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Analysis of RNP Maximum Route Length

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Technical Report

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Technical Report

EXECUTIVE SUMMARY

Maximum Course Length between Required Navigation Performance (RNP) Waypoints

It is the purpose of this analysis to determine a maximum distance between RNP waypoints, which ensure that overall risk to navigation remains at or below an acceptable level.

Ideally, an aircraft flies perfectly centered upon a constructed route. Herein lies two problems: (1) No pilot or autopilot can track an ideal line, and (2) for any route, the geometry of construction may not match the navigation solution and presentation of the route based on the on-board Flight Management System (FMS). The first kind of error is Flight Technical Error (FTE). The second kind of error is Navigation System Error (NSE). The Total System Error (TSE) is a statistical sum of these errors. With the advent of RNP, the TSE must be small enough to ensure that the probability of an aircraft being beyond the protected airspace is not above an acceptable level. One part of the NSE is bias error, due to the difference between the geometrically constructed route and the FMS computed route.

Manufacturers of FMS systems were contacted regarding the computational course solution used in their FMS systems, including satellite navigation. Each indicated that the computations used in each of their respective systems are based upon great circles of the volumetric sphere. The paper cited in the references from Smiths Industries also examines the differences between spherical great circle and geodesic WGS-84 ellipsoid solutions. The results shown are consistent with the analysis contained herein.

For routes with designed primary widths of 8 NM (RNP 2 as an example) constructed using WGS-84 geodetic computations and flown with great circle computations, or vice versa, it is recommended that maximum route length between waypoints be set at 500 NM. If these routes are to be represented on Lambert conformal charts, construction waypoints not more than 100 NM apart should be determined using WGS-84 or volumetric spherical coordinate systems.

Similarly, for routes with designed primary widths of 8 NM constructed using Lambert projections and flown with either great circle or WGS-84 geodetic computations, it is recommended that maximum route length between waypoints be set at 100 NM.

Further, it is recommended that these route lengths or constructions be modified directly by the square root of the ratio of the primary width compared to 8 NM. For example, RNP 3 has a primary width of 12 NM, thus the WGS-84 versus the spherical model maximum route length can be 500 x (12/8)^.5 = 612 NM, and Lambert versus WGS-84 or the sphere can be 100 x (12/8)^.5 = 122 NM. Similarly for RNP 0.3 which has a primary width of 1.2 NM, the recommended maximum route length between waypoints, WGS-84 versus sphere should not exceed 500 x (1.2/8)^.5 = 193 NM, and Lambert versus WGS-84/sphere should not exceed 100 x (1.2/8)^.5 = 38 NM.

Lastly, with an appropriate obstacle assessment, routes constructed and flown using identical computational methods are not limited in their length.

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Maximum Course Length between Required Navigation Performance (RNP) Waypoints

1.0 Introduction

Briefly, maps are flat, the Earth is not. Errors in navigation do result from these differences. It is important to ensure that navigation errors, due to differences between the route structure and the methods of navigation, do not cause the overall level of risk to be unacceptable. Since the magnitude of error increases with distance, it is the purpose of this analysis to determine a maximum distance between RNP waypoints, which ensures that overall risk to navigation remains at or below an acceptable level.

Several projections and earth models are examined in the following paragraphs to present an overview of these projections and models, and to quantify the magnitude of errors among the ellipsoid and sphere models, and Lambert conformal projections.

2.0 Projections

There are several commonly used two-dimensional (flat) projections of the not so flat earth. These are in no particular order: Tangent plan projection, Mercator Cylindrical Projection, and Lambert Conformal Projection.

a. Tangent Plane. A large area (small scale) tangent plan projection is most commonly used in the Polar Regions, with the tangent point being the North or South Pole. However, local radar displays are often tangent plane projections. Tangent plan projections are very accurate for short distances before radial distances exhibit excessive foreshortening due to earth curvature. As compared to the WGS-84 ellipsoid or to the sphere, radial directions from or to the pole of the projection are accurate, however, other radial directions have some error, and these errors increase with distance from the pole.

b. Mercator Cylindrical. A large area (small scale) Mercator projection is most commonly used to represent the globe, sometimes omitting the extreme Polar Regions. These are the maps where Canada, Alaska, Greenland, Siberia, and Antarctica appear disproportionately large. A Mercator projection centered over a particular point with its equator oriented in the direction of an intended route gives the same results as a spherical model with the same radius. Perpendicular to the equator, a route results in the same errors as the tangent plane projection. Directions other than along the equator or perpendicular to the equator will result in both along-track and cross-track errors, which tend to increase with distance as compared to the WGS-84 ellipsoid or the sphere.

