



**Federal Aviation  
Administration**

# **Geometrical Models for Aircraft in Terminal Area Risk Analyses**

**Flight Systems Laboratory  
DOT-FAA-AFS-450-63**

**April 2011**

**Flight Systems Laboratory  
6500 S. MacArthur Blvd.  
Systems Training Building Annex, RM 217  
Oklahoma City, Oklahoma 73169  
Phone: (405) 954-8191**

### **NOTICE**

This document is disseminated under the sponsorship of the U.S. Department of Transportation in the interest of information exchange. The United States Government assumes no liability for the contents or use thereof.

The United States Government does not endorse products or manufacturers. Trade or manufacturers' names appear herein solely because they are considered essential to the objective of this report.

**DOT-FAA-AFS-450-63**

Flight Systems Laboratory  
Flight Technologies and Procedures Division  
Flight Standards Service

**GEOMETRICAL MODELS FOR AIRCRAFT IN TERMINAL AREA RISK  
ANALYSES**

**Reviewed by:**

  
Date 27 April 2011  
Harry Hodges  
Manager, Flight Systems Laboratory, AFS-450

**Released by:**

  
Date 2 May 2011  
Leslie Smith  
Manager, Flight Technologies and  
Procedures Division, AFS-400

**April 2011**

**Technical Report**

# Geometrical Models for Aircraft in Terminal Area Risk Analyses

**DOT-FAA-AFS-450-63**

**April 2011**

Technical Report Documentation Page

|  |                                    |   |
|--|------------------------------------|---|
| <b>1. Report No.</b><br>DOT-FAA-AFS-450-63   | <b>2. Government Accession No.</b> | <b>3. Recipient's Catalog No.</b>                               |
| <b>4. Title and Subtitle</b><br>Geometric Models for Aircraft in Terminal Area Risk Analyses   |                                    | <b>5. Report Date</b><br>April 2011                             |
| <b>6. Author(s)</b><br>Dr. Richard Greenhaw, AFS-450   |                                    | <b>7. Performing Organization Code</b>                          |
| <b>8. Performing Organization Name and Address</b><br>Federal Aviation Administration<br>Flight Systems Laboratory, AFS-450<br>P.O. Box 25082, Oklahoma City, OK 73125   |                                    | <b>9. Type of Report and Period Covered</b><br>Technical report |
| <b>10. Sponsoring Agency Name and Address</b><br>Federal Aviation Administration<br>Flight Systems Laboratory, AFS-450<br>P.O. Box 25082, Oklahoma City, OK 73125  |                                    |   |
| <b>11. Supplementary Notes</b>   |                                    |   |
| <b>12. Abstract</b><br><p>The purpose of this study is to examine the relationship between shapes and sizes of various volume regions such as spheres, cylinders, and boxes used to model aircraft in terminal area safety studies.</p> <p>Various geometrical shapes and sizes have been used to model aircraft in en route and terminal areas for collision risk analyses. And the risk analysis results can be quite sensitive to the specific shapes and sizes used. This study will describe the effects of using various of these shapes and sizes.</p> <p>The study will also develop more general hazard likelihood regions about aircraft for purposes of safety management risk assessment. For example, answering such questions as "what radius sphere would correspond to a 10<sup>-7</sup> extremely remote likelihood region?"</p> <p>For parallel approach studies, we recommend using for the TCV region a cylinder based on the wing semi-spans and tail heights of the aircraft pair. The radius of this cylinder will be the sum of the wing semi-spans and its height will be the sum of the tail heights.</p> <p>Since this cylinder is dependent on knowing the aircraft mix in the operations studies, we add a recommendation for situations in which this mix is unknown or cannot otherwise be used. We recommend a cylinder of radius 265 feet and height 160 feet. The dimensions of this cylinder are conservative in that they will accommodate two Airbus A380 aircraft laterally or vertically.</p> |                                    |   |
| <b>13. Key Words</b><br>Models<br>Geometry<br>Terminal Area<br>500 Foot Sphere<br>Cylinder   |                                    | <b>14. Distribution Statement</b>                               |
| <b>15. Security Classification of This Report</b>  |                                    | <b>16. Security Classification of This Page</b>                 |

## Executive Summary

The purpose of this study is to examine the relationship between shapes and sizes of various volume regions such as spheres, cylinders, and boxes used to model aircraft in terminal area safety studies.

Various geometrical shapes and sizes have been used to model aircraft in en route and terminal areas for collision risk analyses. And the risk analysis results can be quite sensitive to the specific shapes and sizes used. This study will describe the effects of using various of these shapes and sizes.

The study will also develop more general hazard likelihood regions about aircraft for purposes of safety management risk assessment. For example, answering such questions as “what radius sphere would correspond to a  $10^{-7}$  extremely remote likelihood region?”

For parallel approach studies, we recommend using for the TCV region a cylinder based on the wing semi-spans and tail heights of the aircraft pair. The radius of this cylinder will be the sum of the wing semi-spans and its height will be the sum of the tail heights.

Since this cylinder is dependent on knowing the aircraft mix in the operations studies, we add a recommendation for situations in which this mix is unknown or cannot otherwise be used. We recommend a cylinder of radius 265 feet and height 160 feet. The dimensions of this cylinder are conservative in that they will accommodate two Airbus A380 aircraft laterally or vertically.

Table of Contents

**1.0 Introduction: Models for Terminal Area Collision Risk Analysis..... 1**  
**2.0 Shapes to Model Mid Air Collisions .....2**  
**3.0 TLS Regions ..... 19**  
**References .....25**

Table of Figures and Tables

Figure 1 B737-400 in 500 Foot Radius Sphere to Scale..... 1  
 Table 1 Four Aircraft Types Used in CSPO Data Sets..... 2  
 Table 2 CSPO Data Set Tags..... 2  
 Figure 2 B737-400 in 140.6 Foot Radius Sphere to Scale..... 3  
 Figure 3 B737-400 in Small Sphere in Large Sphere..... 4  
 Table 3 CSPO Data Set TCVs for Large Sphere and Small Sphere..... 4  
 Figure 4 Large (500 ft, blue) and Small (140.6 ft, red) Sphere TCVs..... 6  
 Figure 5 B737-400 in 140.6 Foot Radius (Small) Cylinder to Scale..... 7  
 Figure 6 B737-400 in Small Cylinder in Large Sphere..... 8  
 Table 4 CSPO Data Set TCVs for Large Sphere and Small Cylinder..... 8  
 Figure 7 Large Sphere (blue) and Small Cylinder (red) TCVs..... 9  
 Figure 8 B737-400 in 265 Foot Radius (Large) Cylinder to Scale..... 10  
 Figure 9 B737-400 in Large Cylinder in Large Sphere..... 11  
 Table 5 CSPO Data Set TCVs for Large Sphere and Large Cylinder..... 11  
 Figure 10 Large Sphere (blue) and Large Cylinder (red) TCVs..... 12  
 Figure 8 B737-400 in 302.5 by 281.2 by 88.0 Box to Scale..... 13  
 Figure 9 B737-400 in Box in Large Sphere..... 14  
 Table 6 CSPO Data Set TCVs for Large Sphere and Box..... 14  
 Figure 10 Large Sphere (blue) and Box (red) TCVs..... 16  
 Table 7 TCV Summary by Shape..... 17  
 Figure 11 B737-400, Small Cylinder, Box, Small Sphere, Large Cylinder, and Large Sphere..... 18  
 Figure 12 Rayleigh Distribution,  $R(x; 5600)$ ..... 19  
 Table 8 Overall TCV Rates and Sphere Radii ( $p = 1.0 \text{ E-}09$ )..... 21  
 Table 9 Theoretical (Rayleigh) to Empirical (CSPO Data) Comparison..... 22  
 Table 10 Overall TCV Rates and Sphere Radii ( $p = 5.0 \text{ E-}09$ )..... 24  
 Table 11 Overall TCV Rates and Sphere Radii ( $p = 1.0 \text{ E-}08$ )..... 24  
 Table 12 Overall TCV Rates and Sphere Radii ( $p = 4.0 \text{ E-}08$ )..... 24

## 1.0 Introduction: Models for Terminal Area Collision Risk Analysis

Various geometrical shapes and sizes have been used to model aircraft in en route and terminal areas for collision risk analyses. The risk analysis results can be quite sensitive to the specific shapes and sizes used.

