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16. Abstract

The Statistical Discrete Gust (SDG) method is an alternate procedure for estimating and predicting severe gust and severe turbulence loads. This method claims to have the potential to replace the combined Power Spectral Density (PSD) Model of Title 14 Code of Federal Regulations (CFR) 25.341(b) and the Tuned Isolated Discrete Gust (IDG) of 14 CFR 25.341(a), which jointly ensures that an airplane has sufficient structural strength for encounters with continuous turbulence (PSD) and discrete gusts (TDG) of design-level intensity. The SDG method provides such a specification in a form that takes into account the non-Gaussian statistical structure of the more intense turbulence fluctuations and the manner in which these interact with the dynamic response of a flexible aircraft.

This report documents the development of the method, as applied to the response of a linear aircraft, in the following order: historical development, evidence that severe and extreme gust encounters are characterized by short-duration bursts evidence for the 1/6 scaling law examples of calculated loads and comparisons with models in existing requirements, and advantages and limitations of current SDG models.

The SDG method was developed over a period of over 30 years, and during its development, many in industry, research institutions, and academia have expressed divergent viewpoints regarding the suitability of using the SDG method for the derivation of design gust loads. Consequently, the Federal Aviation Administration has invited three experts to express their views and comments, which have been included as an addendum in the documentation of the SDG method contained herein.

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PREFACE

The Federal Aviation Administration has supported the development of the Statistical Discrete Gust (SDG) Method, for use as an alternative procedure of estimating severe gust and turbulence loads. This report reviews and documents the SDG methodology. This methodology was reviewed by a group of international Gust Specialists, who met approximately yearly during a 15-year period.

Aircraft design must ensure sufficient structural strength to survive extreme cases of encounters with atmospheric turbulence, thus requiring a mathematical model of turbulence to be used in the design process.

The Power-Spectral-Density (PSD) model of continuous turbulence, Title 14 Code of Federal Regulations (CFR) 25.341(b), is based on the concept that, at least in patches of limited extent, random turbulence can be represented by a spectrum of harmonic gust waves across the frequency range, with the instantaneous gust velocity having a Gaussian distribution about the mean. In this respect, PSD has limitations as a means of representing the most intense and highly localized wind fluctuations, which are most relevant for structural design, and which tend to be more severe than implied by a Gaussian velocity distribution.

The Tuned Isolated Discrete Gust (IDG) Model, 14 CFR 25.341(a), does address the problem of correctly representing extremely large localized gusts. However, its use of just a single-shape gust-profile severely limits the IDG to relate realistically the gust loads on aircraft with widely differing dynamic response characteristics, which can tune to gust patterns of different shape.

The SDG method provides a specification, which accounts of the non-Gaussian statistical structure of the more intense turbulence fluctuations, and the manner in which these interact with the dynamic response of a flexible aircraft. SDG can be interpreted as a generalization of the existing tuned IDG model to take into account tuning to gust patterns of different shapes. Also, it is expressed in a statistical format that parallels that of the PSD method, being applicable in both Mission Analysis and Design Envelope forms. However, whereas the PSD method neglects the influence of phase correlations in the calculation of critical loads, the SDG representation takes account of the effects of the phase correlations in measured severe turbulence, which result in the associated statistics being highly non-Gaussian. This is achieved by modeling localized discrete fluctuations explicitly in terms of ramp-shaped gust components and expressing the statistical description of severe and extreme turbulence in the form of probability distributions of patterns comprising both single- and multiple-ramp components. Both the scaling law relating gust amplitude to gust gradient distance and the probabilities attached to localized patterns in the form of sequences of ramp gusts containing different numbers of components are based on measured data.

From 1986 to 2001, an international team of specialists, convened by the Federal Aviation Administration, met approximately annually to re-evaluate the gust criteria for future generations of commercial transport aircraft. The goals of this International Ad Hoc Committee have been (1) to reduce the number of design criteria to be met and (2) to recommend a design method with the ability to handle advanced technologies such as active controls and gust load alleviation.

The SDG method has been identified as a possible method, which can handle both discrete gust events and relatively continuous turbulence, and which moreover can be used to evaluate highly nonlinear systems. However, in part as a result of a perceived computational complexity, the SDG has not been recommended by the Gust Specialists Committee for consideration as a revised airworthiness requirement.

DEDICATION

This report is dedicated to the memory of George (J.K.) Zbrozek (Royal Aircraft Establishment, Bedford) and Harry (H.P.Y.) Hitch (British Aerospace Aircraft Group, Weybridge-Bristol Division)

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The author would like to acknowledge the contribution of many of my colleagues in the preparation of this report. Tom Zeiler, Peter van Gelder, and Jurjen Roos all assisted by providing constructive comments leading to significant improvements in this report. Tom Zeiler also provided the present draft of appendix B. John Glaser made a major contribution in the preparation of the draft gust criterion (appendix C) and in the provision of data concerning the turboprop-type aircraft. Mark Hockenhull provided the data concerning the Airbus-type aircraft. The contribution is also acknowledged of Graham Watson of QinetiQ who, under subcontract to Stirling Dynamics, was responsible for the software implementation of SDG1. The Federal Aviation Administration's former Chief Scientist for Flight Loads and Aeroelasticity, Mr. Terry Barnes, chaired all of the meetings, seminars, and communications, which resulted in the publication of this report.

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LIST OF ACRONYMS

AS	Alternating sign
BAe	British Aerospace Aircraft Group
BCAR	British Civil Airworthiness Requirements
DSP	Deterministic spectral procedure
FAA	Federal Aviation Administration
IDG	Tuned isolated discrete gust
NTSB	National Transportation Safety Board
PSD	Power spectral density
rms	Root mean square
SC	Staircase
SDG	Statistical discrete gust
SRF	Staircase reduction factor
TAS	True airspeed
UK	United Kingdom
U.S.	United States

EXECUTIVE SUMMARY

Aircraft design has to ensure sufficient structural strength to survive extreme cases of encounters with atmospheric turbulence. This requires the formulation of a mathematical specification of turbulence to be used in the design process. The Statistical Discrete Gust (SDG) method, documented in this report, provides such a specification in a form that takes into account the non-Gaussian statistical structure of the more intense turbulence fluctuations and the manner in which these interact with the dynamic response of a flexible aircraft.

The Power Spectral Density (PSD) model of turbulence, as formulated in the current design requirements, is based on the concept that at least in patches of limited extent, the turbulence can be modelled as a stationary Gaussian random process. In this respect, it has severe limitations as a means of representing the most intense and highly localized wind fluctuations.

The existing tuned isolated discrete gust (IDG) model, also in the current requirements, does address the problem of representing very localized fluctuations and takes into account, statistically, some properties of extreme turbulence. However, its use of just a single shape of gust profile severely limits its ability to relate, realistically, the gust loads on aircraft having widely differing dynamic response characteristics, which can be shown to tune to gust patterns of different shapes.

It was to meet these deficiencies in the existing requirements that the SDG model of severe-to-extreme turbulence was developed. It can be interpreted as a generalization of the existing tuned IDG model to take into account tuning to gust patterns of different shapes. Also, it is expressed in a statistical format that parallels that of the PSD method, being applicable in both Mission Analysis and Design Envelope forms. However, whereas the PSD method neglects the influence of phase correlations in the calculation of critical loads, the SDG representation takes into account the effects of the phase correlations between the Fourier components in measured severe turbulence, which result in the associated statistics being highly non-Gaussian. This is achieved by modeling localized discrete fluctuations explicitly in terms of ramp-shaped gust components and expressing the statistical description of severe-to-extreme turbulence in the form of probability distributions of patterns, comprising both single- and multiple-ramp components. Both the scaling law relating gust amplitude to gust gradient distance and the probabilities attached to localized patterns in the form of sequences of ramp gusts containing different numbers of components are based on the analysis of measured data, including both turbulence measurements recorded by specially instrumented research aircraft and also records obtained from severe gust encounters during routine operational flying by civil airlines.

During the period 1986-2001, an international team of specialists, convened by the Federal Aviation Administration (FAA), met approximately annually to re-evaluate the gust criteria for future generations of commercial transport aircraft. Included amongst the goals of this International Ad Hoc Committee of Gust Specialists have been

- to reduce the number of design criteria to be met.
- to recommend a design method with the ability to handle advanced technologies such as active controls and gust load alleviation.

In this context, “the SDG method has been identified as the only existing method that can handle both discrete gust events and relatively continuous turbulence and which moreover can be used to evaluate highly nonlinear systems” (Barnes, T.J.: “Overview of the Activities of the Ad Hoc Committee of International Gust Specialists,” Enclosure 9(a) to ANM-105N:94-20, 1994). Although, in part as a result of a perceived computational complexity of the method, it has not been recommended by the Gust Specialists Committee for consideration as a revised airworthiness requirement, it remains the only method with the potential to meet the two goals listed above. A draft proposed airworthiness criterion, based on the SDG method, is included in this report as appendix C.

At a meeting of the Gust Specialists Committee held in Ottawa, Ontario, Canada, June 7-8, 2001, the attendees agreed with the objective of publishing an FAA report, presented here, which pulls together and documents background material concerning those aspects of the SDG method that are applicable when the response of the aircraft can be assumed to be linear. Therefore, it would no longer be necessary to search for reference materials, many of which exist in the form of internal reports. These reports are now to be made available in electronic form on a CD ROM disk, with unlimited distribution.

1. INTRODUCTION TO THE STATISTICAL DISCRETE GUST METHOD.

Examination of records of atmospheric disturbances taken under a wide variety of conditions shows two conflicting trends: one towards order and the other towards disorder. In some instances, isolated discrete gusts stand out in a clearly identifiable manner. In others, the turbulence records appear to have a predominantly irregular or random pattern with little obvious structure. This nature of turbulence has led to alternative ways of attempting to describe it. Its tendency to fluctuate in a chaotic random manner has suggested that the mathematical theory of continuous random processes be taken as a basis; this approach is typified by the power spectral density (PSD) method [1-3]. On the other hand, an impression that the more intense fluctuations could somehow be singled out as individual events was the basis of the earliest approach used by aircraft engineers, and there has persisted a reluctance on the part of some aircraft manufacturers to dispense with a discrete gust model as a means of representing the more severe disturbances [4-7]. This has led to research aimed at clarifying the relationship between the two approaches and at developing a combined gust and turbulence model which takes into account both the random character of air motions and the discrete structures that appear to be particularly relevant to the larger, potentially critical, disturbances.

One consequence of this research has been a realization of the fundamental role played by phase correlations, between Fourier modes, in influencing the statistical properties of turbulence and related aircraft response, particularly in regards to the occurrence of large amplitude fluctuations that are of primary concern in the context of aircraft safety. A basic simplifying assumption made in the PSD method is that the phase distribution is completely random (the power spectrum defining only the amplitude components in the Fourier representation). This assumption of random phase is introduced implicitly in the standard PSD representation of atmospheric turbulence as a sequence of Gaussian patches [3]. It has become clear that the way to take into account both the random character of air motions and the discrete structures associated with potentially critical disturbances is to incorporate the effects of phase correlation into the statistical representation.

This is the approach adopted in the Statistical Discrete Gust (SDG) model of turbulence, which takes the discrete ramp gust as a basic element, or building brick, from which more complex gust patterns are built up and related to probability of occurrence. In mathematical terms, the discrete ramp gust represents a localized wave packet of Fourier components having a highly correlated phase distribution. Other effects of phase correlation appear in the scaling law relating gust amplitude to gust gradient distance, which differs from that applicable when the phase is random, and in the statistics of the ramp gust clusters that form the more complex patterns.

In the SDG method, the assessment of a particular design of aircraft, or control system, involves finding the worst case, or design case, which produces maximum aircraft response, from within a specified family of equiprobable gust patterns. Such an equiprobable gust family is a generalization of the traditional concept of a design gust.

At the same time, an important aspect of the method is the relationship that may be shown to exist (section 6) between such a worst-case and the overall statistical distribution for the rate of occurrence of large fluctuations in aircraft response.

