



Safety Study Report
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Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Segments with Radar Surveillance and Turns (Phase II)

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January 2007

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Flight Operations Simulation and Analysis Branch
Flight Technologies and Procedures Division
Flight Standards Service

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Segments with Radar Surveillance and Turns (Phase II)**

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Technical Report

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Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Segments with Radar Surveillance and Turns (Phase II)

Errata:

The last sentence of the first paragraphs in both the abstract (page ii) and the executive summary (page iii) should be removed. This sentence states: “The study also addresses the difference in risk between scenarios with and without radar surveillance.”

The part of the study that includes separation risk analyses without radar surveillance was moved to a future report.

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12. Abstract Developed by AFS-440, this study provides a risk assessment of lateral en route separation between parallel Area Navigation (RNAV) routes with separation for both opposite-direction and same-direction traffic. The study estimates the risk of collision of en route RNAV aircraft with Flight Management System (FMS) guidance flying adjacent, parallel tracks with turns (in the cases of both opposite-direction and same-direction tracks) when the aircraft of interest is flying adjacent to only one other track (i.e., on an outer track) and when the aircraft of interest is flying between two tracks (i.e., on an inner track). The study also addresses the difference in risk between scenarios with and without radar surveillance. This study develops risk estimates for scenarios in which the target aircraft's route lies between two other routes and for scenarios in which the target aircraft's route lies adjacent to just one other route. The study shows that for same-direction flights with radar surveillance, with 5 nautical miles (NM) longitudinal separation, and with turn angles up to 70°, a rate of up to five turns per hour of flight also meets the 5.0 E-09 target level of safety. However, for the more stringent 1.0 E-09 target level of safety, some turn angles and rates meet the target and some do not.		
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Executive Summary

This study¹ provides a risk assessment of lateral en route separation between parallel Area Navigation (RNAV) routes with separation for both opposite-direction and same-direction traffic. The study estimates the risk of collision of en route RNAV aircraft with Flight Management System (FMS) guidance² flying adjacent, parallel tracks with turns (in the cases of both opposite-direction and same-direction tracks) when the aircraft of interest is flying adjacent to only one other track (i.e., on an outer track) and when the aircraft of interest is flying between two tracks (i.e., on an inner track). The study also addresses the difference in risk between scenarios with and without radar surveillance.

The analysis is based on three types of data: results from a previous study of straight RNAV routes [1], including values specified in AC 90-100 and radar track data; values specified in DO-236 related to turns greater than 15°; and data from flight simulator tests focused on FMS generation of turn radius and starting point. AC 90-100 specifies a value for track-keeping accuracy value for RNAV aircraft.

The study uses the statistical distributions developed in [1] and the en route turn boundaries specified for RNAV FMS-equipped aircraft in DO-236 to model the likelihood of adjacent aircraft intersecting laterally. It also models and estimates the likelihood of aircraft on parallel, high altitude routes with turns becoming adjacent. Using those models, it estimates the probability of collision.

The study develops a general risk model that can be used to determine the probability of collision for aircraft suitably equipped with RNAV and FMS on parallel, adjacent inner or outer routes laterally separated by any track-to-track distance, longitudinally separated by any distance, flying in the opposite or same direction, with straight segments or turns. The study then applies this risk model to various typical operational scenarios including both those with and without radar surveillance.

This study develops risk estimates for scenarios in which the target aircraft's route lies between two other routes and for scenarios in which the target aircraft's route lies adjacent to just one other route. The study shows that for same-direction flights with radar surveillance, with 5 nautical miles (NM) longitudinal separation, and with turn angles up to 70°, a rate of up to five turns per hour of flight also meets the 5.0 E-09 target level of safety. However, for the more stringent 1.0 E-09 target level of safety, some turn angles and rates meet the target and some do not.

¹ This study is a follow-up study to the study *Analysis of Area Navigation (RNAV) En Route Separation along Adjacent Straight Segments with Radar Surveillance (Phase I)* and utilizes its results.

² A subsequent study will assess the risk for RNAV aircraft not equipped with FMS guidance.

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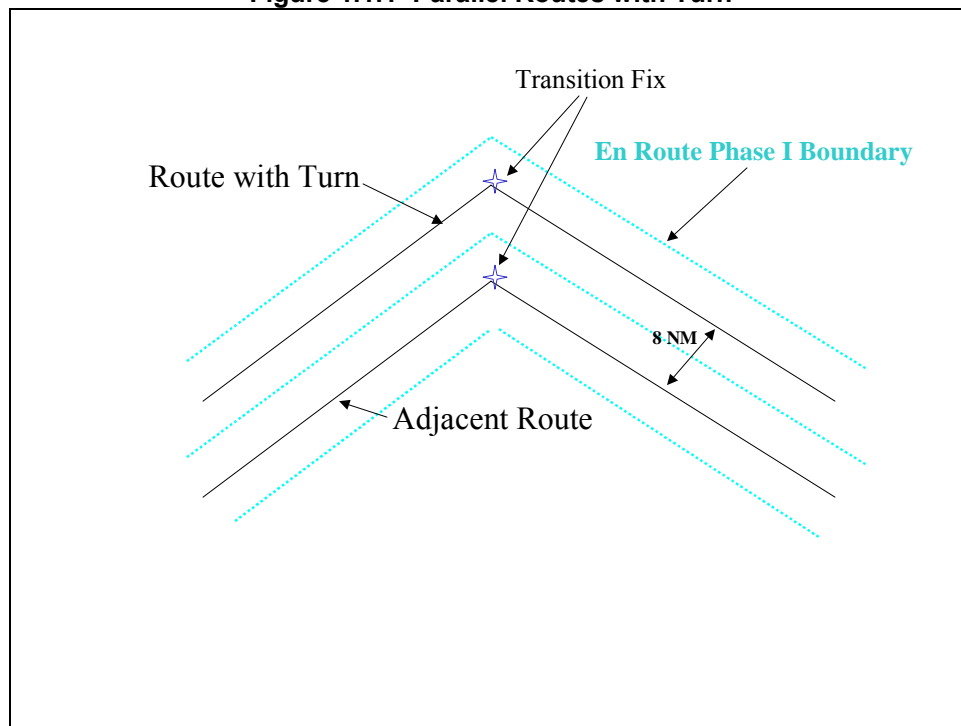
1.0 Introduction

This section of the report describes the purpose and structure of this document, and provides a description of the problem.

1.1 Purpose and Structure of This Document

The purpose of this study is to provide a risk assessment of lateral en route separation between parallel Area Navigation (RNAV) routes with separation for both opposite-direction and same-direction traffic under radar surveillance. The study estimates the risk of collision of en route RNAV aircraft flying adjacent, parallel tracks *with turns* (in the cases of both the opposite and same direction) when the aircraft of interest is flying adjacent to only one other track (i.e., on an outer track) and when the aircraft of interest is flying between two tracks (i.e., on an inner track). (See Figure 1.1.1.)

Figure 1.1.1 Parallel Routes with Turn



1.2 Statement of the Problem

Specifically, this study uses the lateral track deviation of typical RNAV-equipped aircraft³, with Flight Management System (FMS) guidance on en route segments with

³ As defined in ICAO Document 9689-AN/953, First edition, 1998

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turns greater than 15° to determine the probability of collision of two aircraft with a given track-to-track separation distance of 8 nautical miles (NM) under radar surveillance, with both inner and outer tracks, and with the two cases: flying in the same or opposite direction.

This analysis assumes the following basic method of turn completion using RNAV equipment authorized for Q-routes. The FMS or similar equipment, e.g. Flight Management Computer (FMC), computes a turn radius based on ground speed and a software-determined bank angle (typically a function of amount of turn, altitude, and aircraft characteristics). From the computed turn radius, a turn initiation start point is then determined to provide a smooth, seamless transition between route segments. Throughout the turn, the FMS provides guidance to an autopilot or a flight director to automatically affect the turn transition. Radio Technical Commission for Aeronautics (RTCA) DO-236, paragraph 3.2.5.4, provides a boundary for the fly-by transition area based upon maximum ground speed assumptions and typical bank angle assumptions for air carrier type aircraft for both high (above flight level 195) and low altitude transitions. While DO-236 relates primarily to Required Navigation Performance (RNP) systems, the fly-by turn transition boundaries are considered and assumed to be representative of RNAV FMS-equipped systems which provide command guidance throughout a turn transition. Furthermore, this study assumes that all RNAV aircraft, whether FMS-equipped or not, approved for Q-route operation can comply with the DO-236 fly-by turn performance.