For example, parallel approach studies performed by the Flight Systems Laboratory (e.g., [1]) have used a 500 foot radius sphere to model a region about the target aircraft. Penetration of this region has been typically referred to as a Test Criteria Violation (TCV). In such studies the blundering aircraft is represented by a point in three space and the target (or evading) aircraft is represented by the 500 foot radius sphere. A penetration of the sphere by the point represents a TCV which models a collision.

However, the centers of gravity of two commercial aircraft might easily be found to be less 500 feet apart without a collision occurring. The 500 foot radius sphere was, in fact, chosen as a very conservative TCV model (see Figure 1). And it is possible to chose other, more precise models especially for use with smaller target levels of safety (TLS).

**Figure 1 B737-400 in 500 Foot Radius Sphere to Scale**



**2.0 Shapes to Model Mid Air Collisions**

We will examine three shapes for modeling aircraft in collision risk analyses: a sphere (small sphere), a flattened cylinder (small cylinder or hockey puck), a large cylinder, and a rectangular prism (box). For each shape we will estimate the collision risk relative to a 500 foot diameter sphere.

Small Sphere

This is a sphere of radius 140.6 feet. The radius value was calculated based on data from a Closely Spaced Parallel Operations (CSPO) study [2]. Four data sets from that study provide closest points of approach (CPAs) for pairs of blundering / evading aircraft. There are four types of aircraft used: B747-400, A330, B737-800, and an ERJ. Table 1 gives the dimensions assumed for each aircraft type.

**Table 1 Four Aircraft Types Used in CSPO Data Sets**

| Type     | Length (ft) | Wingspan (ft) | Tail Height (ft) |
|----------|-------------|---------------|------------------|
| B747-400 | 231         | 212           | 64               |
| A330     | 193         | 198           | 56               |
| B737-800 | 120         | 118           | 41               |
| ERJ      | 93          | 66            | 22               |

The four data sets represent a combination of two blunder angles and two approach track distances: blunders of 20 and 30 degrees and track distances of 3380 and 3600 feet. Table 2 shows the tags used to name each data set, their blunder angles. The track spacing used, and the number of runs (sample size) used to generate the TCV count in the Monte Carlo simulation used in the study.

**Table 2 CSPO Data Set Tags**

| Tag | Blunder Angle (degrees) | Track Spacing (ft) | Number of Runs |
|-----|-------------------------|--------------------|----------------|
| N20 | 20                      | 3380               | 188,078        |
| N30 | 30                      | 3380               | 188,078        |
| W20 | 20                      | 3600               | 188,149        |
| W30 | 30                      | 3600               | 188,149        |

Figure 2 shows, to scale, a B737-400 target aircraft inside this small sphere. In these models a collision is assumed to occur when the center of gravity of the blundering aircraft penetrates the sphere containing the target aircraft. Figure 3 shows the B737-400 inside the small sphere with both inside the large, 500-foot radius sphere.

Figure 2 B737-400 in 140.6 Foot Radius Sphere to Scale

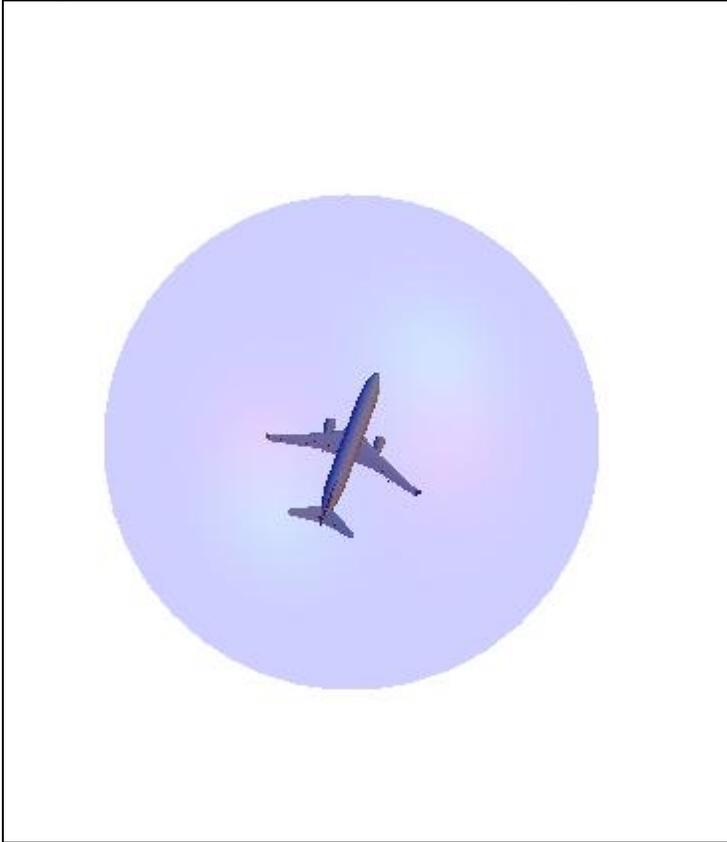


Figure 3 B737-400 in Small Sphere in Large Sphere

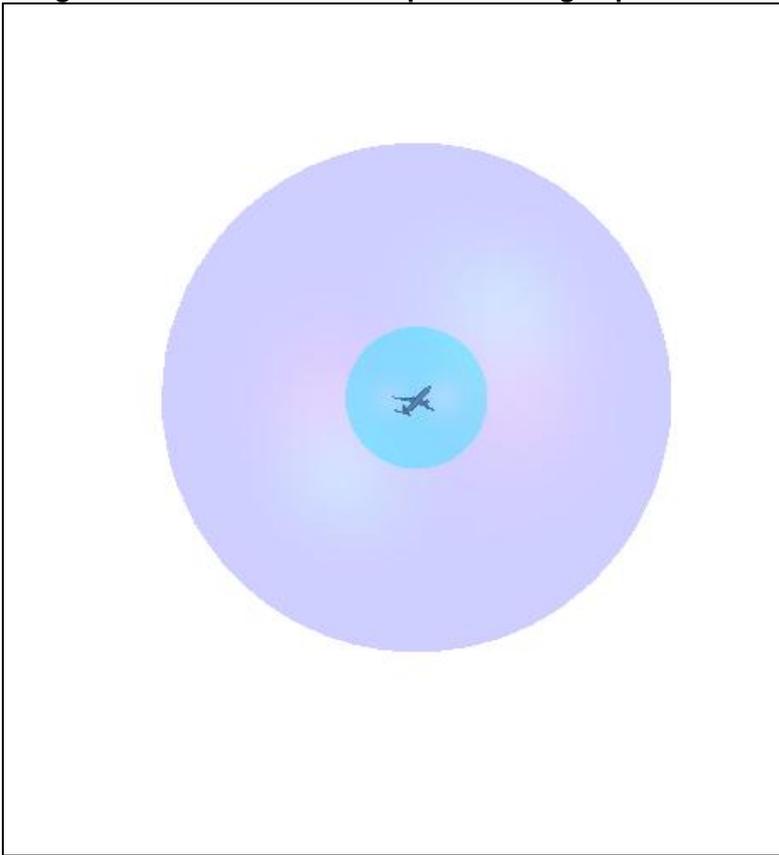


Table 3 gives the number of TCVs counted in the study for each data set. In each case a TCV occurs when the CPA for the two aircraft is less than 500 feet. Table 3 also gives the number of TCVs that would have been counted had the small sphere (of average radius 140.6 feet<sup>1</sup>) been used rather than one of 500 feet. And Figure 3 provides a 3-dimensional scatter graph of the 500 foot and 140.6 foot TCVs to show their geometrical relationship.