To summarize, the point of view underlying the SDG model is that a turbulent flow field, even when apparently random and continuous, in fact contains discrete structures that are more appropriately described in terms of spatial velocity distributions than by transforming (as in the PSD approach) to a spectral or frequency distribution. The discrete structures are represented by means of ramp gusts, either singly or in clusters. The random element is then incorporated by the use of probability distributions to represent a turbulence velocity component as a statistical ensemble of such discrete structures. The existence of such discrete structures in turbulence corresponds mathematically to the existence of strong phase correlations between different components in the Fourier representation—an important feature that is ignored in the PSD approach.

2. HISTORICAL DEVELOPMENT.

2.1 BACKGROUND, 1950-1973.

Until the mid-1950s a discrete gust method for representing fluctuating air velocities for the purpose of aircraft design and assessment was practically universal. In this method, gusts were assumed to take a fixed and relatively simple shape: the ramp gust and the 1-cosine gust (figures 1a and 2 of reference 8). The two variable parameters in this representation are the gust gradient distance and the gust intensity. The gust gradient distance was assumed to take some definite value, generally 100 feet (in the United Kingdom (UK)) or 12.5 wing chords (in the United States (U.S.)).

An alternative approach, the PSD method [1-3], was introduced into gust load studies in the late 1940s. In this method, the gusts are regarded as random fluctuations in a continuous random process, the mathematical theory that was developed mainly in the context of electrical engineering. The PSD is essentially a decomposition of the energy of the random process with respect to frequency or wavelength. Knowledge of the PSD of the fluctuating air velocity together with a specification of the dynamics of the aircraft (frequency response function) allows the PSD of the response to be calculated if the system is linear. On this basis, the average aircraft response may be calculated in terms of its mean-square intensity. However, the question that primarily concerns the aircraft design engineer is how often the relatively large and rare disturbances to aircraft occur. Should these be represented as discrete events, as in the discrete gust method, or as fluctuations occurring at random as part of a continuous random process? In the latter case, there remains the problem of relating the mean-square intensity of response, which can be calculated using the PSD method, to the amplitudes of large peaks. While this step can be taken for a Gaussian process, for which the peaks follow a Rayleigh distribution, there exists no general relationship between mean-square intensity and the amplitudes of large peaks when the input is non-Gaussian. The theoretical relationship that exists, for a Gaussian process, between the mean-square intensity of response and the amplitudes of large peaks depends upon the phases in the Fourier representation being randomly distributed. In a non-Gaussian process, on the other hand, the phases of the Fourier components are correlated and no general relationship between the mean-square intensity and the amplitudes of large peaks exists. As described in section 3, there is strong evidence that even short patches of severe turbulence are generally highly non-Gaussian.

Over the period 1960-1967, in the UK, scientists in government [9-11] and in aircraft manufacturing companies [4 and 6] addressed the advantages and limitations of the mathematical model of turbulence prescribed in the PSD method. Within the same period, operational flight records of encounters with severe turbulence were examined [12] with a view to providing data relevant to the question as to whether patches of severe turbulence could adequately be represented as samples from a Gaussian process.

The position as it stood in 1964 was described in a lecture [9] by J.K. (George) Zbrozek, of the (UK) Royal Aircraft Establishment, to the Royal Aeronautical Society. In this lecture, Zbrozek reviewed the (then-existing) discrete gust approach to aircraft loadings and described the alternative approach to continuous turbulence based on the use of the power spectrum. In conclusion, Zbrozek expressed dissatisfaction with both methods: "The discrete gust technique … can only be applied to aircraft of similar dynamic and aeroelastic properties, and therefore has extremely limited application to more advanced aircraft. The spectral technique, although it allows in principle for the dynamic properties of the aircraft and of turbulence, has still limited application… There are also types of "turbulence" which may be of more organized than random character and therefore not a priori amenable to spectral treatment."

In the same lecture [9], Zbrozek referred to the possibility of establishing an improved model of atmospheric turbulence in terms of discrete gusts, defining them by just two parameters, the gradient distance and the change in gust velocity occurring over this gradient distance. The model would then be completed by a two-dimensional statistical distribution expressed in terms of these two variables. In subsequent work at the Royal Aircraft Establishment, effort was directed towards implementing this concept. This led, in 1968, to a report [13] outlining the principles of what would later be called the Statistical Discrete Gust method. A review of the progress made between 1964 and 1968 was presented by Burnham [11], who further emphasized the limitations of the Gaussian-process assumptions made in the PSD method when applied to relatively extreme and rare events.

In a critique, in 1967, of the model of stationary random turbulence employed in the PSD method, Jones [10] made the point that one of the most important properties of turbulence about which the aircraft engineer requires statistical information is the magnitude of the change in wind velocity over any prescribed distance. To quote from the conclusions of that report:

> "Aircraft responses (loads, etc.) due to turbulence depend to a large extent on the change in turbulence velocity in an interval of the order of the aircraft response time. There is an increasing tendency for the power spectral approach to be used in the estimation of aircraft response and this spectral approach depends on the assumption that samples of turbulence velocity can be adequately approximated by a stationary Gaussian process. In particular, the joint probability distribution (of turbulence velocity) at two points (in space), which determines the gradient properties of the process, is assumed to be Gaussian. On the other hand there is strong experimental evidence in the case of turbulence behind a grid that the joint probability distributions of turbulence velocity at two points are not Gaussian. There is a tendency for a small number of strong gradients to form rather than a uniform distribution of smaller gradients. In view of this it appears that there is a

> strong case for comparing the gradient properties of measured samples of atmospheric turbulence with the gradient properties of the spectral model."

Extensive comparisons of this type were made in subsequent years at the Royal Aircraft Establishment (section 4), and the highly non-Gaussian nature of the statistical distribution of two-point differences, or increments, in measured samples of atmospheric turbulence velocity was confirmed. Similar arguments to the above, backed up by illustrations of non-Gaussian two-point distributions in measured data from a variety of sources, were presented in 1972 by Chen [14]. Furthermore, Chen showed that the basic equation, due to Rice, used in the PSD method to predict response statistics, depends heavily on the assumption that, at least in turbulence patches of limited extent, changes in turbulence velocity over arbitrarily prescribed distances obey Gaussian statistics.

From the point of view of the aircraft manufacturer, James [4] outlined reasons why, at that time, some sections of the British aircraft industry were of the opinion that discrete gust methods were more relevant than PSD methods as a means for establishing critical gust loads. These views were underlined by the results of investigations into the application of the Federal Aviation Administration (FAA) continuous turbulence design procedure. These had shown [4] that at least for the BAC 1-11 and Super VC 10 aircraft, a Design Envelope approach based on the PSD method leads to design loads differing significantly from those produced by the then-established British Civil Airworthiness Requirements (BCAR) discrete gust method. In particular, although on these aircraft the original (1966) U-sigma values (PSD Design Envelope) gave symmetric wing bending moments comparable with those of the BCAR discrete gust method, when applied in antisymmetric cases these U-sigma values led to a significant and unjustified (up to 80%) increase in fin loads over those produced by the BCAR. (Recent results which support the argument that the PSD method can predict overconservative lateral loads are presented in section 7).

Subsequently, in 1971, the FAA design envelope U-sigma values prescribed in the PSD method were reduced to a lower value. However, it was emphasized in reference 4 that for this class of aircraft, while the reduction in U-sigma led to more reasonable requirements regarding fin strength, it also led to significant decreases in symmetric wing-bending moments when compared with the BCAR. Since the BCAR discrete gust model had for many years been used quite satisfactorily for guarding against extreme wing loads due to turbulence, it was concluded that for wing design purposes there is the possibility that the 1971 U-sigma values were too low for the class of aircraft considered in that report. Combining the above results for fins and wings, it could be seen [4] that "the FAA PSD Design Envelope proposal leads to a different distribution of design strength than exists in aircraft with a proven satisfactory safety record. According to how the values are actually chosen, we are led to the conclusion that, in comparison with BCAR, either fins must be (very) much stronger or wings may be weaker. There is no practical evidence to justify either step ... ." Section 7 will demonstrate how the SDG method resolves this problem, restoring the distribution of design strength between wings and fins to something closer to that which existed prior to the introduction of the PSD criterion and its associated implicit assumption that turbulence has a random phase distribution.

On the other hand, the BCAR discrete gust method could be justly criticized in that it was based on an implied statistical model in which the probability distribution of gust velocity was independent of gust wavelength. This property was not consistent with experimental measurements. Furthermore, it was clearly deficient as a means of predicting loads associated with the response of lightly damped modes in patches of continuous turbulence through its failure to take into account the large response amplitudes that could arise as a result of a resonant buildup. In particular, it was questionable as a basis for determining fin strengths in situations where critical fin loads might be associated with lightly damped Dutch Roll oscillations.

It was primarily to meet these limitations of the existing criteria that work was pursued which led to the development of the SDG method as a possible single turbulence model for use in a future airworthiness requirement for aircraft loads.

2.2 DEVELOPMENT OF THE SDG METHOD, 1968-1993.

2.2.1 1968-1978.

The basic principles of the SDG method were first introduced in 1968 [13]. In that report, drawing upon concepts discussed previously by Mandelbrot, turbulence was represented as a non-Gaussian random process having the properties of self-similarity and intermittency. To define (loosely) a self-similar process X(t): if a stretching transformation is performed on a sample of the process, in which the time (t) axis is uniformly expanded by a factor of h, and the X (dependent variable) axis is uniformly expanded by a factor of h^k, for some fixed power-law exponent k, then the probabilities of observing the original and stretched samples are the same.

Intermittency, on the other hand, refers to the degree to which regions of high activity are interspersed by regions of low activity. Turbulence is intermittent in that there is a tendency for a small number of regions of large velocity gradient to form, rather than smaller gradients distributed uniformly. In reference 13 this property was quantified in terms of the non-Gaussian probability distribution of two-point differences, or increments. Compared with that applicable to a Gaussian process, the distribution of two-point differences in an intermittent process is stronger-tailed (high kurtosis), typically exponential.

Incorporating the above properties, reference 13 introduced a model of turbulence comprising an ensemble of discrete gusts and showed how the probability distribution of loads on an aircraft could be expressed as an integral over this ensemble, where individual discrete gusts were assumed to have independent effects and the integral is two-dimensional, over lengths (gradient distances) and amplitudes. A final major step was to exploit the fact that assuming the amplitude distribution of discrete gusts at any particular scale to be exponential, the above integral takes a form for which a standard asymptotic form exists, the Laplace approximation. In this form, the distribution of loads of large amplitude is dominated by the effects of gusts whose lengths (gradient distances) lie in the vicinity of a particular tuned gust length, which depends upon the dynamics of the aircraft. With this simplifying feature, the method became tractable. The resulting theory thus implemented successfully the approach originally proposed by Zbrozek [9], as discussed in the previous section.

The theory was further consolidated in a report in the following year [15] in which it was applied in an analysis of the acceleration response at an aircraft center of gravity. The discrete gust took the form of a single ramp and the response was assumed to be relatively well damped. It was also shown how, by applying the theory to the response of a digital smoothing-and-differencing filter, a method was obtained for analyzing measured turbulence data, displaying the results in a form such that the free parameters in the SDG model could be inferred directly.

In 1971, it was shown [16] how the SDG model could be used to formulate both Design Envelope and Mission Analysis criteria, in a format analogous to that of the PSD method. In the former case, the criterion is specified in terms of a worst-case, the statistics being incorporated implicitly, whereas in the latter case, the statistical formulation is explicit. Preliminary consideration was also given to the possible effects of gust clustering, and the effect of a sequential pair of ramp gusts was discussed in terms of the possibility of resonant peak amplification associated with the combination of gusts (appendix B). However, the step of incorporating gust patterns, comprising clusters of ramps with specified probabilities, was not taken at this stage. In the same year (1971), the SDG model was applied [17] to investigate the statistics of fluctuations in the speed of a controlled aircraft due to turbulence at low altitudes.

The introduction of gust patterns, comprising clusters of ramps with specified probabilities, depending upon the numbers of ramp components, was first described in 1973 [8], when patterns comprising just a sequential pair of ramp components were considered. To achieve equality of probability, it was proposed, on the basis of an investigation [18] by Hawker Siddeley Aviation Ltd. at Hatfield (UK), involving a dynamic model for aircraft loads based on a Trident aircraft, that the amplitudes of the individual ramps in a ramp pair combination should be reduced to 70 per cent of the amplitude of an isolated single ramp. For an aircraft whose response includes both rigid-body and flexible modes, the tuned response to a ramp pair arises typically in a situation in which the first ramp of the pair excites rigid-body response and the second, shorter, ramp excites the flexible response, the spacing between the two ramps being such as to induce the maximum load reinforcement, or transient resonance (appendix B).