For suitably equipped RNAV aircraft, as referenced in AC 90-100 and DO-236, this study answers the following general question:

What is the risk of an aircraft flying a high altitude, en route track segment colliding with an aircraft flying a parallel, adjacent en route track as a function of the following variables:

- Track-to-track route separation
- Longitudinal separation within a route
- Angle of turn
- Density of turns along a route⁴
- Whether adjacent aircraft are flying in the same or opposite direction as the aircraft at risk⁵
- Whether the aircraft at risk is flying an inner route or an outer route⁶

⁴ Density of turns refers to the number of turns per hour of flight along a given route.

⁵ The aircraft at risk is the one for which we are assessing the risk of collision with another aircraft.

⁶ On an outer track, the aircraft at risk flies a route adjacent to only one other route. On an inner track, it flies between two other routes.

We answer this general question by looking at the following specific question:

Given that the previous study of en route segments without turns found that an 8 NM track-to-track separation met the target level of safety (assuming at least 15 NM longitudinal separation), do aircraft flying routes with the same 8 NM track-to-track separation but with turns between 15° and 70° meet the target level of safety? More specifically, what turn angles and turn densities (if any) allow the target level of safety to be met at 8 NM separation?

2.0 Study Methodology

This section of the report provides a description of all of the scenarios used in the study, and summarizes the data used in this study.

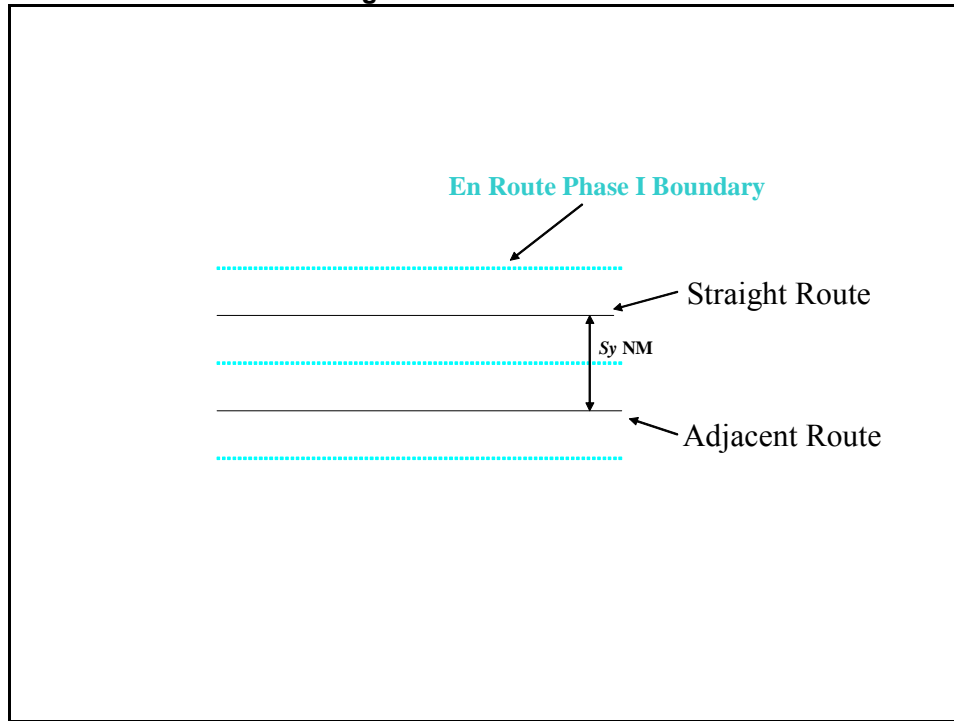
2.1 Model Description

We describe the risk model in terms of two scenarios: Scenario 1, (straight tracks) and Scenario 2, (tracks with turns greater than 15°).

Scenario 1 (Straight Tracks)

This is the overall scenario addressed in the previous study [1]. This scenario addresses straight en route track segments with turns of no more than 15°. There can be parallel, adjacent straight tracks with aircraft on those tracks flying either opposite-direction or same-direction routes. The aircraft at risk can be on an outer track or an inner track. The aircraft are assumed to be under radar surveillance. (See Figure 2.1.1.)

Figure 2.1.1 Scenario 1



The specific Test Criteria Violation (TCV) for this scenario is the combined lateral, longitudinal, and vertical conjunction of the two aircraft (i.e., a collision). This conjunction is modeled by centers of gravity of the aircraft converging within their mean wingspan laterally, within their mean lengths longitudinally, and within their mean heights vertically.

In the previous study [1], we made the following assumptions:

- The aircraft lateral, longitudinal, and vertical deviations are independent.
- The aircraft length and width are both approximately 0.03 NM (182 feet).
- The probability of aircraft on adjacent tracks being at the same altitude is 1.
- The aircraft are contained laterally within 2 NM of the straight track 95% of the time.

This last assumption is based on the AC 90-100 en route requirement. We used a statistical distribution to model the lateral track deviation that would provide 95% containment. The specific distribution used is a mixed Johnson S_B /Double-Exponential distribution. This distribution satisfies both theoretical and empirical reasonableness tests. Theoretically, the distribution models a combination of core (typical) lateral deviation behavior and tail (atypical) lateral deviation behavior. Empirically, the distribution is conservative compared to the results of the three empirical studies ([2], [3], and [4]).

The probability of a TCV for this scenario is the sum of the probabilities of three mutually exclusive events: side-to-side (C_s), top-to-bottom (C_t), or nose-to-nose (C_n) collisions between aircraft on adjacent tracks.

That is,

$$P(TCV) = P(C_s) + P(C_t) + P(C_n). \quad (1)$$

This TCV probability was shown to have a value given by the equation,

$$P(TCV) = P_y(S_y) \left[\frac{1}{\sqrt{2}} + 1 \right], \quad (2)$$

where $P_y(S_y)$ is the probability of a lateral overlap of the two aircraft.

The value for $P_y(S_y)$ can be accurately estimated from a polynomial fit of the natural logarithms of the convolution integral results (see Appendix A, Statistical Distributions Used in the Study). As a result, we can take

$$P_y(S_y) = e^{a_0 + a_1 S_y + a_2 S_y^2}, \quad (3)$$

where $a_0 = 0.11742$, $a_1 = -3.38814$, and $a_2 = 0.00357$, and S_y is the track-to-track separation in nautical miles.

Since the target level of safety for these en route operations is specified in number of collisions per hour of flight, the TCV rate must be converted to the units of collisions per hour of flight. To accomplish this, we multiply the TCV rate by the expected number of encounters of adjacent aircraft per hour of flight. If we assume a mean ground speed of V knots, a mean overtake speed of ΔV knots (for same-direction aircraft), and a mean traffic separation of d NM, then in one hour an aircraft will encounter $2V/d$ other aircraft on an opposite-direction track and $\Delta V/d$ aircraft on a same-direction track.

Therefore, the hourly collision rate, C , for this scenario can be determined from Equation (1),

$$C(S_y, V, \Delta V, d, n_o, n_s) = \left(\frac{2V}{d} n_o + \frac{\Delta V}{d} n_s \right) \left[\frac{1}{\sqrt{2}} + 1 \right] e^{a_0 + a_1 S_y + a_2 S_y^2}. \quad (4)$$

For example, the collision rate assuming a track-to-track separation of 8 NM ($S_y = 8$), a mean ground speed (V) of 500 knots, 5 NM longitudinal separation ($d = 5$), and two

adjacent opposite-direction tracks ($n_s = 0, n_o = 2$, that is, the aircraft at risk is on an inner track with adjacent tracks flying in the opposite direction), is calculated as

$$C = \frac{2(500)}{5} (2 + 0) \left[\frac{1}{\sqrt{2}} + 1 \right] e^{a_0 + a_1 8 + a_2 8^2} = 1.6 \text{ E-}09.$$

Equation (4) forms the basis for the risk models in Scenarios 2 and 3.