c. Lambert Conformal. A large area (small scale) Lambert projection is often used to represent the contiguous 48 states of the United States. Additionally, Lambert projections of Alaska and Hawaii may also be shown as insets, but each will be based upon a different cone. Here the lines of latitude (east-west lines) show a curvature. The amount of curvature depends on which lines of latitude are used to anchor the cone of projection. Lambert projections are 7.5-minute quadrangle charts.

East-west and north-south errors are minor, but in directions other than the cardinal directions, errors can be much larger. Also errors beyond the bounding latitudes of the projection will exhibit even greater errors. On a small area (large scale) map such as a quadrangle, a Lambert projection does an admirable job of balancing distance, direction, and area errors. Quad charts are the projections upon which TERPS approach and missed approach procedures are hand drawn.

3.0 Three Dimensional Earth Models

Eratosthenes was one of the earliest to calculate the circumference of the earth. He did so by knowing that on a certain date at local noon the sun cast no shadow in a vertical well, which was located a known distance to his south, while at the same time at his location the sun was below the vertical by several degrees. Using this angle difference as the angle subtended from the center of the earth and using the known distance between his location and the well, Erastasthenes was able to calculate the circumference of a spherical earth to within about 15 percent of today's accepted value.

Isaac Newton was the first to demonstrate that the earth must be an oblate spheroid (an ellipse of revolution with a greater equatorial diameter than polar diameter). Careful measurement over the years, especially with the help of satellite measurement, has refined the three-dimensional model of the earth. The WGS-84 ellipsoid model has a polar flattening of about 14 km and an equatorial bulge of about 7 km as compared to a sphere of the same volume. The WGS-84 is the best second order fit of the geoid, the actual physical body of the earth. Third order fit induces a slight pear shape to the model. The North Pole is about 16 m higher than the ellipsoid. The South Pole is about 16 m lower than the ellipsoid. The northern mid latitudes are, at most, about 7 m below the ellipsoid, and the southern mid latitudes are, at most, about 7 m above the ellipsoid. Local anomalies account for some additional differences, but the combined effect of third and higher order models and local anomalies are not significant with respect to horizontal positioning.

a. Volumetric Sphere Model. The radius of the volumetric sphere is 6,371,000.79 m. Positional distances and directions on the sphere can be found by using spherical trigonometry or by converting to and from a rectangular coordinate system and using plane trigonometry.

b. WGS-84 Ellipsoid Model. The equatorial radius (semi major axis) of the ellipsoid is 6,378,137 m and the polar radius (semi minor axis) is 6,356,752.3141 m. Positional distances and directions on the ellipsoid present a much more complex problem and are computed using iterative numerical methods.

4.0 Route Differentials

Because each projection and earth model portrays the earth's positional and directional information differently, there are differences in routes constructed between any two known points as well.

> **Note**: The WGS-84 model is the same as the 1983 North American Datum (NAD83) model. All longitudes and latitudes were converted from the NAD27 model (equivalent to the Clarke 1863 model) to NAD83 on October 1st of 1993.

Shifts in recorded positions had changes of a few feet in the contiguous 48 states, a little more in Alaska, but as much as 1,600 feet in Hawaii. Old charts and maps published before October of 1993 may well be in error and should not be used to construct routes.

5.0 The Construction and Navigation Problem

Ideally, an aircraft flies perfectly centered upon a constructed route. Herein lies two problems: (1) No pilot or autopilot can track an ideal line and (2) for any route, the geometry of construction may not match the navigation solution and presentation of the route based upon the on board Flight Management System (FMS). The first kind of error is Flight Technical Error (FTE). The second kind of error is Navigation System Error (NSE). The Total System Error (TSE) is a statistical sum of these errors. With the advent of RNP, the TSE must be small enough to ensure that the probability of an aircraft being beyond the protected airspace is not above an acceptable level. One part of the NSE is bias error, due to the difference between the geometrically constructed route and the FMS computed route. Since the beginning and ending waypoints will match, any construction versus navigation error which occurs must happen between these two points, growing from the beginning waypoint to a maximum somewhere near mid course, then reducing to the ending waypoint.

