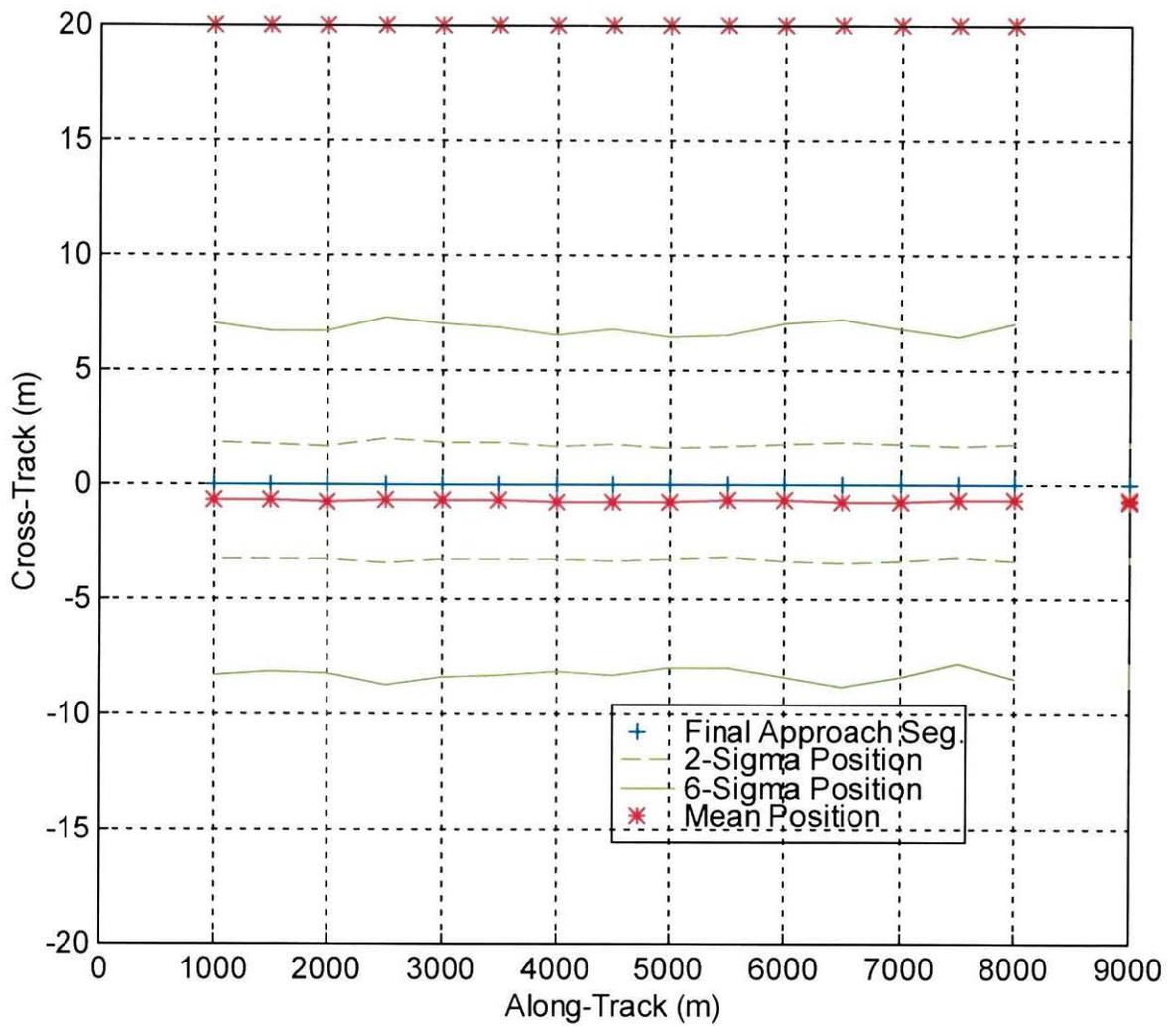


Figure 5: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY C TEST FLIGHTS

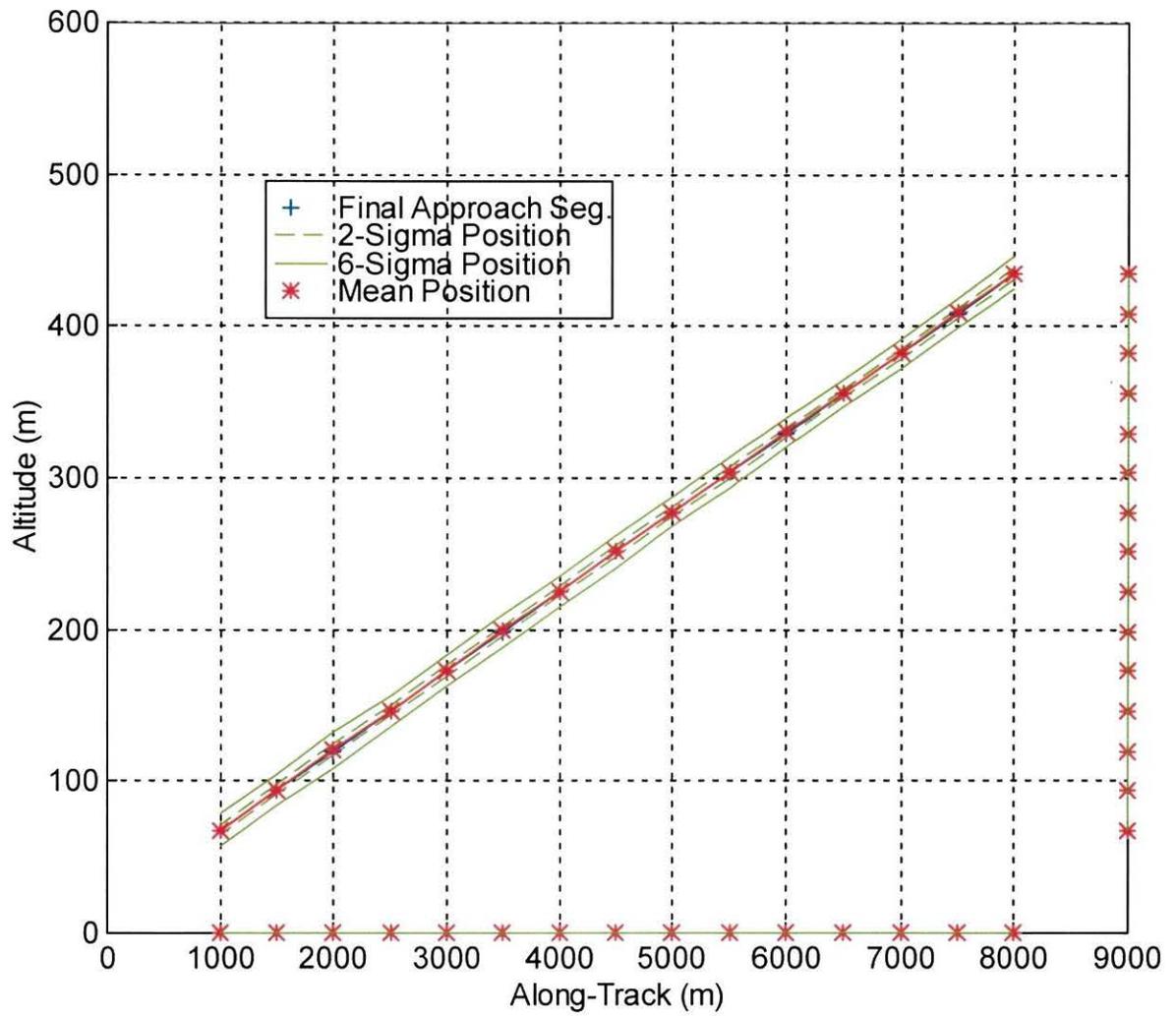


Figure 6: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY C TEST FLIGHTS

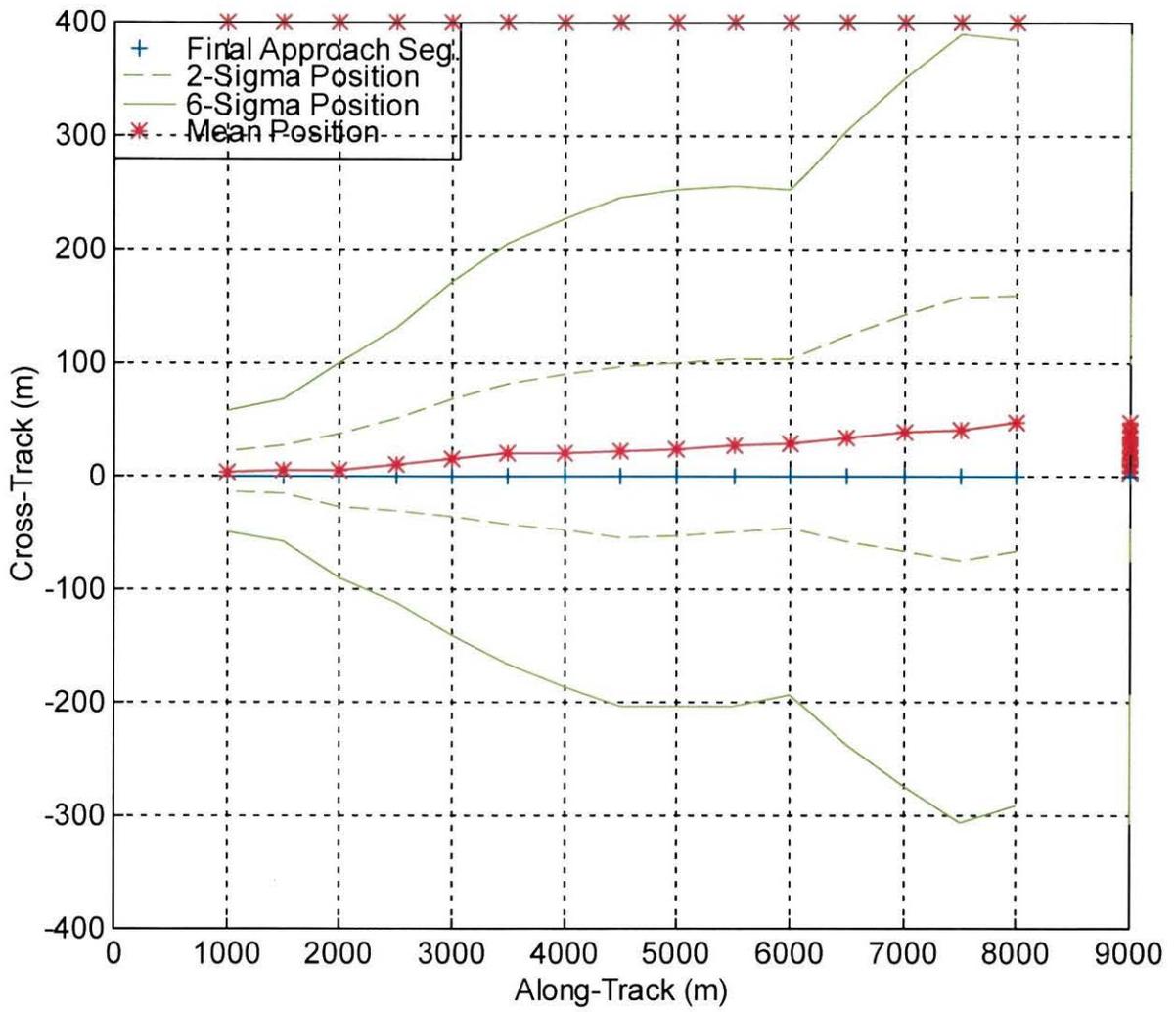


Figure 7: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY C TEST FLIGHTS

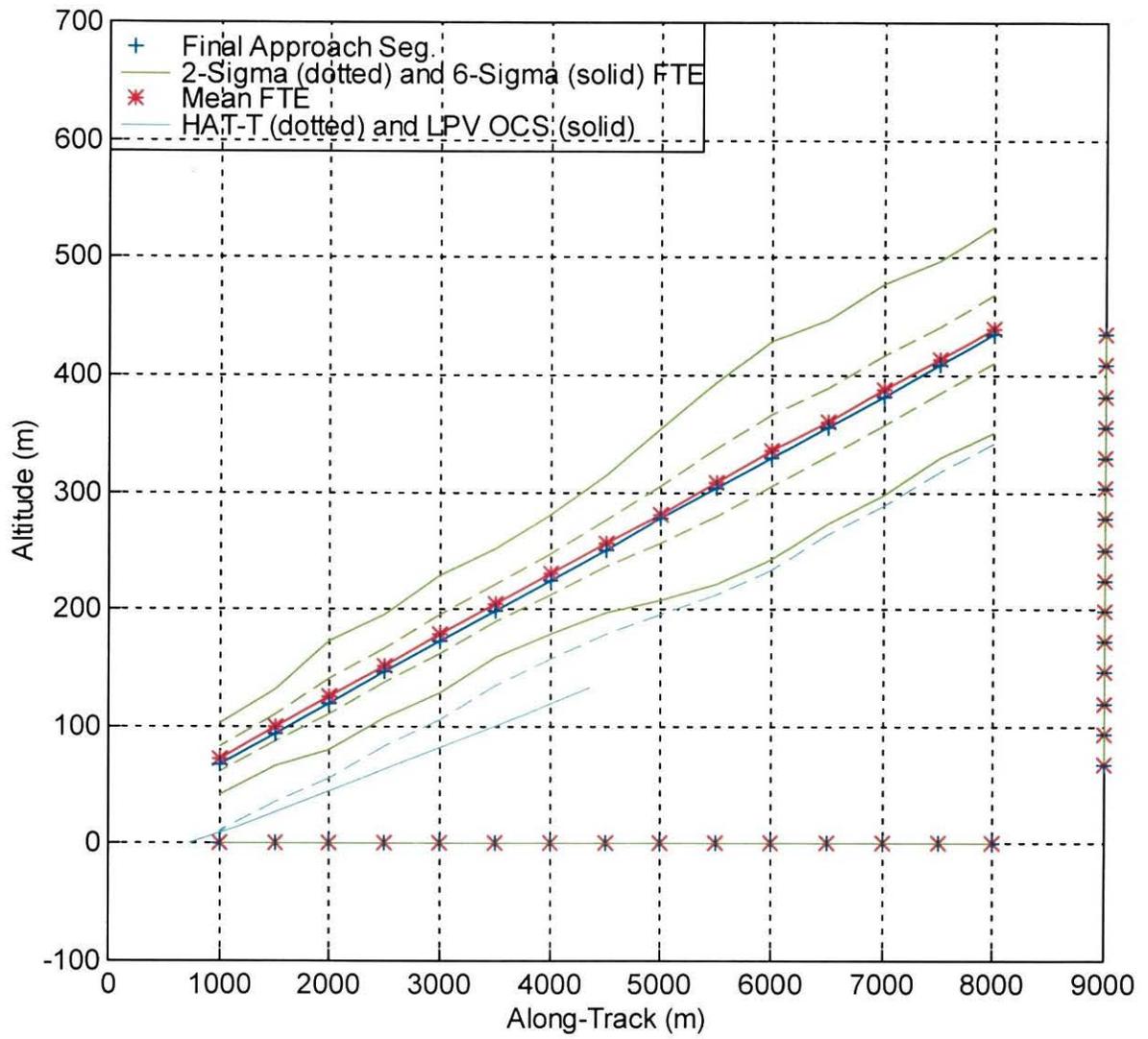


Figure 8: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY C TEST FLIGHTS

GPS/WAAS Precision Approach

Mean Navigation System Error and 2-Sigma Uncertainty Ellipses

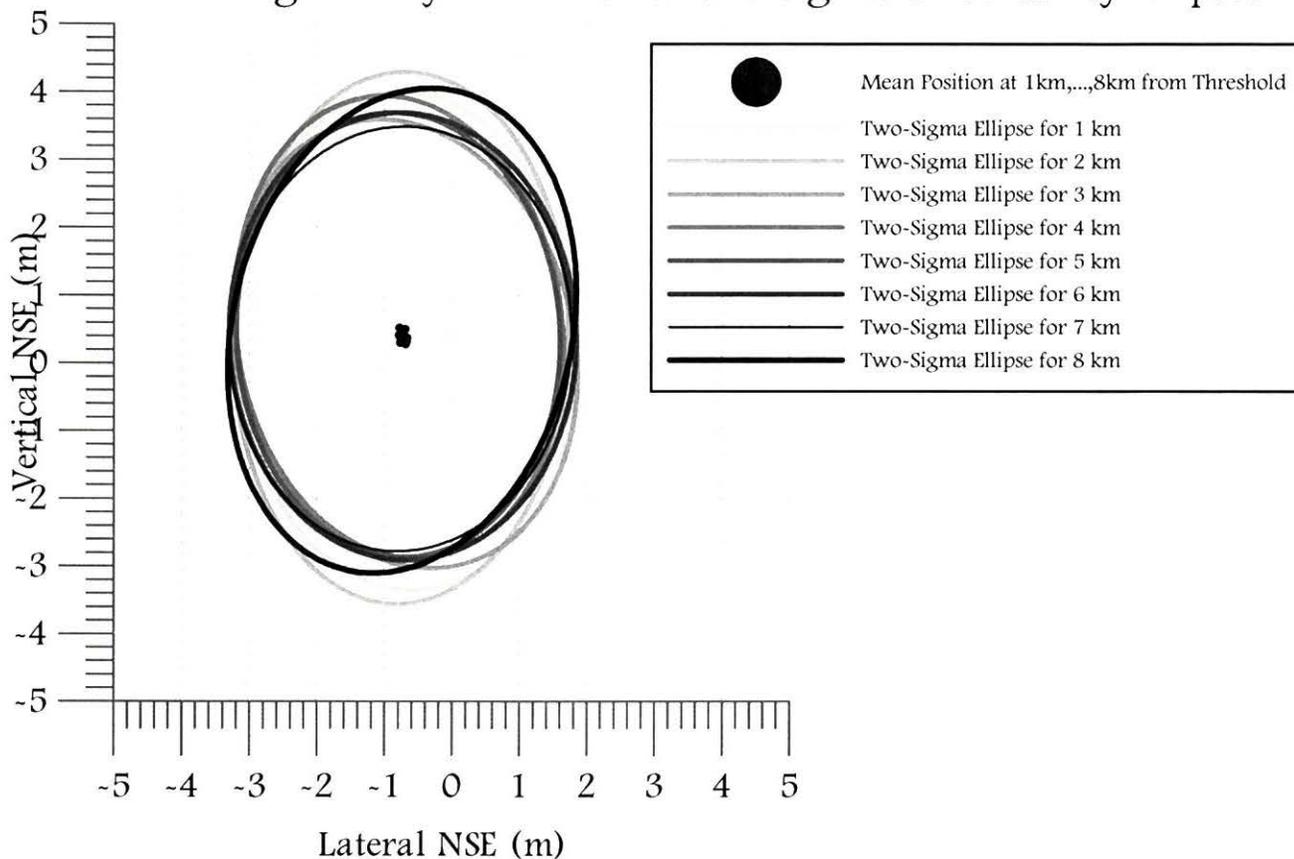


Figure 9: FAATC NAVIGATION SYSTEM ERROR 2-SIGMA ELLIPSES AT 1 KM ALONG-TRACK INTERVALS VIEWED DOWN THE GLIDEPATH

<i>Range Along-Track (m)</i>	<i>Mean TSE Cross-Track (m)</i>	<i>Std. Dev. TSE Cross- Track (m)</i>	<i>Mean TSE Vertical (m)</i>	<i>Std. Dev. TSE Vertical (m)</i>
8000	46.48	56.26	5.34	14.41
7800	45.89	55.09	5.70	13.82
7500	40.32	58.02	5.60	14.16
7000	37.85	52.14	5.55	14.73
6500	32.32	45.07	4.94	14.42
6000	28.21	36.98	6.17	15.46
5500	25.83	38.03	4.93	14.34
5000	23.00	37.90	4.51	12.29
4500	20.56	37.37	5.65	9.85
4200	20.36	35.76	5.93	8.72
4000	20.17	34.42	5.66	8.49
3500	19.22	30.86	7.20	7.76
3000	14.98	25.94	7.02	8.29
2500	8.84	20.20	5.88	7.33
2000	4.54	15.83	6.19	7.46
1500	4.71	10.38	5.55	5.60
1200	4.01	9.24	5.34	5.30
1000	2.98	8.68	4.87	5.07
872	2.68	7.90	4.58	4.98

**Table 1: CATEGORY C TOTAL SYSTEM ERROR
STATISTICS FOR GPS/WAAS TEST APPROACHES**

<i>Range Along-Track (m)</i>	<i>Mean NSE Cross-Track (m)</i>	<i>Std. Dev. NSE Cross-Track (m)</i>	<i>Mean NSE Vertical (m)</i>	<i>Std. Dev. NSE Vertical (m)</i>
8000	-.7412	1.2918	.4745	1.7899
7800	-.6786	1.2210	.5548	2.1141
7500	-.7081	1.1806	.4942	1.5369
7000	-.7588	1.2658	.3534	1.5669
6500	-.7682	1.3363	.3016	1.4687
6000	-.7313	1.2856	.3905	1.6511
5500	-.7190	1.2088	.3914	1.6181
5000	-.7943	1.1994	.4117	1.6404
4500	-.7572	1.2619	.3024	1.7961
