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Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments

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Technical Report

Analysis of RNAV En Route Separation Along Adjacent Straight Segments

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Executive Summary

The purpose of this study is to provide a risk assessment of lateral en route separation between parallel Area Navigation (RNAV) routes, such as Q-routes, with procedural separation for both opposite and same direction traffic. The study will estimate the risk of RNAV aircraft flying straight tracks (tracks with turns of less than 15°) deviating from the nominal track laterally by more than 2, 3, or 4 nautical miles (NM). It will also estimate the risk of collision of en route RNAV aircraft flying adjacent, parallel, straight tracks (both opposite and same directions cases) when the aircraft of interest is flying adjacent to only one other track (on an outer track) and when the aircraft of interest is flying between two tracks (on an inner track). It will not examine risk of collision between an aircraft on an RNAV route and one on a conventional VHF Omnidirectional Range/Distance Measuring Equipment (VOR/DME) airway or between an aircraft on an RNAV route and special use airspace.

The analysis is based on two types of data: values specified in AC 90-100 and data from radar tracks reported in previous RNAV studies. AC 90-100 specifies a value for track-keeping accuracy value for RNAV aircraft. This criterion is the basis for the analysis. There are three studies examined that have used RNAV track data. This study uses the data and results from those studies to validate the criterion-based analysis results.

The study fits statistical distributions to the values from the AC 90-100 criteria and to the data from the studies to model the likelihood of adjacent aircraft intersecting laterally. It also models and estimates the likelihood of aircraft on parallel routes becoming adjacent. Using those models, it estimates the probability of collision.

For outer tracks, the results of this analysis show that the hourly probability of collision for suitably equipped RNAV aircraft on parallel adjacent routes flying <u>opposite directions</u> (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 5 NM on average meets the acceptable level of risk (1.0 E-08). The hourly probability of collision for suitably equipped RNAV aircraft on parallel adjacent routes flying <u>the same direction</u> (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 10 NM on average meets the acceptable level of risk.

For inner tracks, the results of this analysis show that the hourly probability of collision for suitably equipped RNAV aircraft on parallel adjacent routes flying <u>opposite directions</u> (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 10 NM on average meets the acceptable level of risk. The hourly probability of collision for suitably equipped RNAV aircraft on parallel adjacent routes flying <u>the same direction</u> (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 15 NM on average meets the acceptable level of risk.

For track separation of 4 or 6 NM the probability of collision for all cases exceeds this acceptable level of risk.

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1.0 Introduction

1.1 Purpose and Structure of This Document

The purpose of this study is to provide a risk assessment of en route lateral separation between parallel Area Navigation (RNAV) routes, such as Q-routes, with procedural separation for both opposite and same direction traffic. The study will estimate the risk of RNAV aircraft flying straight tracks (tracks with turns of less than 15°) deviating from the nominal track laterally by more than 2, 3, or 4 nautical miles (NM). It will also estimate the risk of collision of en route aircraft flying parallel, adjacent, straight tracks (both opposite and same directions cases). The study will also examine the scenarios in which the aircraft is flying a track between two other parallel tracks (an inner track) and the scenarios in which the aircraft is flying adjacent to only one other track (an outer track).

1.2 Statement of the Problem

Specifically, this study seeks to quantify the lateral track deviation of typical RNAV equipped aircraft[1] on straight en route segments – segments with no turns, or turns of at most 15°. This lateral track deviation will be used to determine the probability that a typical RNAV en route operation deviates laterally from the track by more than certain given distances (each of 2, 3, or 4 NM).

This lateral track deviation will also be used to determine the probability of collision of two aircraft flying parallel, adjacent en route tracks, with given track-to-track separation distances (4, 6, or 8 NM), with both inner and outer tracks, and with the two cases: flying same direction or opposite direction.

For suitably equipped RNAV aircraft, as referenced in AC 90-100, this study will answer the following questions:

1. What is the probability of an aircraft flying a straight en route track segment deviating laterally from that track by more than 2 NM (or 3 NM or 4 NM) during one hour of flight?

2. What is the risk of an aircraft flying a straight en route outer track segment colliding with an aircraft flying a parallel, adjacent en route track, in a direction <u>opposite</u> that of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM)?

3. What is the risk of an aircraft flying a straight en route outer track segment colliding with an aircraft flying a parallel, adjacent en route track, in a direction the <u>same</u> as that of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM)?

[1] As defined in ICAO Document 9689-AN/953, First edition, 1998

4. What is the risk of an aircraft flying a straight en route inner track segment colliding with an aircraft flying a parallel, adjacent en route track, in a direction opposite that of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming aircraft on either side of the inner track aircraft are flying in an opposite direction to that of the inner aircraft?

5. What is the risk of an aircraft flying a straight en route inner track segment colliding with an aircraft flying a parallel, adjacent en route track, in a direction the same as that of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming aircraft on either side of the inner track aircraft are flying in the same direction to that of the inner aircraft?

6. What is the risk of an aircraft flying a straight en route inner track segment colliding with an aircraft flying a parallel, adjacent en route track, in a direction the same as or opposite to that of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming the aircraft on one side of the inner track aircraft is flying in the same direction as and the aircraft on the other side is flying in the opposite direction to that of the inner aircraft?

2.0 Study Methodology

2.1 Model Description

We will describe the models in terms of their scenarios and the associated hazards. As described above, there are three scenarios of interest.

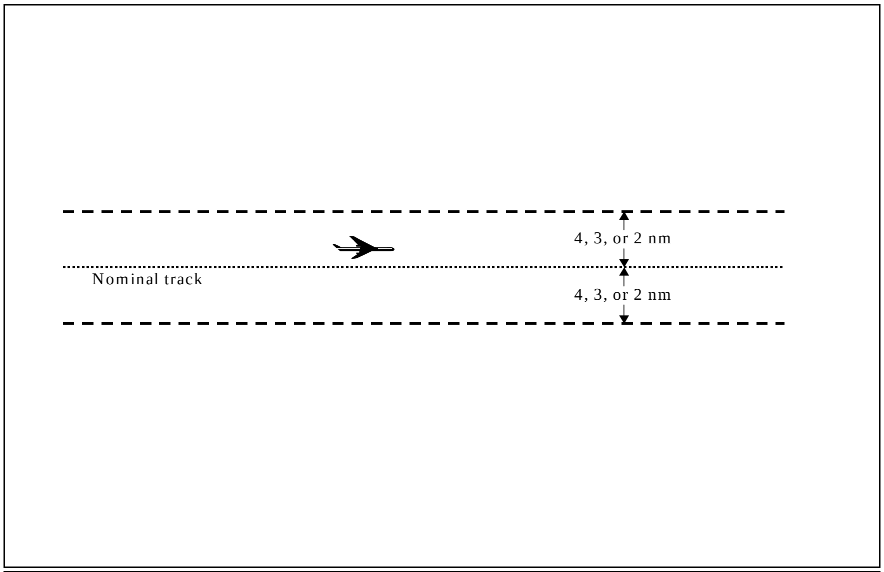
Scenario 1

In this scenario a typical aircraft is flying a straight en route track segment with turns of no more than 15°. The hazard in this scenario is the aircraft deviating laterally from that track by more than 2 NM (or 3 NM or 4 NM) during one hour of flight (Figure 2.1.1). The severity of this hazard is major (see Appendix A, Severity Definitions Based on the Perspective of the Flying Public, for a description of this severity).

The specific Test Criteria Violation (TCV) for this hazard is the deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM or more (or 3 NM or more, or 4 NM or more). We will estimate the probabilities of these TCVs, but will not assess their risk since no actual collision is involved.

We model this scenario by a statistical distribution of lateral aircraft deviations. This distribution will be used to determine the probability of a TCV.

Figure 2.1.1 Scenario 1 Hazards



Scenario 2

In this scenario two aircraft are flying in opposite directions on parallel, adjacent, straight en route track segments with turns of no more than 15° (Figure 2.1.2). The parallel tracks are separated by 4 NM (or 6 NM or 8 NM). The hazard in this scenario is the collision of the aircraft. The severity of this hazard is catastrophic (see Appendix A).