Because of different curvatures based upon the different projections and models, the maximum differential between the constructed and computed routes in not necessarily at the midcourse point, but may occur slightly toward the part of the route which exhibits the greatest curvature. In the case of the spherical model, the route curvature is constant. A Lambert projection in the northern hemisphere has greater east-west curvature (smaller radius) at its northern limit than at its southern limit. The WGS-84 ellipsoid has greatest north-south curvature at the equator. This north-south curvature decreases toward the poles, and along any route the least curvature occurs in the east-west direction. The difference between the course differentials at the point of actual maximum separation and the midcourse separation are not significant and may be ignored for this analysis.

Manufacturers of FMS systems were contacted regarding the computational course solution used in their FMS systems, including satellite navigation. Each indicated that the computations used in each of their respective systems are based upon great circles of the volumetric sphere.

The paper cited in the references from Smiths Industries also examines the differences between spherical great circle and geodesic WGS-84 ellipsoid solutions. The results shown are consistent with the analysis contained herein.

6.0 Data Analysis

Tables 1 through 3 in Appendix I show computed cross-track and along-track differences at the route midpoints for the WGS-84 ellipsoid versus the volumetric sphere (Table 1), the WGS-84 ellipsoid versus three different Lambert conformal projections (Tables 2a, 2b, and 2c), and the volumetric sphere versus the three different Lambert projections (Tables 3a, 3b, and 3c) for route distances of 100, 200, 300, 400, and 500 NM, all using routes generated from and centered upon the WGS-84 midpoint 35° N 70° W. Table 4 shows the endpoints of the 15 different routes used in the computations. Table 5 shows the effect of latitude on east-west routes.

An inspection of each of the tables suggests that midcourse differences increase with the square of the distance, i.e., doubling the route length approximately quadruples the midcourse differences. East-west and north-south routes show the least midcourse differences and northeast-southwest (and equivalently northwest-southeast) routes show the greatest differences. Latitude also is a factor with WGS-84 ellipsoid versus the sphere exhibiting greatest differences near 45° N or S.

7.0 Recommendations

For routes with designed primary widths of 8 NM (RNP 2 as an example) constructed using WGS-84 geodetic computations and flown with great circle computations, or vice versa, it is recommended that maximum route length between waypoints be set at 500 NM. If these routes are to be represented on Lambert conformal charts, construction waypoints not more than 100 NM apart should be determined using WGS-84 or volumetric spherical coordinate systems.

Similarly, for routes with designed primary widths of 8 NM constructed using Lambert projections and flown with either great circle or WGS-84 geodetic computations, it is recommended that maximum route length between waypoints be set at 100 NM, and that these recommended route lengths or constructions be modified directly by the square root of the ratio of the primary width compared to 8 NM. For example, RNP 3 has a primary width of 12 NM, thus the WGS-84 versus the spherical model maximum route length can be 500 x (12/8)^.5 = 612 NM, and Lambert versus WGS-84 or the sphere can be 100 x (12/8)^.5 = 122 NM. Similarly for RNP 0.3 which has a primary width of 1.2 NM, the recommended maximum route length between waypoints, WGS-84 versus sphere, should not exceed 500 x (1.2/8)^.5 = 193 NM, and Lambert versus WGS-84/sphere should not exceed 100 x (1.2/8)^.5 = 38 NM.

Lastly, with an appropriate obstacle assessment, routes constructed and flown using identical computational methods are not limited in their length.

Appendix A. List of Tables

Table 1: WGS-84 Model versus Volumetric Sphere Model

Table 2a: WGS-84 Model versus Lambert Conformal Projection (30° to 40° N)

Table 2b: WGS-84 Model versus Lambert Conformal Projection (25° to 35° N)

Table 2c: WGS-84 Model versus Lambert Conformal Projection (35° to 45° N)

Table 3a: Volumetric Sphere Model versus Lambert Conformal Projection (30° to 40° N)

Table 3b: Volumetric Sphere Model versus Lambert Conformal Projection (25° to 35° N)

Table 3c: Volumetric Sphere Model versus Lambert Conformal Projection (35° to 45° N)