Scenario 2 (Tracks with Turns Greater than 15°)

In this scenario, we address the first question asked in Section 1.2: What turn angles and turn densities (if any) would allow the target level of safety to be met at the 8 NM separation? The turns are nested (see Figure 1.1.1) with 8 NM track-to-track separation.

The specific TCV for this hazard is the combined lateral, longitudinal, and vertical conjunction of the two aircraft on parallel, adjacent tracks with at least one nested turn of greater than 15°. This conjunction is modeled by centers of gravity of the aircraft converging to within their mean wingspan laterally, within their mean lengths longitudinally, and within their mean heights vertically.

We model this scenario using these assumptions:

1. The divergence from the route is due to two factors: track definition and track holding error.
2. Track definition follows the turn transition area requirements of DO-236 Section 3.2.5.4 for transitions between legs.
3. Track holding error (a combination of flight technical error and navigational error) is modeled based on the lateral deviation model used to develop Equation (4) of Scenario 1.

The second assumption requires that transition track segments lie between the worst-case circular arc and linear legs which meet at the transition fix (see Figure 2.1.2). For this model, we include a fourth assumption:

4. If two aircraft are engaged in parallel route turns simultaneously, the outer route aircraft will attempt to follow the worst-case DO-236 radial track. (The worst case for this outer route would be the track farthest from the transition fix.) The inner route aircraft will attempt to follow the worst-case radial track. (The worst-case scenario for the inner route would be the track closest to the transition fix; see Figure 2.1.3.)

With these assumptions, we can model this scenario using these steps:

1. Determine the mean radial track-to-track distance between the two aircraft engaged in a parallel route turn.
2. Determine the amount of time the aircraft are engaged in the turn.
3. Use Equation (4) to determine the straight segment TCV probability based on the track-to-track distance, S_y (typically $S_y = 8$ NM).
4. Use Equation (4) to determine the turn segment TCV probability based on the mean radial track-to-track distance found in step 1.
5. Apportion the straight and turn segment TCV probabilities based on the relative times to fly each segment.

Figure 2.1.2 Transition Track Segment Boundary

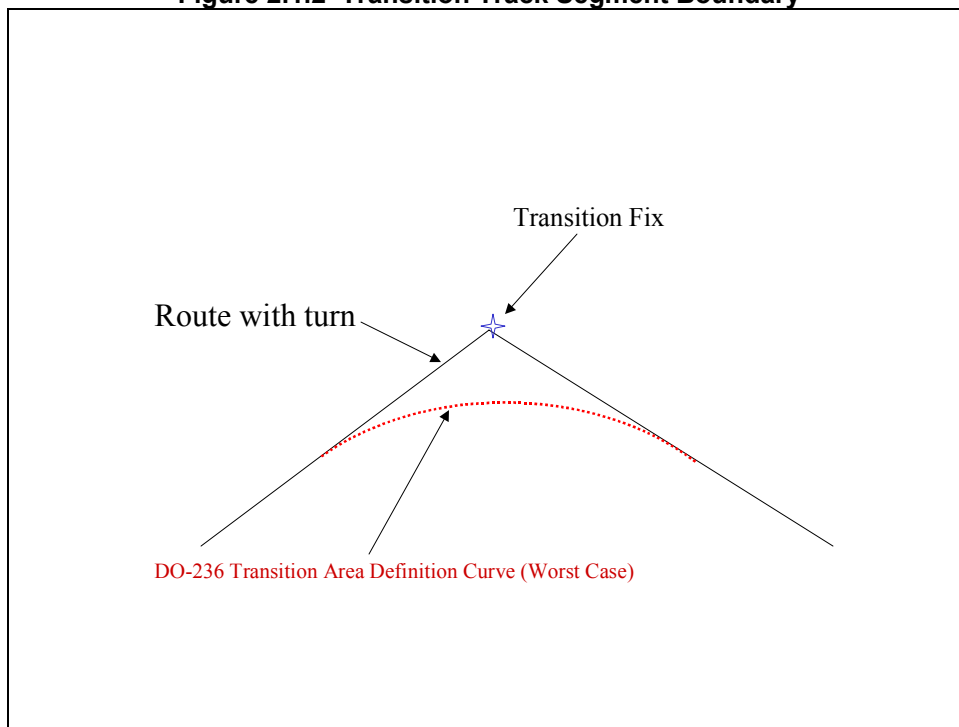
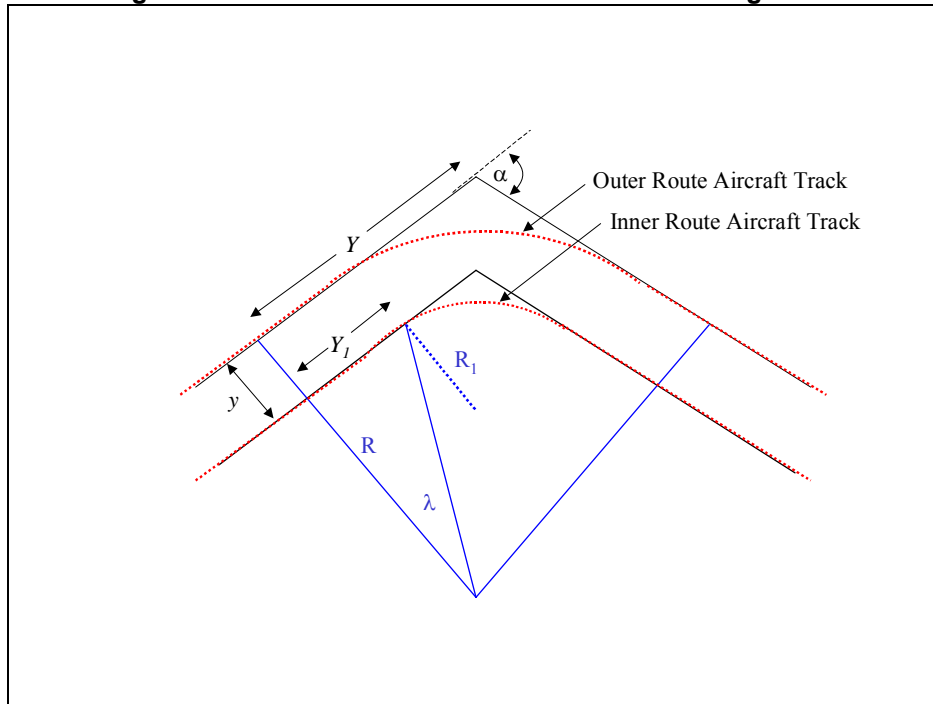


Figure 2.1.3 Outer and Inner Parallel Routes During Turns



The DO-236 transition area definition turn is specified in terms of the turn radius, R , and the turn initiation boundary distance, Y . (See Appendix B, Definitions of Turn Radius (R), Initiation Boundary Distance (Y), and Mean Radial Track-to-Track Distance, for DO-236 operational definitions of R and Y .) In the worst-case scenario, the outer route aircraft track is based on a high-speed aircraft with high tail wind (resulting in a total ground speed of 750 knots) and the inner route aircraft track is based on a low speed aircraft. We use that worst-case scenario as the basis for the calculation of turn segment TCV probability.

If the low speed aircraft is proceeding in the opposite direction of the high-speed aircraft on the outer track, the inner route aircraft track is based on a ground speed that is the difference between the aircraft's true airspeed (TAS) and the maximum wind speed. If we assume the low speed aircraft is flying at Mach 0.6 and flight level 250 and the wind speed is 120 knots, then the ground speed used is 243 knots⁷ (363 TAS minus 120).

On the other hand, if the low speed aircraft is proceeding in the same direction as the high-speed aircraft on the outer track, the inner route aircraft track is based on a ground speed that is the sum of the aircraft's airspeed and the maximum wind speed. If we (again) assume the low speed aircraft is flying at Mach 0.6 and flight level 250 and the wind speed is 120 knots, then the ground speed used is 483 knots (363 TAS plus 120).

⁷ This study assumes omnidirectional winds.