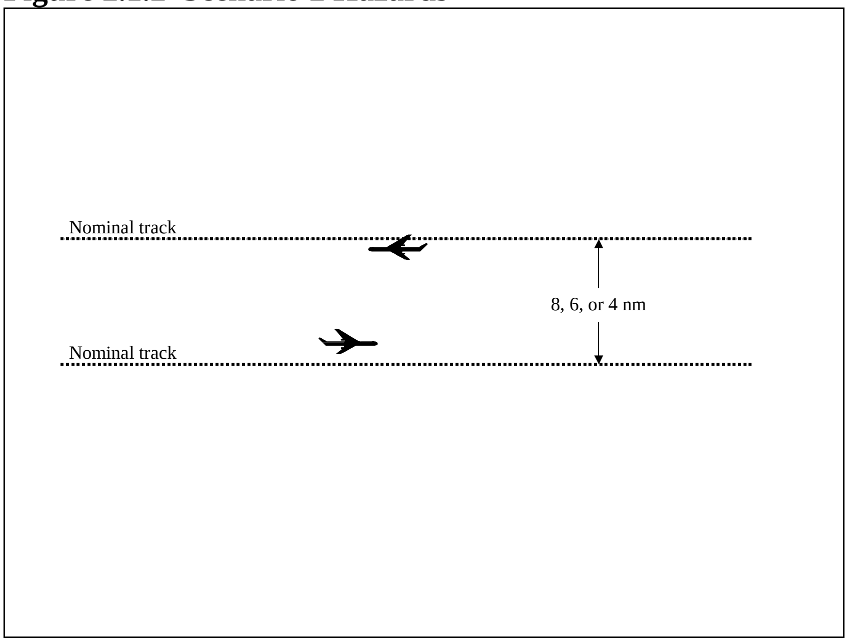
The specific TCV for this hazard is the combined lateral, longitudinal, and vertical conjunction of the two aircraft. This conjunction is modeled by centers of gravity of the aircraft converging to within their mean wingspan laterally, within their mean lengths longitudinally, and within their mean heights vertically.

We model this scenario by statistical distributions for lateral aircraft deviations and by probabilities for longitudinal and vertical convergence of the two aircraft. We assume that the lateral deviation, the vertical deviation, and the longitudinal encounter with the other aircraft are independent. This is a conservative assumption because for these to be dependent would imply that the two aircraft were either trying to avoid each other or trying to collide. For aircraft operating under normal conditions we can eliminate this latter possibility. Therefore, under normal operating conditions dependence implies avoidance. But we assume (conservatively) non-avoidance (the effects of the Traffic Alert and Collision Avoidance System (TCAS) are neglected for this study) and therefore independence.

Under the assumption of independence the probability of a TCV for this scenario is the product of the probabilities of three events: the centers of gravity of the aircraft converging to within their mean wingspan laterally (C_y), the centers of gravity of the aircraft converging to within their mean lengths longitudinally (C_x), and the centers of gravity of the aircraft converging to within their mean heights vertically (C_z). That is,

$$P(TCV) = P(C_y)\ P(C_x)P(C_z).$$

Figure 2.1.2 Scenario 2 Hazards



Scenario 3

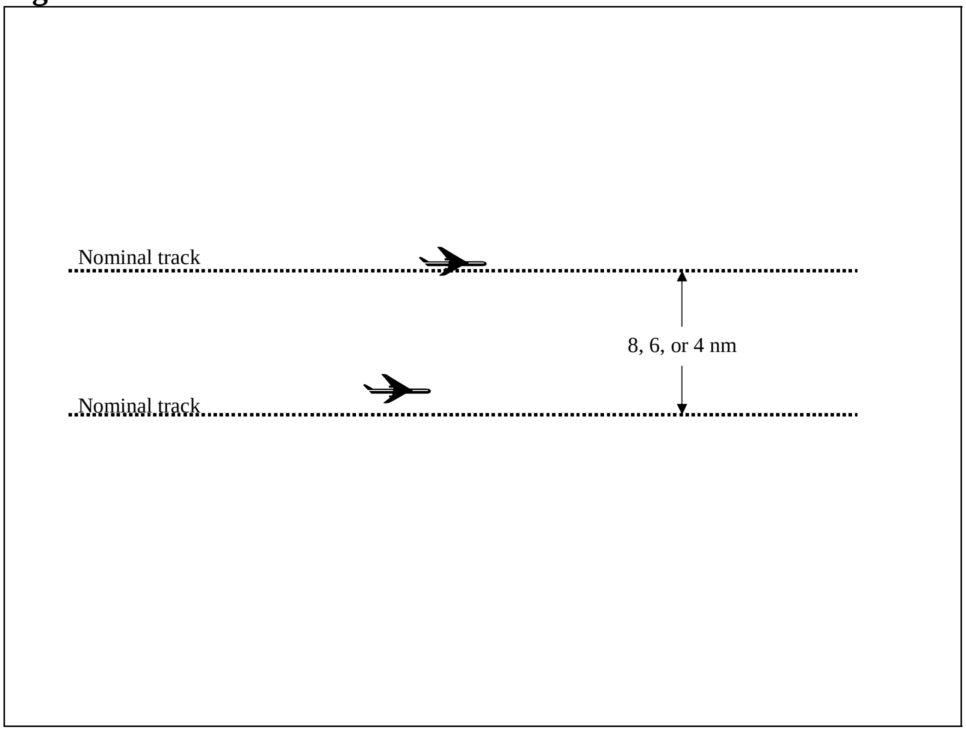
This scenario is the same as Scenario 2 except the two aircraft are flying in the same direction on parallel, adjacent, straight en route track segments with turns of no more than 15° (Figure 2.1.3). The hazards and TCVs are the same. The mathematical model for the calculation of the probability of a TCV is the same except that the probability of C_x, the centers of gravity of the aircraft converging to within their mean lengths longitudinally, will be determined differently since the along-track relationships of same-direction and opposite-direction aircraft are necessarily different.

In other words,

$$P(TCV) = P(C_y)P(C_x)P(C_z)$$

will be used in Scenario 3 as in Scenario 2, with the modification of $P(C_x)$ to account for same direction traffic.

Figure 2.1.3 Scenario 3 Hazards



Scenario 4

This scenario is the same as Scenario 2 except that there are three aircraft rather than two. The aircraft track of interest is the one between the other two. All aircraft are flying in the same direction on parallel, adjacent, straight en route track segments with turns of no more than 15°. The hazards and TCVs are the same as Scenario 2. The mathematical model for the calculation of the probability of a TCV is the same except that the probability of the TCV is twice that of the probability of Scenario 2's TCV.

In other words, if TCV_n denotes the TCV of Scenario n:

$$P(TCV_4) = 2P(TCV_2).$$

Scenario 5

This scenario is the same as Scenario 3 except there are three aircraft rather than two. The aircraft track of interest is the one between the other two. The inner aircraft is flying in the opposite direction to the other two on parallel, adjacent, straight en route track segments with turns of no more than 15° (Figure 2.1.3). The hazards and TCVs are the same as Scenario 3. The mathematical model for the calculation of the probability of a TCV is the same, except the probability of the TCV is twice that of the probability of Scenario 3's TCV.

In other words, $P(TCV_5) = 2P(TCV_3)$.

Scenario 6

This scenario is the same as Scenario 5 except that the inner aircraft is flying in the opposite direction to one of the other aircraft and in the same direction as the other on parallel, adjacent, straight en route track segments with turns of no more than 15° (Figure 2.1.3). The hazards and TCVs are the same as Scenario 5. The mathematical model for the calculation of the probability of a TCV is the same except that the probability of the TCV is given by the equation below.

$$P(TCV_6) = P(TCV_2) + P(TCV_3).$$

2.1 Summary of Data Used

The data used fall into two categories: values specified in AC 90-100 and data from radar tracks reported in previous RNAV studies.

AC 90-100 specifies a value for track-keeping accuracy for RNAV aircraft. This criterion will be the basis for the analysis.