Table 4: Route Beginning and Ending Points

Table 5: 500 NM East-West Route Errors, WGS-84 vs. Volumetric Sphere

Table 1				
WGS-84 vs. Sphere				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length	S mid lat	S mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99998035	70	0	0.0012
200 NM	34.99992421	70	0	0.0045
300 NM	34.99982829	70	0	0.0103
400 NM	34.99969495	70	0	0.0183
500 NM	34.99952211	70	0	0.0286
North-South Route				
Route Length	S mid lat	S mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99994250	70	0.0034	0
200 NM	34.99977000	70	0.0138	0
300 NM	34.99948250	70	0.0310	0
400 NM	34.99908167	70	0.0550	0
500 NM	34.99856500	70	0.0860	0
Northeast-Southwest Route				
Route Length	S mid lat	S mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99996131	70.00002292	0.0024	0.0008
200 NM	34.99984693	70.00009301	0.0097	0.0032
300 NM	34.99965522	70.00020903	0.0219	0.0073
400 NM	34.99938795	70.00037156	0.0389	0.0130
500 NM	34.99904356	70.00058009	0.0607	0.0203

Table 2a				
WGS-84 vs. Lambert (30° to 40° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99998360	70	0	0.0010
200 NM	34.99993812	70	0	0.0037
300 NM	34.99986307	70	0	0.0082
400 NM	34.99976544	70	0	0.0141
500 NM	34.99964965	70	0	0.0210
North-South Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	35.00000014	70	0.0000	0
200 NM	35.00000055	70	0.0000	0
300 NM	35.00000118	70	0.0001	0
400 NM	35.00000365	70	0.0002	0
500 NM	35.00000536	70	0.0003	0
Northeast-Southwest Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99999184	70.00001921	0.0003	0.0010
200 NM	34.99997062	70.00007604	0.0015	0.0041
300 NM	34.99993935	70.00016286	0.0031	0.0082
400 NM	34.99990756	70.00026960	0.0055	0.0133
500 NM	34.99988460	70.00038095	0.0084	0.0182

Table 2b				
WGS-84 vs. Lambert (25° to 35° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99947010	70	0	0.0317
200 NM	34.99788394	70	0	0.1268
300 NM	34.99524030	70	0	0.2851
400 NM	34.99153677	70	0	0.5070
500 NM	34.98680121	70	0	0.7906
North-South Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99946823	70	0.0319	0
200 NM	34.99787301	70	0.1274	0
300 NM	34.99521466	70	0.2867	0
400 NM	34.99149535	70	0.5095	0
500 NM	34.98671332	70	0.7959	0
Northeast-Southwest Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	34.99946916	69.99932189	0.0012	0.0461
200 NM	34.99788013	69.99728671	0.0048	0.1844
300 NM	34.99523660	69.99388685	0.0113	0.4148
400 NM	34.99154926	69.98911219	0.0216	0.7375
500 NM	34.98682903	69.98294740	0.0366	1.1523

Table 2c				
WGS-84 vs. Lambert (35° to 45° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	35.00052919	70	0	0.0317
200 NM	35.00212071	70	0	0.1270
300 NM	35.00477465	70	0	0.2860
400 NM	35.00849900	70	0	0.5091
500 NM	35.01329957	70	0	0.7967
North-South Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	35.00053205	70	0.0319	0
200 NM	35.00212807	70	0.1275	0
300 NM	35.00478768	70	0.2868	0
400 NM	35.00851190	70	0.0599	0
500 NM	35.01329731	70	0.7966	0
Northeast-Southwest Route				
Route Length	L mid lat	L mid lon	Along Track (NM)	Cross Track (NM)
100 NM	35.00053057	70.00059912	0.0016	0.0434
200 NM	35.00212521	70.00239581	0.0065	0.1735
300 NM	35.00478599	70.00538281	0.0151	0.3903
400 NM	35.00852089	70.00955068	0.0281	0.6938
500 NM	35.01333706	70.01488495	0.0462	1.0837

Table 3a				
Sphere vs. Lambert (30° to 40° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0	0.0002
200 NM	midpoints	midpoints	0	0.0008
300 NM	shown in	shown in	0	0.0021
400 NM	Table 1	Table 2a	0	0.0042
500 NM			0	0.0076
North-South Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0035	0
200 NM	midpoints	midpoints	0.0138	0
300 NM	shown in	shown in	0.0311	0
400 NM	Table 1	Table 2a	0.0552	0
500 NM			0.0863	0
Northeast-Southwest Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0012	0.0014
200 NM	midpoints	midpoints	0.0046	0.0058
300 NM	shown in	shown in	0.0136	0.0104
400 NM	Table 1	Table 2a	0.0256	0.0186
500 NM			0.0426	0.0287

