



Safety Study Report
DOT-FAA-AFS-440-24

Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments with Radar Surveillance (Phase I)

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January 2007

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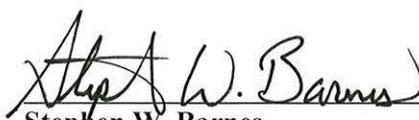
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DOT-FAA-AFS-440-24

Flight Operations Simulation and Analysis Branch
Flight Technologies and Procedures Division
Flight Standards Service

Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent
Straight Segments with Radar Surveillance (Phase I)

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January 2007

Technical Report

**Analysis of RNAV En Route Separation Along Adjacent Straight Segments with Radar Surveillance
(Phase I)**

DOT-FAA-AFS-440-24

January 2007

Technical Report Documentation Page

1. Report No. DOT-FAA-AFS-440-24	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments with Radar Surveillance (Phase I)		5. Report Date January 2007
6. Author(s) Dr. Richard Greenhaw, AFS-440 Gerry McCartor, AFS-440 Dr. Sherri Avery (ATSI) Dr. James Yates (ISI) Carl Moore (ISI)		7. Performing Organization Code
8. Performing Organization Name and Address Federal Aviation Administration Flight Operations Simulation and Analysis Branch, AFS-440 P.O. Box 25082, Oklahoma City, OK 73125		9. Type of Report and Period Covered Technical report
10. Sponsoring Agency Name and Address Federal Aviation Administration Flight Operations Simulation and Analysis Branch, AFS-440 P.O. Box 25082, Oklahoma City, OK 73125		
11. Supplementary Notes		
12. Abstract Developed by AFS-440, this report is intended to replace a technical report released in December 2005, <i>Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments</i> (DOT-FAA-AFS-440-17). This report corrects two problems of the previous study. The previous report did not assume radar surveillance for RNAV en route operations. This study also provides a more accurate method of comparison of Test Criteria Violation (TCV) rates and target level of safety. The results of this analysis show that the hourly probability of collision for suitably equipped RNAV aircraft on parallel, adjacent routes under radar surveillance flying in the same or opposite directions (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 5 NM on average meets the acceptable level of risk established for this study (5.0 E-09 collisions per hour). This is true for a target aircraft adjacent to just one other track (outer track) or for a target aircraft between two other tracks (inner track).		
13. Key Words En Route Separation RNAV RNP	14. Distribution Statement	
15. Security Classification of This Report	16. Security Classification of This Page	

Executive Summary

This report is intended to replace a technical report released in December 2005, *Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments* (DOT-FAA-AFS-440-17). This report corrects two problems of the previous study. The previous report did not assume radar surveillance for RNAV en route operations. This study also provides a more accurate method of comparison of Test Criteria Violation (TCV) rates and target level of safety.

This study provides a risk assessment of lateral, en route separation between parallel Area Navigation (RNAV) routes, such as Q-routes, with separation for both opposite-direction and same-direction traffic *under radar surveillance*. The study estimates the risk of RNAV aircraft flying straight tracks (tracks with turns of less than 15°) deviating from the nominal track laterally by more than 2, 3, or 4 nautical miles (NM). It also estimates the risk of collision of en route RNAV aircraft flying adjacent, parallel, straight tracks (in the cases of both opposite-direction and same-direction tracks) when the aircraft of interest is flying adjacent to only one other track (on an outer track) and when the aircraft of interest is flying between two tracks (on an inner track) and both are under radar surveillance. It does not examine risk of collision between an aircraft on an RNAV route and one on a conventional Very High Frequency (VHF) Omni-directional Range/Distance Measuring Equipment (VOR/DME) airway or between an aircraft on an RNAV route and special use airspace.

The analysis is based on two types of data: values specified in AC 90-100 and data from radar tracks reported in previous RNAV studies. AC 90-100 specifies a value for tracking the accuracy value for RNAV aircraft. This criterion is the basis for the analysis. There are three studies examined that have used RNAV track data. This study uses the data and results from those studies to validate the criterion-based analysis results.

The study fits statistical distributions to the values from the AC 90-100 criteria to model the likelihood of adjacent aircraft intersecting laterally. It also models and estimates the likelihood of aircraft on parallel routes becoming adjacent. Using those models, it estimates the hourly rate of collision.

The results of this analysis show that the hourly probability of collision for suitably equipped RNAV aircraft on parallel, adjacent routes under radar surveillance flying in the same or opposite direction (with turns of less than 15°), laterally separated by a track-to-track distance of at least 8 NM, longitudinally separated by at least 5 NM on average meets the acceptable level of risk established for this study¹ (5.0 E-09 collisions per hour). This is true for a target aircraft adjacent to just one other track (outer track) or for a target aircraft between two other tracks (inner track).

¹ Based on the ICAO target level of safety for en route separation minima established in [6].

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For the more stringent target level of safety from the FAA Safety Management System Manual, v 1.1 (1.0 E-09 collisions per hour), all scenarios at 8 NM separation meet the target level of safety, except for Scenario 4 in which a inner aircraft is flying in a direction opposite to those of the two aircraft on either side. Scenario 4 does meet this target level of safety at 9 NM separation.

For track-to-track separation of 4 or 6 NM, the probability of collision for all cases exceeds this acceptable level of risk.

The methodology of this study uses conservative assumptions for lateral track deviations, vertical track deviations, and longitudinal traffic density.

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1.0 Introduction

This section of the report describes the purpose and structure of this document, and provides a description of the problem.

1.1 Purpose and Structure of This Document

The purpose of this study is to provide a risk assessment of en route lateral separation between parallel Area Navigation (RNAV) routes, such as Q-routes, with separation for both opposite-direction and same-direction traffic under radar surveillance. The study estimates the risk of such RNAV aircraft flying straight tracks (i.e., tracks with turns of less than 15°) deviating from the nominal track laterally by more than 2, 3, or 4 nautical miles (NM). It also estimates the risk of collision of en route aircraft flying parallel, adjacent, straight tracks (in the cases of both opposite-direction and same-direction). The study also examines the scenarios in which the aircraft is flying a track between two other parallel tracks (i.e., an inner track) and the scenarios in which the aircraft is flying adjacent to only one other track (i.e., an outer track).

1.2 Statement of the Problem

Specifically, this study seeks to quantify the lateral track deviation² of typical RNAV-equipped aircraft³ on straight en route segments—segments with no turns, or turns no greater than 15° with radar surveillance. This lateral track deviation is used to determine the probability that a typical RNAV en route operation deviates laterally from the track by more than certain given distances (each of 2, 3, or 4 NM).

This lateral track deviation is also used to determine the probability of collision of two aircraft flying parallel, adjacent en route tracks under radar surveillance, with given track-to-track separation distances (4, 6, or 8 NM), with both inner and outer tracks, and with the two cases: flying in the same or opposite direction.

For suitably equipped RNAV aircraft, as referenced in AC 90-100, this study answers the following questions:

1. What is the probability of an aircraft flying a straight en route track segment under radar surveillance deviating laterally from that track by more than 2 NM (or 3 NM or 4 NM)?

² This study addresses collision risk between aircraft on laterally parallel RNAV routes. It does not attempt to address collision risk between aircraft on vertical parallel routes, that is, routes one above the other.