There are three studies we examined that have used RNAV track data. We will use the data and results from those studies to validate the criterion-based analysis results. That is, these studies will not be used as the basis for determining hazard risk, but rather will be used to validate the model based on the AC 90-100 criterion. The three studies are described below.

1. “Preliminary Re-evaluation of the Probability of Lateral Overlap, Py(0), based on non-Global Positioning Satellite (GPS) and GPS Equipped Aircraft Performance at Entry into North Atlantic Reduced Vertical Separation Minimum Airspace” [1].

This paper was published by the North Atlantic Mathematicians’ Implementation Group as NAT MIG/5-WP/18 in April 1999. The paper’s analysis was based on data collected in 1995 from 11 aircraft flights by 5 operators with 3 aircraft types (B747-200, B747-400, A340). Each aircraft was using GPS navigation on an oceanic route -- 5 flights were North Atlantic routes, 5 were Pacific Oceanic Airspace, and one was a South Atlantic route.

2. “Estimating the Well-Fit Model for the Distribution of Cross Track Deviations of GPS Equipped Aircraft on a North Pacific Route” [2].

This paper was published by the Separation and Airspace Safety Panel (SASP) as SASP-WG/WHL/4-WP/23 in November 2003. The paper’s analysis was based on data collected between December 2001 and May 2002, from *3,150 flights on the North Pacific route R220*. Each aircraft, types B747-400, B777, A340, was using GPS navigation on the route.
3. “Analysis of Lateral Track Deviation along Two Q-Routes” [3].

This paper was published by the FAA's Flight Technologies and Procedures Division (AFS-400) in October 2005. The paper's analysis was based on data collected in February and March 2003, from 865 flights on Q-Routes 100 and 102 in the Gulf of Mexico. Each aircraft was using some type of RNAV navigation, typically GPS or DME/DME IRU on the route.

3.0 Summary of Data Analysis and Risk Evaluation

In this section we determine the probability of the TCVs in each of the three scenarios, use those probabilities along with the hazard severities discussed in Section 2.1 (Model Description) to define the risk for each hazard, and then compare those risks with standard acceptable levels of risk.

3.1 Summary of the TCV Probability Analysis

We examine the TCV probability analysis for each scenario beginning with Scenario 1. The results of the analysis for Scenario 1 can be used in the analyses for the other five scenarios.

<u>Scenario 1 Probability Analysis</u>

The TCV for this hazard is the deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM or more (or 3 NM or more, or 4 NM or more). The purpose of the analysis is to determine the probability of each of these three TCVs associated with the 2, 3, and 4 NM cases. We proceed by basing the analysis on the track-keeping accuracy specified in AC 90-100 for aircraft operating on RNAV routes. Then we compare results from previous en route studies with the results of the AC 90-100 analysis to generate a reasonable set of TCV probabilities.

AC 90-100 Analysis
The track-keeping accuracy specified in AC 90-100 for aircraft operating on RNAV routes is an accuracy "bounded by ±2 NM for 95% of the total flying time". This means that the frequency of an aircraft remaining within the 2 NM boundary is 95%. Using the frequency definition of probability, this translates into a 95% probability of containment within the 2 NM boundary.

This AC 90-100 requirement allows us to describe one or more statistical distributions of lateral deviation. Such a distribution will be symmetric and centered at zero. Also 95% of its area will be contained between –2 and +2 NM. If we specify that the distribution is, say, normal, these requirements allow us to fix the probability density function (PDF) exactly.

Using a <u>normal distribution</u> for en route lateral displacement is reasonable because in the initial Q-Route study [3] we determined that the aircraft already established on those Q-Routes displayed a lateral displacement that was normally distributed.

A normal distribution that satisfies the AC 90-100 requirements (as described above) will have a mean of 0 and a standard deviation of 1.02 and will generate TCV containment probabilities as summarized in Table 3.1.1.

Table 3.1.1 Normal Distribution Containment Probabilities

Lateral Distance from Track	Containment Probability	Probability Not Contained*
±2 NM	0.950	5 E-02
±3 NM	0.997	3 E-03
±4 NM	0.999	9 E-05

*The notation E-02 denotes 10^{-2}. That is, ten to the negative second power.

However, there are indications that a normal distribution such as the one above may be tighter (lower kurtosis, resulting in thinner tails) than RNAV en route tracks might be in reality. The Q-Route study analyzed only aircraft already established on the route. The two other RNAV studies, [1] and [2], develop distributions based on track data that display slightly higher kurtosis (thicker tails) about the nominal track than the normal distribution above. And it is possible to meet the requirements of AC 90-100 by using distributions other than the normal distribution. These distributions can have slightly higher kurtosis similar to the more conservative data from the other studies referenced.

One such set of distributions is the Johnson S_U family. This family of distributions, in addition to meeting the AC 90-100 requirements, has been used quite often to describe lateral deviation of aircraft. Since this is a family of distributions, we can choose specific distributions that satisfy the AC 90-100 requirements but have varying kurtosis values, and thus thinner or fatter tails. (See Appendix B for a detailed discussion of Johnson S_U distributions.)

The Johnson S_U distribution with the largest kurtosis (fattest tails) that satisfies the AC 90-100 requirements has parameter values $\varepsilon = 0$, $\gamma = 0$, $\eta = 0.37$, $\lambda = 0.02$. This distribution contains an extreme amount of area in its tails (2.67 E-02 outside four standard deviations) and is not appropriate for describing the lateral deviations. However, there is another Johnson S_U distribution with kurtosis between that distribution and the normal distribution, which is appropriate. It has parameter values $\varepsilon = 0$, $\gamma = 0$, $\eta = 2.18$, $\lambda = 2.00$.

Figure 3.1.1 shows this latter Johnson S_U distribution (red, dashed line) plotted along with the normal distribution (green, solid line with lower maximum at 0). Figure 3.1.2 shows the detail of the plot for x-values between 3 and 4 (normal below Johnson S_U).

Figure 3.1.1 Johnson S_U and Normal Distributions

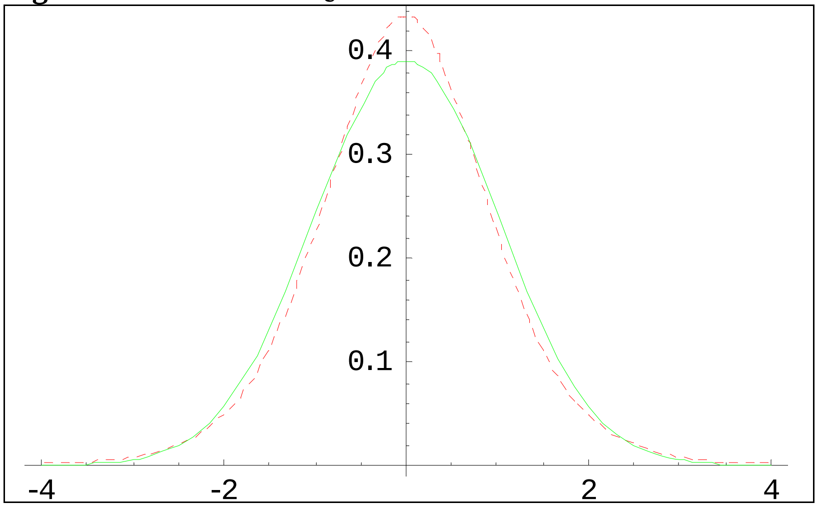
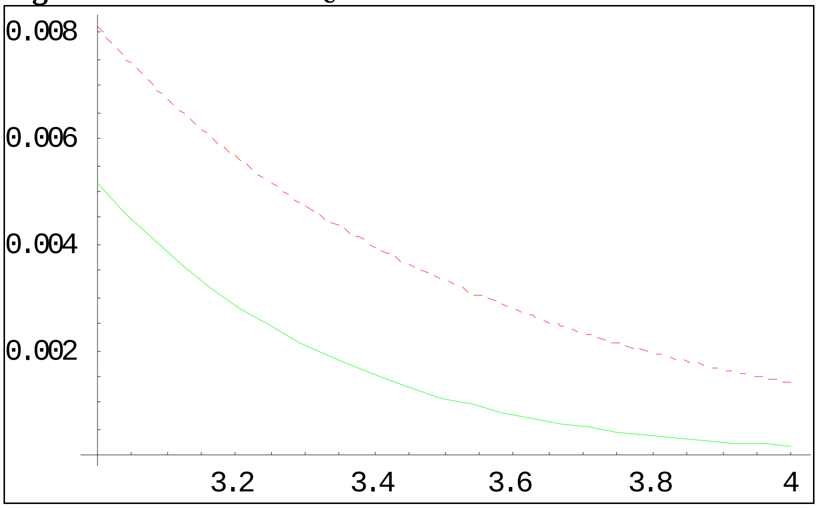