Table 3b				
Sphere vs. Lambert (25° to 35° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0	0.0306
200 NM	midpoints	midpoints	0	0.1222
300 NM	shown in	shown in	0	0.2748
400 NM	Table 1	Table 2b	0	0.4887
500 NM			0	0.7620
North-South Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0284	0
200 NM	midpoints	midpoints	0.1136	0
300 NM	shown in	shown in	0.2557	0
400 NM	Table 1	Table 2b	0.4544	0
500 NM			0.7100	0
Northeast-Southwest Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0036	0.0453
200 NM	midpoints	midpoints	0.0145	0.1811
300 NM	shown in	shown in	0.0332	0.4075
400 NM	Table 1	Table 2b	0.0604	0.7245
500 NM			0.0971	1.1320

Table 3c				
Sphere vs. Lambert (35° to 45° N)				
WGS-84 mid point 35° N 70° W				
East-West Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0	0.0329
200 NM	midpoints	midpoints	0	0.1315
300 NM	shown in	shown in	0	0.2963
400 NM	Table 1	Table 2c	0	0.5274
500 NM			0	0.8253
North-South Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0353	0
200 NM	midpoints	midpoints	0.1413	0
300 NM	shown in	shown in	0.3178	0
400 NM	Table 1	Table 2c	0.5649	0
500 NM			0.8825	0
Northeast-Southwest Route				
Route Length			Along Track (NM)	Cross Track (NM)
100 NM	Spherical	Lambert	0.0040	0.0442
200 NM	midpoints	midpoints	0.0142	0.1768
300 NM	shown in	shown in	0.0234	0.3872
400 NM	Table 1	Table 2c	0.0669	0.7068
500 NM			0.1068	1.1040

Table 4

Route Beginning and Ending Points

	East to West				
	Lat 1	Lon 1		Lat 2	Lon 2
100 NM	34° 59.7457' N	68° 59.1399' W		34° 59.7457' N	71° 00.8601' W
200 NM	34° 58.9831' N	67° 58.2923' W		34° 58.9831' N	72° 01.7077' W
300 NM	34° 57.7124' N	66° 57.4698' W		34° 57.7124' N	73° 02.5302' W
400 NM	34° 55.9344' N	65° 56.6848' W		34° 55.9344' N	74° 03.3152' W
500 NM	34° 53.6499' N	64° 55.9498' W		34° 53.6499' N	75° 04.0502' W
	North to South				
	Lat 1	Lon 1		Lat 2	Lon 2
100 NM	35° 50.0774' N	70° 00.000' W		34° 9.9157' N	70° 00.000' W
200 NM	36° 40.1478' N	70° 00.000' W		33° 19.8246' N	70° 00.000' W
300 NM	37° 30.2112' N	70° 00.000' W		32° 29.7267' N	70° 00.000' W
400 NM	38° 20.2676' N	70° 00.000' W		31° 39.6222' N	70° 00.000' W
500 NM	39° 10.3167' N	70° 00.000' W		30° 49.5111' N	70° 00.000' W
	Northeast to Southwest				
	Lat 1	Lon 1		Lat 2	Lon 2
100 NM	35° 35.2821' N	69° 16.6519' W		34° 24.4601' N	70° 42.7300' W
200 NM	36° 10.2972' N	68° 32.6676' W		33° 48.6715' N	71° 24.8599' W
300 NM	36° 45.0355' N	67° 48.0289' W		33° 12.6429' N	72° 06.4070' W
400 NM	37° 19.4872' N	67° 02.7171' W		32° 36.3828' N	72° 47.3888' W
500 NM	37° 53.6419' N	66° 16.7135' W		31° 59.8993' N	73° 27.8222' W

Table 5	
500 NM East-West Routes	
WGS-84 vs. Sphere	
Latitude	Cross Track (NM)
5° N	0.0052
15° N	0.0153
25° N	0.0233
35° N	0.0286
45° N	0.0305
55° N	0.0287
65° N	0.0234
75° N	0.0152
85° N	0.0053