We assume that the outer track aircraft is following a turn track based on the maximum ground speed used in DO-236 of 750 knots and that the inner track aircraft is following a turn track based on a ground speed, V . These parameters together with the angular amount of turn, α , and the straight segment track-to-track distance, y , determine the mean radial track-to-track distance between the two aircraft engaged in a parallel route turn, $S_y = \bar{y}_{V,\alpha}$. The mean radial track-to-track distance between the two aircraft is the distance (normal to the outer track) between the outer route aircraft track and the inner route aircraft track shown in Figure 2.1.3. Appendix B shows the details of the derivation of $\bar{y}_{V,\alpha} =$

$$R - \frac{2}{\alpha} \left[h \sin(\alpha / 2 - \lambda) + (R - y) \ln |\sec \lambda + \tan \lambda| + \int_{\lambda}^{\alpha/2} \sqrt{R_1^2 - h^2 \sin^2(\alpha / 2 - \theta)} d\theta \right] \quad (5)$$

where, $h = \frac{R - y}{\cos(\alpha / 2)} - \frac{Y_1}{\sin(\alpha / 2)}$, Y_1 is the inner track turn initiation distance, R_1 is the inner track turn radius, R is the outer track turn radius, and λ is the angle determined by Y_1 and R_1 (see Figure 2.1.3.). Note that Y_1 and R_1 are based on the ground speed, V , assumed for the inner track aircraft and calculated according to DO-236.

Assuming a mean ground speed of V knots (for both aircraft taken together), the proportion of an hour that the aircraft is engaged in a turn is approximately $2Y/V$ hours. Therefore, using Equation (4), the collision rate, C , for this scenario (with one turn) is

$$C(S_y, V, \Delta V, d, n_o, n_s, \alpha) = (1 - 2Y/V) C(S_y, V, \Delta V, d, n_o, n_s) + (2Y/V) C(\bar{y}_{V,\alpha}, V, \Delta V, d, n_o, n_s) \quad (6)$$

where $P(TCV: r, y, d, n_o, n_s)$ is defined as in Equation (4).

Since there may be more than one turn during an hour of flight, we generalize Equation (6) to

$$C(S_y, V, \Delta V, d, n_o, n_s, n_\alpha, \alpha) = (1 - 2n_\alpha Y/V) C(S_y, V, \Delta V, d, n_o, n_s) + (2n_\alpha Y/V) C(\bar{y}_{V,\alpha}, V, \Delta V, d, n_o, n_s), \quad (7)$$

where $C(S_y, V, \Delta V, d, n_o, n_s)$ is defined as in Equation (4) and n_α is the number of turns of turn angle α per hour of flight.

In addition, since the angular amount of turn may vary from turn to turn, we generalize Equation (7) to address a set of turns of various angles, α_1 ,

$$C(S_y, V, \Delta V, d, n_o, n_s, n, \alpha_1, \alpha_2, \dots, \alpha_n) =$$

$$(1 - \sum_{i=1}^n [2Y_i/V])C(S_y, V, \Delta V, d, n_o, n_s) + (\sum_{i=1}^n [(2Y_i/V)(\bar{y}_{V,\alpha_i}, V, \Delta V, d, n_o, n_s)], \quad (8)$$

where n is the total number of turns per hour of flight ($n > 0$), Y_i is the turn initiation boundary distance for a turn with angle α_i , and \bar{y}_{V,α_i} is the mean radial track-to-track distance between the two aircraft engaged in a parallel route turn of angle α_i with the inner aircraft at ground speed V .

Equation (8) is the most general form of the risk model we develop for Scenario 2. It can be used to determine the estimated hourly collision rate for any configuration of track-to-track separation, S_y ; longitudinal separation, d ; number of opposite-direction tracks, n_o ; number of same-direction tracks, n_s ; mean ground speed, V ; mean overtake speed, ΔV ; and set of turn angles $\alpha_1, \alpha_2, \dots, \alpha_n$. For example, the hourly collision rate for a configuration of 8 NM track-to-track separation, 5 NM longitudinal separation, no opposite-direction tracks, one opposite-direction track, a mean ground speed of 500 knots, and turn angles of 30° and 45° would be 3.9 E-09 collisions per hour of flight.

2.2 Summary of Data Used

The data for this study are used to validate the risk models, but the models themselves are based on values specified in the documents AC 90-100 and DO-236. AC 90-100 specifies a value for track-keeping accuracy for RNAV aircraft. This criterion is the basis for the lateral deviation model used in the present study. DO-236 specifies worst-case boundaries for turns. These criteria are the bases for this study's turn path definition model.

A previous study [1] examined three sets of data to validate the track-keeping accuracy. We rely on this study as validation for the track-keeping model used for turns in the present study. For data to validate the turn path definition model used in the present study, we rely on data from the study [5] and on supplementary data collected for the present study.

Table 2.2.1 summarizes the turn boundary initiation distance (Y) for various aircraft and turn angles. The data for B747, A320, and MD11 aircraft are taken from the study [5] while the data for the B737 were collected from the B737-800 simulator managed by AFS-440 in Oklahoma City. For the B747, A320, and MD11 aircraft, the boundary initiation distances are estimated from the minimum observed leg lengths, which are generally twice the length of the boundary initiation distances. For the B737 aircraft, the boundary initiation distances are the actual observed mean distances from test runs with sample size $N = 6$ (60°) and $N = 8$ (90°). The wind speed, while constant, varied with the aircraft heading. The indicated airspeed was approximately 250 knots (k).

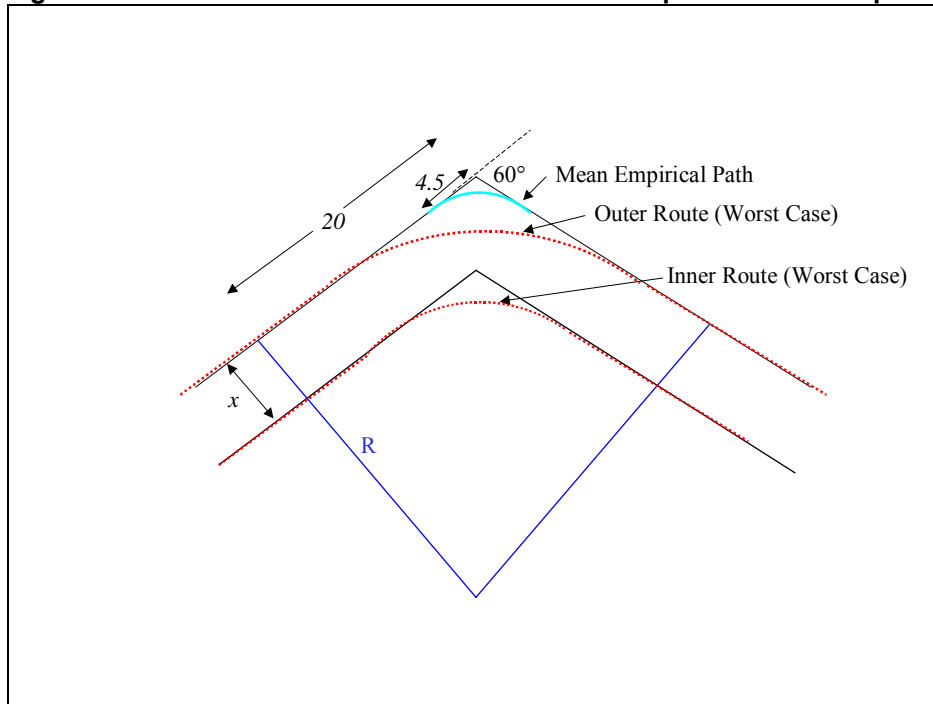
Table 2.2.1 Estimated Boundary Initiation Distance (Y)

Turn Angle (°)	Aircraft	Wind Speed (k)	Estimated Y (NM)	Worst-Case Y (NM) From DO-236
30	B747	60	3.0	20.0
30	A320	60	2.0	20.0
30	MD11	60	2.5	20.0
30	B737	60	2.3	20.0
30	B737	120	2.4	20.0
60	B747	60	4.5	20.0
60	A320	60	4.5	20.0
60	MD11	60	4.5	20.0
60	B737	60	4.6	20.0
60	B737	120	5.5	20.0
90	B747	60	8.0	20.0
90	A320	60	7.5	20.0
90	MD11	60	7.5	20.0
90	B737	60	7.4	20.0
90	B737	120	8.1	20.0

Figure 2.2.1 depicts the relationship between the DO-236 worst-case turn path definition ($Y = 20$ NM for a 60° turn) and the mean empirical turn path definition from Table 2.2.1 ($Y = 4.5$ NM for a 60° turn). These data show that for the aircraft listed the estimated turn initiation boundary distance, Y , is much less (for the reasonable wind speeds studied) than the worst-case value of Y^8 .