The step of introducing gust patterns having greater numbers of components was described in 1976 [19], when patterns comprising up to eight component ramps were considered. For a gust pair, the proposed amplitude reduction factor (now referred to as a complexity factor, or p-factor, section 5) was taken to be 0.85 (compared with the factor of 0.7 used in previous work [8]), the corresponding amplitude reduction factors for patterns comprising respectively of four- and eight-component ramps being 0.6 and 0.4. The use of just this logarithmic sequence of patterns with one, two, four, and eight components was found, in practice, to give sufficient coverage to responses that could strictly tune to intermediate cases. [Note, subsequent work indicated the advisability of including just the single intermediate case of three-ramp patterns.] The above numerical values were provisional estimates based on digital simulation, using measured samples of continuous turbulence of moderate intensity, of the responses of simple dynamic systems including single degree-of-freedom oscillatory systems covering a wide range of damping ratios.

It may be noted that on the basis of a particular measurement made near storm tops, reference 19 also included a proposal to include an explicit class of vortex patterns in a discrete gust model for limit loads. While this proposal has not been followed up in practice, subsequent measurements of severe turbulence [20 and 21] have indicated that it should be given serious consideration (see section 9).

In 1977, applications of the SDG method to aircraft ride quality [22] and autoland systems [23] were described, the model being adapted to wind shear in the latter case, and a review of the method was incorporated in a wide-ranging survey [24] of severe gusts and wind shear, including physical mechanisms and effects on aircraft, presented as a contribution to a course on aircraft accident investigation. The use of the SDG method to represent wind shears for automatic landing assessment was its first application to a strongly nonlinear problem, the response quantities of interest being rate of descent at touchdown and the distance of touchdown from the desired point on the runway. Horizontal wind profiles comprising just a sequential pair of ramps were found (by a global search procedure) to give a typical critical case, the first ramp causing a large airspeed increase, causing the engines to be throttled back, and the second ramp causing a subsequent sharp loss of airspeed at a time when the engine revolutions were uncomfortably low.

Significant progress with the development of the SDG method was made at this time through simulation studies made by British Aerospace (BAe) Aircraft Group at Weybridge. Over the period 1972-1975, BAe performed an investigation of the application of PSD methods to gust design procedures [25]. However, they retained the view that large gusts of design magnitude often stand out as discrete events, above a background of more moderate atmospheric fluctuations [4 and 6]. Thus, it was felt that extreme gust events, pertinent to aircraft limit loads, may be better described by isolated gust procedures. In view of this, BAe initiated a research program to investigate the fundamental assumptions inherent in the SDG model, with a view to developing a procedure for the improved prediction of aircraft gust loads.

Preliminary results were published in 1978 [25]. From a digital simulation study of the SDG model, using samples of measured turbulence of moderate intensity, it was concluded that the effects of oscillatory response required the inclusion of gust patterns comprising two or more ramp components. For aircraft lateral motion, where a lowly damped Dutch Roll mode dominates the response, the critical gust pattern usually consisted of four or eight successive ramps. Provided that such compound gust patterns were taken into account, it was concluded [25] that there existed a sufficient basis for the SDG model to warrant the formulation of a tentative airworthiness requirement based upon this procedure. It was also concluded [25] that the SDG method compared favorably with the PSD method on the basis of numerical simulation studies of both methods using real turbulence samples.

An explicit formulation of the SDG method as a design envelope airworthiness requirement for large structural loads was proposed in 1978 [26], and its relationship to the existing FAA PSD and BCAR discrete gust requirements was discussed. It was demonstrated that both continuous turbulence and relatively isolated discrete gusts could be accommodated, by different choices of amplitude-reduction factors (now referred to as complexity factors, or p-factors) in the SDG model, within a single unified approach.

Further information providing details of the development and application of the SDG method during the period 1968-1978 is provided in references 27-33.

2.2.2 1979-1993.

In the preliminary work by BAe [25], originally published in 1978, the amplitude-reduction, or complexity, factors used in the SDG model were obtained empirically by analyzing the responses of systems with different damping ratios to samples of moderate continuous turbulence. However, in subsequent work [25], originally reported in 1981, studies using data from gust encounters recorded during routine operational flying by civil aircraft [12 and 34] showed that in order to represent such severe turbulence, amplitude-reduction factors were required that were much more strongly attenuated, as the number of ramp components increased, than those appropriate for moderate continuous turbulence, thus reflecting the fact that this more severe turbulence tends to occur in short bursts. With this modification incorporated, it was concluded [25] that “the SDG theory does seem to be a representative and practical atmospheric model for use in aircraft gust design procedures.”

At about the same time, proposals were put forward for a change in the scaling law relating gust amplitude to gust gradient distance. In previous reports on the SDG method, the representation of atmospheric turbulence as a self-similar random process had assumed that discrete gust amplitudes scaled according to a one-third power of gradient distance. However, in 1979 [35] a modification of the SDG model exhibiting a systematic departure from self-similarity was introduced according to which, for the more intense gusts, the power-law exponent takes a value less than one-third. This revised geometrical model, according to the more recently introduced concepts of fractal geometry, was related to the fractal dimension of the active turbulent fluctuations which become concentrated on to ever-smaller proportional regions of space as scale is reduced (so-called scale-dependent intermittency). In reference 35, experimental data were presented that were consistent with this development, and on this basis, it was proposed that at the low level of probability relevant to the prediction of aircraft limit loads, a discrete gust amplitude power-law exponent of one-sixth be used, rather than the value of one-third used previously. Since the experimental data were also consistent with the fact that the root mean square (rms) gust amplitude scaled according to one-third, the implication was that the ratio of the amplitude of an extreme gust, of specified low probability, to the rms gust amplitude, increases as gradient distance is reduced. This non-Gaussian property of extreme gusts contrasts with the Gaussian relationship, assumed in the PSD method, in which the ratio of the amplitude of an extreme gust, of specified low probability, to the rms gust amplitude is a constant, independent of gradient distance.

This proposal was followed up in the same year (1979), by a reappraisal of the statistical characteristics of extreme atmospheric gusts [36], in which the proposed revised scaling law of one-sixth was further examined in the light of available experimental data, and it was shown how the revised power law could be incorporated into a proposal for a design envelope airworthiness requirement expressed in terms of the SDG method. A significant conclusion of this study [36] was that the use of a revised SDG procedure with a one-sixth scaling law for extreme gust amplitudes would not only be consistent with the more recent theories of turbulence (subsequently reviewed in reference 37) but would alleviate existing disparities between traditional and revised requirements for any class of aircraft. An update of progress with the SDG method was issued in 1981 [37]. This presented in detail the statistical theory for predicting aircraft response, in a form applicable to both linear and nonlinear systems, and showed how the concept of scale-dependent intermittency and the associated one-sixth scaling law could be incorporated.

A feature of the SDG method is that system response is evaluated in terms of a worst-case input, in the sense of producing maximum system response, chosen from a family subject to a prescribed constraint related to probability. In reference 38, alternative formulations of worst-case analysis were compared and relationships were illustrated between worst-case deterministic response and the response to stochastic, or random types of input. As pointed out in reference 38, the problem is in fact closely related to the matched-filter problem arising in radar detection. A change in the system characteristics will, in general, lead to a change in the related worst-case input waveform: system and input waveform are in a sense matched to each other.

Following up this idea, considerable effort was devoted, in the early 1980s, to clarifying the relationship between the SDG method and the PSD method. This work involved showing how the latter theory could be reformulated in the time plane and implemented by a variational technique, referred to as the method of equivalent deterministic variables [39-43]. It was also shown how the solution of this variational problem could be interpreted as another instance of finding a worst-case input, which is a most probable input as well. However, the concept of a most probable time history requires that a probability distribution be defined on the sample time histories of the random process under consideration. In references 41 and 42, it was shown how relative probabilities of sample time histories from a Gaussian process, as assumed in the PSD theory, may be quantified and related to the power spectrum of the process.

The conclusions of this work were summarized in reference 44. In particular, an overlap was demonstrated between the SDG and PSD methods such that with appropriate choice of free parameters in the SDG model, the former becomes essentially an approximate numerical implementation of the latter. (In particular, this requires that a one-third scaling law be used in the SDG model.) Particular numerical values for these free parameters were specified, which it was claimed [43-45], would allow the standard PSD response quantity A-bar to be approximated by using an SDG procedure. In retrospect, it can be seen that such a choice of SDG parameters corresponds to approximating the properties associated with random phase and Gaussian distributions.

While the purpose of the claim was simply to enhance confidence in the statistical credentials of the SDG method, subsequent work by NASA [46], which confirmed this claim (within a numerical scatter of about 10%), was in some quarters interpreted as being in support of the SDG method as an alternative computational implementation of the PSD method, proposed for practical use. This was never the case, as was emphasized in reference 44 and clarified in a subsequent statement by NASA (see the discussion in references 47 and 48). The work on the SDG and PSD overlap proposed in references 44 and 45 was intended purely to provide a basis of common ground, in terms of the departures that resulted when the SDG model parameters were matched to real turbulence data (and hence, non-Gaussian properties associated with phase correlation incorporated) would be better understood.

An incidental spin-off from the work described above, concerning the expression of power spectral procedures in the time plane [39-43], was a reformulation of the (PSD-based) Design

Envelope criterion, in the mandatory aircraft limit-load requirements, as a Deterministic Spectral Procedure (DSP) [49 and 50]. In this form, the criterion becomes equally applicable to both linear and nonlinear aircraft. Thus, this procedure provided a proposed means of applying the current PSD requirements to nonlinear aircraft.

Algorithms for fitting the SDG model parameters to measured turbulence data, first introduced in reference 15, were described in detail, with illustrative examples, in reference 51. An application of this analysis method to measured wind shear data was presented in reference 52. The existence of such an SDG-related data analysis procedure, applicable to measured turbulence-velocity time histories, is regarded as an essential component of the overall SDG method. In this context, it may be noted that since the representation of severe gusts is strongly influenced by the effects of phase correlation in the frequency plane ([21] and figure E-1 of appendix E), a property not taken into account in the PSD model, it is a major limitation of this latter model that the most significant information in measured severe gust time histories cannot be used in its calibration or assessment.

The version of the SDG model as it existed in 1989 is summarized in reference 53. In this version, localized gust patterns are represented as clusters of discrete ramp hold elements, with regions of constant wind velocity separating the ramps. Individual ramps in such a cluster are assumed to have equal probability (equipartition of probability). With appropriate parameter settings, this model may be used to provide a statistical representation of either moderate continuous turbulence (random phase) or relatively isolated severe discrete gusts (correlated phase). The statistics of aircraft response, which may be linear or nonlinear, are derived by an application of the Laplace asymptotic approximation and are expressed in a form that shows the dominant influence of a particular tuned, or worst-case, gust pattern, which excites a transient resonance in the aircraft dynamics.

In 1992, the concepts of wavelet analysis were introduced into gust representation [54 and 55]. A wavelet is essentially a highly localized packet of phase-correlated Fourier components. In 1993, it was shown [56] how these techniques could be used to quantify the discrete gust structure in measured atmospheric turbulence, in a manner totally consistent with the SDG model. Further details of the wavelet methodology were presented in reference 57. Subsequently, wavelet analysis was introduced as a basis for the reformulation of the SDG model (see section 2.3).

Further information providing details of the development and application of the SDG method during the period 1979-1993 is provided in references 58-65.

2.3 DEVELOPMENT OF THE SDG METHOD, 1994 TO 1997.

It was proposed at a meeting of gust specialists in 1994 [66] that the SDG model offered the possibility of unifying the current separate requirements for flight in so-called continuous turbulence and for encounters with isolated gusts (appendix E). Such a unification would result in both an overall reduction in the amount of computation required during the design process to validate an aircraft for flight through atmospheric turbulence and an increase in the degree of realism in the resulting model, achieved by incorporating data from routine operational flying.

In the latter case, the model could be subjected to continuous review in light of special events or incidents in the form of severe gust encounters.