4200	-.7226	1.2496	.3543	1.7435
4000	-.7823	1.2181	.5247	1.7055
3500	-.6736	1.2705	.3686	1.8387
3000	-.6919	1.2779	.2784	1.6552
2500	-.7013	1.3285	.3937	1.7111
2000	-.7520	1.2431	.3639	1.9625
1500	-.7041	1.2293	.5003	1.6372
1200	-.6669	1.2024	.4054	1.8101
1000	-.6756	1.2739	.3207	1.8362
872	-.6881	1.2938	.2901	1.8724

Table 2: CATEGORY C NAVIGATION SYSTEM ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

<i>Range Along-Track (m)</i>	<i>Mean FTE Cross-Track (m)</i>	<i>Std. Dev. FTE Cross-Track (m)</i>	<i>Mean FTE Vertical (m)</i>	<i>Std. Dev. FTE Vertical (m)</i>
8000	47.2	56.35	4.9	14.6
7800	46.57	55.12	5.14	13.99
7500	41	58.1	5.1	14.05
7000	38.6	52.15	5.21	14.95
6500	33.1	45.2	4.61	14.55
6000	28.9	37.15	5.81	15.45
5500	26.5	38.2	4.52	14.35
5000	23.8	38	4.12	12.2
4500	21.3	37.5	5.32	9.8
4200	21.08	35.88	5.57	8.65
4000	21	34.4	5.13	8.45
3500	19.9	30.95	6.83	7.65
3000	15.7	25.95	6.74	8.35
2500	9.5	20.3	5.44	7.35
2000	5.3	15.85	5.84	7.7
1500	5.4	10.5	5.05	5.4
1200	4.67	9.38	4.94	5.16
1000	3.7	8.85	4.55	5.15
872	3.4	8.1	4.24	5.05

Table 3: CATEGORY C FLIGHT TECHNICAL ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

2.1.1 CORRELATION OF CATEGORY C ERROR COMPONENTS

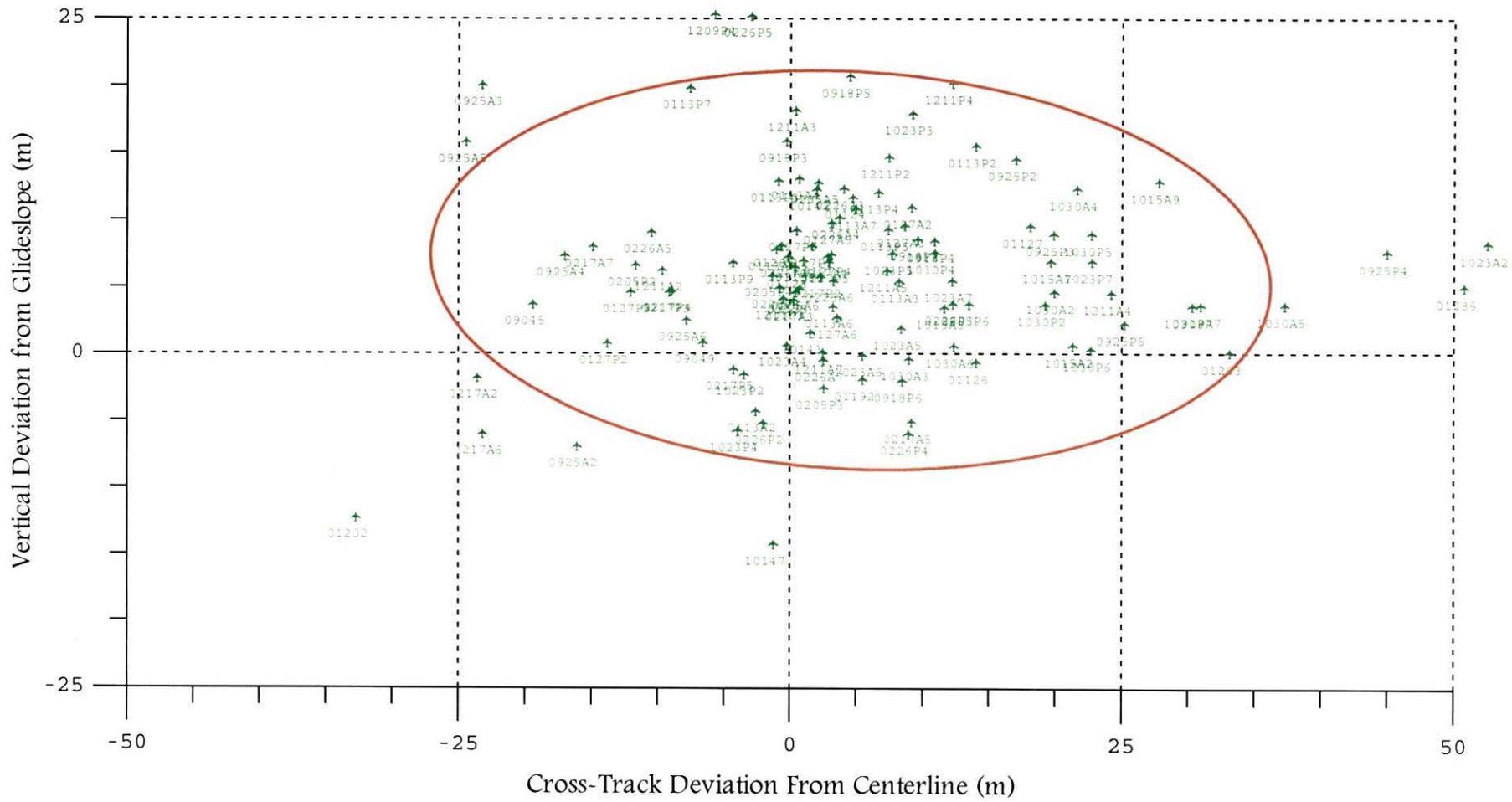
Calculations indicate that the category C horizontal and vertical components of TSE, NSE, or FTE are not substantially correlated (see also (1)). Table 4 lists the correlation coefficients relating y (lateral error) and z (vertical error). Correlation values that are approximately zero (either positive or negative) indicate either weak correlation or no correlation between variables underlying the sample data, while those near one in absolute value indicate a strong correlation. Evidently, components are either uncorrelated or are only weakly correlated. The slight correlation that appears in flight technical error and total system error near decision height is expected due to the inclusion of data from go-around procedures.