Appendix B. BASIC Code for Lambert Conformal Projection Midpoints

```
REM Lambert Conformal Projection Midpoint
CLS

REM Constants
  a# = 6378137#
  b# = 6356752.4141#
  pi# = 4 * ATN(1)

PRINT "Lambert Conformal Projection Midpoint Calculator"
PRINT

REM input upper latitude anchor
 PRINT "Input Anchor Latitudes"
 PRINT
' INPUT "  Upper Anchor Lat degrees ", latupdeg#
' INPUT "  Upper Anchor Lat minutes ", latupmin#
' INPUT "  Upper Anchor Lat seconds ", latupsec#
' latup# = latupdeg# + latupmin# / 60 + latupsec# / 3600
 latup# = 35
 PRINT

REM input lower latitude anchor
' INPUT "  Lower Anchor Lat degrees ", latdndeg#
' INPUT "  Lower Anchor Lat minutes ", latdnmin#
' INPUT "  Lower Anchor Lat seconds ", latdnsec#
' latdn# = latdndeg# + latdnmin# / 60 + latdnsec# / 3600
 latdn# = 25
 PRINT

REM Input Point 1 Latitude and Longitude
 PRINT "Input point 1 Latitude and Longitude"
 PRINT
 INPUT "  Lat 1 degrees? ", lat1deg#
 INPUT "  Lat 1 minutes? ", lat1min#
 INPUT "  Lat 1 seconds? ", lat1sec#
 lat1# = lat1deg# + lat1min# / 60 + lat1sec# / 3600
' lat1# = 34
 PRINT
 INPUT "  Lon 1 degrees? ", lon1deg#
 INPUT "  Lon 1 minutes? ", lon1min#
 INPUT "  Lon 1 seconds? ", lon1sec#
 lon1# = lon1deg# + lon1min# / 60 + lon1sec# / 3600
' lon1# = 69
 PRINT
```

```
REM Input Point 2 Latitude and Longitude
 PRINT "Input Point 2 Latitude and Longitude"
 PRINT
 INPUT "  Lat 2 degrees? ", lat2deg#
 INPUT "  Lat 2 minutes? ", lat2min#
 INPUT "  Lat 2 seconds? ", lat2sec#
 lat2# = lat2deg# + lat2min# / 60 + lat2sec# / 3600
' lat2# = 36
 PRINT
 INPUT "  Lon 2 degrees? ", lon2deg#
 INPUT "  Lon 2 minutes? ", lon2min#
 INPUT "  Lon 2 seconds? ", lon2sec#
 lon2# = lon2deg# + lon2min# / 60 + lon2sec# / 3600
' lon2# = 71
 PRINT

REM Convert to Radians
 latupr# = latup# * pi# / 180
 latdnr# = latdn# * pi# / 180
 lat1r# = lat1# * pi# / 180
 lon1r# = lon1# * pi# / 180
 lat2r# = lat2# * pi# / 180
 lon2r# = lon2# * pi# / 180

REM Find upper and lower anchor radii
 rup# = a# ^ 2 / (a# ^ 2 + b# ^ 2 * (TAN(latupr#)) ^ 2) ^ .5
 rdn# = a# ^ 2 / (a# ^ 2 + b# ^ 2 * (TAN(latdnr#)) ^ 2) ^ .5

REM Find upper and lower z-values
 zup# = b# * (1 - (rup# / a#) ^ 2) ^ .5
 zdn# = b# * (1 - (rdn# / a#) ^ 2) ^ .5

REM Find height, upper slant height, and lower slant height
 h# = rup# * (zup# - zdn#) / (rdn# - rup#)
 sup# = (h# ^ 2 + rup# ^ 2) ^ .5
 sdn# = ((h# + zup# - zdn#) ^ 2 + rdn# ^ 2) ^ .5

REM Find difference in longitudes adjusted to cone
 alpha# = ABS(lon1# - lon2#) * rup# / sup#

REM Find r1, z1, r2 and z2 for ellipsoid
 r1# = a# ^ 2 / (a# ^ 2 + b# ^ 2 * (TAN(lat1r#)) ^ 2) ^ .5
 r2# = a# ^ 2 / (a# ^ 2 + b# ^ 2 * (TAN(lat2r#)) ^ 2) ^ .5
 z1# = b# * (1 - (r1# / a#) ^ 2) ^ .5
 z2# = b# * (1 - (r2# / a#) ^ 2) ^ .5
```

```
REM Determine the adjusted r and z's
 Det# = (-h# / rup# - rup# / h#)
 Dr1# = -(h# + zup#) - (rup# / h# * r1# - z1#)
 Dz1# = (h# / rup# * (rup# / h# * r1# - z1#)) - (rup# / h# * (h# + zup#))
 r1adj# = Dr1# / Det#
 z1adj# = Dz1# / Det#

 Dr2# = -(h# + zup#) - (rup# / h# * r2# - z2#)
 Dz2# = (h# / rup# * (rup# / h# * r2# - z2#)) - (rup# / h# * (h# + zup#))
 r2adj# = Dr2# / Det#
 z2adj# = Dz2# / Det#

REM Determine slant heights
 s1# = sup# / rup# * r1adj#
 s2# = sup# / rup# * r2adj#

REM Law of Cosines to find distance between points
 d# = (s1# ^ 2 + s2# ^ 2 - 2 * s1# * s2# * COS(alpha# * pi# / 180)) ^ .5

REM Find cosine beta
 cbeta# = (s2# ^ 2 + d# ^ 2 - s1# ^ 2) / (2 * s2# * d#)

REM Find slant height and radius and height of midpoint
 sm# = (s2# ^ 2 + d# ^ 2 / 4 - s2# * d# * cbeta#) ^ .5
 rm# = rup# / sup# * sm#
 zm# = -h# / rup# * rm# + h# + zup#

REM Find gamma
 cgamma# = (sm# ^ 2 + s2# ^ 2 - d# ^ 2 / 4) / (2 * sm# * s2#)
 gammar# = ATN((1 - cgamma# ^ 2) ^ .5 / cgamma#)
 IF gammar# < 0 THEN gammar# = gammar# + pi#
 gamma# = gammar# * 180 / pi#

REM find output longitude
 gammadj# = gamma# * sup# / rup#
 IF lon2# > lon1# THEN lonm# = lon2# - gammadj# ELSE lonm# = lon2# + gammadj#

REM Find rajd# by quadratic equation
 aq# = (b# / a#) ^ 2 + (rup# / h#) ^ 2
 bq# = 2 * (rup# / h# * zm# - (rup# / h#) ^ 2 * rm#)
 cq# = zm# ^ 2 + (rm# * rup# / h#) ^ 2 - 2 * rup# * rm# * zm# / h# - b# ^ 2
 rmadj# = (-bq# + (bq# ^ 2 - 4 * aq# * cq#) ^ .5) / (2 * aq#)

REM Find midpoint latitude
 tlatm# = (a# ^ 2 * (a# ^ 2 - rmadj# ^ 2) / (b# * rmadj#) ^ 2) ^ .5
```

```
  latm# = ATN(tlatm#) * 180 / pi#
'PRINT rup#, rdn#, zup#, zdn#, h#, sup#, sdn#, alpha#, r1#
'PRINT r2#, z1#, z2#, r1adj#, r2adj#, z1adj#, z2adj#, s1#, s2#
'PRINT d#, cbeta#, sm#, rm#, zm#, rmadj#, aq#, bq#, cq#
'PRINT rmadj#
PRINT
PRINT latm#, lonm#
```