Subsequent to this proposal, a work program was funded by the FAA to conduct a feasibility study [67] to assess the suitability of the SDG method as a basis for a future airworthiness requirement. This would take an updated form that incorporated recent developments, including wavelet analysis and, for application to nonlinear aircraft response, recently developed stochastic search methods, including genetic algorithms. New SDG algorithms, based on wavelet analysis, for the application of the SDG method to linear systems were implemented in the scientific computer language Matlab [68 and 69]. Two versions of the method were implemented: SDG1 and SDG2. The associated algorithms generate the worst-case responses when the SDG model is matched respectively to atmospheric measurements with relevance to extreme turbulence (phase-correlated) on the one hand and to the existing PSD model (random phase) on the other (exploiting the overlap described in section 2.2.2).

SDG1 employs a one-sixth power law and gust patterns comprising up to four elementary gust components. SDG2 employs a one-third power law and gust patterns comprising up to eight elementary gust components. It should be re-emphasized that SDG2 was never intended as a practical tool but simply as part of a validation exercise for SDG concepts. The results of this feasibility study are described in reference 70.

The basic properties of the SDG1 model are summarized in section 7.2. Comparisons of SDG1 design gust amplitudes with the corresponding design amplitudes resulting from the PSD method and from the tuned Isolated Discrete Gust criterion, using a provisional calibration of SDG1 and a linear aircraft model corresponding to an Airbus-type aircraft, were presented at gust specialist meetings in 1996 [71] and 1997 [72], respectively.

Following a discussion of the results of this feasibility study [70], the majority of members of the Gust Specialists Committee were of the opinion that the proposed replacement of the current separate requirements, for flight in so-called continuous turbulence and for encounters with isolated gusts, by a single requirement would be premature in 1997. Reasons in support of this view were that the established existing requirements are associated with a satisfactory safety record and that the SDG method, as proposed, required specialized wavelet software. As a result, there would be unacceptable costs in implementing a change for which there was at that time no demonstrated need.

Applications of the SDG method to problems outside the field of aircraft loads were also pursued during the period 1994-1997. One example is the application of the SDG model in the context of helicopter flight dynamics, described in reference 73.

2.4 DEVELOPMENT OF THE SDG METHOD, 2000 TO PRESENT.

In September 2000, to take into account the objections raised in 1997 to the adoption of the SDG method as a revised airworthiness requirement, a simplified version of the SDG method was proposed [74]. The implementation involved a return to the use of ramp-hold gust components subject to equipartition of probability among the components in any given gust pattern (as in reference 53. It was demonstrated [21] that following this approach, a gust criterion for limit loads could be formulated in sufficiently simple form, whose implementation avoided the need for the specialized wavelet software used by the SDG1 method.

Examples illustrating the new method [21 and 74] were computed using provisional Matlab algorithms (referred to as SDG-00-AS). An advantage of Matlab is that it not only runs useable code on a personal computer but it is also written at a sufficiently high level that it can act as a detailed specification of the method that could easily, if required, be translated into manufacturers' internal code. Progress with this approach continued since the September 2000 meeting in terms of both technical development and simplification of software implementation. These are discussed below.

The first technical step concerns gust overlap. The code used to compute the examples presented at the September 2000 Gust Specialists' meeting contained parameters that controlled the degree of overlap allowed between component ramps in any given gust pattern. Experiments with the effects of varying these parameters, together with the consideration that the associated airworthiness criterion should be kept as simple as possible, led to the subsequent decision to freeze these parameters such that no overlap between component ramps was allowed. A second, more major, change was the introduction of a Staircase Reduction Factor (SRF) that can be used to scale (reduce) the amplitude of a gust pattern, and the associated load response, in situations where the gust pattern includes a two-ramp staircase, i.e., two consecutive ramps in the same direction (up-up or down-down). With the SRF incorporated, the probabilities of such staircases and of up-down patterns are equalized. The resulting revised set of algorithms is referred to as SDG-AS-SC (described in section 7.3 and reference 75), where the AS standing for alternating sign and SC for staircase. This gust model provides a detailed implementation of a revised proposed gust criterion, appendix C.

Although the code used to implement the simplified SDG method originally proposed in reference 74 (SDG-00-AS) was written in Matlab, it contained files written in C-code (and compiled as .mex or .dll files), which were derived from the software suite used to implement SDG1 [68]. In order to achieve a more visible correspondence between the code and the formulation of the method in the proposed draft criterion (appendix C), all files originally written in C-code have been replaced, in SDG-AS-SC, by code written entirely in Matlab (.m files). In particular, the replacement of all C-code involved completely rewriting the code for generating ramp profiles and for detecting extreme values in the associated response. The resulting code is much simplified; it involves only elementary operations to detect extrema, applied to response evaluations on a fixed two-dimensional grid of ramp gradient distance versus time. In the earlier code, the detection of extrema on a fixed grid was followed by a local search involving points off the grid. To compensate for this increased simplicity, the grid spacing in the revised code has been reduced.

In its current form, the software package SDG-AS-SC (section 7.3, and reference 75) can be used to implement the SDG method on a personal computer. Acting as a detailed specification of the method, it supplements the information contained in the proposed draft criterion, (appendix C). The Matlab code for SDG-AS-SC allows manufacturers (or the certification authorities) to easily make comparisons with existing design loads and also provides a template for implementing the method in in-house code.

3. EVIDENCE THAT SEVERE-TO-EXTREME GUST ENCOUNTERS ARE CHARACTERIZED BY SHORT-DURATION BURSTS OF NON-GAUSSIAN TURBULENCE.

The most visible consequence of non-Gaussian characteristics in the form of phase correlations in severe turbulence lies in the short-duration bursts that are observed in both records from routine operational flights by civil aircraft and in data recorded by specially instrumented aircraft that have flown through, and in the vicinity of, storms and other meteorological sources of severe wind fluctuations.

3.1 EVIDENCE FROM ROUTINE AIRLINE FLIGHTS.

The earliest documentation of the fact that the most severe gusts encountered in operational flights by civil aircraft are characterized by short-duration bursts of non-Gaussian turbulence goes back to the early 1960s, when the British Civil Aircraft Airworthiness Data Recording Program (CAADRP) was initiated. As described in reference 12, continuous trace records of airworthiness data were taken initially from a small number of aircraft in normal airline service beginning in 1962. In this initial study [12], the acceleration traces on a selection of records covering approximately 3000 flying hours were analyzed in terms of peak values. The durations of patches of turbulence were estimated and the most severe of these patches studied in detail.

From an examination of 24 of these severe patches, it was concluded [12] that the largest acceleration in a patch is on average about 30% greater than would be expected from a Gaussian distribution (in some cases the excess is substantially greater than 30%). It was also pointed out that 9 of the 24 severe patches occurred without warning, and hence, although the technique of reducing airspeed in turbulence does benefit fatigue life and passenger comfort, it cannot ensure that the largest gusts are always met at the reduced airspeed.

Tables 6 to 8 of reference 12 provide quantitative evidence of the highly non-Gaussian structure of the most severe patches. Data presented include patch duration and numbers of times specified levels of acceleration were exceeded in each patch. In many cases, the measured patch duration is 30 sec. or less, and in these cases, the maximum recorded peak acceleration can be seen to exceed the magnitude of the second largest recorded peak by an amount (up to a factor of two) significantly greater than would be expected from a Gaussian distribution.

The characteristics of severe turbulence suggested by this initial study of continuous trace records were subsequently tested by examining turbulence encountered over a larger number of flying hours. Records from over 20,000 flying hours were searched for patches of severe turbulence, and those which produced the highest acceleration increments on the aircraft are presented in detail in reference 34. It was found that the largest acceleration increment in a patch is on average 1.3 times the value that would be predicted from a Rayleigh distribution of peaks, (i.e., the peak distribution for a Gaussian process). The two principal conclusions from this study [34] were (1) the characteristics of severe turbulence are non-Gaussian and (2) there is a one in two chance of severe turbulence occurring without warning: the shorter a patch, the less likely it is that any warning is available.

Tables 1 to 5 of reference 34 provide further quantitative evidence of the highly non-Gaussian structure of the most severe patches. Data presented again include patch duration and number of times specified levels of acceleration were exceeded in each patch. The measured patch duration is frequently 30 sec. or less, and in these cases, the maximum recorded peak acceleration can be seen (tables 1 to 5) of reference 34 to exceed the magnitude of the second largest recorded peak by an amount (up to a factor larger than two) much greater than would be expected from a Gaussian distribution.

A further extreme example from the 1960s, illustrating violent g (normal acceleration) excursions confined to just one 3-second period is illustrated in figure 1 of reference 24. This takes the form of a flight recorder data graph (U.S. National Transportation Safety Board (NTSB)) of in-flight turbulence encountered by a Boeing 727 en route from Las Vegas to Los Angeles and includes within this period transient incremental excursions in normal acceleration of -2.3 g and +1.4 g.

More recently, digital flight records from reported clear-air turbulence incidents during operational airline flights have been studied by NASA Ames, in cooperation with the U.S. NTSB [20 and 76], and found to contain many cases of relatively isolated severe gusts occurring downwind of mountains and thunderstorms causing sharp, sudden jolts. Other cases of severe turbulence have been found in strong updrafts above thunderstorm buildups that may be undetected by onboard weather radar. In reconstructed time histories, illustrated in references 20 and 76, the part of the turbulence time history associated with severe loads takes the form of very localized transient fluctuations.

In reference 20, two instances of severe clear air turbulence are described in detail. In each case the normal acceleration records are qualitatively very similar to those recorded in the earlier British study [12 and 34]. In this later study however, the digital records contain sufficient information to allow reconstructions of estimated time histories for the vertical wind. In the first case (figure 6 of reference 20), the most severe vertical wind fluctuations take the form of a sequence of four successive ramp-shaped gust components, with the maximum increment in normal acceleration (-1 g) aligned in the time history with the end of the fourth ramp. In the second example (figure 9 of reference 20), the most severe vertical wind fluctuations take the form of a sequence of two successive ramp-shaped gust components, causing a maximum increment in normal acceleration of approximately -0.8 g. A principle conclusion of reference 20 is that the highly localized wind fluctuations occurring in these incidents can be interpreted as intersections of the aircraft flight path with layers containing discrete vortices.

Additional examples, in which the greatest increments in normal acceleration take the form of sudden sharp jolts associated with wind fluctuations in the form of a small number of successive ramp-shaped components, are illustrated in reference 76. As in reference 20, the conclusion is drawn that in a high proportion of the encounters the most severe wind fluctuations are vortex-induced.

3.2 EVIDENCE FROM RESEARCH FLIGHTS.

Evidence that the most intense turbulence is highly non-Gaussian was also provided by data from research flights performed in the 1960s. These investigations were primarily concerned with turbulence in and around thunderstorms and had as a primary objective the acquisition of information relevant to the interpretation of weather-radar images.

An early example of this work is presented in reference 77. This paper reviews atmospheric turbulence measurements and associated aircraft response measurement obtained generally in thunderstorms at altitudes up to 40,000 ft. The illustrated measurements were made during flight operations of the Weather Bureau National Severe Storms Project in 1960.

In the first example chosen (figure 1 of reference 77) to illustrate the nature of the turbulent flow within a thunderstorm cloud, the time histories of both the vertical and lateral components of true gust velocity are dominated by a severe and sharp ramp-shaped discontinuity (at 50 sec.). In this instance, an associated change of 2.5 g was experienced in normal acceleration. A second example in which the vertical component of true gust velocity contains a large ramp-shaped increment (at approximately 152 sec. in the time history) is shown in figure 2 of reference 77.

The Weather Bureau National Severe Storms Project continued in subsequent years, and in 1965 a research team from the United Kingdom's Royal Aircraft Establishment took part, using specially instrumented aircraft. In one aspect of this work, 100 storm penetrations were made by two specially instrumented aircraft at altitudes between 23,000 and 37,000 ft. In another part of the program, severe gusts were encountered during flight through clear air at altitudes between 40,000 and 45,000 ft above storms. While the primary purpose of these collaborative studies was to investigate correlations between regions of severe turbulence, weather-radar imagery, and associated meteorological conditions, information was also gathered concerning the statistics of aircraft response during the turbulence encounter. In particular, it was concluded [11] that the percentage of runs in which the maximum normal acceleration exceeds a given multiple of the rms is much greater, in both convective cloud and thunderstorm flights, than would be the case with a Gaussian process.