<i>Range Along-Track</i>	<i>TSE yz-Correlation Coefficient</i>	<i>FTE yz-Correlation Coefficient</i>	<i>NSE yz-Correlation Coefficient</i>
8000	.04390	.04394	.16386
7000	.05430	.06371	.02146
6000	-.02237	.00444	-.01831
5000	-.13329	-.12227	-.03281
4000	-.17735	-.17952	-.09627
3000	-.06485	-.08320	-.14605
2000	-.08122	-.06807	.02373
1000	.17586	.15106	-.03571
872	.21866	.19608	-.02958

Table 4: LATERAL/VERTICAL CORRELATION COEFFICIENTS FOR CATEGORY C TEST APPROACH DATA

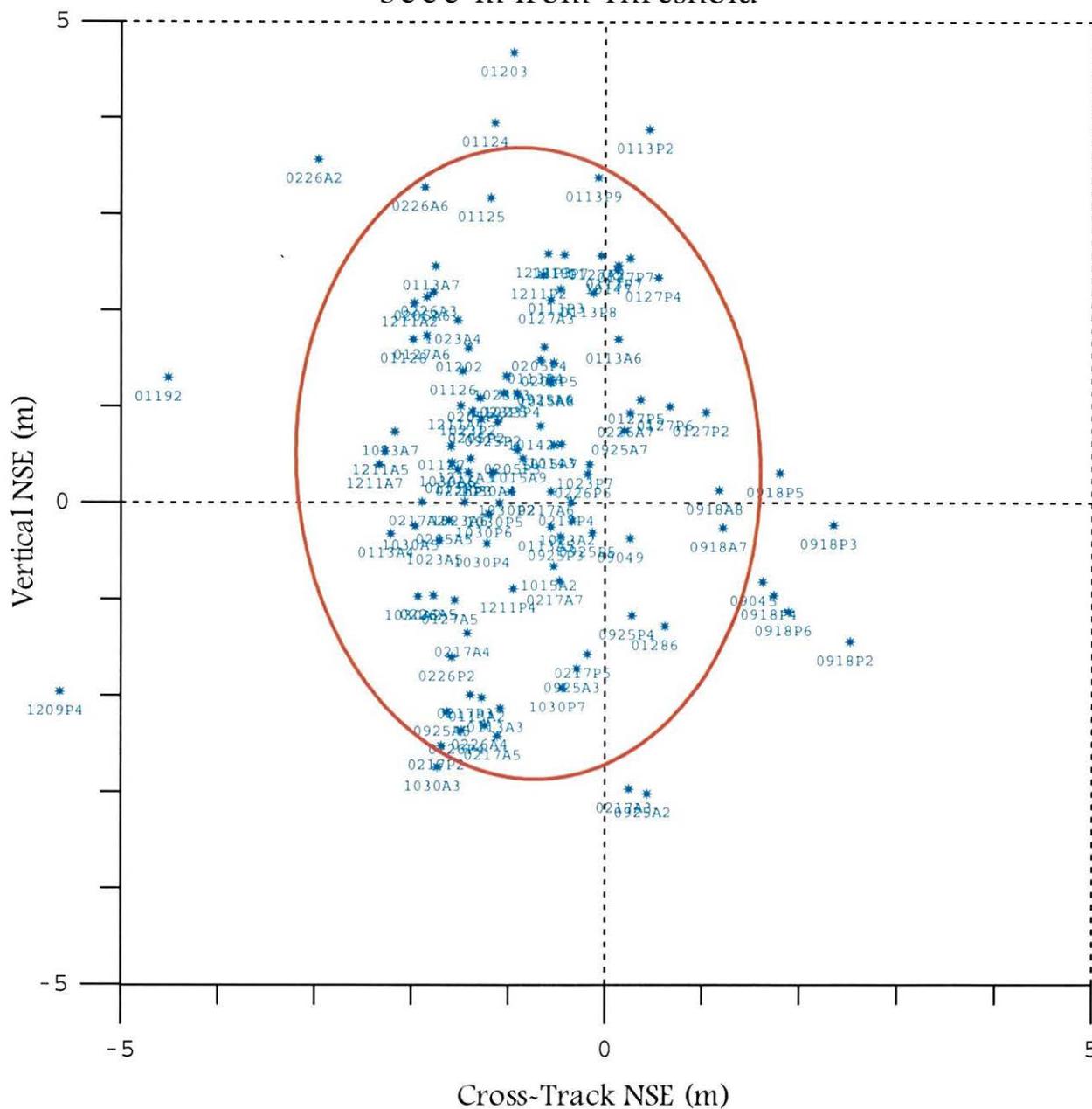
Some TSE and NSE two-sigma ellipses are displayed in figures 10, 11, and 12. The labels indicate the month, day of the month, and procedure number (which may include an A for AM or a P for PM). The fact that the ellipses are not substantially tilted is graphical consequence of the relatively small correlation coefficients.

0925A7



**Figure 11: GPS/WAAS CATEGORY C AIRCRAFT PIERCE POINTS (TSE)
AT 2,000 METERS ALONG-TRACK WITH TWO-SIGMA ELLIPSE**

NSE and 2-Sigma Ellipse for WAAS-Guided Instrument Approaches 5000 m from Threshold



**Figure 12: CATEGORY C GPS/WAAS NSE AT 5,000 METERS
ALONG-TRACK WITH TWO-SIGMA ELLIPSE**

Figure 13 displays the NSE for each approach that was part of a flight containing five or more (admissible) approaches. Approach NSE points within any single flight are plotted with the same symbol and lines are plotted to connect the NSE of sequential approaches within flights. It is found therein that the NSE from approaches that come from any single flight (requiring approximately two and a half hours) are clustered together. Consequently, it may be deduced that *GPS/WAAS has at least one substantial long-term error source (not just high frequency periodic sources)*. In fact, the average standard deviation in NSE over approaches within any single cluster was just .55 meters laterally and 1.1 meters vertically. The standard deviation of the 14 mean cluster positions (points) was 1.0 lateral meters and 1.2 vertical meters.

GPS/WAAS NSE from Flights Containing
Five or More Admissible Approaches at
5000 m from Threshold

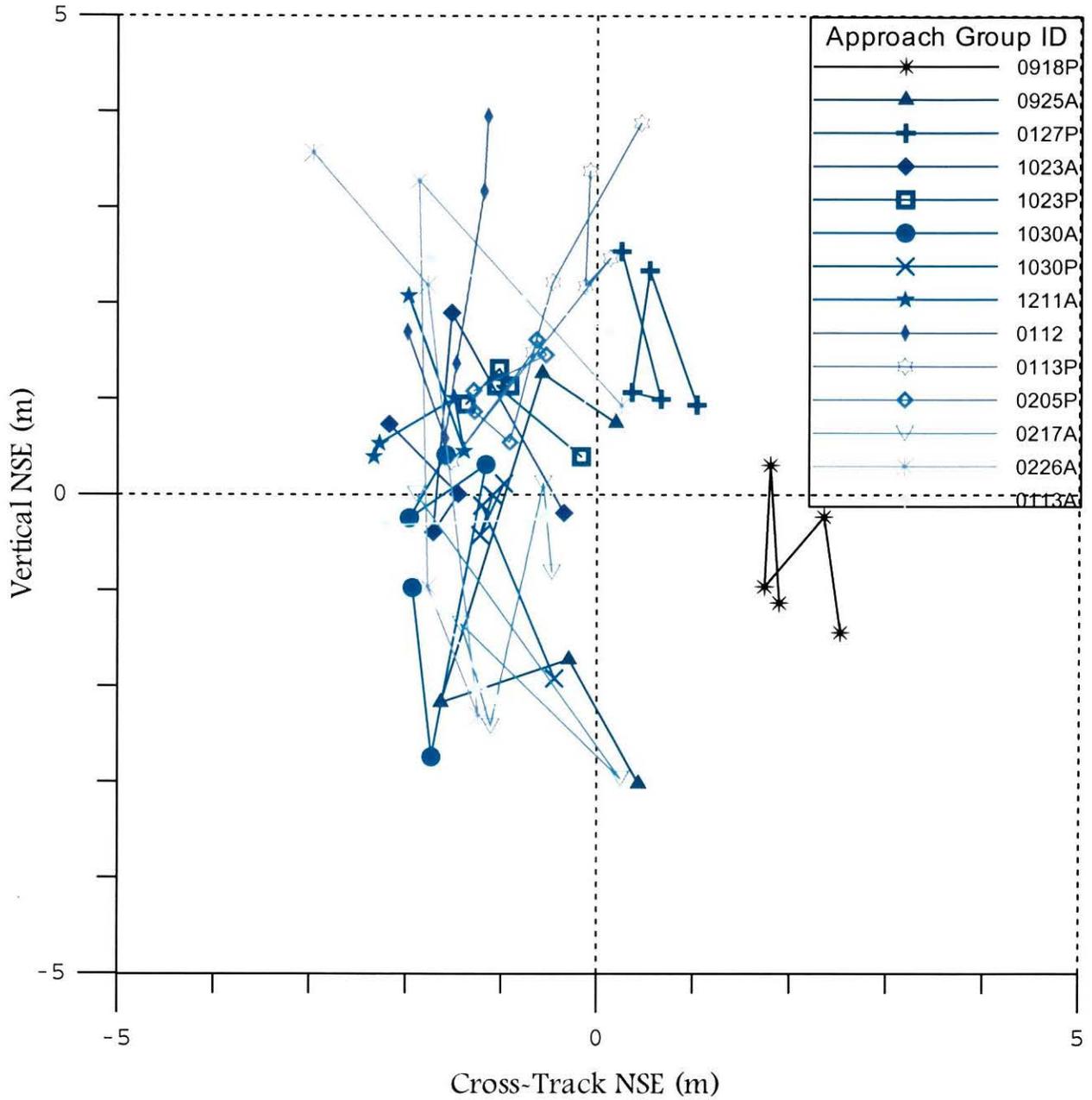


Figure 13: CATEGORY C GPS/WAAS NSE AT 5,000 METERS ALONG-TRACK FOR FLIGHTS CONTAINING FIVE OR MORE ADMISSIBLE APPROACHES

2.1.2 COMPARISON WITH THE COLLISION RISK MODEL

The Collision Risk Model (CRM) (5), whose data is based upon Instrument Landing System (ILS) precision approaches, provides an approximate fit in determining the cumulative probability that a GPS/WAAS-guided aircraft will be to the right by a given number of standard deviations. Figures 14 and 15 illustrate the fit of the CRM cumulative distributions to category C test flight distributions, in which each sample cumulative probability curve is interpolated by the piecewise linear method and linearly extrapolated slightly past the rightmost sample. A larger number of samples is required to compare test flight probabilities with CRM probabilities at greater deviations from the mean.