⁸ Additional data collected for the present study using Airbus A320 simulators and somewhat higher average wind speeds (100 knots average) show larger average boundary initiation distances (Y): for 30° $Y = 6.46$ NM, for 60° $Y = 12.32$ NM, and for 90° $Y = 16.91$ NM.

Figure 2.2.1 DO-236 Worst-Case Path and Mean Empirical Path Compared



The model proposed in the present study assumes the track-holding error for turns to be close to that of straight segments for the class of aircraft considered. This assumption is not contradicted by the study [5] nor by the B737 data collected for the present study. These data demonstrate that the mean lateral track-holding deviation for straight segments (60 knot wind speed) before and after turns is 0.01 NM with a standard deviation of 0.01. The mean lateral track-holding deviation for the turns is 0.02 NM with a standard deviation of 0.01 NM. For 120 knot wind speed, the straight segment mean lateral track-holding deviation before and after turns is 0.02 NM with a standard deviation of 0.02. The mean lateral track-holding deviation for the turns is 0.03 NM with a standard deviation of 0.02 NM.

This result is limited to the B737-800 simulator and may not be representative of aircraft with other FMSs and actual flights. However, the model contains other mitigations that tend to balance the assumption of same track-holding error for turns and straight segments. First, the length of time in a turn is typically somewhat less than the model assumes since the model calculates the aircraft's time in the turn assuming that it flies the two linear segments to and from the transition fix rather than the shorter radial arc. Second, the actual track taken by the aircraft at risk is typically farther from the other aircraft than the track assumed in the model since the model assumes the worst-case radial arc. This worst case can be much closer to the path of the second aircraft than the typical arc determined by an actual FMS (see Figure 2.2.1).

3.0 Summary of Data Analysis and Risk Evaluation

In this section, we determine the probability of the TCVs in each of the three scenarios, use those probabilities along with the hazard severities discussed in Section 2.1, Model Description, to define the risk for each hazard, and then compare those risks with standard acceptable levels of risk.

3.1 Summary of the TCV Probability Analysis

We examine the hourly collision rate probability analysis for each scenario beginning with Scenario 1. The results of the analysis for Scenario 1 can be used in the analyses for the other scenarios.

Scenario 1 Probability Analysis (Straight Segments)

This scenario generalizes the results of the previous study [1]. Using Equation (4) of Section 2.1, we can determine the (straight segment) hourly collision rate for any combination of track-to-track separation (S_y), longitudinal separation (d), number of adjacent opposite-direction tracks (n_o), and number of adjacent same-direction tracks (n_s).

For example, we can determine the hourly collision rate, C , for 7 NM track-to-track separation assuming 20 NM longitudinal separation, and one adjacent opposite-direction track. Using $S_y = 7.0$ NM, $V = 500$ knots, $d = 20$, $n_o = 1$, and $n_s = 0$, Equation (4) gives:

$$C(S_y, V, \Delta V, d, n_o, n_s) =$$

$$\left(\frac{2V}{d} n_o + \frac{\Delta V}{d} n_s \right) \left[\frac{1}{\sqrt{2}} + 1 \right] e^{a_0 + a_1 S_y + a_2 S_y^2} =$$

$$(50 + 0)[1.707] e^{a_0 + a_1 7 + a_2 7^2} = 5.8 \text{ E-09 collisions per hour.}$$

Also, given a desired probability (e.g., 5.0 E-09), Equation (4) can be solved for any of the parameters: S_y , V , ΔV , d , n_o , or n_s .

Scenario 2 Probability Analysis (Tracks with Turns Greater than 15°)

The TCV for this hazard is the collision with another aircraft that is flying at the same altitude in an opposite direction on a parallel RNAV track with at least one turn greater than 15°.

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The rate of collision for this scenario is determined from Equation (8) of Section 2.1. Since this equation allows any combination of track-to-track separation, longitudinal separation, number of adjacent opposite-direction tracks, number of adjacent same-direction tracks, number of turns, and angles of turn, it can be used to answer most questions about en route separation risk including the first question asked in Section 1.2: What turn angles and turn densities (if any) would allow the target level of safety to be met at the 8 NM separation?

We analyze this question by looking at several turn angles in turn (as it were): 15°, 20°, 30°, 45°, 60°, and 70°. In each case, we assume an 8 NM track-to-track separation, a 5 NM longitudinal separation, a mean ground speed of 500 knots, and a mean overtake speed of 100 knots⁹.

Table 3.1.1 demonstrates that for 15° turns, the target level of safety (5.0 E-09 fatal accidents per hour of flight) is met for all variations of adjacent track configuration (one or two, opposite-direction or same-direction tracks) for all the number of turns checked (0 through 5). This is as expected since the straight segment definition allows for turns as large as 15°.

Table 3.1.1 Collision Rates for Turns of 15°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	9.0E-10	9.8E-10	1.1E-09	1.1E-09	1.2E-09
0	2	1.6E-09	1.8E-09	2.0E-09	2.1E-09	2.3E-09	2.5E-09
1	0	8.2E-11	8.9E-11	9.6E-11	1.0E-10	1.1E-10	1.2E-10
2	0	1.6E-10	1.8E-10	1.9E-10	2.1E-10	2.2E-10	2.3E-10
1	1	9.0E-10	9.8E-10	1.1E-09	1.2E-09	1.2E-09	1.3E-09

*The notation E-10 denotes 10⁻⁹, that is, ten to the negative ninth power.

Table 3.1.1.Alt Collision Rates for Turns of 15° (Assuming 150 Knot Overtake Speed, ΔV)

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
1	0	1.2E-10	1.3E-10	1.4E-10	1.5E-10	1.7E-10	1.8E-10
2	0	2.4E-10	2.7E-10	2.9E-10	3.2E-10	3.3E-10	3.5E-10
1	1	9.4E-10	1.0E-09	1.1E-09	1.3E-09	1.3E-09	1.4E-09

⁹ A mean overtake speed of 100 knots is assumed to estimate the number of adjacent aircraft the target aircraft will encounter in an hour. Table 3.1.1.Alt provides a comparison to Table 3.1.1 but with an assumption of a mean overtake speed of 150 knots. The target levels of safety are met similarly in both.

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Table 3.1.2 demonstrates that for 20° turns as for 15° turns, the target level of safety is met for all variations of adjacent track configuration.

Table 3.1.2 Collision Rates for Turns of 20°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	1.0E-09	1.2E-09	1.5E-09	1.7E-09	1.9E-09
0	2	1.6E-09	2.1E-09	2.5E-09	2.9E-09	3.4E-09	3.8E-09
1	0	8.2E-11	1.0E-10	1.2E-10	1.4E-10	1.6E-10	1.8E-10
2	0	1.6E-10	2.0E-10	2.4E-10	2.8E-10	3.2E-10	3.6E-10
1	1	9.0E-10	1.1E-09	1.3E-09	1.6E-09	1.8E-09	2.0E-09

Table 3.1.3 demonstrates that for 30° turns as for 15° turns, the target level of safety is met for all variations of adjacent track configuration.

Table 3.1.3 Collision Rates for Turns of 30°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	1.6E-09	2.4E-09	3.2E-09	4.0E-09	4.8E-09
0	2	1.6E-09	3.2E-09	4.8E-09	6.4E-09	8.0E-09	9.6E-09
1	0	8.2E-11	1.5E-10	2.2E-10	3.0E-10	3.7E-10	4.4E-10
2	0	1.6E-10	3.1E-10	4.5E-10	5.9E-10	7.4E-10	8.8E-10
1	1	9.0E-10	1.7E-09	2.6E-09	3.4E-09	4.2E-09	5.0E-09

Table 3.1.4 demonstrates that for 45° turns, the target level of safety is met for all variations of adjacent track configuration through four turns per hour except for the configuration of two opposite-direction adjacent tracks when there are two or more turns per hour of flight.