While much of the information concerning the nature of turbulence patches, obtained from research flights, took the form of measured records of aircraft normal acceleration, in some cases, measured time histories of true gust velocity are available. Two examples of very short patches of highly non-Gaussian turbulence, measured in clear air near thunderstorm tops, are shown in figure 5 of reference 11. Four examples of relatively isolated gusts from a U.S. High Intensity Gust Investigation, obtained by an F-106A aircraft in thunderstorms, are shown in figure 2 of reference 78. An example of an isolated gust measured near storm tops recorded during the U.S.-UK collaboration is shown in figure 3 of reference 19. In the case of the F-106A data, the first example comprises a single intense ramp, and the second and third cases are dominated by a pair of sequential ramps of opposite sign, with different spacings (compatible with the ramp gust patterns employed in the SDG model). Both the fourth example from the F-106A aircraft, measured in a thunderstorm, and the example measured in clear air near storm tops shown in reference 19 have velocity profiles consistent with the intersection of the aircraft flight path with a vortex core. The possibility of incorporating such a vortex profile explicitly into an SDG gust model, for design purposes, is discussed in reference 19. This question is considered further in section 9.

More recent evidence, from research flights, that the most intense fluctuations in severe turbulence are more strongly localized in space than is the case in a Gaussian process is presented in section 5, where it is demonstrated how this phenomenon is quantified in the SDG model of severe turbulence in terms of complexity factors.

4. EVIDENCE FOR THE SCALING LAW RELATING GUST AMPLITUDE TO GUST GRADIENT DISTANCE.

4.1 HISTORICAL BACKGROUND.

The primary parameters of a discrete gust that influence aircraft response are the maximum change or increment, w, in gust velocity and the distance, H, over which this change occurs. The scaling law relating gust amplitude w to gust gradient distance H, to achieve a constant level of probability, is expressed in the form $w \sim H^k$ in the SDG method (appendix A). The historical background of studies of this scaling law is reviewed in reference 36, from which the following is adapted. It was as early as 1937 that Richard V. Rhode outlined a NACA program examining the relationship between gust intensity and gradient that suggested that the stronger gust velocities have relatively mild gradients. The data were obtained from measurements of the motion of small airplanes in gusts, and the trend of the data showed that, with increasing w, not only did H increase but the gradient w/H actually decreased. An elementary consideration of the transfer of turbulent energy was presented with the conclusion that w should be proportional to $H^{1/3}$. The data fitted this relationship reasonably well, and it was pointed out that families of curves $w \sim H^{1/3}$ could be drawn, each representing a given condition of the roughness of the atmosphere.

This early work appears to have gone largely unnoticed and the basic ideas were subsequently discussed from first principles by independent authors [36]. In particular, the classical reference is generally taken to be Kolmogorov, who in 1941 presented the well-known form (spatial frequency)$^{-5/3}$ for the turbulence energy spectrum and showed that this implied the $w \sim H^{1/3}$ relationship for turbulence-velocity differences or increments, w, over a distance H.

A subsequent step developing the possible use of scaling laws relating w and H was taken by Fletcher, who in 1968 collected extreme turbulence data to illustrate a relationship between gust velocities (increments w) and gradient distances H. On this basis, the $H^{1/3}$ law was tentatively proposed, by Fletcher, for use in studies of aircraft response to discrete gusts.

At about the same time (1967-68), work was in progress at the Royal Aircraft Establishment in the UK which led to the proposal [13] of a two-dimensional (w and H) probability distribution for discrete ramp gusts that incorporated the $w \sim H^{1/3}$ relationship. The form proposed for this probability distribution was influenced by work on the concept of self-similarity due to Mandelbrot (see reference 36 for details). Subsequent data analysis confirmed broad conformity with this proposed distribution [19].

However, while self-similarity of turbulence provided a good first approximation, previous work (1962-64) by Kolmogorov and others (see reference 36 for details) concerning the probability distribution of turbulence velocity had already indicated that the more intense turbulence fluctuations at differing wavelengths in fact show systematic departures from self-similarity and the associated $w \sim H^{1/3}$ scaling law. As reviewed in reference 35, and discussed in reference 37, this work underlined that the $w \sim H^{1/3}$ scaling law is not exact, and that a modified scaling exponent is strictly required by a theory whose objective is to relate fluctuations covering a wide range of scales and intensities.

In particular, Mandelbrot introduced revised scaling laws based on the concept of fractal dimension D where, for turbulence, D lies between 2 and 3. The case D = 3 corresponds to simple self-similarity and the $w \sim H^{1/3}$ scaling law. Values of D < 3, however, more characteristic of measured turbulence, lead to a revised scaling law for turbulence-velocity differences or increments w, of the form $w \sim H^k$ where k takes a value less than one-third. A value of k less than one-third, associated with a value of D less than three, corresponds to the situation in which the strengths of the tails of the probability distributions of velocity differences increase as the gradient distance is reduced; in mathematical terms the kurtosis of the distribution increases as gradient distance is reduced. In engineering terms, a reduction in k increases the relative amplitudes of short gusts.

In 1979 [35], it was shown how a revised scaling law, based on work by Mandelbrot, could be incorporated as a straightforward modification of the SDG method and the consequent implications for predicted aircraft response were discussed. Experimental data were presented that supported the use of a traditional self-similar model (D = 3, k = 1/3) for turbulence fluctuations of low to average intensity but were equally consistent with a fractal (Mandelbrot) representation with D = 2.5, k = 1/6, at the highest intensities. Thus, for the prediction of aircraft limit loads, where extreme levels of turbulence are relevant, it was proposed that consideration be given to the use of a model with k = 1/6, while retaining the self-similar model (k = 1/3) for purposes such as ride quality or handling qualities assessment and possibly structural fatigue.

However, direct evidence for the choice of D and k remained inconclusive at the higher levels of intensity. While k = 1/6 was a limiting value consistent with the most recent theories of turbulence [79], adequate direct evidence for the appropriate scaling laws at the higher intensities in atmospheric turbulence was not available. As a result, subsequent analysis of data from the UK Gnat flight research program (section 4.2) incorporated a study of the scaling relationship between w and H and the dependency of this relationship upon the intensity of the fluctuations [80].

4.2 TURBULENCE MEASUREMENT PROGRAM.

The atmospheric turbulence measurement program, which has been the primary data source for investigating the scaling law relating gust amplitude to gust gradient distance, is described in detail in references 81-85. Measurements of turbulence velocity were made by a specially instrumented small military trainer aircraft (Folland Gnat) at altitudes up to about 1000 feet over a variety of terrain. The aim of the program was to sample the turbulence encountered under a range of atmospheric conditions over various types of terrain and at a number of heights. A total of approximately 400 flights resulted in data that has subsequently been analyzed. In most cases, the pilot made at least three runs along a specified straight track at radio altimeter heights of 250, 500, and 1000 ft. (75, 150, and 300 m). Because maintaining constant radio height over the most rugged (mountainous) terrain was not possible, these nominal heights tended to become minimum heights.

The instrumentation carried by the aircraft is described in detail in references 81-83. The main air data sensors were pairs of Conrad Yawmeters and miniature pitot tubes on the extended nose probe of the aircraft. A Conrad Yawmeter consisted of two hypodermic steel tubes with angled ends. The pressure difference across each double tube was sensed and recorded. In addition to the Conrad Yawmeters on the nose probe, incidence and sideslip were also measured by wind vanes carried on the nose probe and on wing tip probes. Balsa wood vanes were used because it gave better frequency response than metal or carbon-fibre vanes due to their low inertia, at the expense of being less robust.

An instrumentation pack occupied the rear cockpit (the aircraft was basically a two-seat trainer) and contained a MODAS digital tape recorder used for data capture. This gave a 12-bit resolution and the majority of the instruments were recorded at 256 samples per second. The instrument calibration included applying the results from wind tunnel tests to get airflow directions and speed at the nose probe based on data from the Conrad Yawmeters and miniature pitot. The derivation of time histories of the atmospheric turbulence encountered during the measurement runs involved correcting for instrument dynamic behavior and removing the effects of aircraft or, more accurately, sensor motion from the measurements of airflow speed and direction. Full details of the instrument dynamic corrections and the removal of the effects of sensor motion to obtain records of true turbulence velocity are given in references 81-83.

Three measured components of turbulence were obtained, referenced with respect to an aircraft body axis system. These are denoted by u_g (head-on), v_g (side), and w_g (normal). Although they are not components in earth axes, in practice the restriction to a straight track, the fairly small pitch attitudes reached in the turbulence-measuring runs and the pilot's efforts to keep the wings level mean that the normal component, w_g, is very nearly the same as the vertical component. The x direction, in body axes, is along the aircraft forward path and the turbulence field was traversed at an aircraft flight speed of typically 180 m/s. Although the turbulence fluctuations were measured as functions of time, for analysis purposes they were converted to functions of position in space, assuming the standard frozen field (Taylor's) hypothesis and using the mean aircraft forward speed for the run in question.

4.3 DATA ANALYSIS METHOD.

This section outlines the analysis method used to determine numerical values for the scaling exponent k in the SDG model (see appendix A for a definition) from measured data. The procedure depends on the associated theory, described for example in reference 53 and reviewed here in section 6, for relating the statistics of system response to the statistics of the turbulence input. For prescribed values of the defining parameters in the SDG model, including scaling exponents, the theory for linear system response can be used to predict the rate of occurrence of peaks in the response of a prescribed system, as a function of threshold-exceedance level. Conversely, regarding the system as a window through which the turbulence is observed, numerical values for the input model parameters, including scaling exponents, can be inferred by matching measured to predicted response statistics. When measured data are available in the form of records of turbulence velocity, this approach may be applied using systems implemented as digital filters, designed specifically for the detection of ramp gust components, whose response time histories are calculated by standard numerical methods. Specifically, the filter used to detect the occurrence of a ramp gust-shaped profile is a smoothed-difference filter. The methodology involves applying a set of smoothed-difference filters to a measured sample of turbulence velocity, to detect ramp-shaped profiles covering a range of scales, or gradient distances, and processing the resulting set of response peaks to estimate the scaling exponent k [80]. For each gradient distance H, a digital-smoothing filter is applied to remove fluctuations over intervals much shorter than H (i.e., a low pass filter in the frequency domain) and the filtered record is differenced over a length H (i.e., the sample value at a lagged distance H is subtracted from each sample). After these smoothing and differencing operations, the occurrence and amplitude of peaks (and troughs) in the output signal are identified. The result, for each value of H, is an observed cumulative distribution n(H,x) of the number per unit distance of peaks with magnitudes greater than x, for a series of levels of x. The use of this method to fit model parameters depends upon the theoretical relationship (section 6), defined by the SDG method, between the distributions n(H,x) of peaks in the filter outputs and the statistical model of turbulence used to represent the input. Full details are given in reference 51, 80, 81, 83, and 86.

4.4 RESULTS OF DATA ANALYSIS.

Early illustrations of the application of the above analysis method to individual samples of measured turbulence were given in reference 35. Although the results were suggestive that a scaling parameter of k = 1/3 was appropriate at low amplitudes, reducing towards k = 1/6 at the higher gust intensities, they were inconclusive. Subsequent to the availability of data from the Gnat measurement program , however, the multifractal model of turbulence, in which both the fractal dimension D and the scaling exponent k reduce in magnitude as fluctuations of increasing intensity are considered, was proposed [37]. The analysis results for a particular Gnat run, confirming the above trend, were illustrated in reference 87. While the value k = 1/3 was shown to fit the data well at the lower amplitudes, and to match the scale-dependence of the rms value of fluctuation amplitude, a numerical value of k = 0.23 was derived, using a least-squares error technique to fit the theoretical model to the data, over a band of larger fluctuation amplitudes that excluded the range from zero to twice the rms. Another particular example, showing the same trend, was illustrated in reference 56.

Subsequently, the same analysis method was applied [83] to a large number of runs from the Gnat measurement program. By fitting a theoretical statistical model to the measured data over a range of higher intensities, bounded below by twice the rms value of fluctuation amplitude, it was concluded [83] that, averaged over the complete set of 389 records, an overall estimated value of D for the lateral component (which was the least influenced by ground proximity) was about 2.6, with an associated estimated value of the scaling parameter k of 0.2.