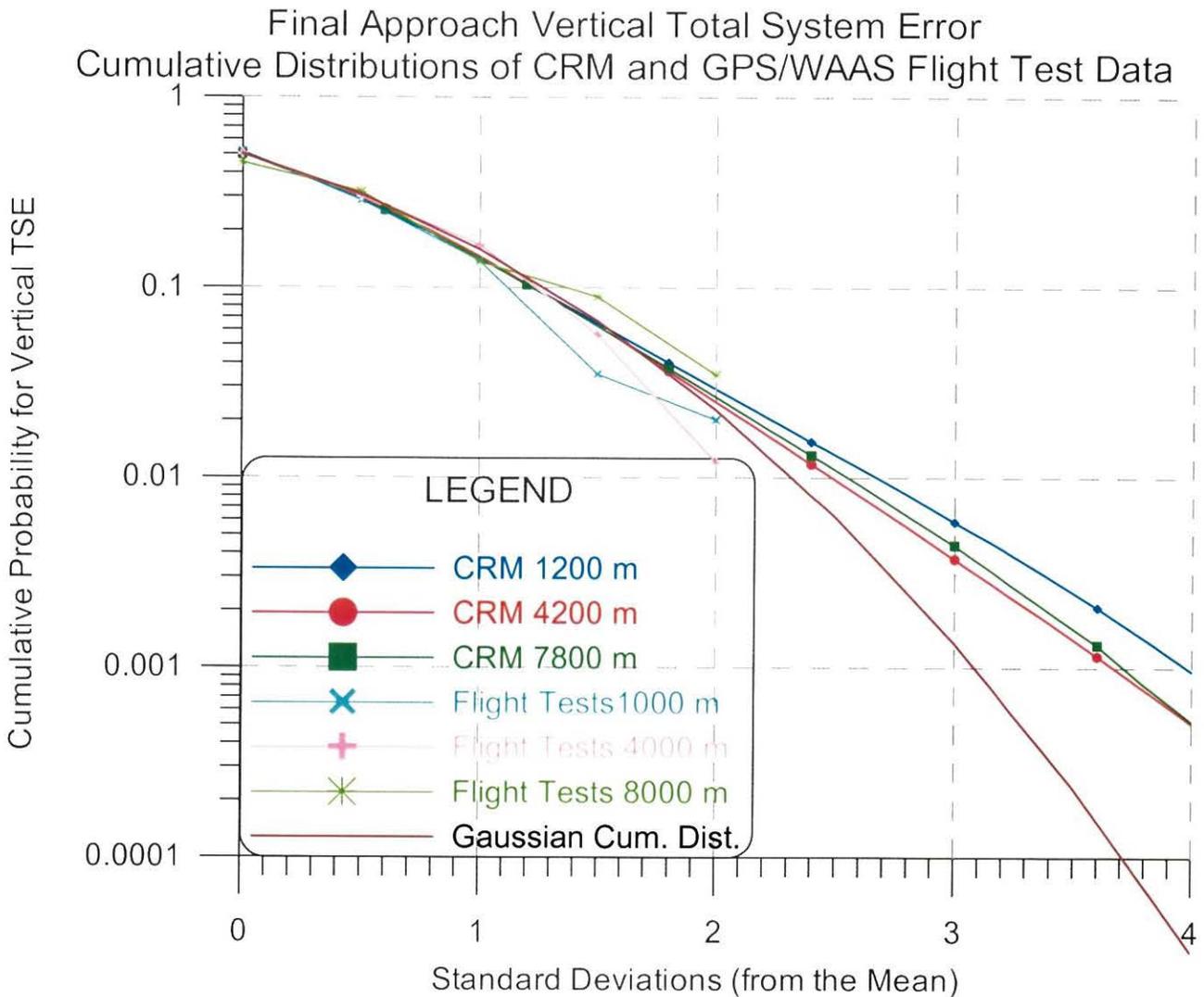


Figure 14: CRM AND CATEGORY C TEST FLIGHT VERTICAL CUMULATIVE DISTRIBUTIONS (SHOWING PROBABILITIES OF BEING ABOVE THE MEAN BY AT LEAST THE AMOUNT SHOWN)

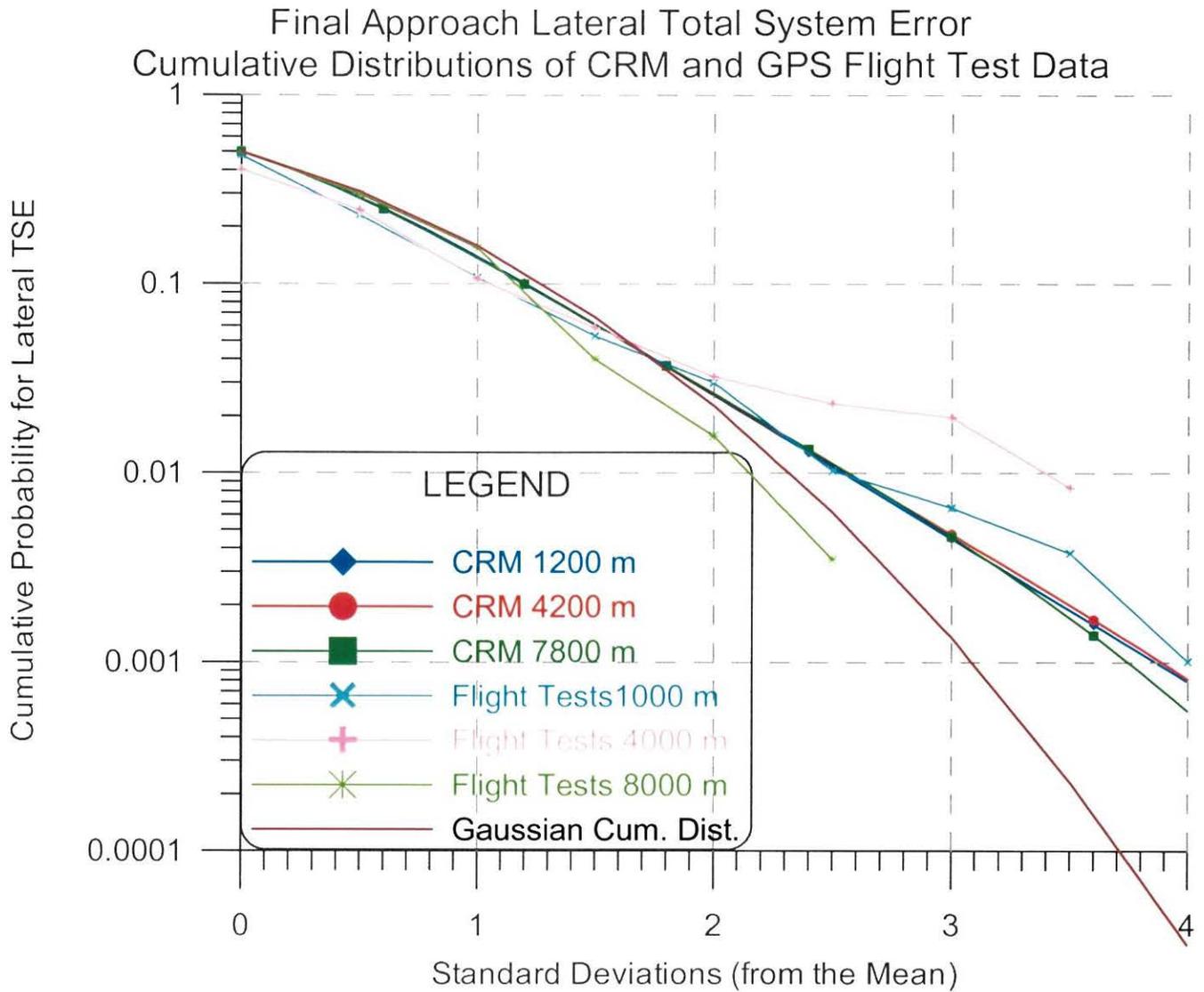


Figure 15. CRM AND CATEGORY C TEST FLIGHT LATERAL CUMULATIVE DISTRIBUTIONS (SHOWING PROBABILITIES OF BEING RIGHT OF THE MEAN BY AT LEAST THE AMOUNT SHOWN)

2.2 CATEGORY B STATISTICS

Category B statistics are presented graphically in figures 16 through 21 based upon the numbers that are compiled in tables 5 through 7. There are some differences from the numbers found in (1) because in that paper many of the statistics were for absolute values of error components or root-sum-squares of along-track and cross-track components.