**Table 3 CSPO Data Set TCVs for Large Sphere and Small Sphere**

| Data Set | Number of Runs | 500 ft Sphere TCV Count | 500 ft Sphere TCV Percent | 140.6 ft Sphere TCV Count | 140.6 ft Sphere TCV Percent |
|----------|----------------|-------------------------|---------------------------|---------------------------|-----------------------------|
| N20      | 188,078        | 278                     | 0.148%                    | 25                        | 0.013%                      |
| N30      | 188,078        | 1342                    | 0.714%                    | 112                       | 0.060%                      |
| W20      | 188,149        | 212                     | 0.113%                    | 14                        | 0.007%                      |
| W30      | 188,149        | 1052                    | 0.559%                    | 87                        | 0.046%                      |
| All      | 752,454        | 2,884                   | 0.38%                     | 238                       | 0.03%                       |

<sup>1</sup> The radius used to count TCVs varied by aircraft pair. It was calculated to be the sum of the two aircraft type wing semi-spans.

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

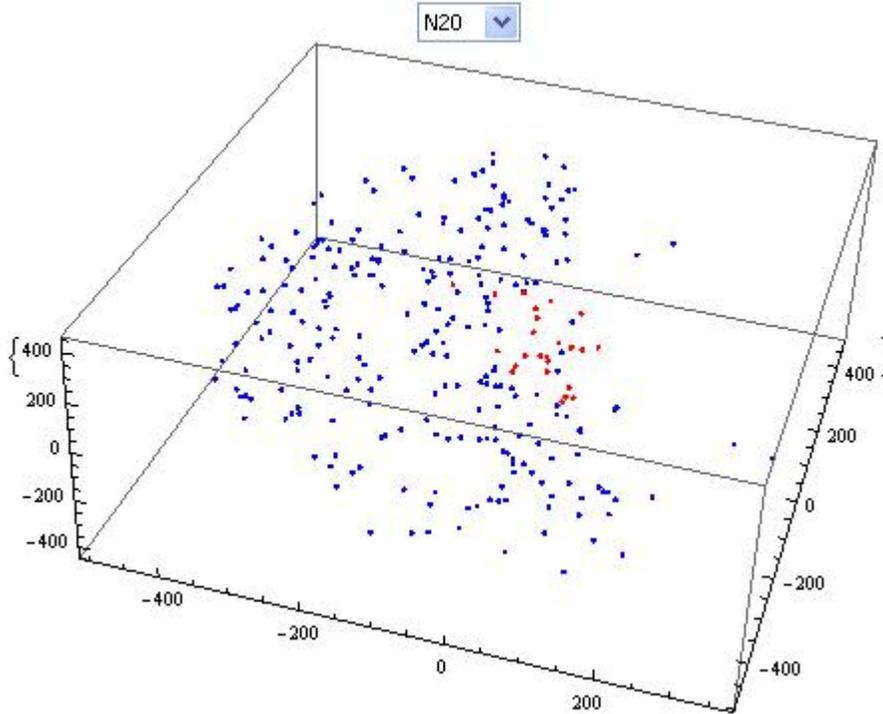
**DOT-FAA-AFS-450-63**

**April 2011**

---

In Figure 4 the blue points represent 500 foot sphere TCVs and the red points represent 140.6 foot sphere TCVs. The number of small sphere TCVs is on the order of one-tenth those of the larger sphere.

Figure 4 Large (500 ft, blue) and Small (140.6 ft, red) Sphere TCVs



Small Cylinder (Hockey Puck)

This is a cylinder of radius 140.6 feet and height 88 feet. These values were, as with the small sphere radius, calculated based on data from the Closely Spaced Parallel Operations (CSPO) study [2].

Figure 5 shows, to scale, a B737-400 target aircraft inside the cylinder. In this model a collision is assumed to occur when the center of gravity of the blundering aircraft penetrates the cylinder containing the target aircraft. Figure 6 shows the B737-400 inside the cylinder with both inside the large, 500-foot radius sphere.

Figure 5 B737-400 in 140.6 Foot Radius (Small) Cylinder to Scale



Figure 6 B737-400 in Small Cylinder in Large Sphere

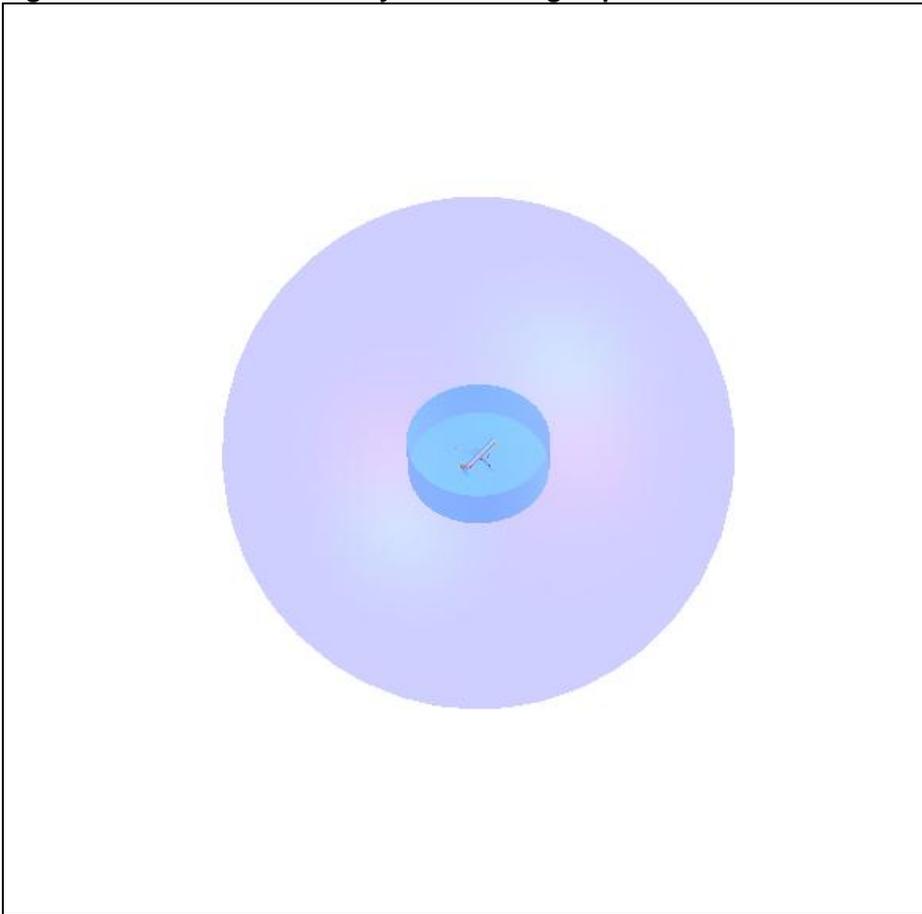


Table 4 gives the number of TCVs counted in the study for each data set. In each case a TCV occurs when the CPA for the two aircraft is less than 500 feet. Table 3 also gives the number of TCVs that would have been counted had a small cylinder (of average radius 140.6 feet and average height 88 feet<sup>2</sup>) been used rather than a sphere of radius 500 feet. And Figure 3 provides a 3-dimensional scatter graph of the 500 foot and 140.6 foot TCVs to show their geometrical relationship.