While the measured values of D and k derived in reference 83 were consistent with the multifractal model of turbulence velocity fluctuations [79], there remained a need for a more detailed analysis of the dependence of these parameters on fluctuation amplitude. This called for a more selective choice of measurement runs in terms of atmospheric conditions.

The approximately 400 runs made by the aircraft covered a range of flight altitudes, conditions of terrain roughness, and wind conditions, resulting in a high degree of variability from run to run. In a study described in reference 80, measurements from runs with similar properties were merged to form data blocks, within each of which the conditions were constrained. A compromise had to be made in that it was desired to make the data blocks as large as possible, in order to improve statistical reliability, while at the same time, constraining the variability within each block. Specifically, the constraints restricted the measured power spectral exponents (spectrum slopes) in both the vertical and lateral components, the average intensities of the runs, and the average intermittency of the runs (according to a strict definition given in reference 80).

Three blocks of data were created, corresponding to low (L), medium (M), and high (H) levels of intermittency. Qualitatively, low intermittency corresponds to turbulence of a very continuous nature, uniform over the whole run. High-intermittency turbulence, on the other hand, shows a much higher degree of variability within the run, with discrete gusts more in evidence.

In the case of the lateral component of turbulence velocity, the results from all three data blocks showed [80] a consistent trend in which the scaling parameters D and k (see section 4.1) both reduced monotonically with increasing fluctuation intensity. The greatest variations in D and k were exhibited by block (H), for which D varied over a range from 3, at low fluctuation intensity, towards a value of 2.5 at the higher intensities, while over the same range of intensities k reduced from approximately 0.375 to 0.2. Similar trends, but with rather smaller variation, were exhibited by the data in blocks (M) and (L).

In contrast to the lateral component, the normal (vertical) component of turbulence velocity exhibited greater variation from run to run and power spectral densities that departed, on average, from the form expected in isotropic turbulence. Ground proximity was proposed as the major factor causing this difference between the two turbulence components. Only the values of D and k from block (H) showed a degree of variation, with increasing fluctuation intensity, comparable with that measured for the lateral component.

The overall conclusion was drawn in reference 80 that the scaling exponent k, which relates discrete gust amplitude to gust gradient distance, has been demonstrated to exhibit a variation with the intensity of the fluctuations, reducing from a value close to one-third at the lower amplitudes to a value of approximately 0.2 for the tails of the measured distributions. However, although derived from severe turbulence data, these measured values correspond to gust amplitudes well below the extreme levels that would cause critical loading conditions. To quote from reference 80, "extrapolation to larger amplitudes is an uncertain operation; nevertheless, the measured trends associated with varying amplitude are consistent with a lower bound of k = 1/6, the value prescribed (for isolated discrete gusts) in the requirements." An associated extrapolated value of D for these extreme intensities is of the order of 2.2.

4.5 OVERVIEW.

The following conclusions may be drawn for the law $w \sim H^k$ relating turbulence velocity differences or increments (ramp gust amplitudes) w to gust gradient distance H for a constant level of probability in atmospheric turbulence. Consistent with the classical Kolmogorov theory of turbulence, the rms amplitude of velocity differences scales approximately according to k = 1/3. This is also the scaling law for velocity differences at any prescribed level of probability in a stochastic process having a spectral density proportional to $(\text{frequency})^{-5/3}$ and a random phase distribution, as assumed in the PSD method.

However, the phase correlations that exist in real turbulence cause the probability distribution for velocity differences w, over any distance H, to be non-Gaussian, with kurtosis (which reflects the strength of the tail of the distribution) greater than that of a Gaussian distribution. Moreover, the tails of the distribution become stronger, i.e., the kurtosis increases, as H is reduced [87 and 79]. As a result, the ratio of the amplitude w of a discrete gust, having specified low probability, to the magnitude of the rms increases as H is reduced. Combining this result with the $w \sim H^{1/3}$ law for the rms, it follows that a scaling law of the form $w \sim H^k$ for a discrete gust at a specified low value of the probability will have a scaling exponent $k < 1/3$. The theoretical background and experimental research reviewed in section 4 indicate that a scaling law of the form $w \sim H^k$ with an extrapolated value of $k = 1/6$ is appropriate at the very low values of probability associated with the most extreme fluctuations. An associated extrapolated fractal dimension for these extreme fluctuations is of the order of $D = 2.2$.

5. EMPIRICAL EVIDENCE IN SUPPORT OF COMPLEXITY FACTORS TO REPRESENT SHORT-DURATION BURSTS.

5.1 HISTORICAL BACKGROUND.

In the SDG model of turbulence, the complexity of a discrete gust pattern is defined in terms of the number of elementary ramp components in the pattern. The dependence of gust amplitude upon complexity, for a given level of probability of occurrence, is expressed in terms of the complexity factors, or p-factors, p_i which depend upon the number i of components in the gust pattern. In the form of the model described in appendix A, taking the single ramp to be the reference case, $p_1 = 1$ and the p_i decrease monotonically with i.

Numerical values for the p-factors were originally obtained by computer simulation of the response of oscillatory systems [19 and 25], covering a wide range of frequencies and damping ratios, using measured samples of continuous turbulence of moderate intensity. In reference 25, the results were further substantiated by computer simulation of the loads response at various points in an aircraft structure, using a full range of flexible modes. The particularly simple empirical result was obtained that for patterns comprising nonoverlapping ramps of alternating sign, p_i depends to a good approximation only on the number of components in the pattern, as follows:

$$p_1 = 1 \text{ (datum case)},\ p_2 = 0.85,\ p_4 = 0.60,\ p_8 = 0.40$$

These values are quite close to the values:

$$p_1 = 1,\ p_2 = 0.81,\ p_4 = 0.57,\ p_8 = 0.40$$

shown subsequently [43 and 45] to be the values that define the relative amplitudes of patterns comprising ramp components for a prescribed level of probability in a Gaussian process, which has a random phase distribution. This result was the basis of the prediction [43, 44, and 45] that for continuous turbulence of moderate intensity, an equivalence, or overlap, exists between the results of a (suitably calibrated) SDG analysis and a PSD analysis (see discussion in section 2.2.2).

An implication of the assumption that the complexity factors p_i associated with a gust pattern comprising nonoverlapping ramp components depend only on the number i of components is that the probability of occurrence of the gust pattern is independent of the spacing between the component ramps. In fact, for a Gaussian process, while this can be shown theoretically [40] to be the case for a process whose PSD is proportional to $(\text{frequency})^{-2}$, for a process with PSD proportional to $(\text{frequency})^{-5/3}$, more representative of atmospheric turbulence, there is a small interaction between the ramp components such that, for a prescribed level of probability, the amplitudes actually increase by a small amount as the spacing tends to zero (figure 10 of reference 40). This result will be addressed in section 5.3 when the effect of gust spacing on probability is discussed in relation to measurements made in severe atmospheric turbulence.

The numerical values of p-factors given above were obtained from computer-simulation studies using inputs in the form of measured samples of continuous turbulence of moderate intensity, for which the phase distribution may be assumed to be approximately random. In subsequent work [25], however, by means of a technique involving numerical simulation of the response of digitally implemented oscillators, reflecting data recorded during routine flying by civil aircraft, values of complexity factors p_i were derived for gust encounters of sufficient intensity to produce a special event in which the aircraft response was at least 0.75 g. By curve fitting to the results at the higher level of intensity (in figure 23 of reference 25), the following values were derived:

$$p_1 = 1,\ p_2 = 0.705,\ p_4 = 0.42$$

(gust patterns with larger numbers of components having negligible effect).

Compared with the values derived for continuous turbulence, which have been shown to give results consistent with the results of a PSD analysis, the p_i for $i > 1$ can be seen to take numerical values that are significantly smaller. Thus, as a result of phase correlations, the very intense fluctuations tend to occur predominantly as gusts that are relatively isolated and, for this class of high-intensity events, a divergence occurs between the results of an SDG analysis and those of a PSD analysis. In effect, the SDG model quantifies the empirical result that the more intense fluctuations tend to occur in short bursts (section 3).

To complement the measured p-factors derived from data recorded during routine flying by civil aircraft [25], a more detailed study has recently been made [87] using severe turbulence data from the Gnat flight measurement program, discussed in section 4.2. The results of this analysis are discussed in section 5.3.

5.2 DATA ANALYSIS METHOD.

The procedure whereby the complexity factors are determined from measured turbulence data follows closely that for determining scaling exponents, see section 4.3. For prescribed values of the defining parameters in the SDG model, including the complexity factors, the SDG theory for linear system response can be used to predict the rate of occurrence of peaks in the response of a prescribed system, as a function of threshold-exceedance level (section 6). Conversely, regarding the system as a window through which the turbulence is observed, and using systems implemented as digital filters whose response time histories are calculated by standard computer methods, numerical values for the complexity factors may be inferred by matching measured to predicted response statistics.

As described in section 4.3, the filter used to detect the occurrence of a single ramp gust-shaped profile is a smoothed-difference filter. It requires only a simple extension of the above method [79] to detect gust patterns represented as linear combinations of ramp-shaped gusts, by means of associated linear combinations of smoothing and differencing filters, and hence to measure associated statistical distributions. As for single ramps, multiple-ramp patterns are detected as local maxima and minima in the associated filter outputs.

5.3 RESULTS OF DATA ANALYSIS.

5.3.1 Results for Severe Turbulence.

In reference 87, and using the source of data described in section 4.2, the above data analysis method has been applied to measure complexity factors for a range of pattern shapes and complexities in severe turbulence. The pattern shapes comprise linear combinations of nonoverlapping ramp components, including ramps of differing gradient distances, where the ramp components within each pattern are taken to have equal probability. The main results are summarized below.

The first set of patterns to be considered in reference 87 consists of up-down sequences of two ramps having equal gradient distances, where different individual patterns within the set have different spacing between the two ramps. Both vertical and lateral components of turbulence are investigated and complexity factors, denoted by p_2, are measured for this class of two-ramp patterns for gust gradient distances in the range 10 to 160 m. A trend is apparent [87] in which p_2 takes an approximately constant value, with an average of 0.646, for gust spacings over the range from 2H to 8H. However, as spacing reduces below 2H, there is a trend in which p_2 increases, the largest increase occurring as the spacing is reduced from H to zero. An average value of p_2 for zero spacing exhibits an increase of 18% over the average for ramp spacing ranging from 2H to 8H.

The second set of patterns to be considered in reference 87 consists of up-down-up-down sequences of four ramps having equal gradient distances and equal distances between the ramp components. Measured complexity factors, denoted by p_4, for this class of four-ramp patterns follow the trend already described for two-ramp patterns. p_4 takes an approximately constant value, with an average of 0.409, over the range of spacings from 2H to 8H. However, following the trend already observed for two-ramp patterns, as spacing reduces below 2H, p_4 increases, the largest increase occurring as the spacing is reduced from H to zero, the average value of p_4 for zero spacing exhibiting an increase of 28% over the average for ramp spacing ranging from 2H to 8H.

Similar results are obtained in reference 87 for patterns comprising 8- and 16-ramp components. In each case, the average value of p_n for zero spacing exhibits a significant increase over the average for larger ramp spacing.

In the particular case of two-ramp patterns with the two component ramps having equal gradient distances, a comparison is also made in reference 87 between the complexity factors associated with an up-down pattern and those associated with a two-ramp up-up, or staircase, pattern. It is demonstrated that over the range of ramp spacing ranging from 2H to 8H the average value of the complexity factor is unchanged by the change in polarity of the second ramp. However, as the ramp spacing tends to zero, whereas the complexity factor shows an increase in the case of the up-down pattern, for the staircase pattern it exhibits a decrease.

The significance of the results for the SDG model are discussed in section 5.3.3.

5.3.2 Results for Random-Phase Signal.

In reference 87, comparable results are also described for a set of related surrogate Gaussian signals, obtained by randomizing the phase of the Fourier coefficients of the measured turbulence velocity components for each of the Gnat runs (section 4.2) used in the study. This provides a means of checking to what extent the phenomena observed in severe turbulence are associated with the non-Gaussian statistics associated with phase correlation and to what extent they remain after the phase randomization.

The transformation applied involves taking the Fourier transform of each measured turbulence velocity component, retaining the amplitude component of this Fourier transform but replacing the true phase component by a purely random phase component, and finally applying an inverse Fourier transform to generate the required surrogate signal. It should be noted that the resulting Gaussian signals have identical power spectral densities to the measured turbulence velocities. A Matlab routine to perform the required transformation is given in reference 87.