³ As defined in [4], ICAO Document 9689-AN/953, First edition, 1998

2. What is the risk of an aircraft flying a straight en route outer track segment under radar surveillance colliding with an aircraft flying a parallel, adjacent en route track, in the *opposite* direction of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM)?
3. What is the risk of an aircraft flying a straight en route outer track segment under radar surveillance colliding with an aircraft flying a parallel, adjacent en route track, in the *same* direction as the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM)?
4. What is the risk of an aircraft flying a straight en route inner track segment under radar surveillance colliding with an aircraft flying a parallel, adjacent en route track, in the *opposite* direction of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming aircraft on either side of the inner track aircraft are flying in an opposite direction to that of the inner aircraft?
5. What is the risk of an aircraft flying a straight en route inner track segment under radar surveillance colliding with an aircraft flying a parallel, adjacent en route track, in the *same* direction as the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming aircraft on either side of the inner track aircraft are flying in the same direction to that of the inner aircraft?
6. What is the risk of an aircraft flying a straight en route inner track segment under radar surveillance colliding with an aircraft flying a parallel, adjacent en route track, in the *same* as or *opposite* direction of the other aircraft, with given track-to-track separation distance of 4 NM (or 6 NM or 8 NM) assuming the aircraft on one side of the inner track aircraft is flying in the same direction as and the aircraft on the other side is flying in the opposite direction to that of the inner aircraft?

2.0 Study Methodology

This section of the report provides a description of all of the scenarios used in the study, and summarizes the data from external reports that was used to validate the criterion-based analysis results.

2.1 Model Description

We describe the models in terms of their scenarios and the associated hazards. As described above, there are three scenarios of interest established in Scenarios 1, 2, and 3. Scenarios 4, 5, and 6 increase the number of aircraft involved in the scenarios.

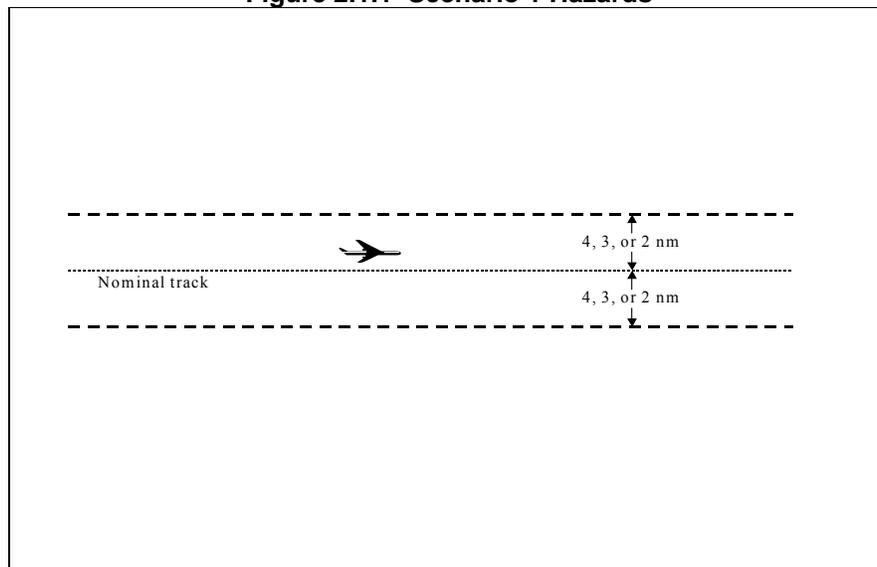
Scenario 1

In this scenario, a typical aircraft is flying a straight en route track segment under radar surveillance with turns of no greater than 15°. The hazard in this scenario is the aircraft deviating laterally from that track by more than 2 NM (or 3 NM or 4 NM) during one hour of flight (see Figure 2.1.1). The severity of this hazard is major (for a description of this severity, refer to Appendix A, Severity Definitions Based on the Perspective of the Flying Public).

The specific Test Criteria Violation (TCV) for this hazard is the deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM or more (or 3 NM or more, or 4 NM or more). We estimate the probabilities of these TCVs, but do not assess their risk since no actual collision is involved.

We model this scenario by a statistical distribution of lateral aircraft deviations. This distribution is used to determine the probability of a TCV.

Figure 2.1.1 Scenario 1 Hazards



Scenario 2

In this scenario, two aircraft are flying in opposite directions on parallel, adjacent, straight en route track segments under radar surveillance with turns of no greater than 15° (Figure 2.1.2). The parallel tracks are separated by 4 NM (or 6 NM or 8 NM). The hazard in this scenario is the collision of the aircraft. The severity of this hazard is catastrophic (see Appendix A).

The specific TCV for this hazard is the combined lateral, longitudinal, and vertical conjunction of the two aircraft (i.e., a collision). This conjunction is modeled by centers of gravity of the aircraft converging to within their mean wingspan laterally, within their mean lengths longitudinally, and within their mean heights vertically.

Assumption 1: Independence

We model this scenario by statistical distributions for lateral aircraft deviations and by probabilities for longitudinal and vertical convergence (or overlap) of the two aircraft. We assume that the lateral deviation, the vertical deviation, and the longitudinal encounter with the other aircraft are independent. This is a conservative assumption because for these to be dependent would imply that the two aircraft were either trying to avoid each other or trying to collide. For aircraft operating under normal conditions, we can eliminate the latter possibility. Therefore, under normal operating conditions dependence implies avoidance. But we assume (conservatively) non-avoidance and therefore, independence. (This study does not include the effects of the Traffic Alert and Collision Avoidance System (TCAS).)

Assumption 2: Mutual Exclusivity

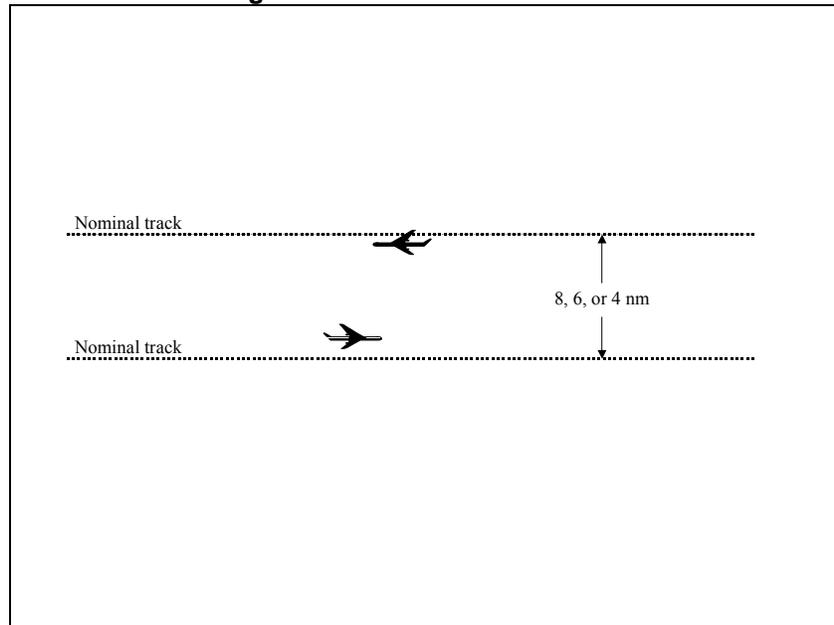
We also assume that for aircraft flying in opposite directions, a collision can occur in only one of three ways: side-to-side, top-to-bottom, or nose-to-nose.