**Table 4 CSPO Data Set TCVs for Large Sphere and Small Cylinder**

| Data Set | Number of Runs | 500 ft Sphere TCV Count | 500 ft Sphere TCV Percent | Small Cylinder TCV Count | Small Cylinder TCV Percent |
|----------|----------------|-------------------------|---------------------------|--------------------------|----------------------------|
| N20      | 188,078        | 278                     | 0.148%                    | 16                       | 0.009%                     |
| N30      | 188,078        | 1342                    | 0.714%                    | 59                       | 0.031%                     |
| W20      | 188,149        | 212                     | 0.113%                    | 3                        | 0.002%                     |
| W30      | 188,149        | 1052                    | 0.559%                    | 45                       | 0.024%                     |

<sup>2</sup> The radius used to count TCVs varied by aircraft pair. As with the small sphere it was calculated to be the sum of the two aircraft type wing semi-spans. The small cylinder height was calculated to be the average of the two aircraft tail heights.

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

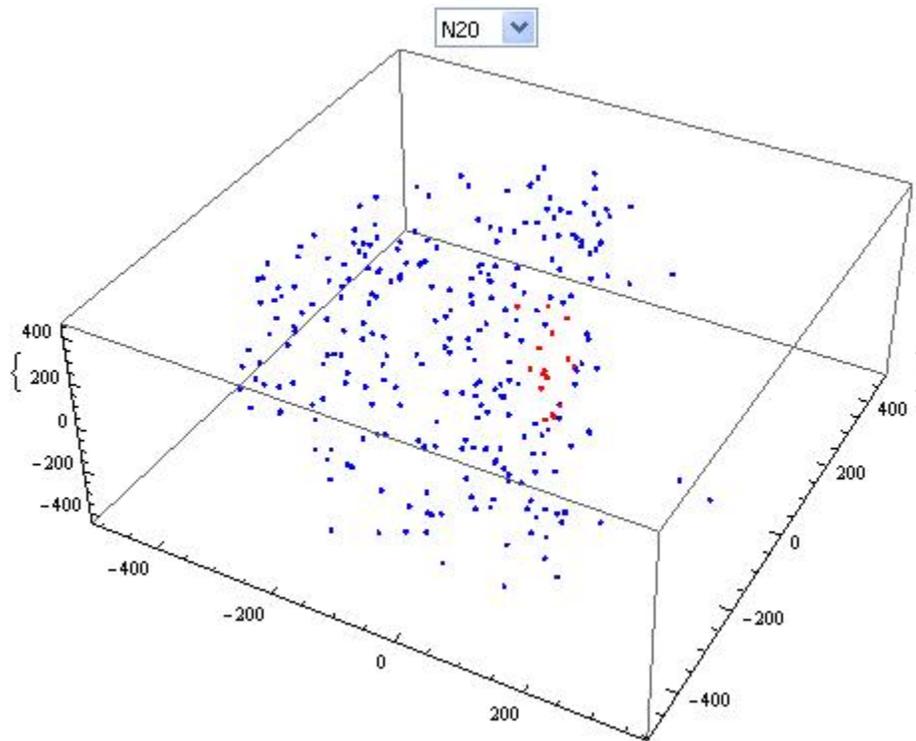
DOT-FAA-AFS-450-63

April 2011

|     |         |       |       |     |       |
|-----|---------|-------|-------|-----|-------|
| All | 752,454 | 2,884 | 0.38% | 123 | 0.02% |
|-----|---------|-------|-------|-----|-------|

In Figure 7 the blue points represent 500 foot sphere TCVs and the red points represent the small cylinder TCVs. The number of small cylinder TCVs is on the order of one-twentieth those of the large sphere.

**Figure 7 Large Sphere (blue) and Small Cylinder (red) TCVs**



### Large Cylinder (Large Hockey Puck)

This is a cylinder of radius 265 feet and height 160 feet. These values were calculated<sup>3</sup> based on a worst case scenario for an encounter between two Airbus A380 aircraft.

Figure 8 shows, to scale, a B737-400 target aircraft inside the large cylinder. In this model a collision is assumed to occur when the center of gravity of the blundering aircraft penetrates the cylinder containing the target aircraft. Figure 9 shows the B737-400 inside the large cylinder with both inside the large, 500-foot radius sphere.

<sup>3</sup> An A380 wingspan is 262 feet and its tail height (gear up) is 80 feet.