The analysis technique described in section 5.2 has been applied [87] to the resulting Gaussian surrogate signals exactly as for the measured severe turbulence velocity components and complexity factors measured for up-down sequences of two ramps in the same format as for the turbulence records. A trend was apparent, in all the results, in which p_2 takes an approximately constant value, with average 0.715, over the range of spacings from 2H to 8H. However, as spacing is reduced below 2H, as in the case of severe turbulence there is a trend in which p_2 increases, with most of the increase occurring as the spacing is reduced from H to zero, the average value of p_2 for zero spacing exhibiting an increase of 11% over the average for ramp spacing ranging from 2H to 8H.

The above results may be compared with theoretical results applicable to a Gaussian process. In reference 40, it is shown how Gaussian processes with power spectral densities of the form $(\text{frequency})^{-B}$ (so-called self-similar Gaussian processes) may be modelled approximately by sets of ramp-shaped profiles. The simplest case having relevance to aeronautical applications is that where B = 2. This case (technically a Brownian process) corresponds to the higher-frequency asymptotic region of the Dryden spectrum. In this case, two fundamental theoretical results can be derived [40]. First, the p-factors for nonoverlapping ramp components, and in particular p_2, are shown to be independent of the spacing between successive ramps. Secondly, the theoretical value of p_2 for an up-down pair of ramps having equal gradient distances is shown to be $1/2^{1/2} = 0.707$. If the result for zero spacing is excluded, both of these theoretical predictions are satisfied, to a good approximation, by the measured values (average 0.715) for the random phase process.

To understand the increase in p_2 that occurs when the spacing between the component ramps is reduced to zero, it is necessary to consider the alternative self-similar Gaussian process for which the power spectral exponent B = 5/3. This case corresponds to the higher-frequency asymptotic region of the von Karman spectrum, a better representation of atmospheric turbulence than the Dryden spectrum. This case has also been considered theoretically in reference 40, where it is shown (figure 10 of reference 40) that the value of p_2 for zero spacing exceeds the average for ramp spacing ranging from 2H to 8H by approximately 8%. This is somewhat less than the excess (11%) measured for the random phase signal but is, nevertheless, within the range of measurement scatter and is indicative of the source of the increase. As shown in reference 40, the increase in p_2 that occurs, in a Gaussian process with power spectral density of the form $(\text{frequency})^{-5/3}$ as the ramp spacing is reduced to zero, is a consequence of the two-point spatial correlations (not to be confused with phase correlations in the Fourier representation) that exist between closely spaced increments in a Gaussian process when the spectral density departs from the form $(\text{frequency})^{-2}$.

Apart from their relevance to the interpretation of the measurements made in severe turbulence, to be discussed in section 5.3.3, the results described above exhibit a consistency between the analysis of measured data and theory which lends support to the validity of the data analysis method outlined in section 5.2. Further validation of the data analysis method, described in reference 87, comes from its application to synthetic Gaussian Brownian noise, for which theoretical results are reproduced almost exactly.

In reference 87, similar conclusions are drawn from the analysis of the surrogate Gaussian process for other gust patterns, including up-down-up-down sequences of four ramps having equal gradient distances and equal distances between the ramp components. In reference 87, measured complexity factors are described for this class of four-ramp patterns for gust spacings ranging from 0 to 8H. As for two-ramp patterns, in all the results, p_4 takes an approximately constant value, now with average 0.504, over the range of spacings from 2H to 8H, but as the spacing is reduced below 2H, there is a trend in which p_4 increases, with most of the increase occurring as the spacing is reduced from H to zero. An average value of p_4 for zero spacing exhibits an increase of 12% over the average for ramp spacing ranging from 2H to 8H.

As for the two-ramp pattern, the above results may be compared with theoretical results applicable to a Gaussian process [40]. In the case of Gaussian processes with power spectral densities of the form $(\text{frequency})^{-2}$, corresponding to the higher-frequency asymptotic region of the Dryden spectrum, it is again the case that the p-factor, in this case p_4, for nonoverlapping ramp components is independent of the spacing between successive ramps. Moreover, the theoretical value of p_4 for an up-down-up-down pattern of four ramps having equal gradient distances is 1/2 = 0.5. If the result for zero spacing is excluded, both of these theoretical predictions are satisfied, to a very good approximation, by the results (average 0.504) for the random phase process described above.

The increase in p_4 (of 12%) that occurs when the spacing between the component ramps is reduced to zero is almost exactly the same as the corresponding increase (11%) observed in the case of two-ramp patterns, and has the same theoretical explanation that involves the two-point spatial correlations that exist between closely spaced increments in a Gaussian process when the spectral density departs from the form $(\text{frequency})^{-2}$.

5.3.3 Discussion.

The above results differ, in respect to the increase in p-factors that has been shown to occur at zero spacing between ramp components, from the broad conclusion reached in previous work using data recorded during routine flying by civil aircraft (section 5.1), which was that the complexity factors p_i depend only on the number of components in the pattern and are thus independent of ramp spacing. One principal difference between the two data analysis programs is that, whereas in the more recent analysis [87], it has been possible to compare statistical distributions corresponding to highly constrained families of gust pattern profiles in which gust spacing is the only variable parameter. The conclusions of earlier work were based on comparisons of statistical distributions corresponding to gust pattern shapes in which spacing and gradient distances were varied simultaneously. On this basis, the conclusions reached, concerning the effects of gust spacing, in the more systematic analysis in recent work are believed to be the more reliable.

Denoting the average complexity factors for two- and four-ramp patterns (excluding the zero-spacing results) respectively by p_2 and p_4, and the corresponding factors in the case of zero spacing by p_{2p} and p_{4p}, the following ratios were measured [87]:

in severe turbulence

$$p_{2p}/p_2 = 1.18, \; p_{4p}/p_4 = 1.28$$

in the related Gaussian random phase process

$$p_{2p}/p_2 = 1.11, \; p_{4p}/p_4 = 1.12$$

and in synthetic Gaussian Brownian noise

$$p_{2p}/p_2 = 1, \; p_{4p}/p_4 = 1$$

As discussed in section 5.3.2, the results for synthetic Brownian noise (a self-similar Gaussian process with spectral density proportional to $(\text{frequency})^{-2}$) are theoretically exact, and the increases in the ratios that occur in the case of the Gaussian random phase process are consistent with theoretical results for a self-similar Gaussian process whose power spectral exponent is different from two. There remains to be explained the large further increase in the ratios in the case of the measurements in severe turbulence.

The interpretation of these results, proposed in reference 87, is that the measured data for severe turbulence are consistent with a gust model which contains a periodic component of significant amplitude, in combination with other non-Gaussian gust patterns which satisfy, to a good approximation, the property that the probability is independent of ramp gust spacing.

Further evidence for the existence of periodic gust components is provided in reference 87 by the results for 8- and 16-ramp patterns, for which the following ratios were measured in severe turbulence

$$p_{8p}/p_8 = 1.23, p_{16p}/p_{16} = 1.38$$

While the measured values of p_8 (= 0.256) and p_{16} (= 0.159) are sufficiently small for the associated gust patterns to be excluded from the SDG model of severe turbulence, the measured factors p_{8p} and p_{16p} are of sufficient magnitude for the associated periodic components (with complexity factors subject to the standard 1/0.88 correction) to be included in the latest implementations of the model (section 7).

The proposed explanation of the existence of periodic gust patterns in severe turbulence is that they are induced by the existence of vortices. This conclusion is supported by the analysis of the Gnat turbulence data in reference 80, where the detection of vortex cores is described explicitly, and it is proposed that while the results in that report predominantly reflect turbulence properties associated with sheet-like structures carrying large transverse velocity gradients, in a small number of instances intersections of the line of flight of the aircraft with tube-, or vortex-like structures influence the extreme tails of the measured statistical distributions. Comparable results have been presented in references 20 and 76 based on records of airline incidents involving passenger or crew injury in clear air turbulence. In particular, reference 20 refers to severe turbulence having a "periodic, deterministic nature" attributing the measured oscillatory vertical wind components (for example figures 7 and 10 of reference 20) to vortex-induced flows.

A comparison may be made between the measured average p-factors in reference 87 and the values previously derived in reference 25 (see section 5.1). However, in order to obtain comparable results, a standard correction factor, determined previously (appendix A of reference 40) to take the value 1/0.88, must be applied to the p_i (i > 1) derived for severe turbulence in reference 87. As explained in reference 40, this factor compensates for the underestimation of aircraft response that occurs, on average, when the ramp components in the SDG gust model are constrained to have equal probability. Applying this factor to the average measured values of p_2 (= 0.646) and p_4 (= 0.409) presented in section 5.3.1, there is obtained

in severe turbulence:

$$p_1 = 1 \text{ (reference value)}, p_2 = 0.734, p_4 = 0.465$$

These values may be compared with those (section 5.1) derived previously by curve fitting to special event data in reference 25

$$p_1 = 1, p_2 = 0.705, p_4 = 0.42$$

Taking into account the differences between data sources and methods of data analysis, the agreement between these two sets of results is within the margins of expected error. Further reference to these results is made in section 7.

5.4 OVERVIEW.

As stated at the beginning of section 3, a characteristic of the non-Gaussian statistics of severe turbulence lies in the short-duration bursts that are observed in both records from routine operational flights by civil aircraft and in data recorded by specially instrumented aircraft. In the above, it has been explained how this property of severe turbulence is characterized in the SDG model in terms of complexity factors, or p-factors, which reflect the reduced probabilities of gust patterns comprising extended sequences of ramp gust components. Through the incorporation of these factors, gust patterns comprising a large number of component fluctuations occur in the SDG model with lower probability than would be predicted by the PSD method on the basis of Gaussian assumptions.

Attention has been drawn to a data analysis technique whereby statistical results derived for a set of surrogate Gaussian signals, obtained by randomizing the phase of Fourier coefficients of measured turbulence velocity components (following an algorithm specified in appendix B of reference 87) are compared with analogous results derived directly from the turbulence records. This provides a means of identifying specifically the effects of phase correlation in the Fourier representation.

In particular, it has been explained how the complexity factors obtained by fitting the SDG model to severe turbulence differ from those that result when the model is fitted to a random phase (or Gaussian) process. In consequence of the differences between the complexity factors applicable to these two classes of processes, when applied to severe turbulence a divergence occurs between the results of an SDG analysis and those of a PSD analysis.

The probabilities of gust patterns comprising a sequence of ramp components have been found to be independent of the spacing between the component ramps, with the exception of configurations in which the spacing between the ramps tends to zero. The complexity factors (p-factors) in the case of zero spacing have been shown to take numerical values that are significantly higher than the values applicable when the spacing is greater than the ramp gradient distance. These results are consistent with a gust model for severe turbulence that contains periodic components of significant amplitude, in combination with other non-Gaussian gust patterns which satisfy, to a good approximation, the property that the probability is independent of ramp gust spacing. The proposed explanation of the existence of these periodic gust patterns in severe turbulence is that they are induced by the existence of vortices.

6. STATISTICAL DISCRETE GUST METHODOLOGY FOR THE CALCULATION OF THE RESPONSE OF LINEAR AIRCRAFT.

The SDG method is concerned with the occurrence of large excursions in aircraft dynamic response when the excitation is represented by a prescribed statistical distribution of discrete gust patterns. The statistical distribution of response peaks whose magnitude exceeds a prescribed threshold is expressed as an integral, over discrete gust patterns, whose asymptotic form for large threshold amplitudes is evaluated by the Laplace approximation [13, 37, and 53]. To this approximation, for any prescribed dynamic system, the large response peaks are associated statistically with input patterns whose configuration lies in the neighborhood of a particular pattern that is matched, or tuned, to the system in question.

The procedure for finding the tuned input pattern corresponding to a given system (which may be linear or nonlinear) involves a variational problem [37 and 53] in which the system peak response is maximized with respect to input patterns subject to a constraint related to their probability of occurrence (appendix A). Alternatively, the procedure for finding the tuned input pattern may be expressed in an equivalent dual form in which the probability of occurrence is maximized, subject to a constraint on response peak amplitude.