Under the assumption of mutual exclusivity, the probability of a TCV for this scenario, $P(TCV_2)$, is the sum of the probabilities of collision for each way:

$$P(TCV_2) = P(C_s) + P(C_t) + P(C_n), \quad (1)$$

Where $P(C_s)$ represents the probability of a side-to-side collision, $P(C_t)$ represents the probability of a top-to-bottom collision, and $P(C_n)$ represents the probability of a nose-to-nose collision.

Figure 2.1.2 Scenario 2 Hazards



Scenario 3

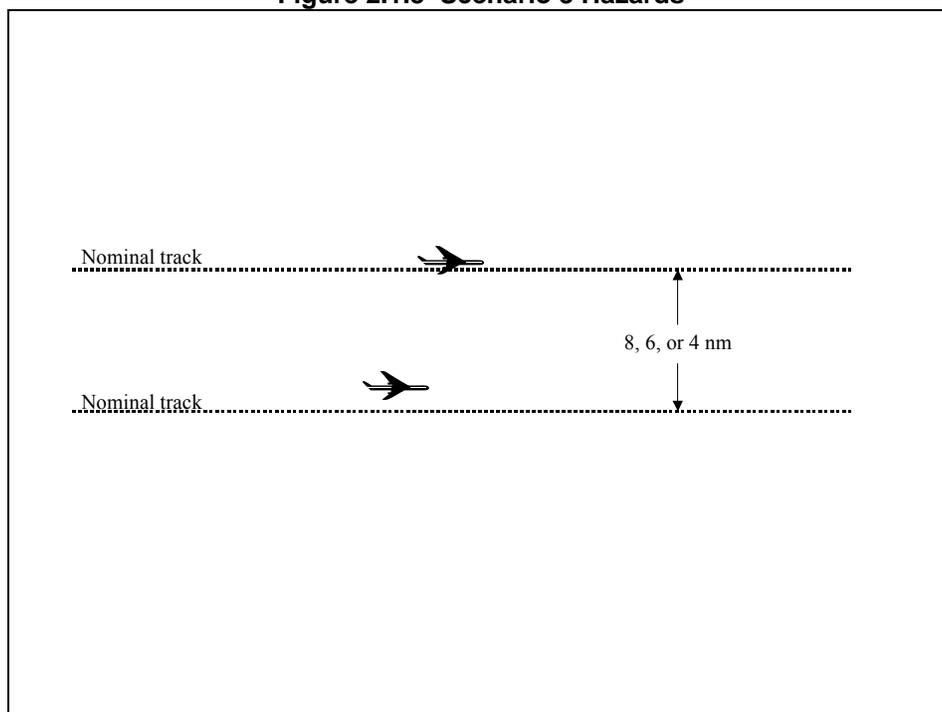
This scenario is the same as Scenario 2, except the two aircraft are flying in the same direction on parallel, adjacent, straight en route track segments under radar surveillance with turns of no greater than 15° (Figure 2.1.3). The hazards and TCVs are the same. The mathematical model for the calculation of the probability of a TCV is similar to that of Scenario 2. We assume both independence and mutual exclusivity.

Under the assumption of mutual exclusivity, the probability of a TCV for this scenario, $P(\text{TCV}_3)$, is the sum of the probabilities of collision for each way:

$$P(\text{TCV}_3) = P(D_s) + P(D_t) + P(D_n), \quad (2)$$

Where $P(D_s)$ represents the probability of a side-to-side collision; $P(D_t)$ represents the probability of a top-to-bottom collision; and $P(D_n)$ represents the probability of a nose-to-tail collision.

Figure 2.1.3 Scenario 3 Hazards



Scenario 4

This scenario is the same as Scenario 3 except that there are three aircraft rather than two. The aircraft track of interest is the one between the other two. All aircraft are flying in the same direction on parallel, adjacent, straight en route track segments under radar surveillance with turns of no greater than 15° . The hazards and TCVs are the same as in Scenario 3. The mathematical model for the calculation of the probability of a TCV is the same. However, the number of encounters with adjacent aircraft is generally twice that of Scenario 3.

Scenario 5

This scenario is the same as Scenario 2 except there are three aircraft rather than two. The aircraft track of interest is the one between the other two. The inner aircraft is flying in the opposite direction of the other two on parallel, adjacent, straight en route track segments under radar surveillance with turns of no greater than 15° (Figure 2.1.3). The hazards and TCVs are the same as Scenario 2. The mathematical model for the calculation of the probability of a TCV is the same. However, the number of encounters with adjacent aircraft is generally twice that of Scenario 2.

Scenario 6

This scenario is the same as Scenario 5 except that the inner aircraft is flying in the opposite direction to one of the other aircraft and in the same direction as the other on parallel, adjacent, straight en route track segments with turns of no greater than 15° (Figure 2.1.3). The hazards and TCVs are the same as Scenario 5. The mathematical model for the calculation of the probability of a TCV is the same. However, the number of encounters with adjacent aircraft is, in general, the sum of the encounters in Scenarios 2 and 3.

2.1 Summary of Data Used

The data used fall into two categories: values specified in AC 90-100 and data from radar tracks reported in previous RNAV studies.

AC 90-100 specifies a value for track-keeping accuracy for RNAV aircraft. This criterion is the basis for the analysis.

We examined three studies that used RNAV track data. We used the data and results from those studies to validate the criterion-based analysis results. That is, these studies do not provide the basis for determining hazard risk, but rather validate the model based on the AC 90-100 criterion. The three studies are described below.

1. “Preliminary Re-evaluation of the Probability of Lateral Overlap, $P_y(0)$, based on non-Global Positioning Satellite (GPS) and GPS-Equipped Aircraft Performance at Entry into North Atlantic Reduced Vertical Separation Minimum Airspace” [1].

This paper was published by the North Atlantic Mathematicians’ Implementation Group as NAT MIG /5-WP/18 in April 1999. The paper’s analysis was based on data collected in 1995 from 11 aircraft flights by five operators with three aircraft types (B747-200, B747-400, A340). Each aircraft was using GPS navigation on an oceanic route—five flights were North Atlantic routes, five were Pacific Oceanic airspace, and one was a South Atlantic route.

2. “Estimating the Well-Fit Model for the Distribution of Cross Track Deviations of GPS Equipped Aircraft on a North Pacific Route” [2].

This paper was published by the Separation and Airspace Safety Panel (SASP) as SASP-WG/WHL/4-WP/23 in November 2003. The paper’s analysis was based on data collected between December 2001 and May 2002, from *3,150 flights on the North Pacific route R220*. Each aircraft, types B747-400, B777, A340, was using GPS navigation on the route.

3. “Analysis of Lateral Track Deviation along Two Q-Routes” [3].