Figure 8 B737-400 in 265 Foot Radius (Large) Cylinder to Scale



Figure 9 B737-400 in Large Cylinder in Large Sphere

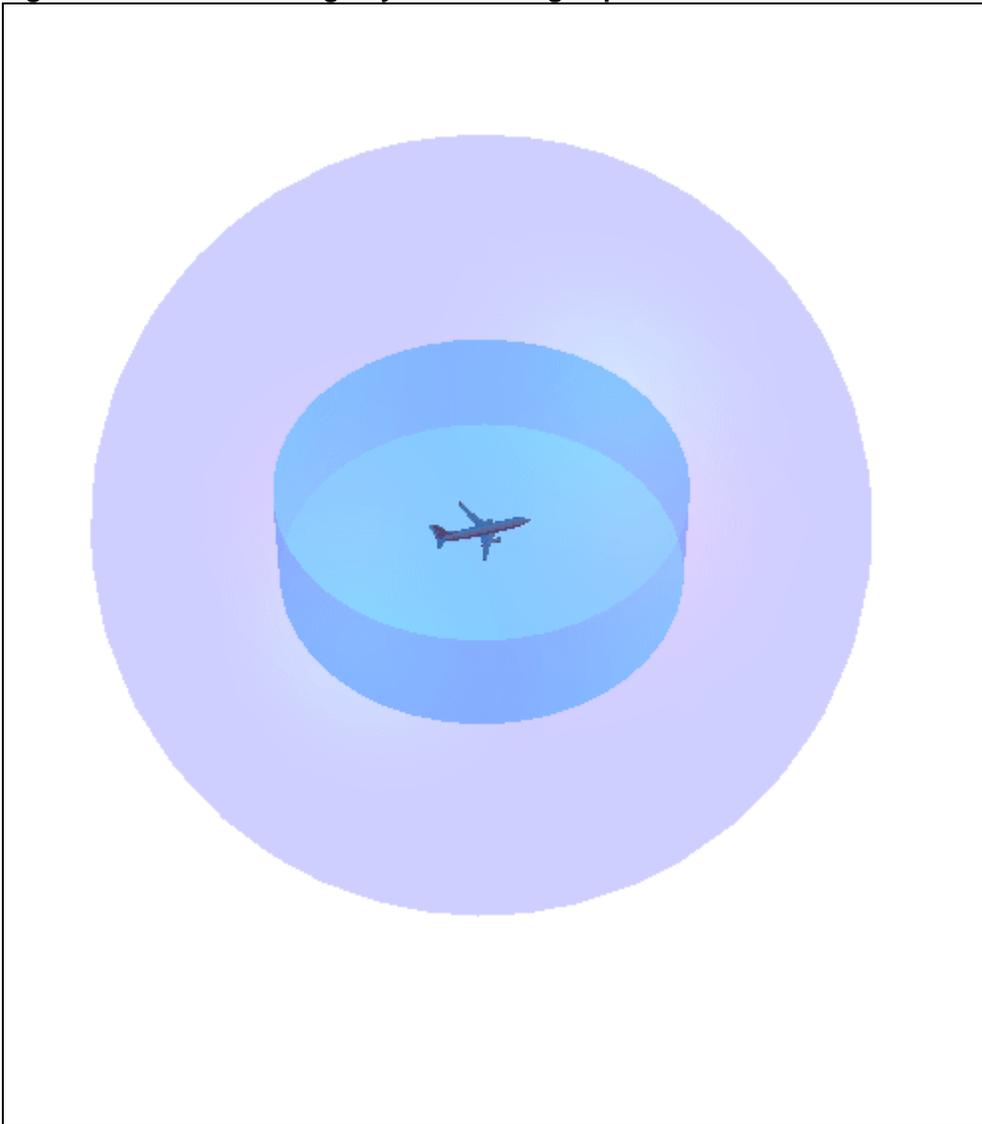


Table 5 gives the number of TCVs counted in the study for each data set. In each case a TCV occurs when the CPA for the two aircraft is less than 500 feet. Table 3 also gives the number of TCVs that would have been counted had a large cylinder (of radius 265 feet and height 160 feet) been used rather than a sphere of radius 500 feet. And Figure 3 provides a 3-dimensional scatter graph of the 500 foot sphere and 265 foot cylinder TCVs to show their geometrical relationship.

**Table 5 CSPO Data Set TCVs for Large Sphere and Large Cylinder**

| Data Set | Number of Runs | 500 ft Sphere TCV Count | 500 ft Sphere TCV Percent | Large Cylinder TCV Count | Large Cylinder TCV Percent |
|----------|----------------|-------------------------|---------------------------|--------------------------|----------------------------|
| N20      | 188,078        | 278                     | 0.148%                    | 29                       | 10.432%                    |
| N30      | 188,078        | 1342                    | 0.714%                    | 137                      | 10.209%                    |
| W20      | 188,149        | 212                     | 0.113%                    | 20                       | 9.434%                     |

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

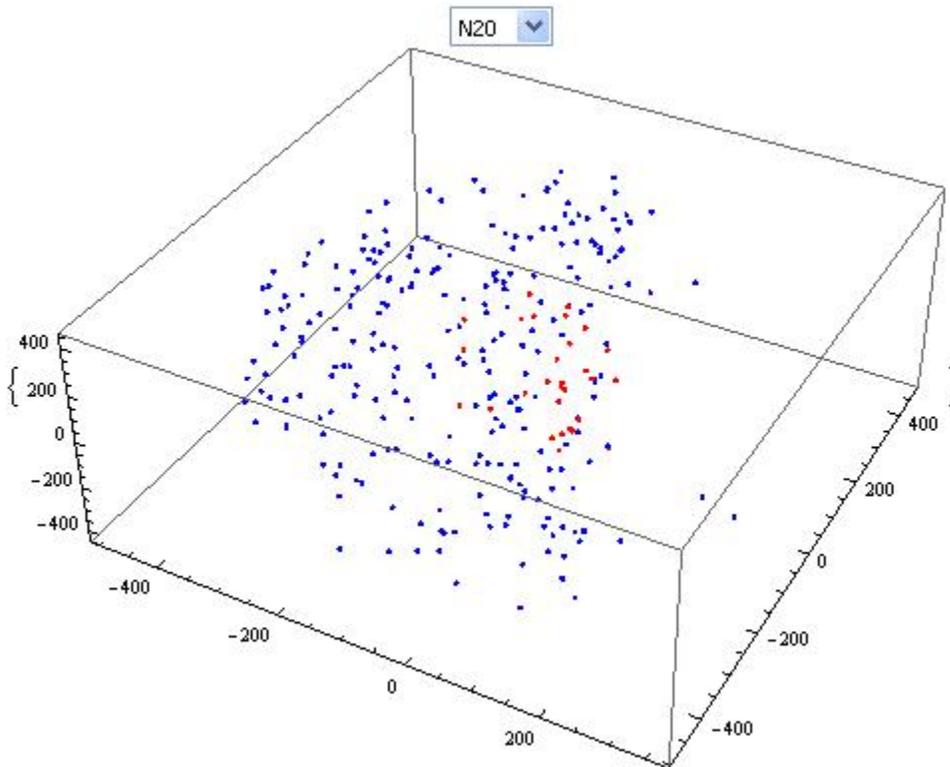
DOT-FAA-AFS-450-63

April 2011

|     |         |       |        |     |         |
|-----|---------|-------|--------|-----|---------|
| W30 | 188,149 | 1052  | 0.559% | 114 | 10.837% |
| All | 752,454 | 2,884 | 0.38%  | 300 | 10.402% |

In Figure 10 the blue points represent 500 foot sphere TCVs and the red points represent the small cylinder TCVs. The number of large cylinder TCVs is on the order of one-tenth those of the large sphere.

**Figure 10 Large Sphere (blue) and Large Cylinder (red) TCVs**



### Box

This is a box of length 302.5 feet, width 281.2 feet, and height 88.0 feet. These values were, as with the small sphere radius and cylinder radius and height, calculated based on data from the Closely Spaced Parallel Operations (CSPO) study [2].

Figure 8 shows, to scale, a B737-400 target aircraft inside the box. In this model a collision is assumed to occur when the center of gravity of the blundering aircraft penetrates the box containing the target aircraft. Figure 9 shows the B737-400 inside the box with both inside the large, 500-foot radius sphere.