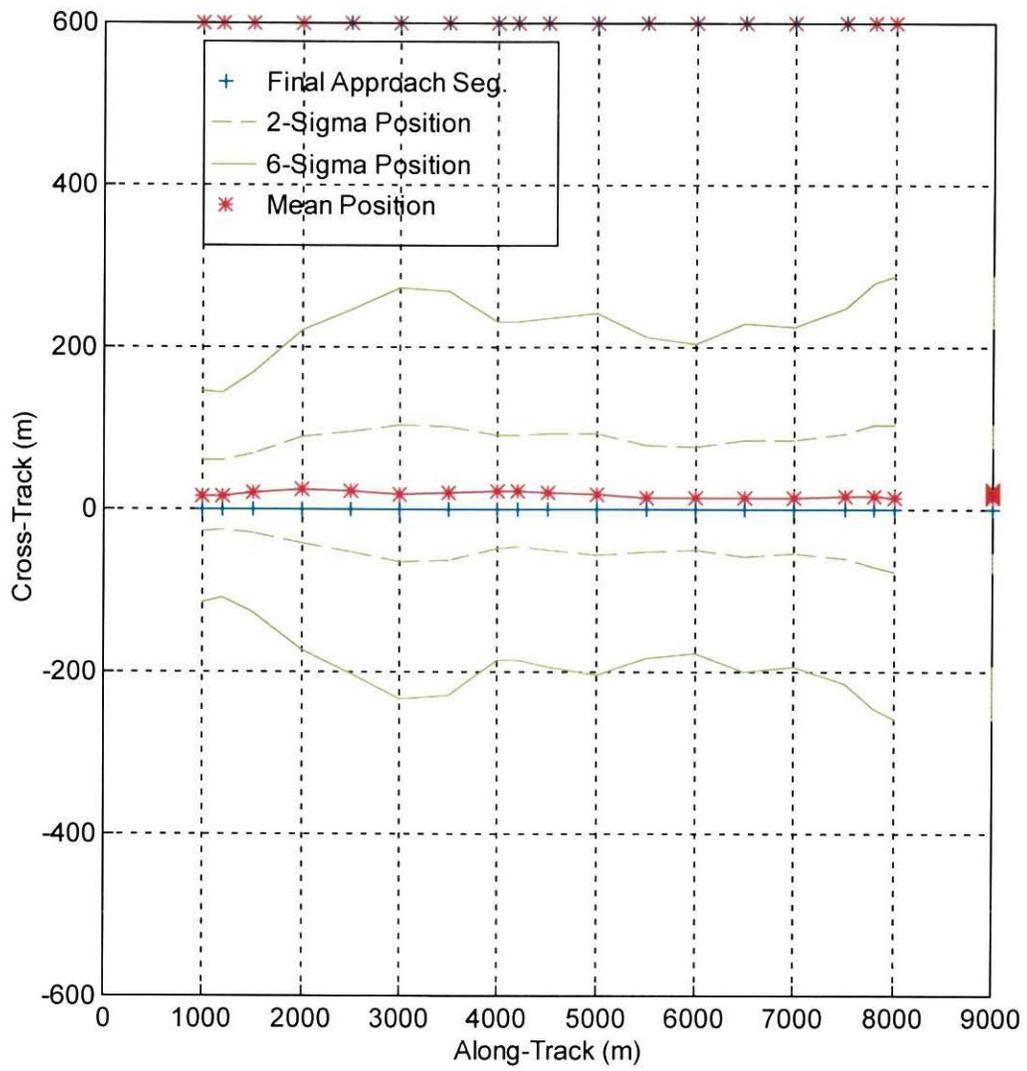


Figure 16: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY B TEST FLIGHTS

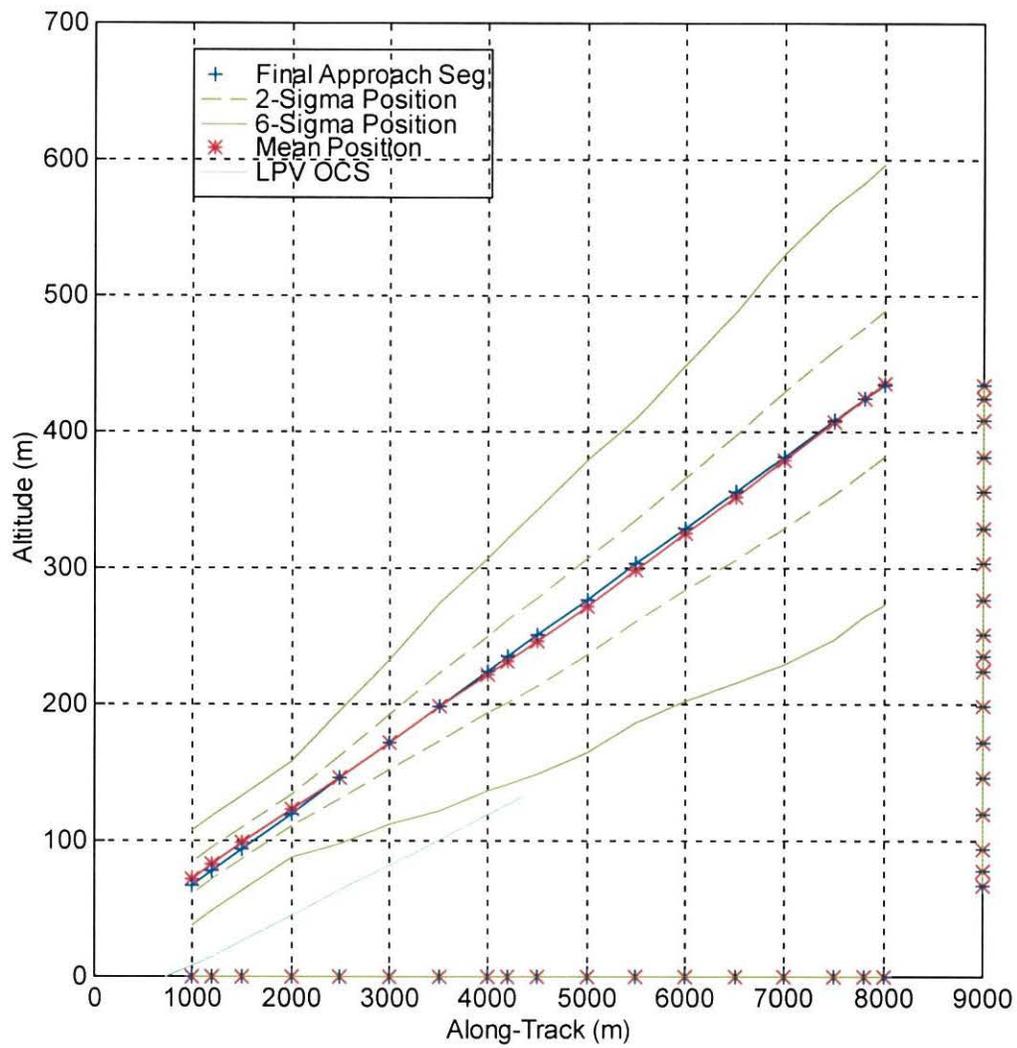


Figure 17: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY B TEST FLIGHTS

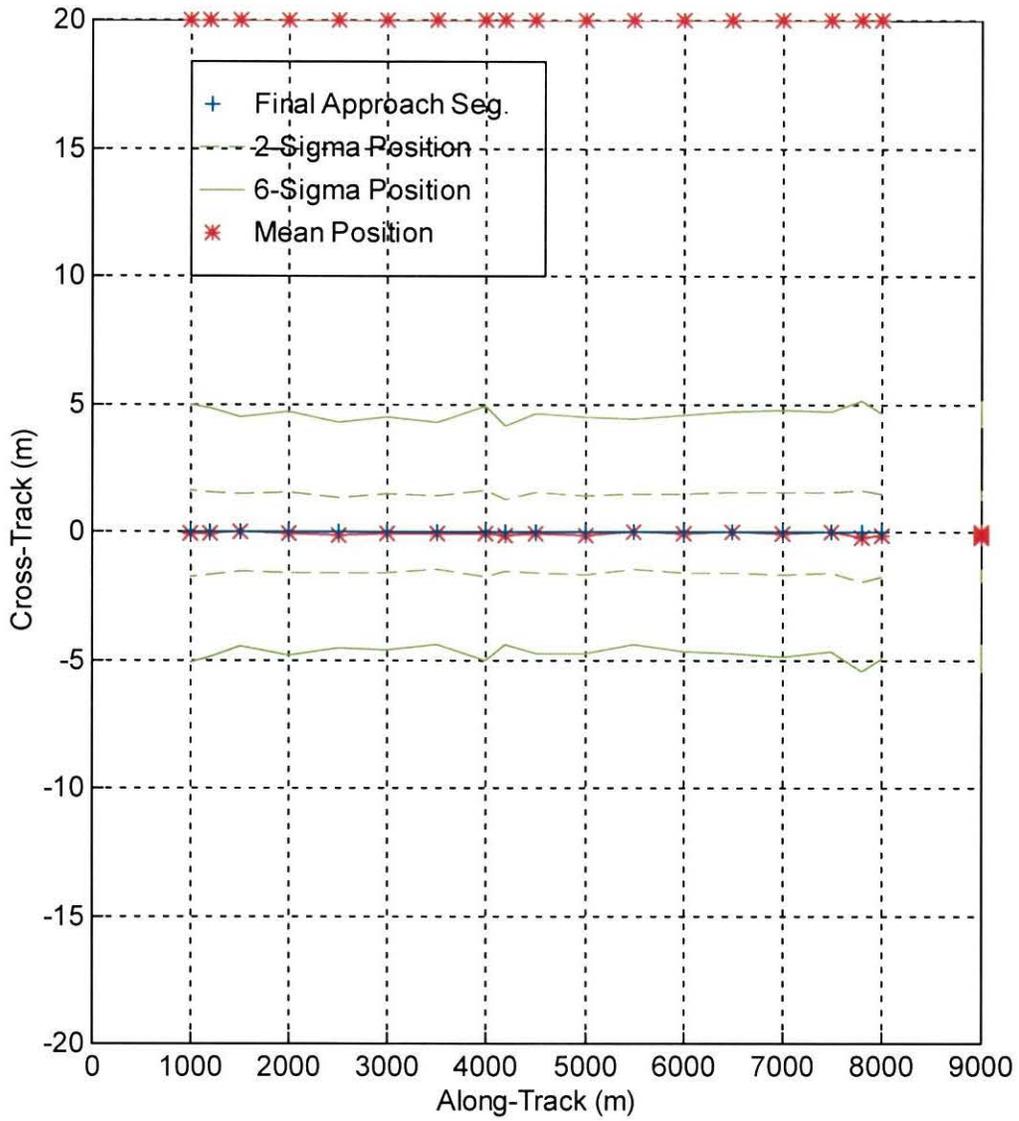


Figure 18: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY B TEST FLIGHTS

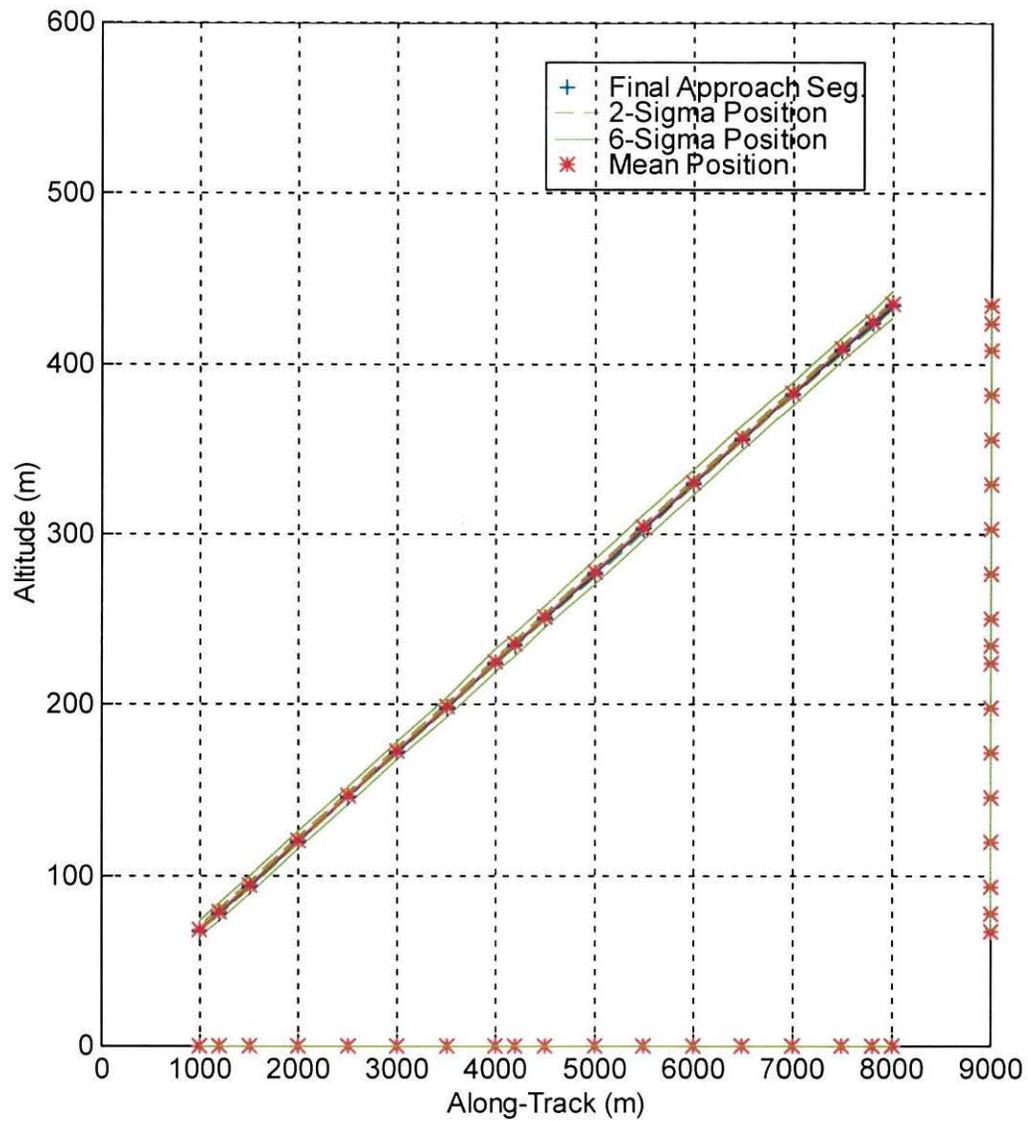


Figure 19: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY B TEST FLIGHTS

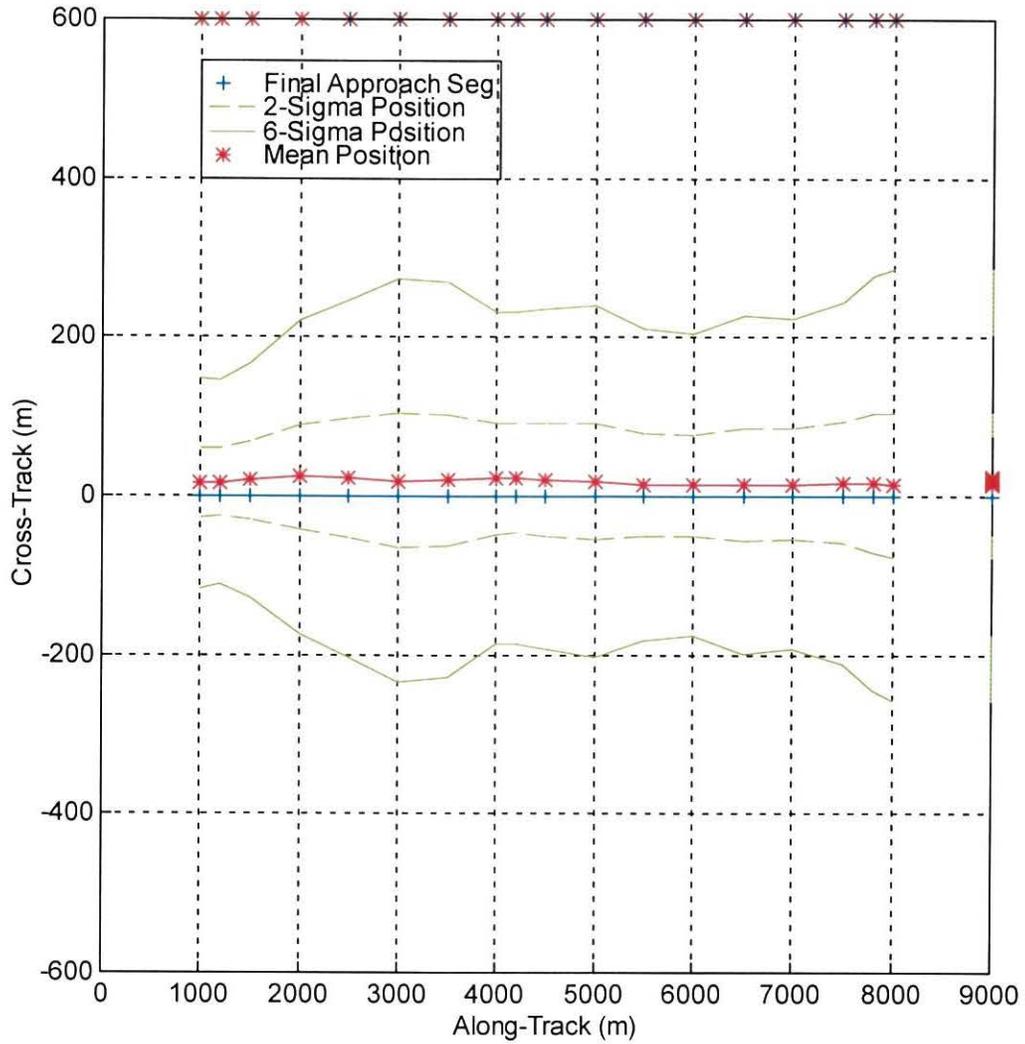


Figure 20: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY B FLIGHTS

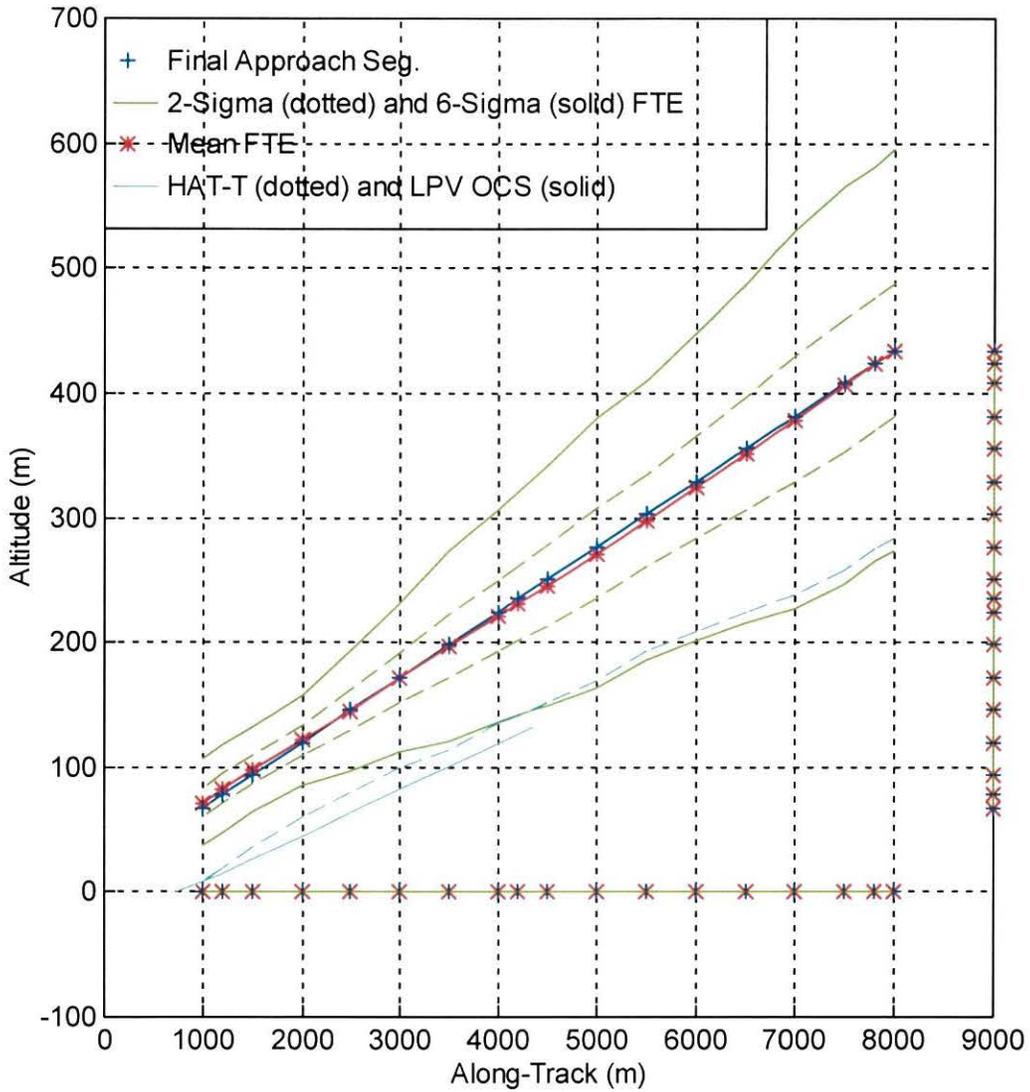


Figure 21: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY B TEST FLIGHTS

<i>Range Along-Track (m)</i>	<i>Mean TSE Cross-Track (m)</i>	<i>Std. Dev. TSE Cross-Track (m)</i>	<i>Mean TSE Vertical (m)</i>	<i>Std. Dev. TSE Vertical (m)</i>
8000	14.07	45.42	0.43	26.87
7800	16.54	43.74	-0.23	26.44
7500	16.72	38.39	-1.25	26.52
7000	15.44	34.89	-2.65	25.08
6500	13.76	35.75	-3.88	22.70
6000	14.17	31.79	-4.43	20.50
5500	14.49	32.83	-5.15	18.55
5000	18.74	37.02	-5.45	17.92
4500	21.20	35.82	-5.14	16.18
4200	23.00	34.74	-3.63	14.96
4000	22.85	34.88	-2.95	14.22
3500	20.21	41.41	-0.41	12.73
3000	19.35	42.13	-0.04	10.08
2500	22.32	37.25	-0.11	8.12
2000	24.26	32.74	2.76	5.89
1500	20.20	24.59	4.56	5.74
1200	17.26	21.10	5.29	5.81
1000	15.97	21.73	4.82	5.77
870	14.24	22.51	4.33	5.33

Table 5. CATEGORY B TOTAL SYSTEM ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