This paper was published by the FAA’s Flight Technologies and Procedures Division (AFS-400) in October 2005. The paper’s analysis was based on data collected in February and March 2003, from 865 flights on Q-routes 100 and 102 in the Gulf of Mexico. Each aircraft was using some type of RNAV navigation, typically GPS or DME/DME IRU on the route.

3.0 Summary of Data Analysis and Risk Evaluation

In this section, we determine the probability of the TCVs in each of the three scenarios, use those probabilities along with the hazard severities discussed in Section 2.1 (Model Description) to define the risk for each hazard, and then compare those risks with standard acceptable levels of risk.

3.1 Summary of the TCV Probability Analysis

We examine the TCV probability analysis for each scenario beginning with Scenario 1. The results of the analysis for Scenario 1 can be used in the analyses for the other five scenarios.

Scenario 1 Probability Analysis

The TCV for this hazard is the deviation of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM or more (or 3 NM or more, or 4 NM or more). The purpose of the analysis is to determine the probability of each of these three TCVs associated with the 2, 3, and 4 NM cases. We proceed by basing the analysis on the track-keeping accuracy specified in AC 90-100 for aircraft operating on RNAV routes. Then we compare results from previous empirical en route studies with the results of the AC 90-100 analysis to generate a reasonable set of TCV probabilities.

AC 90-100 Analysis

The track-keeping accuracy specified in AC 90-100 for aircraft operating on RNAV routes is an accuracy “bounded by ± 2 NM for 95% of the total flying time.” This means that the frequency of an aircraft remaining within the 2 NM boundary is 95%. Using the frequency definition of probability, this translates into a 95% probability of containment within the 2 NM boundary.

This AC 90-100 requirement allows us to describe one or more statistical distributions for lateral deviation. Such a distribution is symmetric and centered at zero. Also 95% of its area is contained between -2 and $+2$ NM. If we specify that the distribution is, say,

normal, these requirements allow us to fix the Probability Density Function (PDF) exactly. However, since there are multiple distributions that fit the 95% criterion, we use a set of reasonable criteria to find an appropriate distribution.

Criteria for Lateral Deviation Distribution Selection

1. The distribution should be symmetric, centered at zero with 95% of its area contained within -2 and $+2$ NM as specified in AC 90-100.
2. The distribution should be consistent with current en route separation analysis practice. There are existing en route analyses (for example, oceanic studies such as [1] and [2]) that used certain types of distributions for lateral deviation.
3. The distribution should take radar surveillance into account.
4. The distribution should be conservative compared to existing empirical data for en route separation.

A natural initial choice for a lateral deviation distribution is a normal distribution because it is symmetric and can be centered at zero and made to fit 95% of its area within 2 NM. In the Q-route study [3], we determined that the aircraft already established on those Q-routes displayed a lateral displacement that was normally distributed.

A normal distribution that satisfies the AC 90-100 requirements (as described above) has a mean of 0 and a standard deviation of 1.02 and generates TCV containment probabilities as summarized in Table 3.1.1.

Table 3.1.1 Normal Distribution Containment Probabilities

Lateral Distance from Track	Containment Probability	Probability Not Contained*
± 2 NM	0.950	5 E-02
± 3 NM	0.997	3 E-03
± 4 NM	0.999	9 E-05

*The notation E-02 denotes 10^{-2} . That is, ten to the negative second power.

However, a single normal distribution does not explicitly take radar surveillance into account. Since en route air traffic controllers are required to prevent aircraft on adjacent route from approaching within 5 NM, a distribution that models lateral separation under radar surveillance should display at least a partial bound to account for the surveillance effect.

The en route studies [1] and [2] use a mixed distribution to model lateral deviation. They both use a combination of normal and double-exponential distributions. The normal distribution is the primary model for the typical (or core) behavior, and the double-exponential distribution accounts (for the most part) for the atypical (or tail) behavior.

However, the studies do not attempt to model radar surveillance effects, so there is no attempt to model bounded (or partially bounded) behavior.

Based on the first three criteria above, we propose a mixed distribution similar to the normal/double-exponential of [1] and [2], but with the normal distribution replaced by a bounded distribution, a symmetric Johnson S_B distribution⁴. The distribution is of the form:

$$f(y) = \frac{\alpha e^{-|y/\delta|}}{2\delta} + \frac{(1-\alpha)\eta\lambda e^{-0.5\eta^2 \text{Ln}\left[\frac{y-\varepsilon}{\varepsilon-y+\lambda}\right]^2}}{\sqrt{2\pi}(y-\varepsilon)(\varepsilon-y+\lambda)} \quad (3)$$

Where the first term represents the double-exponential and the second represents the Johnson S_B distribution. The parameter, α , is the proportionality factor for the mix of distributions. We set $\alpha = 0.0566$ to match the proportion used in the studies [1] and [2]. We set $\delta = 0.3$ and $\eta = 1.2$ to satisfy the 95% criterion. We set $\lambda = 6$ and $\varepsilon = -3$ to provide partial bounding within 3 NM of the center (zero) to reflect the surveillance effect. (If the adjacent tracks are separated by, say, 8 NM and aircraft on the adjacent track do not deviate from the nominal, we can assume that controllers will typically attempt to prevent the target aircraft from deviating toward the adjacent aircraft by more than 3 NM so as to maintain 5 NM separation.) We note that while the Johnson S_B distribution is bounded, the overall distribution is unbounded due to the double-exponential contribution, which is intended to account for atypical lateral deviations.

Next, we compare this distribution with the empirical results of the three studies ([1], [2], and [3]). Since for measuring the likelihood of a collision the critical values are in the tails of the distributions, we compare tail areas of the distributions used in the three studies with the distribution developed above. Figure 3.1.1 depicts the areas we evaluate. And Table 3.1.2 lists the areas to the right of the line at d NM for each of the four distributions.

⁴ The Johnson S_B distribution is a transformation of the normal distribution and has been used frequently to model lateral track deviations.

Figure 3.1.1 Two Distribution Tail Areas

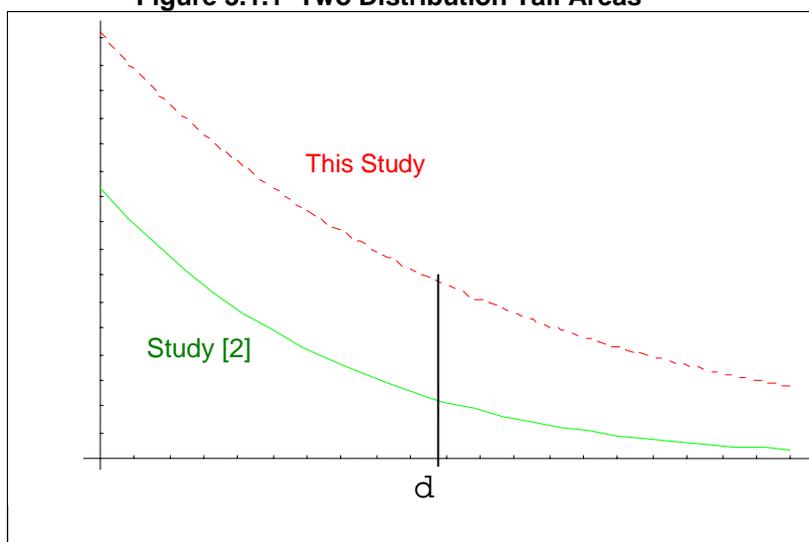


Table 3.1.2 Distribution Areas to the Right of d NM

d	Study [1] (N = 11)*	Study [2] (N = 3150)	Study [3] (N = 865)	f(y) (This Study)	Normal (Criterion 1)
1	2.0 E-02	1.0 E-13	1.0 E-08	1.9 E-01	1.6 E-01
2	1.0 E-02	4.0 E-25	< 1.0 E-50	2.5 E-02	2.5 E-02
4	7.0 E-03	5.0 E-48	< 1.0 E-50	4.6 E-08	4.4 E-05
6	3.0 E-03	< 1.0 E-50	< 1.0 E-50	5.8 E-11	2.0 E-09

*N represents the sample size for each empirical study.