Figure 8 B737-400 in 302.5 by 281.2 by 88.0 Box to Scale

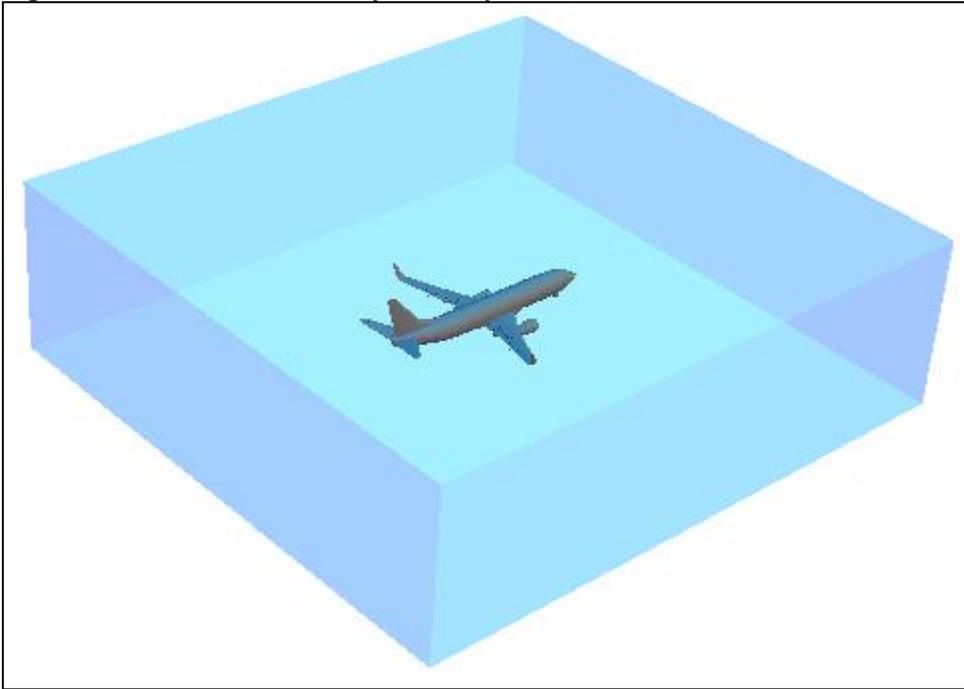


Figure 9 B737-400 in Box in Large Sphere

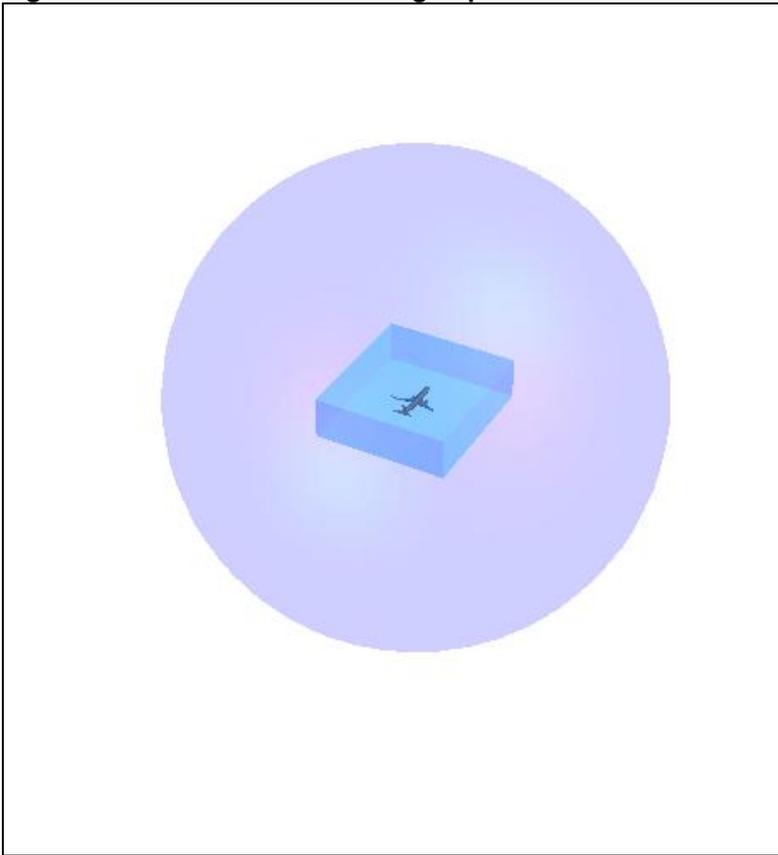


Table 6 gives the number of TCVs counted in the study for each data set. In each case a TCV occurs when the CPA for the two aircraft is less than 500 feet. Table 6 also gives the number of TCVs that would have been counted had a box (of average height 88.0 average width 281.2 feet, and average length 302.5 feet<sup>4</sup>) been used rather than one of 500 feet. And Figure 10 provides a 3-dimensional scatter graph of the 500 foot sphere and box TCVs to show their geometrical relationship.

**Table 6 CSPO Data Set TCVs for Large Sphere and Box**

| Data Set | Number of Runs | 500 ft Sphere TCV Count | 500 ft Sphere TCV Percent | Box TCV Count | Box TCV Percent |
|----------|----------------|-------------------------|---------------------------|---------------|-----------------|
| N20      | 188,078        | 278                     | 0.148%                    | 16            | 0.009%          |
| N30      | 188,078        | 1342                    | 0.714%                    | 72            | 0.038%          |
| W20      | 188,149        | 212                     | 0.113%                    | 4             | 0.002%          |
| W30      | 188,149        | 1052                    | 0.559%                    | 56            | 0.030%          |
| All      | 752,454        | 2,884                   | 0.38%                     | 148           | 0.02%           |

<sup>4</sup> The box dimensions used to count TCVs varied by aircraft pair. The heights, lengths, and widths were calculated as the sum of each dimension of those of the two aircraft.

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

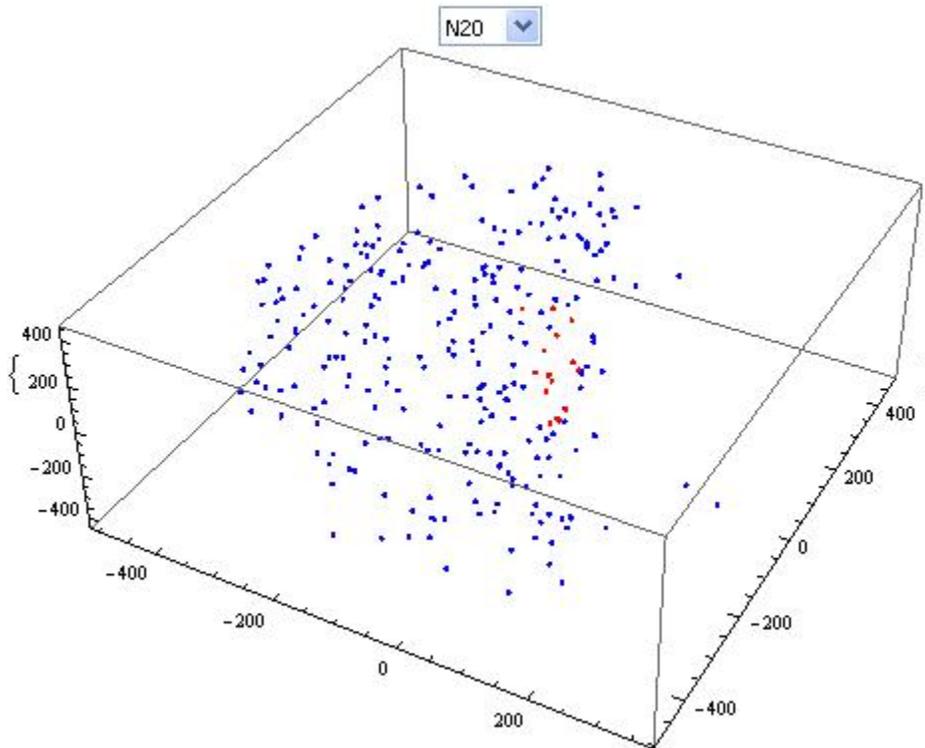
**DOT-FAA-AFS-450-63**

**April 2011**

---

In Figure 10 the blue points represent 500 foot sphere TCVs and the red points represent box TCVs. The number of box TCVs is (again) on the order of one-twentieth those of the large sphere.

Figure 10 Large Sphere (blue) and Box (red) TCVs



Shape Summary

Table 7 below gives a summary of the TCV results for the four shapes as compared with the large 500 foot sphere. The TCV ratios of the shapes to those of the large sphere are closer to the area ratios than to the volume ratios (the areas for these ratios are the cross sectional areas – height and length - rather than the plan view areas).

Also, the TCV ratios for the box and small sphere are very similar (their cross sectional areas are almost the same) and are about half those of the small sphere. The TCV metric using the small or large cylinder is likely a better measure of the metal-to-metal collision rate for parallel approaches than for either the spheres or the box.

Figure 11 shows the nested B737-400, cylinder, box, small sphere, and large sphere.