Figure 3.1.2 Johnson S_U and Normal Distribution Detail



The TCV containment probabilities for the Criterion-Based Johnson S_U distribution are summarized in Table 3.1.2. The ±4 NM containment for this distribution falls (logarithmically[2]) almost exactly between that of the normal distribution (9 E-05) and that of the maximum kurtosis distribution (2.7 E-02). Therefore, since this Johnson S_U distribution satisfies the AC 90-100 containment requirements and displays ±4 NM containment mediating the normal and maximum kurtosis distributions that also satisfy the AC 90-100 containment requirements, it appears to be a good candidate for a criterion-based distribution that can be used to estimate the probability of a TCV for this scenario.

[2] The mean of the natural logarithms of the normal and maximum kurtosis distributions is –6.44. The natural logarithm of the Criterion-Based Johnson distribution is –6.48.

Table 3.1.2 Johnson S_U Distribution Containment Probabilities

Lateral Distance from Track	Containment Probability	Probability Not Contained
±2 NM	0.950	5 E-02
±3 NM	0.991	9 E-03
±4 NM	0.998	2 E-03

One question remains: How does this criterion-based distribution match empirical distributions based on RNAV track data?

Empirical Distribution Analysis

Each of the two studies [2] and [3] uses the same distribution family, but with different parameter values. The distribution family is based, not on the normal or Johnson, but on a mixed distribution – a mixture of the normal and double exponential distributions. This distribution family, which uses parameters α, σ, and λ is described in detail in Appendix B. Table 3.1.3 gives the parameter values for each of the mixed distributions from the two studies. The PDF for this mixed distribution is:

$$f_M(x) = \frac{1-\alpha}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2}\right] + \frac{\alpha}{2\lambda}\exp\left[-\frac{|x|}{\lambda}\right].$$

Table 3.1.3 Study Mixed Distribution Parameters

Study	α	σ	λ
SASP-WG/WHL/4-WP/23	0.0564	0.0232	0.0380
NAT MIG/5 WP/18	0.0567	0.0347	2.7786

The α and σ values for the two distributions are close, whereas the λ values are quite different. The λ parameter of the NAT study is much larger than that of the SASP study. The λ parameter is directly related to the dispersion of atypical lateral error and the σ parameter is the standard deviation of typical lateral error.

These empirical mixed distributions based on the parameters in Table 3.1.3 generate TCV containment probabilities as summarized in Table 3.1.4 along with those of the Johnson S_U distribution. For the ±3 NM and ±4 NM containment probabilities, the criterion-based Johnson S_U distribution provides containment probabilities between those of the two empirical distributions. This validates the choice of the Johnson S_U with parameter values $\varepsilon = 0$, $\gamma = 0$, $\eta = 2.18$, $\lambda = 2.00$ as the criterion-based distribution since it shows that this criterion-based distribution, while meeting the ±2 NM containment probability requirements of AC 90-100 will also provide containment results for ±3 NM and ±4 NM intermediate to those based on the data from the two studies.

Table 3.1.4 Empirical Distribution Containment Probabilities

Lateral Distance from Track	SASP Study Containment Probability	Johnson S_U Containment Probability	NAT Study Containment Probability
±2 NM	0.999+	0.950	0.972
±3 NM	0.999+	0.991	0.981
±4 NM	0.999+	0.998	0.987

Scenario 1 Summary

Using the criterion-based Johnson S_U distribution, we can estimate the probabilities for the TCVs for this scenario's hazards (the deviations of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM, 3 NM, or 4 NM). Table 3.1.5 lists those probabilities.

Table 3.1.5 Scenario 1 TCV Probabilities

Lateral Distance from Track	TCV Probability
±2 NM	5 E-02 (5.0%)
±3 NM	9 E-03 (0.9%)
±4 NM	2 E-03 (0.2%)

Scenario 2 Probability Analysis

The TCV for this hazard is the collision with another aircraft that is flying in an opposite direction on a straight parallel RNAV track at the same altitude and at a lateral track distance of 4, 6, or 8 NM from the first aircraft's track.

The probability of a TCV for this scenario is the product of the probabilities of three events involving the two aircraft:

1. the centers of gravity of the aircraft converging to within their mean wingspan laterally (C_y),
2. the centers of gravity of the aircraft converging to within their mean lengths longitudinally (C_x),
3. and the centers of gravity of the aircraft converging to within their mean heights vertically (C_z).

That is, $P(TCV) = P(C_y)\ P(C_x)P(C_z)$.

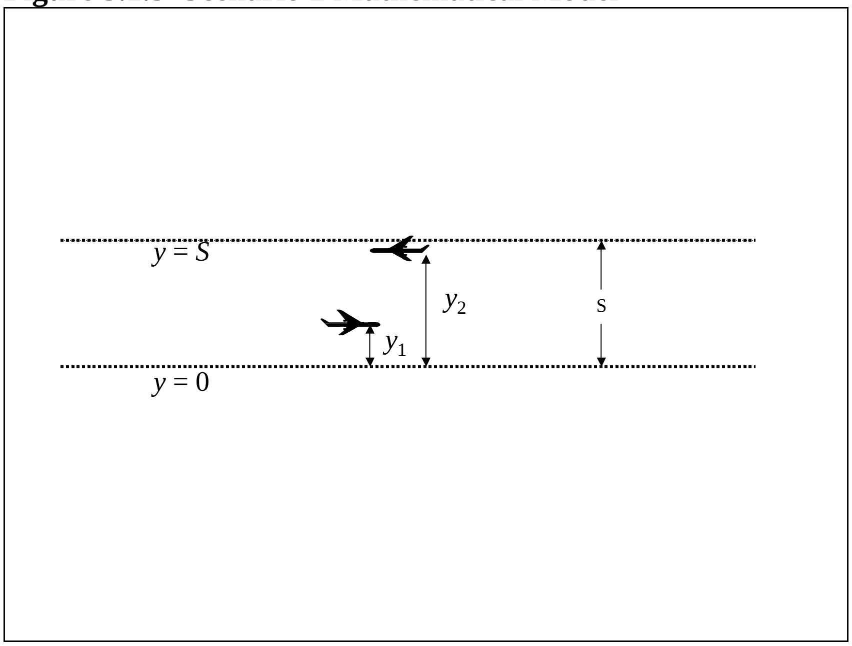
We will determine each of the three probabilities separately. Then we will calculate their product, the probability of the TCV, for this scenario.

The Probability that Two Aircraft Converge Laterally, $P(C_y)$

We assume that there are two aircraft executing RNAV operations on parallel, adjacent, tracks either 4, 6, or 8 NM apart and that each aircraft displays a lateral deviation from it track that can be described by the criterion-based Johnson S_U distribution developed in the Scenario 1 analysis.

Let the wingspan (in NM) of each aircraft we denoted by *W*. Let the first aircraft's intended track be the $y = 0$ axis and (assuming the tracks are *S* NM apart), the second aircraft's intended track will be the line $y = S$. (See Figure 3.1.3.)