The tail values for the distribution developed above, $f(y)$, are larger than those of studies [2] and [3]. This means that the distribution developed above gives a larger estimate for collision probability than either study [2] or [3], and is therefore conservative in comparison. It gives a smaller probability estimate than study [1], but this is not unreasonable considering that study [1] does not account for the radar surveillance effect. It should also be noted that the sample size for study [1] is very small compared to those of the other two studies. (It turns out that $f(y)$ is closer to a normal distribution that satisfies criterion 1, the 95% criterion, but not the radar surveillance effect criterion, than any of the other distributions.)

Given the relative sample sizes and the lack of radar surveillance consideration of the three comparison studies, it is reasonable to conclude that the distribution, $f(y)$, developed above provides a conservative estimate of en route lateral deviation behavior under radar surveillance compared to the other studies, and therefore satisfies all four criteria for lateral distribution selection. Given the empirical results available, this distribution appears to be conservative. It may be refined, however, as more empirical results from RNAV en route operations become available.

Scenario 1 Summary

Using the criterion-based distribution, $f(y)$, we can estimate the probabilities for the TCVs for this scenario's hazards (i.e., the deviations of the center of gravity of the aircraft from the nominal track by a lateral distance of 2 NM, 3 NM, or 4 NM or more). Table 3.1.3 lists those probabilities.

Table 3.1.3 Scenario 1 TCV Probabilities

Lateral Distance from Track	TCV Probability
±2 NM	5.0 E-02
±3 NM	2.6 E-06
±4 NM	9.2 E-08

Scenario 2 Probability Analysis

The TCV for this hazard is the collision with another aircraft that is flying in an opposite direction under radar surveillance on a straight, parallel RNAV track at the same altitude and at a lateral track distance of 4, 6, or 8 NM from the first aircraft's track.

The probability of a TCV for this scenario is the sum of the probabilities of the three mutually exclusive types of collision of the two aircraft⁵:

1. A side-to-side collision (C_s),
2. A top-to-bottom collision (C_t),
3. A nose-to-nose collision (C_n).

That is, $P(TCV_2) = P(C_s) + P(C_t) + P(C_n)$.

First, we determine each of the three probabilities separately. Then we calculate their sum, which is the probability of the TCV for this scenario.

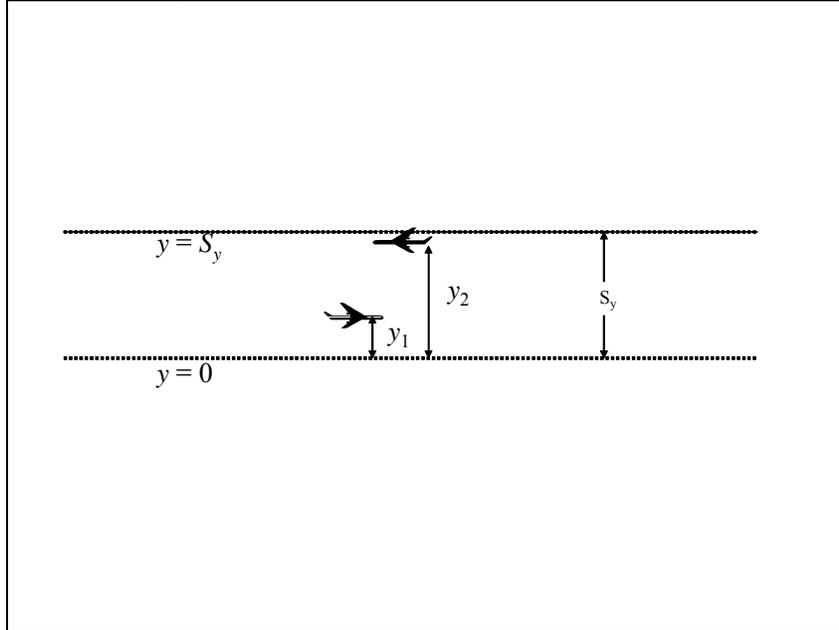
The Probability of a Side-To-Side Collision, $P(C_s)$

We assume that there are two aircraft executing RNAV operations on parallel, adjacent, tracks either 4, 6, or 8 NM apart and that each aircraft displays a lateral deviation from its track that can be described by the criterion-based distribution developed in the Scenario 1 analysis.

⁵ This probability analysis follows that of Moek [4] for lateral separation, which in turn is based on the Reich Model [5] and is also the methodology recommended in the ICAO "Manual on Airspace Planning Methodology for Determining Separation Minima" [6].

Let the target (first) aircraft's intended track be the $y = 0$ axis and (assuming the tracks are S NM apart), the adjacent (second) aircraft's intended track is the line $y = S_y$. (See Figure 3.1.2.) so that the tracks are separated by S_y NM.

Figure 3.1.2 Scenario 2 Mathematical Model



Let V_1 and V_2 denote the target and adjacent aircraft ground speeds respectively. And assume that the wingspan and length of each aircraft is λ NM. Therefore, when they pass each other, the aircraft are adjacent for a period of $2\lambda/(V_1+V_2)$ hours.

A side-to-side collision occurs only when the aircraft move into lateral overlap during that period of adjacency, and also happen to be in vertical overlap when they move into lateral overlap. Since the aircraft motion in the three dimensions is assumed to be independent, the probability of a side-to-side collision, $P(C_s)$, can be taken to be the product of the following:

- The duration of the period of (longitudinal) adjacency: $2\lambda/(V_1+V_2)$ hours
- The rate of entry into lateral overlap: $N_y(S_y)$ occurrences per hour
- The probability of vertical overlap: $P_z(0)$

$$\text{That is, } P(C_s) = \frac{2\lambda}{V_1 + V_2} N_y(S_y) P_z(0). \quad (4)$$

A top-to-bottom collision occurs only when the aircraft move into vertical overlap during that period of adjacency, and also happen to be in lateral overlap when they move into vertical overlap. Since the aircraft motion in the three dimensions is assumed to be

independent, the probability of a top-to-bottom collision, $P(C_t)$, can be taken to be the product of the following:

- The duration of the period of adjacency: $2\lambda/(V_1+V_2)$ hours
- The rate of entry into vertical overlap: $N_z(0)$ occurrences per hour
- The probability of lateral overlap: $P_y(S_y)$