Table 3.1.4 Collision Rates for Turns of 45°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	3.1E-09	5.4E-09	7.7E-09	9.9E-09	1.2E-08
0	2	1.6E-09	6.2E-09	1.1E-08	1.5E-08	2.0E-08	2.4E-08
1	0	8.2E-11	2.8E-10	4.8E-10	6.7E-10	8.7E-10	1.1E-09
2	0	1.6E-10	5.6E-10	9.5E-10	1.3E-09	1.7E-09	2.1E-09
1	1	9.0E-10	3.2E-09	5.6E-09	7.9E-09	1.0E-08	1.3E-08

Table 3.1.5 demonstrates that for 60° turns, the target level of safety is met for all variations of adjacent track configuration for one turn per hour of flight (except for two opposite-direction flights). However, the target level of safety is not met for some of the other configurations involving more than one turn per hour of flight.

Table 3.1.5 Collision Rates for Turns of 60°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	5.7E-09	1.1E-08	1.6E-08	2.1E-08	2.5E-08
0	2	1.6E-09	1.1E-08	2.1E-08	3.1E-08	4.1E-08	5.1E-08
1	0	8.2E-11	4.2E-10	7.6E-10	1.1E-09	1.4E-09	1.8E-09
2	0	1.6E-10	8.4E-10	1.5E-09	2.2E-09	2.9E-09	3.6E-09
1	1	9.0E-10	5.4E-09	9.9E-09	1.4E-08	1.9E-08	2.3E-08

Table 3.1.6 demonstrates that for 70° turns as with 60° turns, the target level of safety is met for all variations of adjacent track configuration for one turn per hour of flight (except for two opposite-direction flights). However, the target level of safety is not met for some of the other configurations involving more than one turn per hour of flight.

Table 3.1.6 Collision Rates for Turns of 70°

Number of Adjacent Tracks		Number of Turns					
Same Direction	Opposite Direction	0	1	2	3	4	5
0	1	8.2E-10	7.7E-09	1.5E-08	2.2E-08	2.8E-08	3.5E-08
0	2	1.6E-09	1.5E-08	2.9E-08	4.3E-08	5.7E-08	7.1E-08
1	0	8.2E-11	4.4E-10	8.0E-10	1.2E-09	1.5E-09	1.9E-09
2	0	1.6E-10	8.8E-10	1.6E-09	2.3E-09	3.0E-09	3.7E-09
1	1	9.0E-10	6.4E-09	1.2E-08	1.7E-08	2.3E-08	2.8E-08

3.2 Summary of Acceptable Level of Risk

As in the previous study [1], the acceptable level of risk is a collision rate of 5.0 E-09 or less per hour of flight.

However, the FAA Safety Management System Manual, v 1.1 sets the probability of a catastrophic level event at 1.0 E-09 or less. Both levels of safety are included in this study for comparison.

4.0 Results and Conclusions

This section summarizes the key results, the scenario risk evaluation, and the conclusions of the study.

This study develops a risk model, Equation (8), that can be used to determine hourly collision rates for both scenarios described, including Scenario 1, which generalizes the results of the previous study [1]. The results are consistent with those of the previous study and its addendum. That is, the hourly collision rates calculated by means of Equation (8) for straight segments with turns less than 15° are the same as those of the previous study.

While the previous study proposed a track-to-track separation of 8 NM, the risk model developed in the present study allows for any track-to-track separation and longitudinal separation assumptions. The results below, however, focus on 8 NM track-to-track separation and 5 NM longitudinal separation to facilitate comparison with the previous study.

The results are expressed in terms of number of turns per hour of flight. They could also be expressed in terms of number of turns per distance, say, 500 NM by using a reasonable average ground speed assumption.

4.1 Results Assuming Radar Surveillance and 5 NM Longitudinal Separation

The results below assume 5 NM longitudinal separation and radar surveillance. The values in bold face are those that meet the more stringent 1.0 E-09 collisions per flight hour target level of safety.

One Turn per Hour of Flight (Radar Surveillance)

Table 4.1.1 demonstrates that for one turn per hour of flight under radar surveillance and assuming 5 NM longitudinal separation, all angles of turn and configurations meet the target level of safety.

Table 4.1.1 Collision Rates for One Turn (Radar Surveillance)

Number of Adjacent Tracks		Turn Angle					
Same Direction	Opposite Direction	15°	20°	30°	45°	60°	70°
0	1	9.0E-10	1.0E-09	1.6E-09	3.1E-09	5.7E-09	7.7E-09
0	2	1.8E-09	2.1E-09	3.2E-09	6.2E-09	1.1E-08	1.5E-08
1	0	8.9E-11	1.0E-10	1.5E-10	2.8E-10	4.2E-10	4.4E-10
2	0	1.8E-10	2.0E-10	3.1E-10	5.6E-10	8.4E-10	8.8E-10
1	1	9.8E-10	1.1E-09	1.7E-09	3.2E-09	5.4E-09	6.4E-09

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Two Turns per Hour of Flight (Radar Surveillance)

Table 4.1.2 demonstrates that for two turns per hour of flight under radar surveillance and assuming 5 NM longitudinal separation, all angles of turn and configurations meet the target level of safety.

Table 4.1.2 Collision Rates for Two Turns (Radar Surveillance)

Number of Adjacent Tracks		Turn Angle					
Same Direction	Opposite Direction	15°	20°	30°	45°	60°	70°
0	1	9.8E-10	1.2E-09	2.4E-09	5.4E-09	1.1E-08	1.5E-08
0	2	2.0E-09	2.5E-09	4.8E-09	1.1E-08	2.1E-08	2.9E-08
1	0	9.6E-11	1.2E-10	2.2E-10	4.8E-10	7.6E-10	8.0E-10
2	0	1.9E-10	2.4E-10	4.5E-10	9.5E-10	1.5E-09	1.6E-09
1	1	1.1E-09	1.3E-09	2.6E-09	5.6E-09	9.9E-09	1.2E-08

Three or More Turns per Hour of Flight (Radar Surveillance)

Tables 4.1.3, 4.1.4, and 4.1.5 demonstrate that for three, four, or five turns per hour of flight under radar surveillance and assuming 5 NM longitudinal separation, all angles of 30° or less of turn and configurations meet the target level of safety.

Table 4.1.3 Collision Rates for Three Turns (Radar Surveillance)

Number of Adjacent Tracks		Turn Angle					
Same Direction	Opposite Direction	15°	20°	30°	45°	60°	70°
0	1	1.1E-09	1.5E-09	3.2E-09	7.7E-09	1.6E-08	2.2E-08
0	2	2.1E-09	2.9E-09	6.4E-09	1.5E-08	3.1E-08	4.3E-08
1	0	1.0E-10	1.4E-10	3.0E-10	6.7E-10	1.1E-09	1.2E-09
2	0	2.1E-10	2.8E-10	5.9E-10	1.3E-09	2.2E-09	2.3E-09
1	1	1.2E-09	1.6E-09	3.4E-09	7.9E-09	1.4E-08	1.7E-08

Table 4.1.4 Collision Rates for Four Turns (Radar Surveillance)

Number of Adjacent Tracks		Turn Angle					
Same Direction	Opposite Direction	15°	20°	30°	45°	60°	70°
0	1	1.1E-09	1.7E-09	4.0E-09	9.9E-09	2.1E-08	2.8E-08
0	2	2.3E-09	3.4E-09	8.0E-09	2.0E-08	4.1E-08	5.7E-08
1	0	1.1E-10	1.6E-10	3.7E-10	8.7E-10	1.4E-09	1.5E-09
2	0	2.2E-10	3.2E-10	7.4E-10	1.7E-09	2.9E-09	3.0E-09
1	1	1.2E-09	1.8E-09	4.2E-09	1.0E-08	1.9E-08	2.3E-08

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Table 4.1.5 Collision Rates for Five Turns (Radar Surveillance)

Number of Adjacent Tracks		Turn Angle					
Same Direction	Opposite Direction	15°	20°	30°	45°	60°	70°
0	1	1.2E-09	1.9E-09	4.8E-09	1.2E-08	2.5E-08	3.5E-08
0	2	2.5E-09	3.8E-09	9.6E-09	2.4E-08	5.1E-08	7.1E-08
1	0	1.2E-10	1.8E-10	4.4E-10	1.1E-09	1.8E-09	1.9E-09
2	0	2.3E-10	3.6E-10	8.8E-10	2.1E-09	3.6E-09	3.7E-09
1	1	1.3E-09	2.0E-09	5.0E-09	1.3E-08	2.3E-08	2.8E-08