<i>Range Along-Track (m)</i>	<i>Mean NSE Cross-Track (m)</i>	<i>Std. Dev. NSE Cross-Track (m)</i>	<i>Mean NSE Vertical (m)</i>	<i>Std. Dev. NSE Vertical (m)</i>
8000	-0.15	0.80	0.60	1.29
7800	-0.18	0.88	0.53	1.18
7500	-0.02	0.78	0.44	1.16
7000	-0.07	0.81	0.60	1.12
6500	-0.03	0.79	0.58	1.17
6000	-0.07	0.77	0.54	1.18
5500	0.00	0.74	0.47	1.25
5000	-0.12	0.77	0.45	1.21
4500	-0.05	0.78	0.56	1.08
4200	-0.15	0.71	0.47	1.14
4000	-0.07	0.83	0.71	1.14
3500	-0.06	0.72	0.45	0.94
3000	-0.08	0.76	0.50	0.92
2500	-0.14	0.74	0.73	0.82
2000	-0.06	0.79	0.61	0.86
1500	-0.01	0.75	0.64	0.84
1200	-0.05	0.81	0.51	0.85
1000	-0.05	0.84	0.63	0.79
870	-0.04	0.77	0.62	0.87

Table 6. CATEGORY B NAVIGATION SYSTEM ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

<i>Range Along-Track (m)</i>	<i>Mean FTE Cross- Track (m)</i>	<i>Std. Dev. FTE Cross-Track (m)</i>	<i>Mean FTE Vertical (m)</i>	<i>Std. Dev. FTE Vertical (m)</i>
8000	14.22	45.35	-0.18	26.73
7800	16.71	43.53	-0.76	26.30
7500	16.74	38.15	-1.69	26.55
7000	15.51	34.63	-3.25	25.20
6500	13.79	35.54	-4.45	22.72
6000	14.24	31.69	-4.97	20.60
5500	14.49	32.82	-5.62	18.59
5000	18.86	36.99	-5.90	18.00
4500	21.25	35.71	-5.70	16.14
4200	23.15	34.76	-4.10	14.94
4000	22.92	34.99	-3.66	14.16
3500	20.27	41.53	-0.86	12.72
3000	19.43	42.31	-0.54	9.86
2500	22.46	37.50	-0.84	8.02
2000	24.33	32.86	2.14	6.02
1500	20.21	24.60	3.92	5.70
1200	17.31	21.43	4.78	5.94
1000	16.02	22.08	4.18	5.76
870	14.28	22.90	3.71	5.09

Table 7. CATEGORY B FLIGHT TECHNICAL ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

2.3 CATEGORY A STATISTICS

Category A statistics for TSE, NSE, and FTE are presented in figures 22 through 27, and tables 8 through 10.