That is,
$$P(C_t) = \frac{2\lambda}{V_1 + V_2} N_z(0) P_y(S_y).$$

A nose-to-nose collision occurs only when the aircraft are in lateral and vertical overlap at the moment they become adjacent. The probability of a nose-to-nose collision, $P(C_n)$, can be taken to be the product of the following:

- The probability of vertical overlap: $P_z(0)$
- The probability of lateral overlap: $P_y(S_y)$

That is,
$$P(C_n) = P_z(0) P_y(S_y).$$

Since, from Equation (1), the probability of a TCV for this scenario is $P(TCV_2) = P(C_s) + P(C_t) + P(C_n)$, then

$$P(TCV_2) = \frac{2\lambda}{V_1 + V_2} N_y(S_y) P_z(0) + \frac{2\lambda}{V_1 + V_2} N_z(0) P_y(S_y) + P_z(0) P_y(S_y). \quad (4)$$

Let $|\dot{y}(S_y)|$ denote the lateral passing speed of the two aircraft, that is their relative lateral approach speed. Therefore, $\frac{2\lambda}{|\dot{y}(S_y)|}$ is the average duration of a lateral overlap in hours.

Since $N_y(S_y)$ is the hourly rate of entry into lateral overlap and $P_y(S_y)$ is the probability that the two aircraft are in lateral overlap, then

$$P_y(S_y) \approx N_y(S_y) \frac{2\lambda}{|\dot{y}(S_y)|}.$$

Therefore,
$$N_y(S_y) \approx P_y(S_y) \frac{|\dot{y}(S_y)|}{2\lambda}.$$

So Equation (4) can be written as:

$$P(\text{TCV}_2) = \frac{2\lambda}{V_1 + V_2} P_y(S_y) \frac{|\dot{y}(S_y)|}{2\lambda} P_z(0) + \frac{2\lambda}{V_1 + V_2} N_z(0) P_y(S_y) + P_z(0) P_y(S_y). \quad (5)$$

To evaluate $P(\text{TCV}_2)$ we find values for the individual factors. To find $P_y(S_y)$, let the lateral positions of the aircraft be given by the variables y_1 and y_2 respectively. The aircraft is assumed to be in lateral overlap when their centers of gravity are within λ , that is, when $|y_2 - y_1| < \lambda$. Therefore, the probability of lateral overlap is calculated as:

$$P_y(S_y) = P(|y_2 - y_1| < \lambda).$$

But, $P(|y_2 - y_1| < \lambda) = P(-\lambda < y_2 - y_1 < \lambda)$. This probability can be found by integrating the PDF describing $(y_2 - y_1)$ between $-\lambda$ and λ .

The PDF describing $(y_2 - y_1)$ is the convolution of the two PDFs of the two variables, y_2 and $-y_1$. The PDF for each variable is that of the criterion-based distribution, $f(y)$ developed for Scenario 1. Appendix B gives the details for the convolution of these two PDFs and of the integration that yields the lateral overlap probability. Table 3.1.4 gives the lateral overlap probability, $P_y(S_y)$, for each of the three values of S : 4, 6, and 8 NM.

Table 3.1.4 Scenario 2 Lateral Overlap Probabilities

Track-to-Track Distance (S_y)	$P_y(S_y)$
4 NM	3.7 E-04
6 NM	1.9 E-09
8 NM	2.4 E-12

To deal with the speeds, we let V be the mean of V_1 and V_2 . Therefore, $V_1 + V_2 = 2V$. Also, the lateral passing speed of the two aircraft, $|\dot{y}(S_y)|$, can be estimated by assuming that the aircraft are converging at a 45° angle, so that

$$|\dot{y}(S_y)| \approx \frac{2V}{\sqrt{2}}.$$

Equation (5) then becomes,

$$P(\text{TCV}_2) = P_y(S_y) \left[\frac{1}{\sqrt{2}} P_z(0) + \frac{\lambda}{V} N_z(0) + P_z(0) \right]. \quad (6)$$

We make the conservative assumption⁶ that the probability of vertical overlap, $P_z(0)$, is 1. This implies that the aircraft cannot collide top-to-bottom (i.e., $N_z(0) = 0$). So Equation (6) becomes,

$$P(\text{TCV}_2) = P_y(S_y) \left[\frac{1}{\sqrt{2}} + 1 \right]. \tag{7}$$

Scenario 2 Summary

The probability of a TCV for this scenario, $P(\text{TCV}_2)$, is, therefore, based on the probability of lateral overlap, $P_y(S_y)$. Table 3.1.5 summarizes these probabilities and the corresponding probabilities of the Scenario 2 TCVs.

**Table 3.1.5 Scenario 2
Lateral Overlap and TCV Probabilities**

Track-to-Track Distance (S)	$P_y(S_y)$	$P(\text{TCV}_2)$
4 NM	3.7 E-04	6.3 E-04
6 NM	1.9 E-09	3.2 E-09
8 NM	2.4 E-12	4.1 E-12

Scenario 3 Probability Analysis

The TCV for this hazard is the collision with another aircraft that is flying in the same direction on a straight, parallel RNAV track under radar surveillance at the same altitude and at a lateral track distance of 4, 6, or 8 NM from the first aircraft’s track.

As with Scenario 2, the probability of a TCV for this scenario is the sum of the probabilities of the three mutually exclusive types of collision of the two aircraft:

1. A side-to-side collision (C_s),
2. A top-to-bottom collision (C_t),
3. Or a nose-to-tail collision (C_n)

That is, $P(\text{TCV}_2) = P(C_s) + P(C_t) + P(C_n)$. Note that $P(C_n)$ now represents a nose-to-tail rather than a nose-to-nose collision since the aircraft are flying in the same direction.

⁶ As navigation and guidance systems become more accurate, vertical overlap will likely become more probable. Because of this fact, we assume that the navigation and guidance systems of the aircraft in this study are quite accurate, and, in fact, that the probability of vertical overlap is 1.0.

The probability analysis for this scenario is similar to that of Scenario 2 except that we substitute the term, ΔV , for the relative velocity, V_1+V_2 of that analysis, where ΔV is the average speed of overtake. Equation (5) becomes,

$$P(\text{TCV}_3) = \frac{2\lambda}{\Delta V} P_y(S_y) \frac{|\dot{y}(S_y)|}{2\lambda} P_z(0) + \frac{2\lambda}{\Delta V} N_z(0) P_y(S_y) + P_z(0) P_y(S_y). \quad (8)$$

Also, the lateral passing speed of the two aircraft, $|\dot{y}(S_y)|$, can be estimated by assuming that the aircraft are converging at a (conservative⁷) 45° angle, so that

$$|\dot{y}(S_y)| \approx \frac{\Delta V}{\sqrt{2}}.$$

Equation (8) then becomes,

$$P(\text{TCV}_3) = P_y(S_y) \left[\frac{1}{\sqrt{2}} P_z(0) + \frac{2\lambda}{\Delta V} N_z(0) + P_z(0) \right]. \quad (9)$$

If we (again) assume that the probability of vertical overlap, $P_z(0)$, is 1. This implies that the aircraft cannot collide top-to-bottom (i.e., $N_z(0) = 0$). So Equation (9) becomes,

$$P(\text{TCV}_3) = P_y(S_y) \left[\frac{1}{\sqrt{2}} + 1 \right], \quad (10)$$

the same probability as for Scenario 2. This is reasonable since the approach speeds, the only difference in the two, in each case cancelled out of the final equation.