Appendix A: Statistical Distributions Used in the Study

The Mixed Johnson S_B and Double-Exponential Distribution Probability Density Function (PDF)

$$f(x) = \begin{cases} \frac{\alpha \exp(-|x/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \text{Ln}\left[\left(\frac{x-\varepsilon}{-x+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(x-\varepsilon)(-x+\varepsilon+\lambda)}, & x < \varepsilon + \lambda \text{ \& \& } x > \varepsilon \\ \frac{\alpha \exp(-|x/\delta|)}{2\delta}, & x \leq \varepsilon \text{ \& } x \geq \varepsilon + \lambda \end{cases} \quad (\text{A3})$$

The Convolution of Variables y_2 and y_1 and the Probability of $|y_2 - y_1| < W$

The PDF describing $(y_2 - y_1)$ is the convolution of the two PDFs of the two variables, y_2 and $-y_1$. The convolution of two variables y_2 and $+y_1$ is defined as the integral

$$f(u) = \int_{-\infty}^{\infty} f_1(y_1)f_2(u - y_1)dy_1, \quad (2)$$

where $u = y_1 + y_2$. If f_1 and f_2 are PDFs of y_1 and y_2 , then f is the PDF of $u = y_1 + y_2$.

Also, if the PDF of y_1 is symmetric about zero, then the convolution of y_2 and y_1 is equivalent to the convolution of y_2 and $-y_1$. Therefore, f is also the PDF of $u = y_2 - y_1$.

This means that the probability of $|y_2 - y_1| < W$ is the integral of f between $-W$ and W . That is,

$$P(|y_2 - y_1| < W) = \int_{-W}^W f(x)dx. \quad (3)$$

But $f(x)$ is defined in Equation (2), where f_1 and f_2 are both Johnson S_U PDFs defined in Equation (1). Therefore,

$$P(|y_2 - y_1| < W) = \int_{-W}^W \int_{-\infty}^{\infty} f_1(y_1)f_2(x - y_1)dy_1dx, \quad (4)$$

where f_1 and f_2 are defined in Equation (1).

Substituting the definitions of f_1 and f_2 , and letting¹⁰ $\varepsilon = 0$ in f_1 and $\varepsilon = S$ in f_2 , Equation (4) becomes

$$P(|y_2 - y_1| < W) = \int_{-W}^W \int_{-\infty}^{\infty} f_1(y_1) f_2(x - y_1) dy_1 dx \quad (5)$$

Where,

$$f_1(y) = \begin{cases} \frac{\alpha \exp(-|y/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \text{Ln}\left[\left(\frac{y-\varepsilon}{-y+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(y-\varepsilon)(-y+\varepsilon+\lambda)}, & y < \varepsilon + \lambda \ \& \ y > \varepsilon \\ \frac{\alpha \exp(-|y/\delta|)}{2\delta}, & y \leq \varepsilon \ | \ y \geq \varepsilon + \lambda \end{cases}$$

And,

$$f_2(x-y) = \begin{cases} \frac{\alpha \exp(-|(x-y-8)/\delta|)}{2\delta} + \frac{(1-\alpha)\eta\lambda \exp\left(-0.5\eta^2 \text{Ln}\left[\left(\frac{(x-y-8)-\varepsilon}{-(x-y-8)+\varepsilon+\lambda}\right)^2\right]\right)}{\sqrt{2\pi}(y-\varepsilon)(-y+\varepsilon+\lambda)}, & (x-y-8) < \varepsilon + \lambda \ \& \ (x-y-8) > \varepsilon \\ \frac{\alpha \exp(-|(x-y-8)/\delta|)}{2\delta}, & (x-y-8) \leq \varepsilon \ | \ (x-y-8) \geq \varepsilon + \lambda \end{cases}$$

¹⁰ Since ε is the location parameter, the PDF for y_1 uses $\varepsilon = 0$ and the PDF for y_2 uses $\varepsilon = S$, where the two tracks are S NM apart.

Appendix B: Definitions of Turn Radius (R), Initiation Boundary Distance (Y), and Mean Radial Track-to-Track Distance (\bar{y}_α)

From DO-236, Section 3.2.5.4

$$\text{Radius of Turn } R = (V + W)^2 * (\tan(\Phi))^{-1} * 1.458(10^{-5}) \text{ NM}$$

$$\text{Turn Initiation Boundary Distance } Y = R * \tan(0.5\alpha),$$

where:

- $V + W$ is the maximum ground speed assumed for the transition (750 knots for high altitude transitions).
- α is the track change, in degrees.
- Φ is the maximum aircraft bank angle (5° for high altitude transitions).
- If the 5° value for Φ results in $Y > 20$ NM, then set $Y = 20$ and $R = 20/\tan(0.5 * \alpha)$.

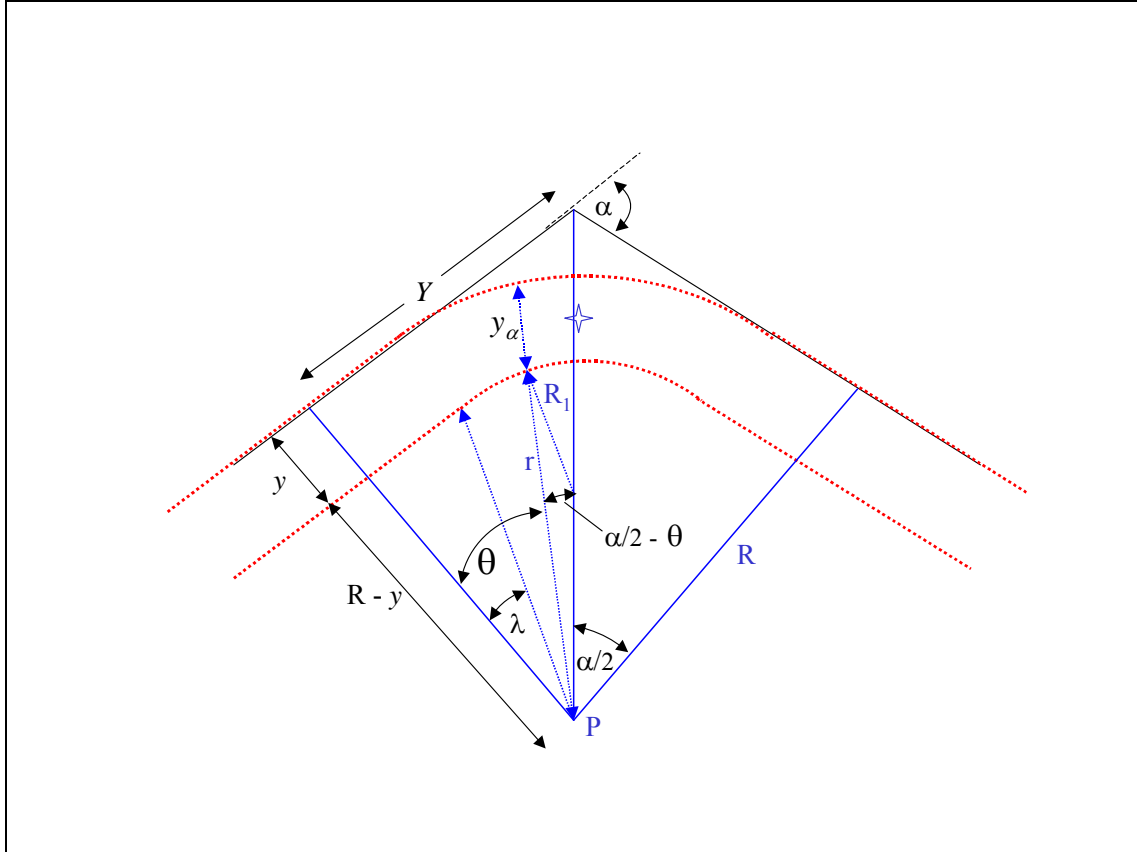
Mean Radial Track-to-Track Distance, \bar{y}_α Derivation

We are interested in the mean distance, y_α , between the radial arc track and the linear angular track in Figure B.1 as the angle θ increases from 0 to $\alpha/2$. The method for finding the average value of such a polar function is to integrate the function from the lower angle to the upper angle and then divide by the angular distance (in radians).