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

**DOT-FAA-AFS-450-63**

**April 2011**

| Shape          | Volume<br>(ft <sup>3</sup> ) | Volume<br>Ratio to<br>Large<br>Sphere | Area<br>Ratio to<br>Large<br>Sphere | Volume<br>Ratio to<br>Small<br>Sphere | TCV<br>Count | TCV Ratio<br>to Large<br>Sphere | TCV<br>Ratio to<br>Small<br>Sphere |
|----------------|------------------------------|---------------------------------------|-------------------------------------|---------------------------------------|--------------|---------------------------------|------------------------------------|
| large sphere   | 5.20E+08                     | 1.000                                 | 1.000                               | 44.966                                | 2884         | 1.000                           | 12.118                             |
| large cylinder | 2.55E+07                     | 0.049                                 | 0.083                               | 2.19                                  | 300          | 0.104                           | 1.261                              |
| small sphere   | 1.20E+07                     | 0.022                                 | 0.079                               | 1.000                                 | 238          | 0.083                           | 1.000                              |
| box            | 7.50E+06                     | 0.014                                 | 0.034                               | 0.643                                 | 148          | 0.051                           | 0.622                              |
| small cylinder | 5.50E+06                     | 0.010                                 | 0.032                               | 0.469                                 | 123          | 0.043                           | 0.517                              |

**Table 7 TCV Summary by Shape**

Figure 11 B737-400, Small Cylinder, Box, Small Sphere, Large Cylinder, and Large Sphere



## 3.0 TLS Regions

In this section we will calculate the overall TCV rates relative to the size of various volume regions centered on the target aircraft. The type of question this analysis is intended to answer is: “if a collision (a catastrophic event) has an extremely improbable likelihood of  $p$ , then what regions about the aircraft would overall TCV rates associated with extremely remote likelihoods?”

### Extremely Improbable TLS $p = 1.0 \text{ E-}09$

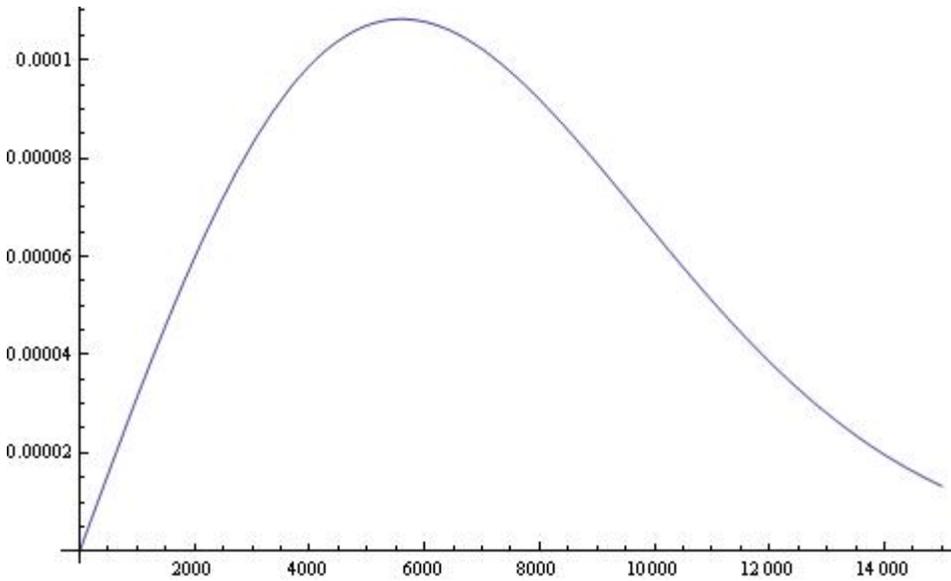
For example, if  $p$  is taken to be  $1.0 \text{ E-}09$ , as the extremely improbable TLS, and this is associated with a cylinder TCV rate, then what volume region would be associated with an extremely remote likelihood between  $1.0 \text{ E-}07$  and  $1.0 \text{ E-}09$ ?

Since the TCV rate is based on the CPAs within a given volume region, we will model the distribution of CPAs in order to associate a volume region with a TCV likelihood.

The distribution of CPAs for an aircraft pair, one blundering, is theoretically unbounded above and zero below: unbounded above since the blundering aircraft may blunder away from the target aircraft and zero below since the CPA is never less than zero. Also, this distribution is likely a combination of normal-like distributions for lateral, vertical, and longitudinal position differences of the two aircraft.

For these reasons, a Rayleigh distribution,  $R(x; \sigma)$  is an appropriate choice. If the typical distance between approach paths is about 3500 feet, then Rayleigh distribution with parameter value  $\sigma = 5600$  would be reasonable as shown below. We represent this distribution by  $R(x; \sigma) = R(x; 5600)$ . Figure 12 is a graph of this distribution.

**Figure 12 Rayleigh Distribution,  $R(x; 5600)$**



We can verify that this distribution is reasonable by comparing the TCV rates it provides with those from the CSPO data sets. First, the likelihood of a CPA less than 500 feet can be predicted by this Rayleigh distribution. This value is calculated to be

$$\int_0^{500} R(x;5600)dx = 0.00398 .$$

The TCV rate for the large sphere (radius 500 feet) from the 4 CSPO data sets averages 0.00383. The two TCV rates are quite close.

Second, the ratio of the TCV rate of the small sphere (radius 140.6 feet) and the large sphere (radius 500 feet) can be calculated based on the Rayleigh distribution to be

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

**DOT-FAA-AFS-450-63**

**April 2011**

$$\frac{\int_0^{140.6} R(x;5600)dx}{\int_0^{500} R(x;5600)dx} = \frac{0.000315}{0.00398} \approx 0.08$$

The TCV ratio of the small sphere to the large sphere (from Table 7) is 0.083. Again, the two values are quite close.

Now we use this Rayleigh distribution to determine what volume region corresponds to an overall TCV rate between 1.0 E-07 and 1.0 E-09. We distinguish between “TCV rate” and “overall TCV rate” because the TCV rates from the CSPO data sets represent at risk rates, and not usual rates per approach. The overall TCV rates used to compare with TLS vales should be usual rates per approach. Therefore, the CSPO TCV rates must be multiplied by a factor to accommodate the at risk nature of the data.

1. Let a collision be given a likelihood corresponding to that of an extremely improbable event: 1.0 E-09.
2. Assume that a small cylinder TCV represents a collision as recommended above.
3. The overall likelihood of a small cylinder TCV should then be 1.0 E-09. But the TCV rate for the small cylinder, from Table 4, is 0.0002. Therefore, this rate should be multiplied by a factor of 5.0 E-06 to give 1.0 E-09. (The TCV rate of 0.0002 from the CSPO test data is for at risk blunders, and does not include most typical parallel approach behaviors, non-blunder behavior.)
4. Next we determine the radius, r, of a sphere whose volume will give a TCV rate, T, such that T times 5.0 E-06 will equal 1.0 E-07. Thus, T must be 0.02. So that

$$\int_0^r R(x;5600)dx = \frac{1.0E-07}{1.0E-09} (0.0002) = 0.02$$

5. From the equation above, r must equal 1125 feet. So that a spherical region of radius 1125 feet gives an overall TCV rate of 1.0 E-07.

Using this methodology, we can describe spheres about the target aircraft for various overall TCV rates. Table 8 provides these results for various overall TCV rates between 1.0 E-09 and 1.0 E-7. Rates above 1.0 E-07 do not correspond to radius distances that are reasonable for close parallel approaches.