Scenario 3 Summary

The probability of a TCV for this scenario, $P(\text{TCV}_3)$, is, therefore, as with Scenario 2, based on the probability of lateral overlap, $P_y(S_y)$. Table 3.1.6 summarizes these probabilities and the corresponding probabilities of the Scenario 3 TCVs.

⁷ A 60° angle would result in a TCV probability about 9% greater than that for a 45° angle. The 60° angle value for the 8NM case in Table 3.1.6 would be 4.5 E-12 rather than 4.1 E-12.

**Table 3.1.6 Scenario 3
Lateral Overlap and TCV Probabilities**

Track-to-Track Distance (S)	$P_y (S_y)$	$P(TCV_3)$
4 NM	3.7 E-04	6.3 E-04
6 NM	1.9 E-09	3.2 E-09
8 NM	2.4 E-12	4.1 E-12

Scenarios 4, 5, and 6 Probability Analysis

The TCV probabilities for Scenarios 4, 5, and 6 are the same as those of Scenarios 2 and 3. The differences are in the number of encounters per hour given in the results section below.

3.2 Summary of Acceptable Level of Risk

This analysis applies to Scenarios 2 and 3 because those deal with risk of collision while Scenario 1 deals only with probability of boundary penetration. The purpose of this section is to recommend an acceptable level of risk for Scenarios 2 and 3 based upon standards, operational experience, and accepted practices within the National Airspace System (NAS) and to develop a basis for comparing the estimated TCV risk with this acceptable level of risk.

The guidelines for this study established a target level of safety⁸ rate of 5.0 E-09 collisions per hour of flight. However, the FAA Safety Management System Manual, v 1.1 sets the probability of a catastrophic level event at 1.0 E-09 or less.

The TCV probabilities calculated in the previous sections were probabilities of collision when a target aircraft encountered (became adjacent to) an aircraft on an adjacent track. To compare those risks to the target level of safety (an hourly rate), we must transform the TCV probabilities per encounter into hourly rates. Since each TCV encounter probability is very small, this can be accomplished by multiplying the TCV probability by the estimated number of encounters in an hour.

The number of encounters per hour is a function of the relative longitudinal approach speed of the target and adjacent aircraft and the spacing density of the aircraft on the adjacent track. The assumptions of higher approach speeds and denser spacing are conservative, that is, they result in higher hourly collision rates.

⁸ Based on the ICAO target level of safety for en route separation minima established in [6]

For opposite-direction aircraft (Scenario 2), we assume a longitudinal approach speed, V_1+V_2 , of 1000 knots⁹. For same-direction aircraft (Scenario 3), we assume a longitudinal approach speed, ΔV , of 100 knots¹⁰. For the spacing density, we assume a minimum 5 NM longitudinal spacing of aircraft on the adjacent route consistent with FAA Order 7110.65.

These assumptions result in (appropriately conservative) values of 200 encounters per hour for Scenario 2 and 20 encounters per hour for Scenario 3.

4.0 Results and Conclusions

This section summarizes the key results, the scenario risk evaluation, and the conclusions of the study.

4.1 Summary of Results

Tables 4.1.1 through 4.1.6 summarize the key results for Scenarios 1 through 6. For Scenarios 2 through 6 the TCV probability (the probability of collision per encounter) is multiplied by the estimated number of encounters per hour to arrive at an estimated hourly collision rate that can be compared to the target level of safety (5.0 E-09 collisions per hour or 1.0 E-09 collisions per hour). The hourly collision rates that meet the 1.0 E-09 collisions per hour level are set in bold.

Table 4.1.1 Scenario 1 Key Results

Scenario Description	Track Width	TCV Probability
A suitably equipped RNAV aircraft is flying a straight en route track segment with turns of no greater than 15° under radar surveillance.	±2 NM	5.0 E-02
	±3 NM	2.6 E-06
	±4 NM	9.2 E-08

⁹ This approach speed (1000 knots) is reasonably conservative for FL 180 and above. It is quite conservative for lower flight levels.

¹⁰ This overtake speed of 100 knots is conservative.

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Table 4.1.2 Scenario 2 Key Results

Scenario Description	Track-to-Track Separation	TCV Probability Per Encounter	Estimated Number of Encounters per Hour	Estimated Hourly Collision Rate
Two suitably equipped RNAV aircraft are flying in opposite directions on parallel straight en route track segments with turns of no greater than 15° under radar surveillance.	4 NM	6.3 E-04	200	1.3 E-01
	6 NM	3.2 E-09	200	6.4 E-07
	8 NM	4.1 E-12	200	8.2 E-10

Table 4.1.3 Scenario 3 Key Results

Scenario Description	Track-to-Track Separation	TCV Probability Per Encounter	Estimated Number of Encounters per Hour	Estimated Hourly Collision Rate
Two suitably equipped RNAV aircraft are flying in the same direction on parallel straight en route track segments with turns of no greater than 15° under radar surveillance.	4 NM	6.3 E-04	20	1.3 E-02
	6 NM	3.2 E-09	20	6.4 E-08
	8 NM	4.1 E-12	20	8.2 E-11

Table 4.1.4 Scenario 4 Key Results

Scenario Description	Track-to-Track Separation	TCV Probability Per Encounter	Estimated Number of Encounters per Hour	Estimated Hourly Collision Rate
An inner aircraft is flying in the opposite direction between two other aircraft on parallel straight en route track segments with turns of no greater than 15° under radar surveillance.	4 NM	6.3 E-04	400	2.6 E-01
	6 NM	3.2 E-09	400	1.3 E-06
	8 NM	4.1 E-12	400	1.6 E-09
	9 NM	1.5 E-13	400	6.0 E-11

Table 4.1.5 Scenario 5 Key Results

Scenario Description	Track-to-Track Separation	TCV Probability Per Encounter	Estimated Number of Encounters per Hour	Estimated Hourly Collision Rate
An <i>inner</i> aircraft is flying in the <i>same</i> direction between two other aircraft on parallel straight en route track segments with turns of no greater than 15° under radar surveillance.	4 NM	6.3 E-04	40	2.6 E-02
	6 NM	3.2 E-09	40	1.3 E-07
	8 NM	4.1 E-12	40	1.6 E-10

Table 4.1.6 Scenario 6 Key Results

Scenario Description	Track-to-Track Separation	TCV Probability Per Encounter	Estimated Number of Encounters per Hour	Estimated Hourly Collision Rate
An <i>inner</i> aircraft is flying between two other aircraft, one flying in the <i>same</i> direction and the other flying in the <i>opposite</i> direction on parallel straight en route track segments with turns of no greater than 15° under radar surveillance.	4 NM	6.3 E-04	220	1.4 E-01
	6 NM	3.2 E-09	220	7.0 E-07
	8 NM	4.1 E-12	220	9.0 E-10

4.2 Scenario Risk Evaluation and Conclusions

For Scenarios 2 through 6, we evaluate the risk of collision with an adjacent aircraft. This evaluation requires us to compare the estimated hourly collision rate with the corresponding acceptable level of risk (target level of safety).