So, if $f(\theta) = y_\alpha$, then the mean value of y_α (\bar{y}_α) between 0 and $\alpha/2$ is:

$$\bar{y}_\alpha = \frac{2}{\alpha} \int_0^{\alpha/2} f(\theta) d\theta .$$

Figure B.1



To represent y_α as a function of angle θ , that is, to find $f(\theta)$, notice that in Figure B.1 $\cos(\theta) = (R - y)/(R - y_\alpha)$. Therefore, for $0 < \theta \leq \lambda$, $y_\alpha = f(\theta) = R - (R - y)/\cos(\theta)$.

But for $\lambda < \theta \leq \alpha/2$, $y_\alpha = f(\theta) = R - r$, where r is the distance from the radial center point, P , to the inner turn circular arc. This inner turn arc is defined by the radius R_1 and the inner turn initiation boundary distance, Y_1 . (See Figure B.2.)

It can be shown that $r = h \cos(\alpha/2 - \theta) + \sqrt{R_1^2 - h^2 \sin^2(\alpha/2 - \theta)}$,

where $h = \frac{R - y}{\cos(\alpha/2)} - \frac{Y_1}{\sin(\alpha/2)}$.

So that

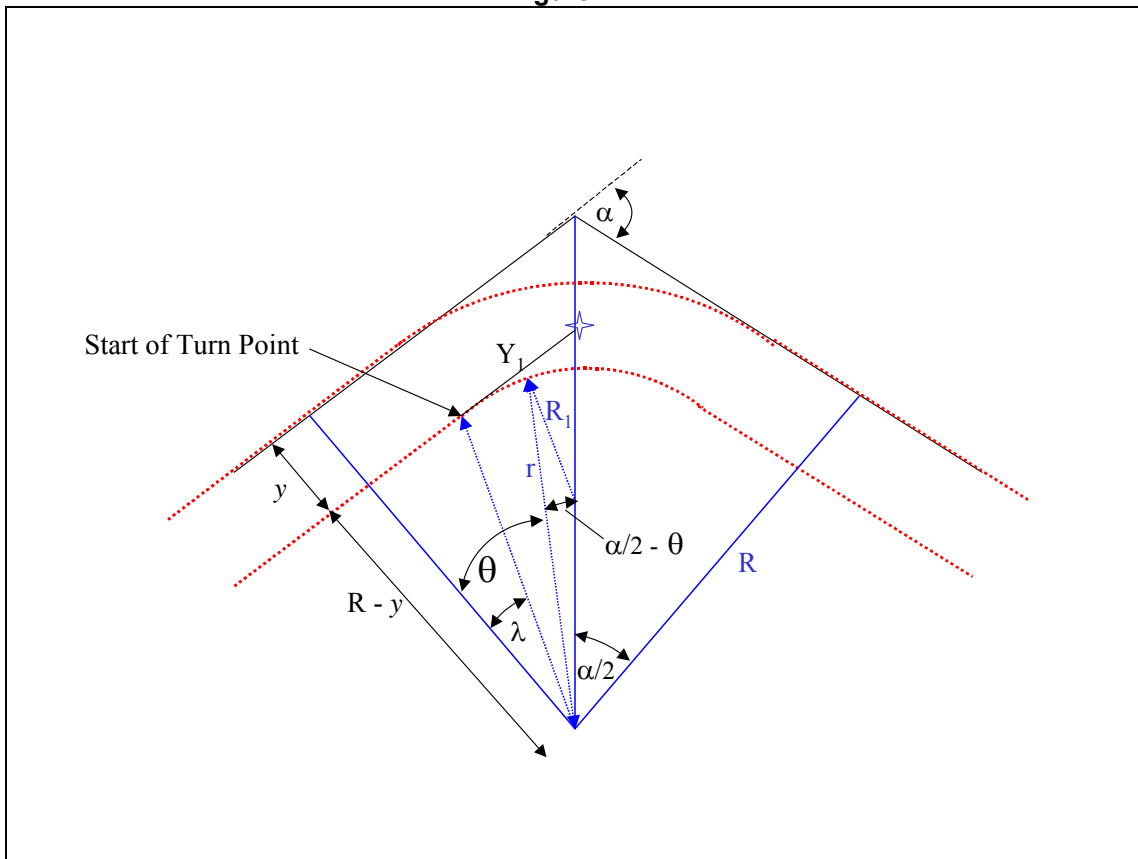
$$\bar{y}_\alpha = \frac{2}{\alpha} \int_0^{\alpha/2} f(\theta) d\theta = \frac{2}{\alpha} \int_0^\lambda \left[R - \frac{R - y}{\cos(\theta)} \right] d\theta + \frac{2}{\alpha} \int_\lambda^{\alpha/2} [R - r] d\theta .$$

Therefore, $\bar{y}_\alpha =$

$$R - \frac{2}{\alpha} \left[h \sin(\alpha / 2 - \lambda) + (R - y) \ln |\sec \lambda + \tan \lambda| + \int_{\lambda}^{\alpha / 2} \sqrt{R_1^2 - h^2 \sin^2(\alpha / 2 - \theta)} d\theta \right].$$

The integral in the expression above is an elliptic integral and therefore not expressible in closed form. Its value must be determined from numerical techniques.

Figure B.2



References

- [1] “Analysis of Area Navigation (RNAV) En Route Separation Along Adjacent Straight Segments.”

AFS-400 Safety Study Report number DOT-FAA-AFS-440-17

This paper was published by the FAA’s Flight Technologies and Procedures Division (AFS-400) in December 2005. The paper’s analysis was confined to RNAV en route segments with no turns greater than 15° (and therefore assumed to be straight).

- [2] “Preliminary Re-evaluation of the Probability of Lateral Overlap, $P_y(0)$, based on non-Global Positioning Satellite (GPS) and GPS-Equipped Aircraft Performance at Entry into North Atlantic Reduced Vertical Separation Minimum Airspace.”

North Atlantic Mathematicians’ Implementation Group
NAT MIG/5-WP/18
Atlantic City, April 1999

This paper was published by the North Atlantic Mathematicians’ Implementation Group as NAT MIG/5-WP/18 in April 1999. The paper’s analysis was based on data collected in 1995 from 11 aircraft flights by five operators with three aircraft types (B747-200, B747-400, A340). Each aircraft was using GPS navigation on an oceanic route—five flights were North Atlantic routes, five were Pacific Oceanic airspace, and one was a South Atlantic route.

- [3] “Estimating the Well-Fit Model for the Distribution of Cross Track Deviations of GPS-Equipped Aircraft on a North Pacific Route.”

Separation and Airspace Safety Panel (SASP)
SASP-WG/WHL/4-WP/23
Honolulu, United States, 10-21 November 2003

This paper was published by the Separation and Airspace Safety Panel (SASP) as SASP-WG/WHL/4-WP/23 in November 2003. The paper’s analysis was based on data collected between December 2001 and 23 May 2002, from 3,150 flights on the North Pacific route R220. Each aircraft, types B747-400, B777, A340, was using GPS navigation on the route.

- [4] “Analysis of Lateral Track Deviation along Two Q-Routes.”

AFS-400 Safety Study

This paper was published by the FAA’s Flight Technologies and Procedures Division (AFS-400) in October 2005. The paper’s analysis was based on data collected between February 19 and March 6, 2003, from 865 flights on Q-routes 100 and 102 in the Gulf of Mexico. Each aircraft was using some type of RNAV navigation, typically GPS or Distance Measuring Equipment/Distance Measuring Equipment (DME/DME) on the route.

Analysis of RNAV En Route Separation Along Adjacent Segments with Radar Surveillance and Turns (Phase II)

DOT-FAA-AFS-440-25

January 2007

[5] “Honeywell Flight Management System (FMS) Lateral Navigation (LNAV) Investigation.”

FAA Safety Study number DOT-FAA-AVN-200-67

This paper was published by the FAA’s Flight Standards Development Branch in January 1993. The paper analyzed the ability of Honeywell FMS on various aircraft to fly fly-by and fly-over waypoints.