Figure 3.1.3 Scenario 2 Mathematical Model



Let the two aircrafts' lateral positions be given by the variables y_1 and y_2 respectively. The aircraft will be assumed to collide when their centers of gravity are within *W*. That is, when $| y_2 - y_1| < W$. And therefore, the probability of lateral convergence,

$$P(C_y) = \mathrm{P}(| y_2 - y_1| < W).$$

But, $\mathrm{P}(| y_2 - y_1| < W) = \mathrm{P}(-W < y_2 - y_1 < W)$. And this probability can be found by integrating the PDF describing $(y_2 - y_1)$ between $-W$ and W.

The PDF describing $(y_2 - y_1)$ is the convolution of the two PDFs of the two variables, y_2 and $-y_1$. The PDF for each variable is that of the criterion-based Johnson S_U distribution. Appendix B gives the details for the convolution of these two PDFs and of the integration that yields the lateral convergence probability. Table 3.1.6 gives the lateral convergence probability for each of the three values of S: 4, 6, and 8 NM.

Table 3.1.6 Scenario 2 Lateral Convergence Probabilities

Track-to-Track Distance (S)	P(Cy)
4 NM	4.4 E-04
6 NM	2.2 E-05
8 NM	1.2 E-06

The Probability that Two Aircraft Converge Longitudinally, $P(C_x)$

Given p_1 and p_2, a pair of parallel RNAV tracks at the same altitude, we are interested in the *frequency per hour* that an aircraft flying along p_1 will be flying adjacent to an aircraft flying in an opposite direction along p_2. Assume all aircraft have length L NM, traveling average ground speed V knots; and let r_i denote the hourly rate at which the aircraft enter p_i.

We are concerned primarily with the portion of an hour in which any two aircraft are adjacent to one another. A single aircraft along p_1 will be adjacent to at most r_2 aircraft traveling along p_2 during one hour.

The aircraft are adjacent to one another for a distance of $2L$ NM. The separation of the two aircraft occurs at a rate of 2V knots. To find the time duration of the adjacency, we solve for t in the equation: ($2V$ knots)(t hours) = $2L$ NM. Thus, $t = (L / V)$ hours.

Each aircraft traveling along p_1 will be adjacent to r_2 aircraft for (L / V) hours. Therefore, each aircraft along p_1 is adjacent to an aircraft along the parallel flight path for $(r_2 L / V)$ hours. We are concerned with the probability (or frequency) that a specific aircraft along p_1 is flying adjacent to an aircraft along p_2. Therefore,

$$\text{P(Aircraft 1 and 2 occupy adjacent airspace per hour)} = \frac{r_2 L}{V}. \qquad (2)$$

For RNAV equipped aircraft with procedural separation, the typical en route longitudinal separation in the National Airspace System (NAS) is at least 20 NM [5]. Therefore, in the worst-case scenario; $r_2 = \frac{V}{20NM}$; Let $L = 0.03$ NM. Replacing these values[3],

$$\text{P(Aircraft 1 and 2 occupy adjacent airspace per hour | opposite direction)} = \frac{r_2 L}{V} = \frac{(V/20)L}{V} = \frac{L}{20} = 0.0015.$$

Since other, less typical, longitudinal separation values such as 5 and 10 NM are possible under certain conditions [5], Table 3.1.7 provides the probabilities of longitudinal convergence for 5 and 10 in addition to the typical 20 NM separation.

[3] The typical aircraft length, L, and wingspan, are assumed to be 0.03 NM. See [2].

Table 3.1.7 Scenario 2 Longitudinal Convergence Probabilities, P(Cy)

Longitudinal Separation Assumption	$P(C_y)$
5 NM	0.0060
10 NM	0.0030
20 NM	0.0015

The Probability that Two Aircraft Converge Vertically, $P(C_z)$

We assume that the two aircraft in Scenarios 2 and 3 are nominally flying at the same altitude. The probability that they actually are at the same altitude (that they converge vertically) is therefore a function of their combined vertical deviations about the nominal altitude.

In the case of lateral deviations about the nominal track, the larger the deviation, the more probable the lateral convergence, and therefore, the more probable a collision. But in the case of vertical deviations about the nominal altitude, the larger the deviation, the less probable the vertical convergence, and therefore, the less probable a collision.

As navigation and guidance systems become more accurate, vertical convergence will likely become more probable. Because of this fact, we will assume that the navigation and guidance systems of the aircraft in this study are quite accurate, and, in fact, that the probability of vertical convergence is 1.0.

Scenario 2 Summary

We have derived the probabilities of :

1. the centers of gravity of the aircraft converging to within their mean wingspan laterally (C_y),
2. the centers of gravity of the aircraft converging to within their mean lengths longitudinally (C_x),
3. and the centers of gravity of the aircraft converging to within their mean heights vertically (C_z).

The probability of a TCV for this scenario is the product of the probabilities of three events involving the two aircraft:

$$P(TCV) = P(C_y)\, P(C_x) P(C_z).$$

Table 3.1.8 summarizes these probabilities and the corresponding probabilities of the Scenario 2 TCVs.

Table 3.1.8 Scenario 2 Convergence and TCV Probabilities

Track-to-Track Distance (S)	Longitudinal Separation	$P(C_y)$	$P(C_x)$	$P(C_z)$	P(TCV)
4 NM	5 NM	4.4 E-04	6.0 E-03	1	2.6 E-06
6 NM	5 NM	2.2 E-05	6.0 E-03	1	1.3 E-07
8 NM	5 NM	1.2 E-06	6.0 E-03	1	7.2 E-09
4 NM	10 NM	4.4 E-04	3.0 E-03	1	1.3 E-06
6 NM	10 NM	2.2 E-05	3.0 E-03	1	6.5 E-08
8 NM	10 NM	1.2 E-06	3.0 E-03	1	3.6 E-09
4 NM	20 NM	4.4 E-04	1.5 E-03	1	6.5 E-07
6 NM	20 NM	2.2 E-05	1.5 E-03	1	3.3 E-08
8 NM	20 NM	1.2 E-06	1.5 E-03	1	1.8 E-09

Scenario 3 Probability Analysis

The TCV for this hazard is the collision with another aircraft that is flying in the same direction on a straight parallel RNAV track at the same altitude and at a lateral track distance of 4, 6, or 8 NM from the first aircraft's track.

As with Scenario 2, the probability of a TCV for this scenario is the product of the probabilities of three events involving the two aircraft:

1. the centers of gravity of the aircraft converging to within their mean wingspan laterally (C_y),
2. the centers of gravity of the aircraft converging to within their mean lengths longitudinally (C_x),
3. and the centers of gravity of the aircraft converging to within their mean heights vertically (C_z).

That is, $P(TCV) = P(C_y)\,P(C_x)P(C_z)$.

And as with Scenario 2, in this scenario we will determine each of the three probabilities separately, and then calculate their product, the probability of the TCV for this scenario.

The Probability that Two Aircraft Converge Laterally, $P(C_y)$

We assume that the direction that the aircraft are flying relative to each other does not affect their lateral track deviation. Therefore, the probability of lateral convergence in Scenario 3 will be exactly the same as that of Scenario 2. Table 3.1.9 summarizes those probabilities.

Table 3.1.9 Scenario 3 Lateral Convergence Probabilities

Track-to-Track Distance (S)	P(Cy)
4 NM	4.4 E-04
6 NM	2.2 E-05
8 NM	1.2 E-06

The Probability that Two Aircraft Converge Longitudinally, $P(C_x)$

Given p_1 and p_2, a pair of parallel RNAV tracks at the same altitude, we are interested in the *frequency per hour* that an aircraft flying along p_1 will be flying adjacent to an aircraft flying in the same direction along p_2. Assume all aircraft have length *L* NM, traveling average ground speed *V* knots; and let r_i denote the hourly rate at which the aircraft enter p_i.

Consider a section of flight paths p_i that aircraft would traverse for duration of one hour, length equal *V* NM. Let the single p_1 aircraft be defined as a point, and the length of the p_2 aircraft (possibly multiple aircraft) is doubled for appropriate spatial analysis.

$2r_2L$ NM = Quantity of airspace occupied by aircraft along p_2, for path length *V* NM.