For the target level of safety established for this study (5.0 E-09 collisions per hour of flight) the risk evaluation results are:

- For all scenarios at 4 NM separation: the target level of safety is exceeded.
- For all scenarios at 6 NM separation: the target level of safety is exceeded.
- For all scenarios at 8 NM separation: the target level of safety is met.

For the more stringent target level of safety (1.0 E-09 collisions per hour of flight) the risk evaluation results are:

- For all scenarios at 4 NM separation: the target level of safety is exceeded.
- For all scenarios at 6 NM separation: the target level of safety is exceeded.
- For all scenarios at 8 NM separation: the target level of safety is met except for Scenario 4 in which a inner aircraft is flying in a direction opposite to those of the two aircraft on either side.
- For Scenario 4, the target level of safety is met at 9 NM separation.

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Appendix A: Severity Definitions Based on the Perspective of the Flying Public

(FAA Safety Management System Manual, Version 1.0, July 24, 2003)

No Safety Effect	Minor	Major	Hazardous	Catastrophic
<p>No effect on flight crew</p> <p>Has no effect on safety</p> <p>Inconvenience</p>	<p>Slight increase in workload such as flight plan changes</p> <p>Slight reduction in safety margin or functional capabilities</p> <p>Minor illness, environmental damage, or system damage</p> <p>Some physical discomfort to occupants of aircraft (except operators)</p>	<p>Significant increase in flight crew workload</p> <p>Significant reduction in safety margin or functional capability</p> <p>Major illness, injury, environmental damage, or system damage</p> <p>Physical distress to occupants of aircraft (except flight crew) including injuries</p>	<p>Large reduction in safety margin or functional capability</p> <p>Serious or fatal injury to small number of persons (other than flight crew)</p> <p>Physical distress/ Excessive workload such that flight crew cannot be relied upon to perform required tasks accurately or completely</p>	<p>Outcome would result in:</p> <ul style="list-style-type: none"> - Hull loss - Multiple fatalities - Fatal injury or incapacitation

Appendix B: Statistical Distributions Used in the Study

The Johnson S_B Distribution PDF

$$f_B(x) = \frac{\eta\lambda}{\sqrt{2\pi}} \frac{1}{(x-\varepsilon)(-x+\varepsilon+\lambda)} \exp\left[-\frac{1}{2}\left(\eta^2 \ln\left\{\left(\frac{x-\varepsilon}{-x+\varepsilon+\lambda}\right)^2\right\}\right)\right] \quad (A1)$$

where $\varepsilon < x < \varepsilon + \lambda$, $-\infty < \gamma < \infty$, $-\infty < \varepsilon < \infty$, $\eta > 0$, $\lambda > 0$. The location parameter is ε . The scale parameter is λ . The shape (including skewness) parameters are γ and η .

The Double Exponential Distribution PDF

$$f_D(x) = \frac{1}{2\delta} \exp\left[-\frac{|x|}{\delta}\right] \quad (A2)$$

The Mixed Johnson S_B and Double Exponential Distribution PDF

$$f(x) = \begin{cases} \frac{\alpha \exp(-|x/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \ln\left[\left(\frac{x-\varepsilon}{-x+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(x-\varepsilon)(-x+\varepsilon+\lambda)}, & x < \varepsilon + \lambda \text{ \& } x > \varepsilon \\ \frac{\alpha \exp(-|x/\delta|)}{2\delta}, & x \leq \varepsilon \text{ \& } x \geq \varepsilon + \lambda \end{cases} \quad (A3)$$

The Convolution of Variables y_2 and y_1 and the Probability of $|y_2 - y_1| < W$

The PDF describing $(y_2 - y_1)$ is the convolution of the two PDFs of the two variables, y_2 and $-y_1$. The convolution of two variables y_2 and $+y_1$ is defined as the integral

$$f(u) = \int_{-\infty}^{\infty} f_1(y_1)f_2(u - y_1)dy_1 \quad (A4)$$

where $u = y_1 + y_2$. If f_1 and f_2 are PDFs of y_1 and y_2 , then f is the PDF of $u = y_1 + y_2$.

Also, if the PDF of y_1 is symmetric about zero, then the convolution of y_2 and y_1 is equivalent to the convolution of y_2 and $-y_1$. Therefore, f is also the PDF of $u = y_2 - y_1$.

This means that the probability of $|y_2 - y_1| < W$ is the integral of f between $-W$ and W . That is,

$$P(|y_2 - y_1| < W) = \int_{-W}^W f(x) dx. \quad (A5)$$

But $f(x)$ is defined in (2) where f_1 and f_2 are both Johnson S_U PDFs defined in (1).
Therefore,

$$P(|y_2 - y_1| < W) = \int_{-W}^W \int_{-\infty}^{\infty} f_1(y_1) f_2(x - y_1) dy_1 dx \quad (A6)$$

where f_1 and f_2 are defined in (1).

Substituting the definitions of f_1 and f_2 , and letting¹¹ $\varepsilon = 0$ in f_1 and $\varepsilon = S$ in f_2 , Equation (4) becomes

$$P(|y_2 - y_1| < W) = \int_{-W}^W \int_{-\infty}^{\infty} f_1(y_1) f_2(x - y_1) dy_1 dx \quad (A7)$$

Where,

$$f_1(y) = \begin{cases} \frac{\alpha \exp(-|y/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \text{Ln}\left[\left(\frac{y-\varepsilon}{-y+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(y-\varepsilon)(-y+\varepsilon+\lambda)}, & y < \varepsilon + \lambda \ \& \ y > \varepsilon \\ \frac{\alpha \exp(-|y/\delta|)}{2\delta}, & y \leq \varepsilon \ | \ y \geq \varepsilon + \lambda \end{cases}$$

And,

$$f_2(x-y) = \begin{cases} \frac{\alpha \exp(-|(x-y-8)/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \text{Ln}\left[\left(\frac{(x-y-8)-\varepsilon}{-(x-y-8)+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(y-\varepsilon)(-y+\varepsilon+\lambda)}, & (x-y-8) < \varepsilon + \lambda \ \& \ (x-y-8) > \varepsilon \\ \frac{\alpha \exp(-|(x-y-8)/\delta|)}{2\delta}, & (x-y-8) \leq \varepsilon \ | \ (x-y-8) \geq \varepsilon + \lambda \end{cases}$$

¹¹ Since ε is the location parameter, the PDF for y_1 uses $\varepsilon = 0$ and the PDF for y_2 uses $\varepsilon = S$, where the two tracks are S NM apart.

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NAT MIG/5-WP/18
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This paper was published by the North Atlantic Mathematicians’ Implementation Group as NAT MIG/5-WP/18 in April 1999. The paper’s analysis was based on data collected in 1995 from 11 aircraft flights by five operators with three aircraft types (B747-200, B747-400, A340). Each aircraft was using GPS navigation on an oceanic route—five flights were North Atlantic routes, five were Pacific Oceanic airspace, and one was a South Atlantic route.

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AFS-400 Safety Study

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