**Table 8 Overall TCV Rates and Sphere Radii (p = 1.0 E-09)**

| Overall TCV Rate | Sphere Radius (ft) |
|------------------|--------------------|
| 1.0 E-09         | 112                |
| 1.6 E-09         | 140.6              |
| 6.0 E-09         | 265                |
| 1.0 E-08         | 355                |
| 2.0 E-08         | 500                |
| 5.0 E-08         | 795                |

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

**DOT-FAA-AFS-450-63**

**April 2011**

|          |      |
|----------|------|
| 1.0 E-07 | 1125 |
|----------|------|

As a further reasonableness check, the CSPO data provides TCV values of 133, 718, and 1441 for spheres of radius 112 feet, 265 feet, and 355 feet respectively. These empirical TCV values translate into overall TCV rates of 8.8 E-10, 4.8 E-09, and 9.6 E-09. These, in turn, compare very favorably with the overall TCV rates above (1.0 E-09 = 10.0E-10, 5.0 E-09, and 1.0E-08 = 10.0 E-09) calculated from the Rayleigh distribution. Table 9 summarizes these comparisons.

**Table 9 Theoretical (Rayleigh) to Empirical (CSPO Data) Comparison**

| Sphere Radius (ft) | Rayleigh TCV Rate | Empirical TCV Rate |
|--------------------|-------------------|--------------------|
| 112                | 1.0 E-09          | 0.88 E-09          |
| 265                | 6.0 E-09          | 4.80 E-09          |
| 355                | 1.0 E-08          | 0.96 E-08          |

There are two other independent studies of parallel approach blunder rates that use variously shaped TCV regions. These studies may be used to check the reasonableness of our results. Unfortunately, these studies do not use blunder scenarios or region sizes strictly comparable to those of the CSPO data sets used here (for example, the definition of at-risk may vary considerably). However, since each of these studies compares cylinder to sphere TCV rates giving cylinder to sphere ratios, it is possible to compare those ratios with the cylinder to sphere ratios presented here.

A study by the Mitre CAASD organization [3] analyzed parallel approaches with 4000 foot spacing and PRM sensors. The at-risk TCV rate<sup>5</sup> it found using a 500 foot radius sphere was 0.1166 and the at-risk TCV rate for a 500 foot radius, 100 foot height cylinder was 0.0391. The cylinder to sphere TCV ratio is then  $0.0391 / 0.1166 = 0.335$ . The TCV ratio of our 140.6 foot radius, 100 foot height cylinder to our 140.6 foot radius sphere is 0.043 to 0.083 or 0.518. Though the dimensions of our regions are somewhat smaller than those of the Mitre study, the two ratios are comparable.

A 2009 study by Air Traffic Simulation, Inc. (ATSI) [4] analyzed parallel approaches with 2500, 3000, 3500, and 4000 foot spacing and PRM sensors<sup>6</sup>. In the 3500 foot spacing analysis, the at-risk TCV rate it found using a 500 foot radius sphere was 0.1028 and the at-risk TCV rate for a 175 foot radius, 150 foot height cylinder was 0.0068. The cylinder to sphere TCV ratio is then  $0.0068 / 0.1028 = 0.066$ . The TCV ratio of our 140.6 foot radius small cylinder to our 500 foot sphere is 0.043. However, the cross-sectional area of our cylinder is about one-half that of the ATSI cylinder. So, again, the results are comparable.

<sup>5</sup> These rates are for scenarios with level flight and climb and turn evasive maneuvers.

<sup>6</sup> This ATSI study analyzed TCV rates using other sensors also, but the PRM sensors are the ones comparable to our results. Also, we used the 20% heavy ATSI analysis for comparison with our values.

Extremely Improbable TLS  $p = \text{Other Values}$

The Extremely Improbable TLS probability,  $p$ , may be some value other than 1.0 E-09. In that case, we can generalize the methodology presented above.

In step 4 of the methodology, the equation to calculate the sphere radius,  $r$ , based upon the overall TCX rate, 1.0 E-07, and the TLS probability, 1.0 E-09 is

$$\int_0^r R(x;5600)dx = \frac{1.0E-07}{1.0E-09}(0.0002) = 0.02 .$$

To generalize, we can replace 1.0 E-07 by a general TCX rate of  $t$ , and we can replace 1.0 E-09 by a general TLS probability of  $p$ . The equation then becomes

$$\int_0^r R(x;5600)dx = \frac{t}{p}(0.0002) . \tag{1}$$

The Rayleigh cumulative distribution function given by  $\int_0^r R(x;5600)dx$  can be written as  $1 - e^{-\frac{r^2}{2\sigma^2}}$  where  $\sigma = 5600$ . Equation (1) above therefore becomes

$$1 - e^{-\frac{r^2}{2\sigma^2}} = \frac{t}{p}(0.0002) . \text{ This equation can be solved for the radius, } r \text{ so that}$$

$$r = \sigma \sqrt{\ln\left(1 - \frac{0.0002t}{p}\right)^{-2}} , \text{ where } \sigma = 5600 . \tag{2}$$

## Geometrical Models for Aircraft in Terminal Area Risk Analyses

DOT-FAA-AFS-450-63

April 2011

Table 8 can be extended to other TLS probabilities,  $p$ , using equation (2) above. Tables 10, 11, and 12 show these results for  $p = 5.0 \text{ E-}09$ ,  $p = 1.0 \text{ E-}08$ , and  $p = 4.0 \text{ E-}08$  respectively. Other such tables can be calculated from this equation.

**Table 10 Overall TCV Rates and Sphere Radii ( $p = 5.0 \text{ E-}09$ )**

| Overall TCV Rate, $t$ | Sphere Radius (ft) |
|-----------------------|--------------------|
| 5.00E-09              | 112                |
| 1.00E-08              | 158                |
| 2.00E-08              | 224                |
| 5.00E-08              | 354                |
| 1.00E-07              | 501                |
| 5.00E-07              | 1126               |

**Table 11 Overall TCV Rates and Sphere Radii ( $p = 1.0 \text{ E-}08$ )**

| Overall TCV Rate, $t$ | Sphere Radius (ft) |
|-----------------------|--------------------|
| 1.00E-08              | 112                |
| 2.00E-08              | 158                |
| 5.00E-08              | 251                |
| 1.00E-07              | 354                |
| 5.00E-07              | 794                |
| 1.00E-06              | 1126               |

**Table 12 Overall TCV Rates and Sphere Radii ( $p = 4.0 \text{ E-}08$ )**

| Overall TCV Rate, $t$ | Sphere Radius (ft) |
|-----------------------|--------------------|
| 4.00E-08              | 112                |
| 5.00E-08              | 125                |
| 1.00E-07              | 177                |
| 5.00E-07              | 396                |
| 1.00E-06              | 561                |
| 5.00E-06              | 1260               |

**References**

- [1] “Safety Study Report on Triple Simultaneous Parallel ILS Approaches and RNAV/RNP Approaches at KIAH”, DOT-FAA-AFS-440-16, December 2005.
- [2] Unpublished Closely Spaced Parallel data collection study. AFS-450, July 2010.
- [3] “Effectiveness of Climb-Only versus Climb-and-Turn Evasion Maneuvers During Simultaneous Instrument Approaches”, S.V. Massimini, Mitre CAASD, (No Date), Project Number 02964A01AA.
- [4] “Revision of Test Criteria (TC) for Near Mid-Air Collision (N-MAC): ASATng Runs & Analysis”, Air Traffic Simulation study ATSI-2009-05-04, May 18, 2009.