Appendix C. Spherical Midpoint Equations

Given

Lat 1, Lon 1
Lat 2, Lon 2

Convert to Radian Measure

Lat 1r = Lat 1 x pi/180, Lon 1r = Lon 1 x pi/180
Lat 2r = Lat 2 x pi/180, Lon 2r = Lon 2 x pi/180

Convert to (x, y, z) coordinate system

x1 = cos(lat 1r) x cos(lon 1r)
y1 = cos(lat 1r) x sin(lon 1r)
z1 = sin(lat 1r)

x2 = cos(lat 2r) x cos(lon 2r)
y2 = cos(lat 2r) x sin(lon 2r)
z2 = sin(lat 2r)

Find straight line midpoint

x = (x1 + x2)/2
y = (y1 + y2)/2
z = (z1 + z2)/2

Unitize lengths for midpoint

xm = x/(x^2 + y^2 + z^2)^.5
ym = y/(x^2 + y^2 + z^2)^.5
zm = z/(x^2 + y^2 + z^2)^.5

Convert to latitude and longitude in radians

latr mid = arccos(xm^2 + ym^2)^.5
lonr mid = arctan(ym/xm)

Convert to degrees

lat mid = latr mid x 180/pi
lon mid = lonr mid x 180/pi

References:

Smiths Industries: FMCS Earth Model Accuracy Analysis – November 13, 1995

Williams, Ed: Great Circle Calculator – http://williams.best.vwh.net/gccalc.htm,