P(Aircraft 1 and 2 occupy adjacent airspace per hour | same direction)

$$= P(C_x)$$

$$= 1 - P(\text{Aircraft 1 and 2 do NOT occupy adjacent airspace per hour})$$

$$= 1 - \frac{\mathit{Quantity\ Of\ Airspace\ Available\ For\ No\ Intersection}}{\mathit{Quantity\ Of\ Airspace}}$$

$$= 1 - \frac{V - 2r_2L}{V}. \qquad (1)$$

For RNAV equipped aircraft with procedural separation, the typical en route longitudinal separation in the NAS is at least 20 NM [5]. Therefore, in the "worst-case" scenario $r_2 = \frac{V}{20nm}$; Let *L* = 0.03 NM. Replacing these values,

$P(C_x)$ = P(Aircraft 1 and 2 occupy adjacent airspace per hour |same direction) =

$$= 1 - \frac{V - 2\frac{V}{20}L}{V} = \frac{2L}{20} = 0.003.$$

This value is exactly twice the adjacency probability found in Scenario 2 for aircraft flying in opposite directions.

As with Scenario 2, other, less typical, longitudinal separation values such as 5 and 10 NM are possible under certain conditions. Table 3.1.10 provides the probabilities of longitudinal convergence for 5, 10, and 20 NM separation.

Table 3.1.10 Scenario 3 Longitudinal Convergence Probabilities, P(Cy)

Longitudinal Separation Assumption	$P(C_y)$
5 NM	0.0120
10 NM	0.0060
20 NM	0.0030

The Probability that Two Aircraft Converge Vertically, $P(C_z)$

As in Scenarios 2 we assume that the probability of vertical convergence is 1.0.

Scenario 3 Summary

As with Scenario 2 we have derived the probabilities of :

1. the centers of gravity of the aircraft converging to within their mean wingspan laterally (C_y),
2. the centers of gravity of the aircraft converging to within their mean lengths longitudinally (C_x),
3. and the centers of gravity of the aircraft converging to within their mean heights vertically (C_z).

The probability of a TCV for this scenario is the product of the probabilities of three events involving the two aircraft:

$$P(TCV) = P(C_y)\, P(C_x) P(C_z).$$

Table 3.1.11 summarizes these probabilities and the corresponding probabilities of the Scenario 3 TCVs.

Table 3.1.11 Scenario 3 Convergence and TCV Probabilities

Track-to-Track Distance (S)	Longitudinal Separation	$P(C_y)$	$P(C_x)$	$P(C_z)$	P(TCV)
4 NM	5 NM	4.4 E-04	1.2 E-02	1	5.3 E-06
6 NM	5 NM	2.2 E-05	1.2 E-02	1	2.6 E-07
8 NM	5 NM	1.2 E-06	1.2 E-02	1	1.4 E-08
4 NM	10 NM	4.4 E-04	6.0 E-03	1	2.7 E-06
6 NM	10 NM	2.2 E-05	6.0 E-03	1	1.3 E-07
8 NM	10 NM	1.2 E-06	6.0 E-03	1	7.2 E-09
4 NM	20 NM	4.4 E-04	3.0 E-03	1	1.3 E-06
6 NM	20 NM	2.2 E-05	3.0 E-03	1	6.5 E-08
8 NM	20 NM	1.2 E-06	3.0 E-03	1	3.5 E-09

Scenario 4 Probability Analysis

Since the probability for a Scenario 4 TCV is twice that of a Scenario 2 TCV, the probabilities for this scenario are calculated directly from those of Scenario 2. Table 3.1.12 summarizes the probabilities.

Table 3.1.12 Scenario 4 TCV Probabilities (Based on Scenario 2)

Track-to-Track Distance (S)	Longitudinal Separation	Scenario 2 P(TCV)	Scenario 4 P(TCV)
4 NM	5 NM	2.6 E-06	5.3 E-06
6 NM	5 NM	1.3 E-07	2.6 E-07
8 NM	5 NM	7.2 E-09	1.4 E-08
4 NM	10 NM	1.3 E-06	2.7 E-06
6 NM	10 NM	6.5 E-08	1.3 E-07
8 NM	10 NM	3.6 E-09	7.2 E-09
4 NM	20 NM	6.5 E-07	1.3 E-06
6 NM	20 NM	3.3 E-08	6.5 E-08
8 NM	20 NM	1.8 E-09	3.5 E-09

Scenario 5 Probability Analysis

Since the probability for a Scenario 5 TCV is twice that of a Scenario 3 TCV, the probabilities for this scenario are calculated directly from those of Scenario 3. Table 3.1.13 summarizes the probabilities.

Table 3.1.13 Scenario 5 TCV Probabilities (Based on Scenario 3)

Track-to-Track Distance (S)	Longitudinal Separation	Scenario 3 P(TCV)	Scenario 5 P(TCV)
4 NM	5 NM	5.3 E-06	1.1 E-05
6 NM	5 NM	2.6 E-07	5.2 E-07
8 NM	5 NM	1.4 E-08	2.8 E-08
4 NM	10 NM	2.7 E-06	5.4 E-06
6 NM	10 NM	1.3 E-07	2.6 E-07
8 NM	10 NM	7.2 E-09	1.4 E-08
4 NM	20 NM	1.3 E-06	2.6 E-06
6 NM	20 NM	6.5 E-08	1.3 E-07
8 NM	20 NM	3.5 E-09	7.0 E-09

Scenario 6 Probability Analysis

Since the probability for a Scenario 6 TCV is the sum of those of Scenarios 2 and 3, the probabilities for this scenario are calculated directly from those of Scenarios 2 and 3. Table 3.1.14 summarizes the probabilities.

Table 3.1.14 Scenario 6 TCV Probabilities (Based on Scenarios 2 and 3)

Track-to-Track Distance (S)	Longitudinal Separation	Scenario 2 P(TCV)	Scenario 3 P(TCV)	Scenario 6 P(TCV)
4 NM	5 NM	2.6 E-06	5.3 E-06	7.9 E-06
6 NM	5 NM	1.3 E-07	2.6 E-07	3.9 E-07
8 NM	5 NM	7.2 E-09	1.4 E-08	2.1 E-08
4 NM	10 NM	1.3 E-06	2.7 E-06	4.0 E-06
6 NM	10 NM	6.5 E-08	1.3 E-07	2.0 E-07
8 NM	10 NM	3.6 E-09	7.2 E-09	1.1 E-08
4 NM	20 NM	6.5 E-07	1.3 E-06	2.0 E-06
6 NM	20 NM	3.3 E-08	6.5 E-08	9.8 E-08
8 NM	20 NM	1.8 E-09	3.5 E-09	5.3 E-09

3.2 Summary of Acceptable Level of Risk

This analysis applies to scenarios 2 and 3 because those deal with risk of collision while scenario 1 deals only with probability of boundary penetration. The purpose of this section is to recommend an acceptable level of risk for Scenarios 2 and 3 based upon standards, operational experience, and accepted practices within the NAS.

Typically, Flight Standards has recommended risk levels based on past performance.
The overriding principle being, as in the Hippocratic Oath, to cause no harm by means of a change in the NAS; and therefore, that the risk associated with the scenario under study should not cause the overall operational risk to increase.
Scenarios 2 and 3 deal with en route operations. So the acceptable level of risk should be one that does not increase the en route risk. Table 3.2 shows data from a previous study [6] that break fatal aircraft accident rates into phases of flight. For our purposes, we classify the phases Climb, Cruise, and Descent as possible en route phases.