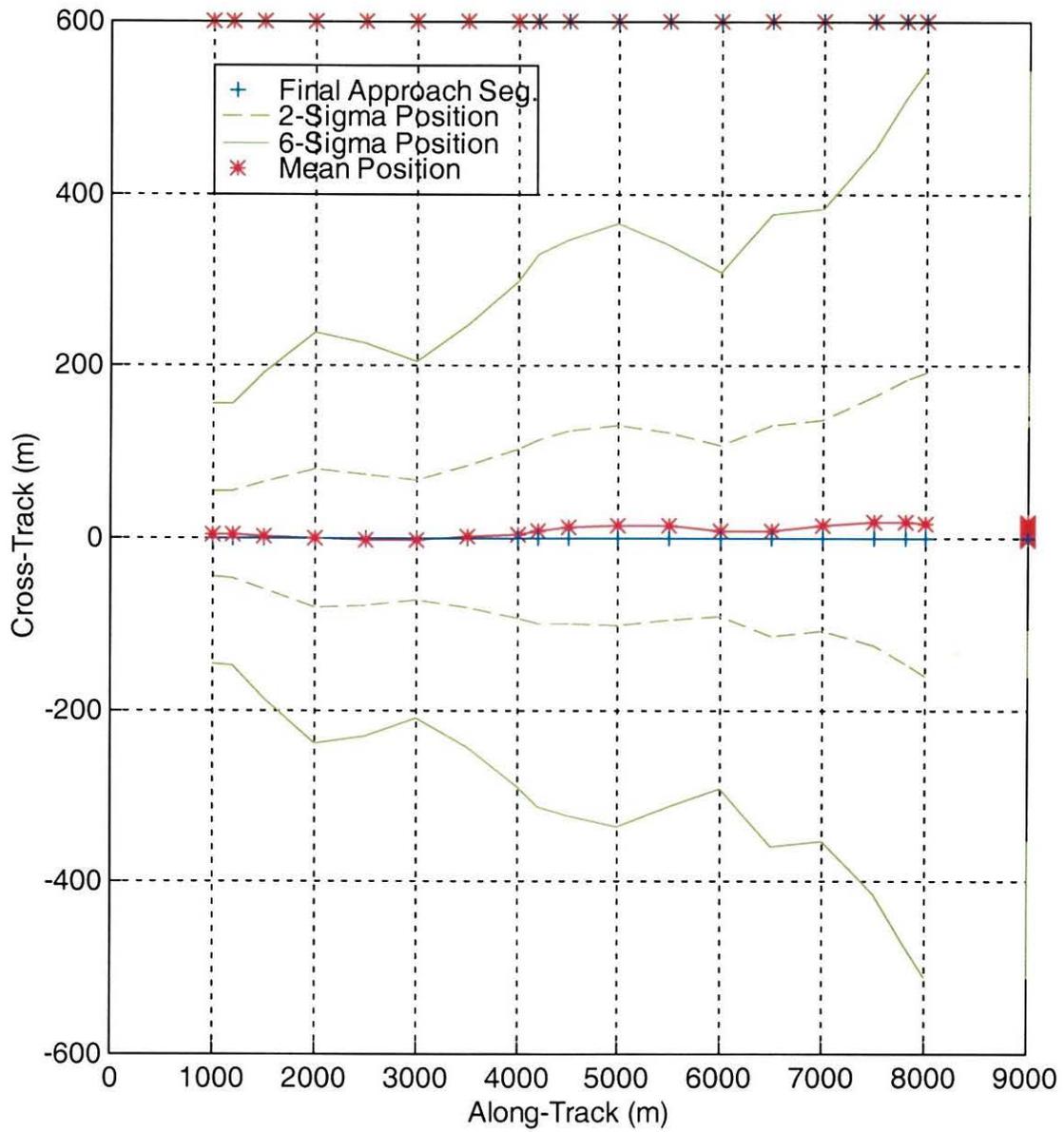


Figure 22: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY A TEST FLIGHTS

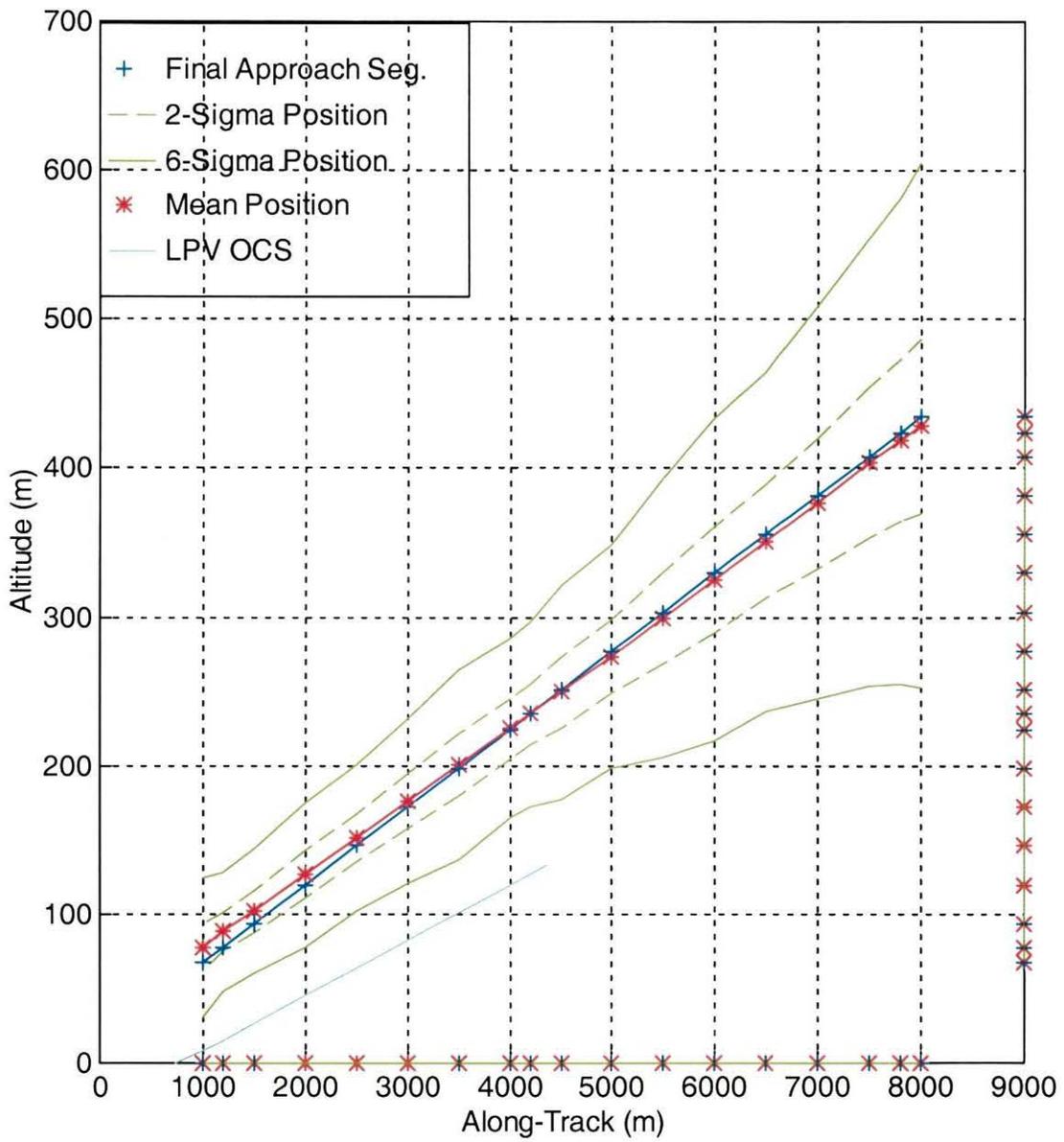


Figure 23: TOTAL SYSTEM ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY A TEST FLIGHTS

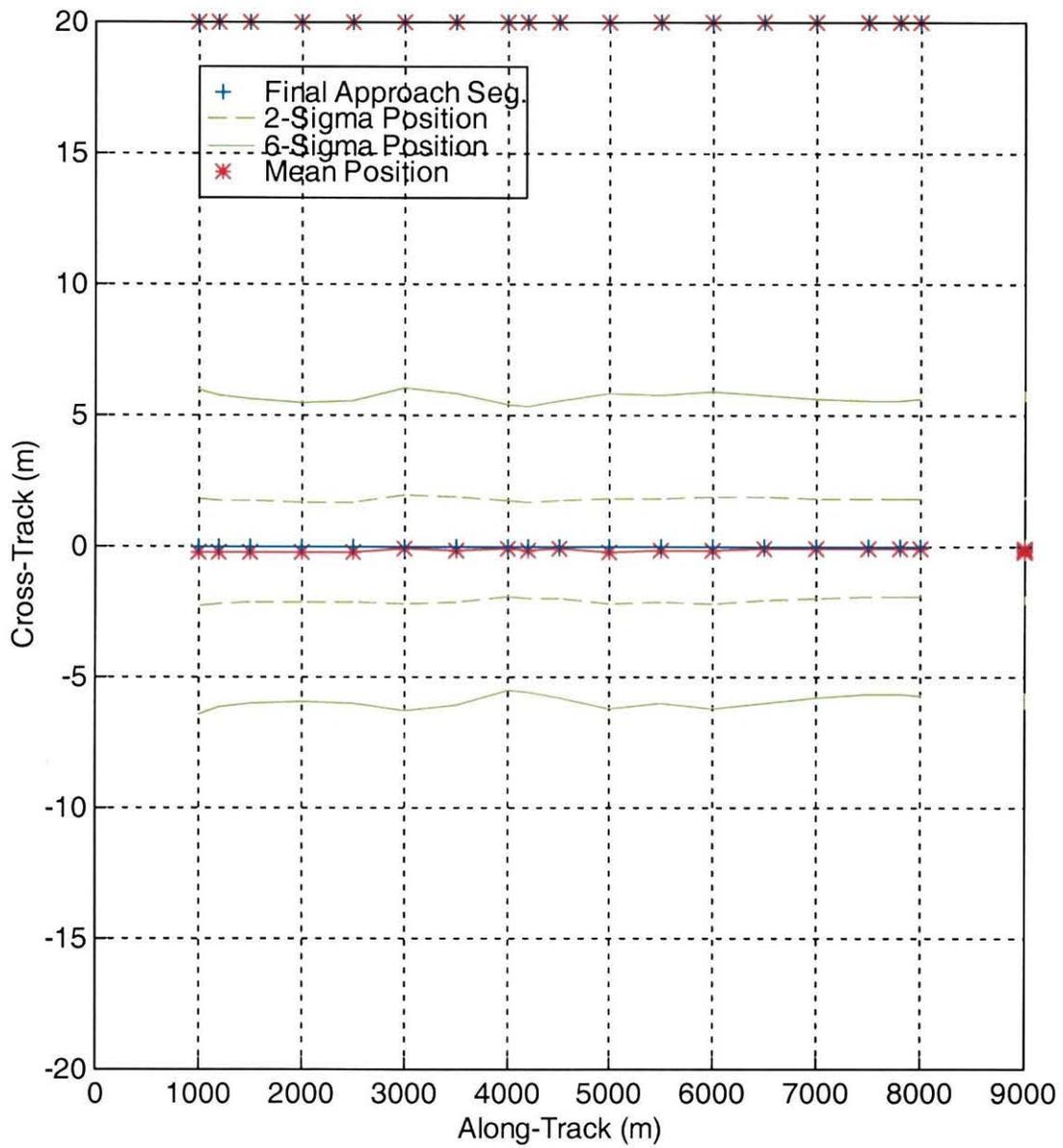


Figure 24: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY A TEST FLIGHTS

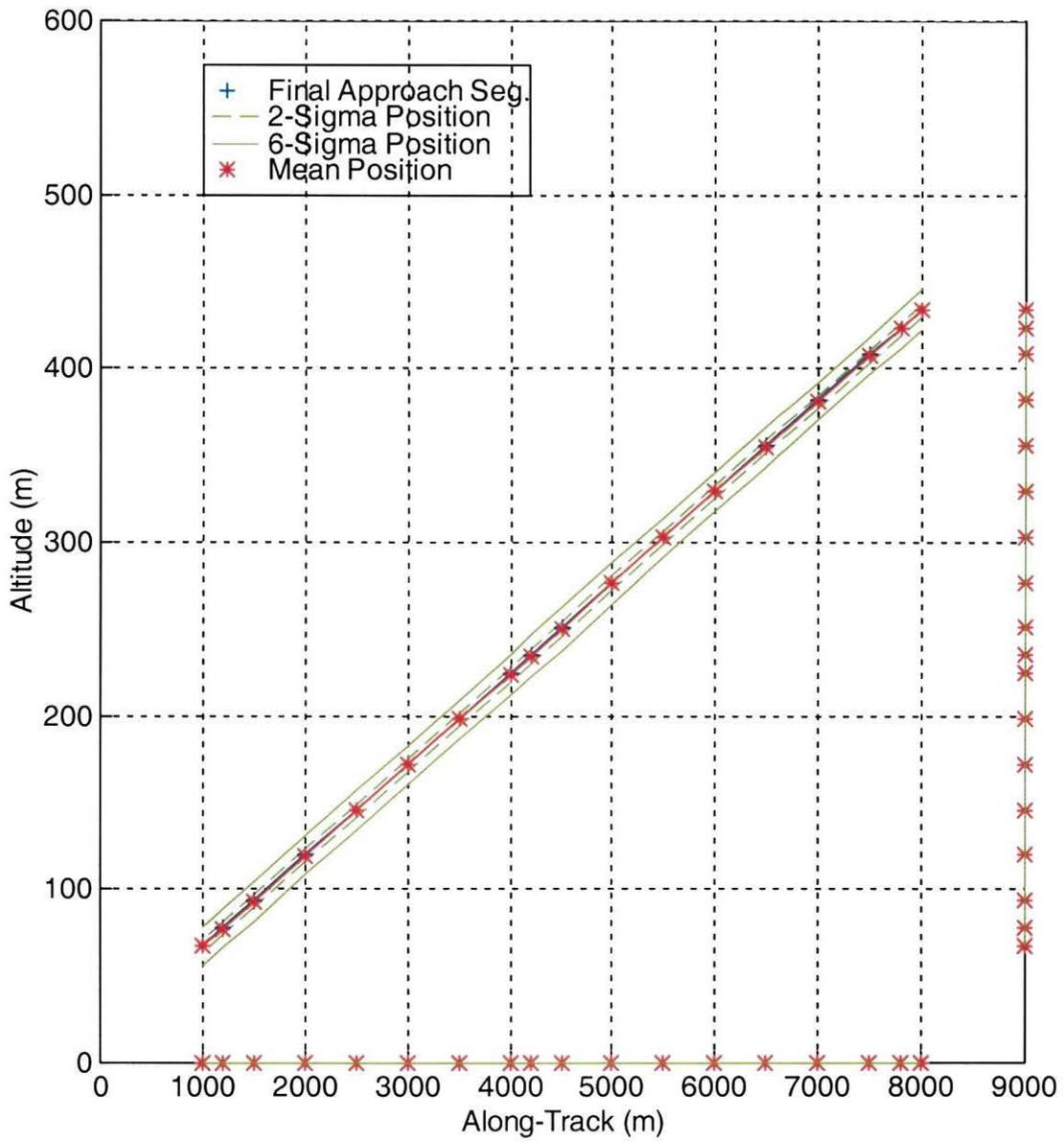


Figure 25: NAVIGATION SYSTEM ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY A TEST FLIGHTS

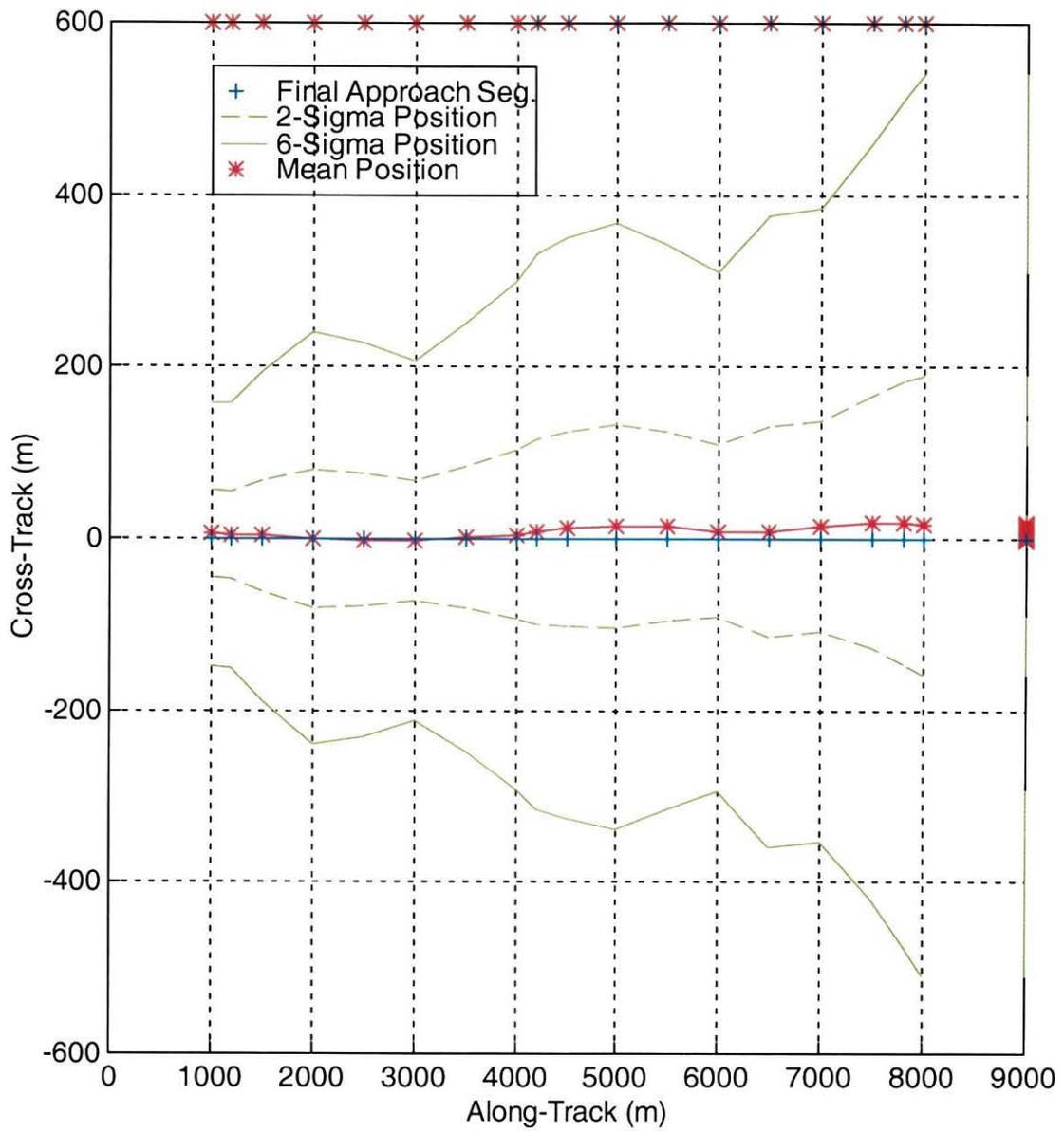


Figure 26: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA FOOTPRINT FOR CATEGORY A TEST FLIGHTS