Table 3.2 Fatal Accidents by Phase of Flight Out of 33.3 Million Air Carrier Flights

Phase of Flight	NTSB Fatal Accident Count 1983-1988	Fatal Accident Rate (Count / 33.3 x 10^6)
Start and Taxi	1	3.0 E-08
Take-off	6	1.8 E-07
Climb*	0	0
Cruise*	3	9.0 x E-08
Descent*	1	3.0 x E-08
Approach	2	6.0 x E-08
Landing	1	3.0 x E-08
Total	14	4.2 x E-07
***En Route**	**4**	**1.2 x E-07**

Therefore, we should not exceed a rate of 1.2 E-07 fatal accidents per flight during en route operations including all causes. There are causes of fatal accidents during en route flight other than collision with adjacent aircraft. If we assume that an en route fatal accident may be caused by one of four events – engine failure, aircraft structural failure, aircraft systems failure, collision with an adjacent aircraft – then it is reasonable (barring more exact information) to allot one-fourth of the risk to each event. Thus, the rate of collisions with adjacent aircraft for en route operations should not exceed one-fourth of 1.2 E-07 fatal accidents per flight. That is, 3.0 E-08 fatal accidents per flight.

To translate that rate to one in terms of fatal accidents per hour, we can assume a mean of 2 hours per en route flight resulting in a not-to-exceed rate of 1.5 E-08 fatal accidents per hour. This rate, of course, is not an exact limit. But it is reasonable given existing data; and it is consistent with previous Flight Standards methodology.

In addition, this not-too-exceed rate is consistent with the (slightly more conservative) rate of 1.0 E-08 previously established for this study and based on ICAO Document 9689-AN/953 [4]. Therefore, we recommend use of the previously established target level of safety rate of 1.0 E-08 fatal accidents per hour of flight.

4.0 Results and Conclusions

This section summarizes the key results, the scenario risk evaluation, and the conclusions of the study.

4.1 Summary of Results

Tables 4.1.1 through 4.1.6 summarize the key results for Scenarios 1 through 6. The probabilities in bold face (beginning with Table 4.1.2) are those meeting the target level of safety.

Table 4.1.1 Scenario 1 Key Results

Scenario	TCV	Probability of TCV
Scenario 1: A suitably equipped RNAV aircraft is flying a straight en route track segment with turns of no more than 15°	The deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of at least 2 NM during 1 hour	5 E-2 (5.0%)
	The deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of at least 3 NM during 1 hour	9 E-3 (0.9%)
	The deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of at least 4 NM during 1 hour	2 E-3 (0.2%)

Table 4.1.2 Scenario 2 Key Results

Scenario	TCV	Probability of TCV
Scenario 2: Two suitably equipped RNAV aircraft are flying in <u>opposite</u> directions on parallel straight en route track segments with turns of no more than 15°. We assume each of 5, 10, and 20 NM longitudinal separation minima.	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 4 NM during 1 hour.	2.6 E-06 at 05 NM Separation 1.3 E-06 at 10 NM Separation 6.5 E-07 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 6 NM during 1 hour.	1.3 E-07 at 05 NM Separation 6.5 E-08 at 10 NM Separation 3.3 E-08 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 8 NM during 1 hour.	**7.2 E-09 at 05 NM Separation** **3.6 E-09 at 10 NM Separation** **1.8 E-09 at 20 NM Separation**

Table 4.1.3 Scenario 3 Key Results

Scenario	TCV	Probability of TCV
Scenario 3: Two suitably equipped RNAV aircraft are flying in the same direction on parallel straight en route track segments with turns of no more than 15°. We assume each of 5, 10, and 20 NM longitudinal separation minima.	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 4 NM during 1 hour.	5.3 E-06 at 05 NM Separation 2.7 E-06 at 10 NM Separation 1.3 E-06 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 6 NM during 1 hour.	2.6 E-07 at 05 NM Separation 1.3 E-07 at 10 NM Separation 6.5 E-08 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 8 NM during 1 hour.	1.4 E-08 at 05 NM Separation **7.2 E-09 at 10 NM Separation** **3.5 E-09 at 20 NM Separation**

Table 4.1.4 Scenario 4 Key Results

Scenario	TCV	Probability of TCV
Scenario 4: An inner aircraft is flying in the opposite direction between two other aircraft on parallel straight en route track segments with turns of no more than 15°. We assume each of 5, 10, and 20 NM longitudinal separation minima.	The combined lateral, longitudinal, and vertical conjunction two of the aircraft with track-to-track separation of 4 NM during 1 hour.	5.2 E-06 at 05 NM Separation 2.6 E-06 at 10 NM Separation 1.3 E-06 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of two of the aircraft with track-to-track separation of 6 NM during 1 hour.	2.6 E-07 at 05 NM Separation 1.3 E-07 at 10 NM Separation 6.6 E-08 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of two of the aircraft with track-to-track separation of 8 NM during 1 hour.	1.4 E-08 at 05 NM Separation **7.2 E-09 at 10 NM Separation** **3.5 E-09 at 20 NM Separation**

Table 4.1.5 Scenario 5 Key Results

Scenario	TCV	Probability of TCV
Scenario 5: An inner aircraft is flying in the same direction between two other aircraft on parallel straight en route track segments with turns of no more than 15°. We assume each of 5, 10, and 20 NM longitudinal separation minima.	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 4 NM during 1 hour.	1.1 E-05 at 05 NM Separation 5.4 E-06 at 10 NM Separation 2.6 E-06 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 6 NM during 1 hour.	5.2 E-07 at 05 NM Separation 2.6 E-07 at 10 NM Separation 1.3 E-07 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of the two aircraft with track-to-track separation of 8 NM during 1 hour.	2.8 E-08 at 05 NM Separation 1.4 E-08 at 10 NM Separation **7.0 E-09 at 20 NM Separation**

Table 4.1.6 Scenario 6 Key Results

Scenario	TCV	Probability of TCV
Scenario 6: An inner aircraft is flying between two other aircraft, one flying in the same direction and the other flying in the opposite direction on parallel straight en route track segments with turns of no more than 15°. We assume each of 5, 10, and 20 NM longitudinal separation minima.	The combined lateral, longitudinal, and vertical conjunction of two of the aircraft with track-to-track separation of 4 NM during 1 hour.	7.9 E-06 at 05 NM Separation 4.0 E-06 at 10 NM Separation 2.0 E-06 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of two of the aircraft with track-to-track separation of 6 NM during 1 hour.	3.9 E-07 at 05 NM Separation 2.0 E-07 at 10 NM Separation 9.8 E-08 at 20 NM Separation
	The combined lateral, longitudinal, and vertical conjunction of two of the aircraft with track-to-track separation of 8 NM during 1 hour.	2.1 E-08 at 05 NM Separation 1.1 E-08 at 10 NM Separation **5.3 E-09 at 20 NM Separation**

4.2 Scenario Risk Evaluation

We evaluate the risk of collision with an adjacent aircraft, that is, the risks for each scenario. This evaluation requires us to compare the probability of occurrence of the scenario's TCVs with the corresponding acceptable level of risk.

The acceptable level of risk for each scenario, established in section 3.2, is on the order of 1.0 E-08. For both scenarios the TCV probability for either 4 or 6 NM exceeds this level for all longitudinal separation assumptions (5, 10, and 20 NM). However, for Scenario 2 the TCV probability for 8 NM (and 5 NM longitudinal separation) is 7.2 E-09, and the 10 and 20 NM longitudinal separation probabilities at 8 NM are even smaller. Therefore, all 8 NM TCV probabilities lie within the acceptable level of risk.

For Scenarios 3 and 4 the TCV probabilities for 8 NM (and 5 NM longitudinal separation) are both 1.4 E-08, which lies just outside the acceptable level of risk. However, the 10 and 20 NM longitudinal separation probabilities both lie within the acceptable level of risk.

For Scenario 5 the TCV probability for 8 NM (and 5 NM longitudinal separation) is 2.8 E-08, which lies outside the acceptable level of risk. And the probability for 8 NM (and 10 NM longitudinal separation) is 1.4 E-08, which lies outside the acceptable level of risk. However, the 20 NM longitudinal separation probability at 8 NM is 7.0 E-09 and which lies within the acceptable level of risk.

For Scenario 6 the TCV probability for 8 NM (and 5 NM longitudinal separation) is 2.1 E-08, which lies outside the acceptable level of risk. And the probability for 8 NM (and 10 NM longitudinal separation) is 1.1 E-08, which lies just outside the acceptable level of risk. However, the 20 NM longitudinal separation probability (5.3 E-09) lies within the acceptable level of risk.

Of the six scenarios, Scenario 5 is of particular interest[4] since its TCV probability for 8 NM with 10 NM longitudinal separation (1.4 E-08) lies just outside the acceptable level of risk, it would be useful to determine the minimum longitudinal separation necessary to meet the acceptable level of risk (1.0 E-08). The equation relating TCV to acceptable level of risk based on the previous analysis is:

$$P(TCV_5) = 2P(TCV_3) = 2P(C_y)P(C_x)P(C_z),$$

where $P(C_z) = 1$ as before, and for the 8 NM case, $P(C_y) = 1.2$ E-06, and $P(C_x) = 2L / D$, where L is the aircraft length and D is the longitudinal separation. Using P(TCV) = 1.0 E-08 and L = 0.03 NM, this equation becomes:

$$1.0 \text{ E-08} = (2)(1.2 \text{ E-06})(0.06/D)(1)$$

Therefore, D = (2)(1.2 E-06)(0.06) / (1.0 E-08) = 14.4 NM.

[4] Scenario 5 has the highest TCV probability at 10 NM longitudinal separation of all the scenarios, so it is the controlling scenario.

This means that a longitudinal separation of no more than 15 NM would meet the target level of safety for all scenarios assuming an 8 NM track-to-track distance.

4.3 Conclusions

In this study we analyzed the risk of collision due to failure of either of the aircraft to hold the track laterally. We assume the AC 90-100 criteria provides for adequate signal space support and therefore that the risk due to loss of integrity or continuity is covered in the criteria and our otherwise conservative assumptions.

For Scenarios 2 and 3, two suitably equipped RNAV aircraft (the outer track case) with procedural separation, flying in opposite directions (Scenario 2) or the same direction (Scenario 3) on parallel, straight en route track segments with turns of no more than 15° and longitudinal separation of at least 10 NM on average, the risk of collision appears to meet the acceptable level of risk as long as the track-to-track separation of the routes is at least 8 NM.

For Scenarios 4, 5, and 6, a suitably equipped RNAV aircraft with procedural separation, flying in the same direction or opposite directions between two other aircraft (the inner track case) on parallel, straight en route track segments with turns of no more than 15° and longitudinal separation of at least 15 NM on average, the risk of collision appears to meet the acceptable level of risk as long as the track-to-track separation of the routes is at least 8 NM.

For track-to-track separation of 6 or 4 NM, the risk appears to exceed the acceptable level of safety for all assumed longitudinal separation distances.

A final caveat. The risk calculations in this study assume typical longitudinal separations (such as the 20 NM separation, for example) based on air traffic control criteria, but are not intended to evaluate the likelihood of such longitudinal separations. That is, the typical longitudinal separations used in the calculations are based on what should be the case in the NAS assuming standard air traffic control criteria are being followed. We have not evaluated whether or not these separations are actually being maintained.

In addition, the typical separation values used in this study assume that the aircraft are operating under procedural separation rules. It is possible that aircraft are operating under radar surveillance rules, and therefore with typically closer longitudinal separation (as close as 5 NM in some cases). But we assume that in these cases the risk of collision is mitigated by the radar surveillance. This study, therefore, has examined only the procedural separation cases.

Appendices

Appendix A

Severity Definitions Based on the Perspective of the Flying Public

(FAA Safety Management System Manual, Version 1.0, July 24, 2003)

No Safety Effect	Minor	Major	Hazardous	Catastrophic
No effect on flight crew. Has no effect on safety. Inconvenience.	Slight increase in workload such as flight plan changes. Slight reduction in safety margin or functional capabilities. Minor illness, environmental damage, or system damage. Some physical discomfort to occupants of aircraft (except operators).	Significant increase in flight crew workload. Significant reduction in safety margin or functional capability. Major illness, injury, environmental damage, or system damage. Physical distress to occupants of aircraft (except flight crew) including injuries.	Large reduction in safety margin or functional capability. Serious or fatal injury to small number of persons (other than flight crew). Physical distress/ Excessive workload such that flight crew cannot be relied upon to perform required tasks accurately or completely.	Outcome would result in: - Hull loss - Multiple fatalities - Fatal injury or incapacitation

Appendix B

Statistical Distributions Used in the Study

The Johnson SU Distribution PDF

$$f_J(x) = \frac{\eta}{\sqrt{2\pi}} \frac{1}{\sqrt{(x-\varepsilon)^2 + \lambda^2}} \exp\left[-\frac{1}{2}\left(\gamma + \eta \ln\left\{ \left(\frac{x-\varepsilon}{\lambda}\right) + \left[\left(\frac{x-\varepsilon}{\lambda}\right)^2 + 1 \right]^{\frac{1}{2}} \right\} \right)^2 \right] \quad (1)$$

where $-\infty < x < \infty$, $-\infty < \gamma < \infty$, $-\infty < \varepsilon < \infty$, $\eta > 0$, $\lambda > 0$. The location parameter is ε. The scale parameter is λ. And the shape (including skewness) parameters are γ and η.

The Mixed Normal – Double Exponential Distribution PDF

$$f_M(x) = \frac{1-\alpha}{\sigma\sqrt{2\pi}} \exp\left[-\frac{x^2}{2\sigma^2} \right] + \frac{\alpha}{2\lambda} \exp\left[-\frac{|x|}{\lambda} \right]$$

The Convolution of Variables y_2 and y_1 and the Probability of $|y_2 - y_1| < W$

The PDF describing ($y_2 - y_1$) is the convolution of the two PDFs of the two variables, y_2 and $-y_1$. The convolution of two variables y_2 and $+y_1$ is defined as the integral

$$f(u) = \int_{-\infty}^{\infty} f_1(y_1) f_2(u - y_1) dy_1 \quad (2)$$

where $u = y_1 + y_2$. If f_1 and f_2 are PDFs of y_1 and y_2, then f is the PDF of $u = y_1 + y_2$.

Also, if the PDF of y_1 is symmetric about zero, then the convolution of y_2 and y_1 is equivalent to the convolution of y_2 and $-y_1$. Therefore, f is also the PDF of $u = y_2 - y_1$.

This means that the probability of $|y_2 - y_1| < W$ is the integral of f between $-W$ and W. That is,

$$P(|y_2 - y_1| < W) = \int_{-W}^{W} f(x) dx . \quad (3)$$

But *f(x)* is defined in (2) where f_1 and f_2 are both Johnson S_U PDFs defined in (1). Therefore,

$$P(\,|y_2 - y_1| < W\,) = \int_{-W}^{W}\int_{-\infty}^{\infty} f_1(y_1) f_2(x - y_1)\,dy_1\,dx \qquad (4)$$

where f_1 and f_2 are defined in (1).

Substituting the definitions of f_1 and f_2, and letting[5] $\varepsilon = 0$ in f_1 and $\varepsilon = S$ in f_2, equation (4) becomes

$$\mathbf{P(\,|y_2 - y_1| < W\,)} = \int_{-W}^{W}\int_{-\infty}^{\infty} f_1(y_1) f_2(x - y_1)\,dy_1\,dx =$$

$$\int_{-W}^{W}\int_{-\infty}^{\infty} \frac{c\eta^2 \exp\left(-0.5\left(\gamma + \eta \ln\left[\left(1 + \frac{(x - y_1 - S)^2}{\lambda^2}\right)^{1/2} + \frac{x - y_1 - S}{\lambda}\right]\right)^2 - 0.5\left(\gamma + \eta \ln\left[\left(1 + \frac{y_1^{\,2}}{\lambda^2}\right)^{1/2} + \frac{y_1}{\lambda}\right]\right)\right)}{\left(\left[(x - y_1 - S)^2 + \lambda^2\right]^{1/2}\left[y_1^{\,2} + \lambda^2\right]^{1/2}\right)}\,dy_1\,dx$$

Where c = 0.159154943.

[5] Since ε is the location parameter, the PDF for y_1 uses ε = 0 and the PDF for y_2 uses ε = S, where the two tracks are S NM apart.

References

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Atlantic City, April 1999

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