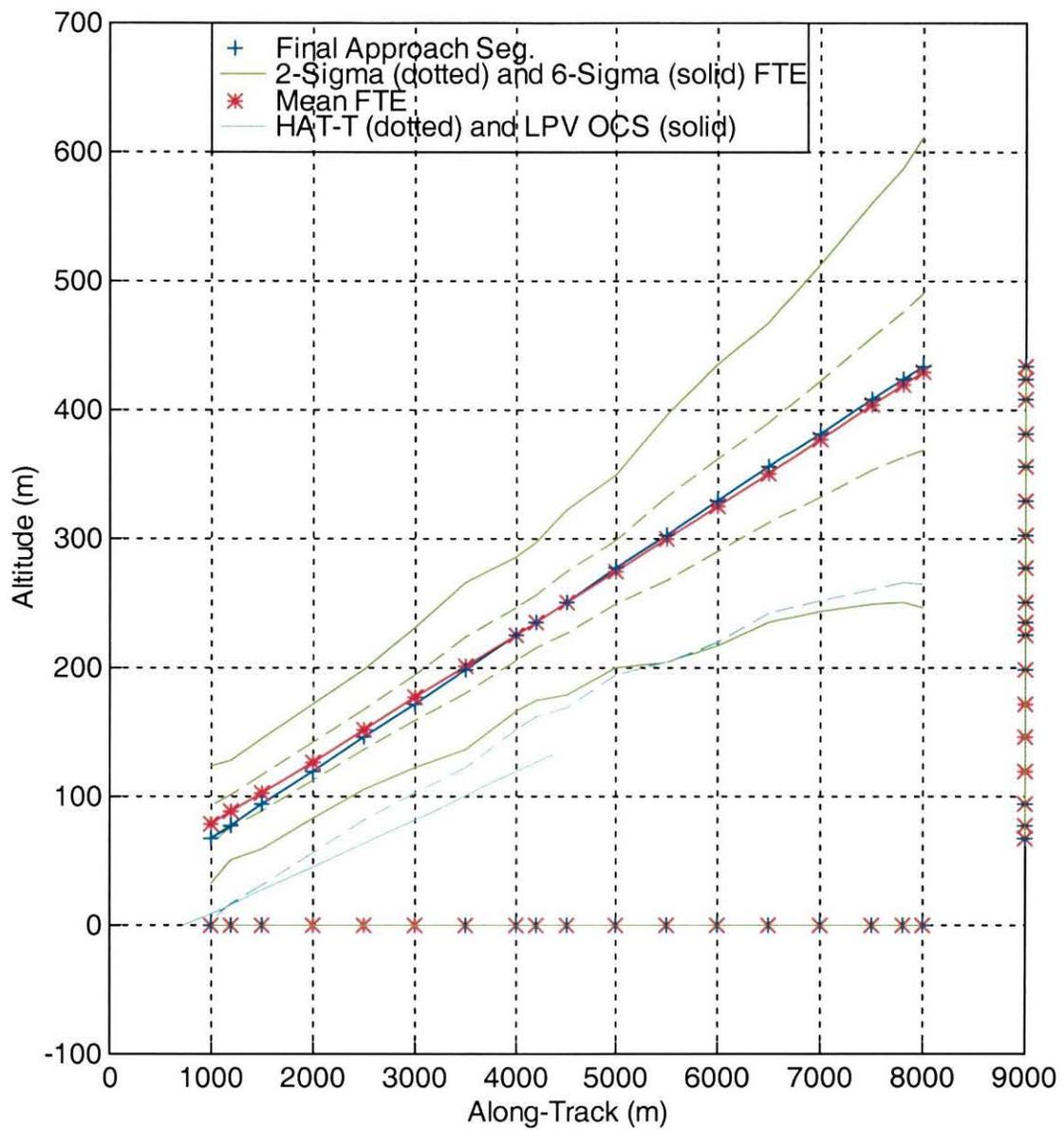


Figure 27: FLIGHT TECHNICAL ERROR 2-SIGMA AND 6-SIGMA PROFILE FOR CATEGORY A TEST FLIGHTS

<i>Range Along-Track (m)</i>	<i>Mean TSE Cross- Track (m)</i>	<i>Std. Dev. TSE Cross-Track (m)</i>	<i>Mean TSE Vertical (m)</i>	<i>Std. Dev. TSE Vertical (m)</i>
8000	15.85	87.72	-5.87	29.39
7800	18.76	82.22	-5.29	27.17
7500	19.54	72.25	-4.02	25.02
7000	15.34	61.33	-5.06	21.89
6500	8.84	61.17	-5.17	19.08
6000	8.43	50.03	-4.29	18.06
5500	13.89	54.46	-4.02	15.69
5000	14.47	58.40	-3.77	12.47
4500	11.78	55.94	-1.21	12.02
4200	8.17	53.40	-0.37	10.32
4000	4.81	48.96	0.37	10.09
3500	1.94	40.99	2.45	10.63
3000	-2.18	34.54	3.69	9.24
2500	-1.26	38.04	5.10	8.16
2000	-0.12	40.04	6.55	8.13
1500	4.71	27.96	7.50	6.33
1200	5.86	22.59	9.01	5.91
1000	5.69	23.02	8.43	6.68
870	5.51	25.02	8.01	6.95

**Table 8. CATEGORY A TOTAL SYSTEM ERROR
STATISTICS FOR GPS/WAAS TEST APPROACHES**

<i>Range Along-Track (m)</i>	<i>Mean NSE Cross- Track (m)</i>	<i>Std. Dev. NSE Cross-Track (m)</i>	<i>Mean NSE Vertical (m)</i>	<i>Std. Dev. NSE Vertical (m)</i>
8000	-0.04	0.94	-0.68	1.94
7800	-0.05	0.93	-0.75	1.94
7500	-0.02	0.94	-0.83	1.76
7000	-0.10	0.95	-0.82	1.82
6500	-0.09	0.98	-0.66	1.96
6000	-0.15	1.01	-0.63	1.84
5500	-0.14	0.98	-0.71	1.89
5000	-0.18	1.00	-0.70	2.05
4500	-0.10	0.94	-0.73	2.12
4200	-0.12	0.91	-0.78	2.03
4000	-0.06	0.91	-0.72	1.98
3500	-0.11	0.99	-0.60	1.83
3000	-0.09	1.03	-0.69	1.85
2500	-0.20	0.96	-0.74	1.96
2000	-0.22	0.96	-0.47	1.89
1500	-0.17	0.97	-0.51	2.02
1200	-0.08	0.98	-0.74	1.92
1000	-0.11	1.01	-0.59	1.88
870	0.04	1.04	-0.69	1.79

**Table 9. CATEGORY A NAVIGATION SYSTEM ERROR
STATISTICS FOR GPS/WAAS TEST APPROACHES**

<i>Range Along-Track (m)</i>	<i>Mean FTE Cross-Track (m)</i>	<i>Std. Dev. FTE Cross-Track (m)</i>	<i>Mean FTE Vertical (m)</i>	<i>Std. Dev. FTE Vertical (m)</i>
8000	15.90	87.60	-5.18	30.38
7800	18.81	82.10	-4.54	28.07
7500	19.60	72.88	-3.42	26.01
7000	15.44	61.43	-4.24	22.40
6500	8.93	61.26	-4.51	19.27
6000	8.58	50.24	-3.66	18.24
5500	14.03	54.82	-3.30	15.94
5000	14.65	58.83	-3.06	12.47
4500	11.88	56.38	-0.48	12.05
4200	8.30	53.77	0.41	10.17
4000	4.88	49.31	1.09	10.06
3500	2.05	41.43	3.05	10.83
3000	-2.09	34.81	4.38	8.95
2500	-1.06	38.20	5.81	7.64
2000	0.10	40.23	7.02	7.42
1500	4.88	28.37	7.92	6.42
1200	5.94	23.06	9.70	5.66
1000	5.80	23.46	9.01	6.46
870	5.47	25.27	8.68	6.56

Table 10. CATEGORY A FLIGHT TECHNICAL ERROR STATISTICS FOR GPS/WAAS TEST APPROACHES

3.0 SUMMARY

GPS/WAAS precision approach test procedures have been evaluated for 116 category C final approaches, 43 category B, and 53 category A final approaches. In the category C tests, an NSTB WAAS emulation was provided at the FAA Technical Center in Atlantic City, New Jersey. SA was turned on, and the system relied upon an approximately worst geometrical case for its wide area reference stations. The category A and B tests took place in Norman, Oklahoma, and category B tests employed the true (experimental testing stage) WAAS signal with SA turned off.

Navigation system errors were slightly different in each category of tests but appeared to be independent of range from threshold. In the category C tests, navigation system error data mean component values were about -.7 meters horizontally and .4 meters vertically (for a magnitude of about .8 meters) and component standard deviations were about 1.3 meters laterally and about 1.7 meters vertically. In the category B navigation system error statistics mean components were about -.1 meters horizontally and .5 meters vertically, and standard deviations were approximately .6 meters laterally and 1.0 meters vertically. Category A tests yielded mean components of -.1 meters horizontally and -.7 meters vertically, with standard deviations of 1.0 meters and 1.9 meters.

Further analysis of category C navigation system error components showed that they had no detectable correlation and contained both short-term and long-term errors. The short-term

navigation system errors (computed during approaches in a single test flight period of about two and a half hours) had component standard deviations which averaged only .55 meters laterally and 1.1 meters vertically, while long-term component standard deviations were about 1.0 meters laterally and 1.2 meters vertically.

Total system error and flight technical error component statistics were relatively close in value (since navigation system errors were consistently much smaller) and no detectable correlation between lateral and vertical components were found. As the aircraft crossed the 7,800 meters, 4,200 meters, and 1,200 meters ranges from threshold, the category C vertical standard deviation in total system error decreased from 14 meters to 9 meters to 5 meters. The corresponding standard deviations in the lateral component were 55 meters, 36 meters, and 9 meters. In the category B tests, the vertical total system error standard deviation decreased from 26 meters to 15 meters to 6 meters while the lateral standard deviation decreased from 44 meters to 35 meters to 21 meters. Category A vertical standard deviation in total system error decreased from 27 meters to 10 meters to 6 meters while lateral standard deviation decreased from 82 meters to 53 meters to 23 meters.

Visual comparison of the category C total system error component probability distributions, with those predicted by the Collision Risk Model (for the Instrument Landing System), indicated a reasonable agreement. However, a larger number of flight tests would be necessary to validate the tails of the CRM as a model for GPS/WAAS precision approach errors.

BIBLIOGRAPHY

1. "The ILS Equivalency Methodology and SBAS Flight Test Results," a paper presented to the Obstacle Clearance Panel (OCP) Working Group Meeting, of the International Civil Aviation Organization (ICAO), in Brussels, Belgium, April 8-19, 2002, by Lynn Boniface.
2. LPV Approach Procedure Construction Criteria, FAA Doc 8260.LPV, to appear 5/02.
3. "Global Positioning System/Wide Area Augmentation System (GPS/WAAS) Final Approach Error Analysis, "Proceedings of the 2002 National Technical Meeting, Institute of Navigation (ION) CD ROM (1/2802-1/30/02).
4. Requirements and Technical Concepts for Aviation, RTCA/DO-229B, "Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment" (10/6/99).
5. International Civil Aviation Organization, ICAO Doc 9274-AN/904, "Manual on the Use of The Collision Risk Model (CRM) for ILS Operations," 1st Ed. (1980).