In the case of a linear system, the basic principles of the method may be illustrated by first considering just the response to a family of single ramp gusts [53]. A response function $\gamma(H)$ is defined to be the magnitude of the peak response to a ramp gust of length (gradient distance) H and amplitude $w = H^k$, where k is chosen such that this equation represents a constraint on probability (section 4). The ramp length H is then varied, to find the tuned gust length H = $\overline{H}$ at which $\gamma(H)$ attains its maximum value $\overline{\gamma} = \gamma(\overline{H})$. In the more general situation, the family of single-ramp gusts is replaced by a family of more general gust patterns, subject to a constraint on probability, to which the overall maximum response $\overline{\gamma}$ is found. The relative probabilities of gust patterns comprising different numbers of ramp gust components are expressed in terms of complexity factors, or p-factors (section 5). Taking advantage of the assumed linearity of the system, the calculation of the overall maximum response $\overline{\gamma}$ is much simplified by evaluating the response to a general gust pattern as the linear superposition of responses to a set of individual ramps (appendix B). A summary of the implementation of this procedure is demonstrated in appendix A of reference 25.

As is the case with the standard PSD method, the SDG method may be expressed [16] in either Mission Analysis or Design Envelope forms. In each case, $\overline{\gamma}$ plays a role in the SDG method analogous to that of the standard response quantity $\overline{A}$ in the PSD method [3]. For the Mission Analysis, $\overline{\gamma}$ and $\overline{A}$ play equivalent roles in an explicit statistical expression for the rate of occurrence of response values as a function of the threshold exceeded. For the Design Envelope formulation, $\overline{\gamma}$ and $\overline{A}$ are each multiplied by design gust intensities, respectively U_0 (or U_{ref}) and U_σ, to obtain quantities such as design loads.

A description of SDG methodology, for the calculation of aircraft response, as it stood in 1980 is given in reference 25. More recently, since 1993, the implementation has been further developed by the use of Matlab code and of wavelet analysis. In reference 88, an interpretation of the SDG model was described in which the discrete gusts are elementary wavelets. The standard SDG analysis procedure, in which the (linear) aircraft response is calculated for a set of ramp-shaped discrete gusts covering a range of gust lengths, corresponds to the calculation of a wavelet transform of the aircraft impulse response function using an analyzing wavelet [57] in the form of a ramp gust. The process of finding maximum peak amplitudes as gust length is varied becomes a problem of finding local extreme values, with respect to position and scale, in the wavelet surface. This wavelet-based method was subsequently implemented in the form of Matlab code and, for its application to severe-to-extreme gusts, designated as SDG1. This software implementation and the subsequent development of a simplified model (SDG-AS-SC), which requires no special-purpose wavelet software, are described in section 7.

7. THE SDG1 AND SDG-AS-SC GUST MODELS.

7.1 INTRODUCTION.

The first step (module 1 in software implementations) in the application of the linear SDG method is the identification of local maximum and minimum values, with respect to both time and gradient distance, in the responses to a family of single-ramp gusts whose members are constrained, by means of a prescribed amplitude/gradient distance scaling relationship (section 4), to have equal probability. In the case of aircraft response containing flexible modes, depending upon the damping ratios of the modes, there may be a very large set of such local extreme values.

Each local extreme value contains a ramp gust of particular gradient distance and amplitude, resulting in an associated set of candidate ramps for use as components in the step 2.

The second step (module 2 in software implementations) consists of the synthesis of compound gust patterns, each comprising the sum of a subset of ramp components drawn from the overall set of ramp gusts identified in step one. In each such compound gust pattern, the component ramps are displaced in time such that their associated extreme values of response occur at the same instant and their signs are chosen such that the response values reinforce one another (appendix B). Because of this synthesis procedure, each chosen subset of ramps leads to a unique compound gust pattern. The role of system linearity in this second step lies in the fact that, just as the input comprises a sum of component ramp inputs, so the response to the compound gust pattern simply comprises the sum of the associated component ramp responses. Each such compound gust pattern and its associated response are then scaled in amplitude, by a factor that depends primarily upon the number of ramp components (the complexity factor, or p-factor), such that all compound gust patterns have the same probability (section 5). The overall maximum response is then the SDG tuned response and the particular compound gust pattern with which it is associated is the SDG tuned input.

While the above paragraphs define the principles of the linear SDG method, there remains in practice some freedom of choice with regards to the specific implementation. To achieve a practicable method, a simplified procedure is called for in which the class of patterns considered is not all-inclusive but is nevertheless sufficiently representative of the severe or extreme gust patterns that exist and should be considered in aircraft design.

Since the SDG method was first introduced, several such simplified procedures have been proposed. In particular, two alternative formulations of the SDG method for application to severe or extreme turbulence, SDG1 and SDG-AS-SC, have been implemented in Matlab. Of these, SDG1 is the more comprehensive analytical version having a minimum of simplifying assumptions and SDG-AS-SC is a much simplified version, now proposed [75] as the basis for a viable design Criterion (appendix C), whose implementation for a linear aircraft requires no special-purpose software. Other implementations of the SDG method have been introduced by D.L. Hull [89-93].

7.2 THE SDG1 GUST MODEL.

Compared with the formulation of the SDG method presented [53] in 1989, the principal changes incorporated in the SDG1 gust model [67-69] are as follows.

a. A family of equiprobable elementary gust components is generated by passing a set of pulses of prescribed energy through a modified von Karman shaping filter. For pulse lengths significantly less than the scale length, this produces an initial ramp followed by a more gradual decay. This contrasts with the ramp-hold model of previous SDG implementations [53]. For pulse lengths of the order of, or longer than, the scale length, it modifies the ramp shape in a way consistent with the energy distribution in the standard PSD model.

b. For gust gradient distances significantly less than the scale length, the modified von Karman shaping filter produces a family of discrete gusts satisfying approximately a one-sixth power law. This is consistent with the results of analysis of severe atmospheric gusts (section 4, reference 81). Subsequent to the issue of the original documentation of SDG1, it was pointed out by D.L. Hull that the approximation to the one-sixth law could be improved by an adjustment of numerical parameters in the SDG1 algorithms. Details of how to implement this update are given in reference 94.

c. An advantage of using a family of pulse-shaped profiles as inputs to a prescribed shaping filter is that it allows an interpretation of the procedure for evaluating linear aircraft response in terms of wavelet analysis [54, 68, and 69]. This allows the removal of complications in the implementation previously associated with peak tracking and gust overlap. Overlapping gust components are allowed, thus widening the class of available gust patterns.

d. For application to atmospheric gusts of severe or extreme intensity, it is only necessary to consider gust patterns comprising up to four elementary components. This assumption is supported by analysis of data from measured severe gust encounters obtained during routine operational flying by civil airlines (section 3) and by the analysis of severe turbulence data obtained by a specially instrumented aircraft (section 5).

e. Complexity weighting factors (p factors) that define the amplitudes of the component gusts in higher-order gust patterns, as defined in reference 53, are replaced by equivalent energy reduction factors (e factors), applied to the overall energy of the pattern of pulses used as input to the modified von Karman shaping filter.

f. A feature of the use of energy reduction factors, as described above, is that it is not necessary to introduce the constraint of equipartition of probability between the individual components of a gust pattern, as used in reference 53. This further widens the class of available gust patterns.

g. Following the recent identification of periodic gust patterns in measured severe turbulence data [79], it is now recommended that SDG1 be amended to incorporate families of such patterns, following exactly the procedures laid down for SDG-AS-SC (see h. of 7.3).

7.3 THE SDG-AS-SC GUST MODEL.

The most recent SDG model for severe turbulence is the SDG-AS-SC model [75], where AS refers to alternating sign, and SC refers to staircase. While the full specification and explanation of the method are provided in the associated draft criterion (appendix C) and the associated Explanatory Information (appendix D), the basic principles may be summarized as follows.

a. There is a return to the ramp-hold gust, of early SDG implementations (as in reference 53), as the elementary component of the model.

b. The compound gust patterns considered are synthesized from no more than four ramp-hold gust components.

c. The component ramps are sequential, i.e., nonoverlapping.

d. No more than two successive ramps in a pattern may act in the same direction (a succession of two ramps in the same direction, i.e., up-up or down-down, is referred to as a two-ramp SC).

e. o account for the differences in probability that occur, at small values of the gust spacing, between two-ramp up-down patterns and up-up SC patterns, an SRF has been introduced for gust patterns possessing staircases.

f. The subset of ramp components from which any compound pattern is synthesized comprises, at most, two up-ramps and two down-ramps.

g. The component ramps are selected successively according to an AS procedure, applied to their associated response values, and the no-overlap condition is implemented, following the selection of any ramp, by the deletion from the list of candidate ramps of all those having any overlap with the selected ramp.

h. The above primary set of patterns is supplemented by particular cases of alternating sign sequences in which the gradient distances of the component ramps are taken to be equal to one another and the spacing between the end of any component ramp and the onset of the following ramp is taken to be zero. The only variable quantities in these periodic gust patterns are, thus, the gradient distance and the associated scaled amplitude. Periodic patterns comprising 2, 4, 8 and 16 successive ramp components are included.

The implementation of the above principles, a. to g., follows an unambiguous algorithmic procedure in which the responses to successively selected ramps are denoted by E_1, E_2, E_3, and E_4, where (following the AS rule) E_1 and E_3 have one sign (e.g., both positive) and E_2 and E_4 have the opposite sign (e.g., both negative). The maximum response to a pattern comprising a single ramp is then based on E_1, the maximum response to an up-down pattern of two ramps is based on the combination of E_1 and E_2, the maximum response to a pattern of three ramps is derived by comparing the combination of E_1, E_2, and E_3 with the combination of E_1, E_2, and E_4. The maximum response to a pattern of four ramps is based on the combination of E_1, E_2, E_3, and E_4. The maximum response to a two-ramp staircase pattern is derived by comparing the combination of E_1 and E_3 with the combination of E_2 and E_4.

Finally, the maximum responses to the families of periodic gust patterns (h. above) are evaluated. The systematic evaluation of the above cases leads to a list of associated maxima from which the overall maximum response value is then selected and the corresponding tuned gust pattern identified.

It may be noted that, in addition to their occurrence as two-ramp patterns, staircase gust sequences (d. above) can arise as two-ramp sub-components of three- or four-ramp patterns generated by the AS procedure (the fact that an AS procedure is applied to the magnitudes of the peak responses, in the selection of candidate ramp components, does NOT imply that the resulting compound gust pattern comprises an AS sequence of ramps in time). The need to include staircase gust sequences in the family of gust patterns included in the SDG-AS-SC gust model arises from both theoretical and empirical evaluations of the relative probabilities of, for example, up-up and up-down patterns [87].

A similar consideration applies to the need to include the periodic gust patterns. As demonstrated in reference 87, and discussed in section 5, for the particular periodic configurations that arise when the component gradient distances in an AS sequence become equal, and the spacing between successive ramps tends to zero, the pattern amplitude for a given level of probability shows a significant increase.

The rationale behind the SRF can be explained with reference to figure D-1 of appendix D. Case 1(a) and case 2(a) illustrate two staircase gust patterns, which differ only in the spacing B C between the two sequential ramps. In case 1(a), the dashed line A to D joins the beginning of the first ramp to the end of the second ramp. In appendix D it is shown how the probability of this transition from A to D can be calculated.

The need for the SRF becomes apparent when one considers the situation, case 2(a) in figure D-1, in which the spacing between the two sequential ramps becomes small. As demonstrated in appendix D, the probability of the transition, or pseudo-ramp, A to D then becomes a dominant factor in determining the probability of the overall pattern. Thus, for small spacing, the existence of the gust velocity transition from A to D causes the staircase gust pattern to lie outside the class of gust patterns at the probability level specified implicitly by the proposed draft criterion (appendix C). To remove this anomaly, the SRF has been introduced as a scaling factor whose application reduces the overall amplitude of the staircase gust pattern such that it becomes consistent with other gust patterns at the specified level of probability.

7.4 COMPARISON OF LOADS FROM SDG1 AND SDG-AS-SC WITH PSD LOADS.

In this section, relationships between the SDG1 and SDG-AS-SC gust models are illustrated in terms of the calculated responses of two different aircraft types, representative of an Airbus, with underwing engines and a turbo-prop, respectively.

7.4.1 Airbus-Type Aircraft.

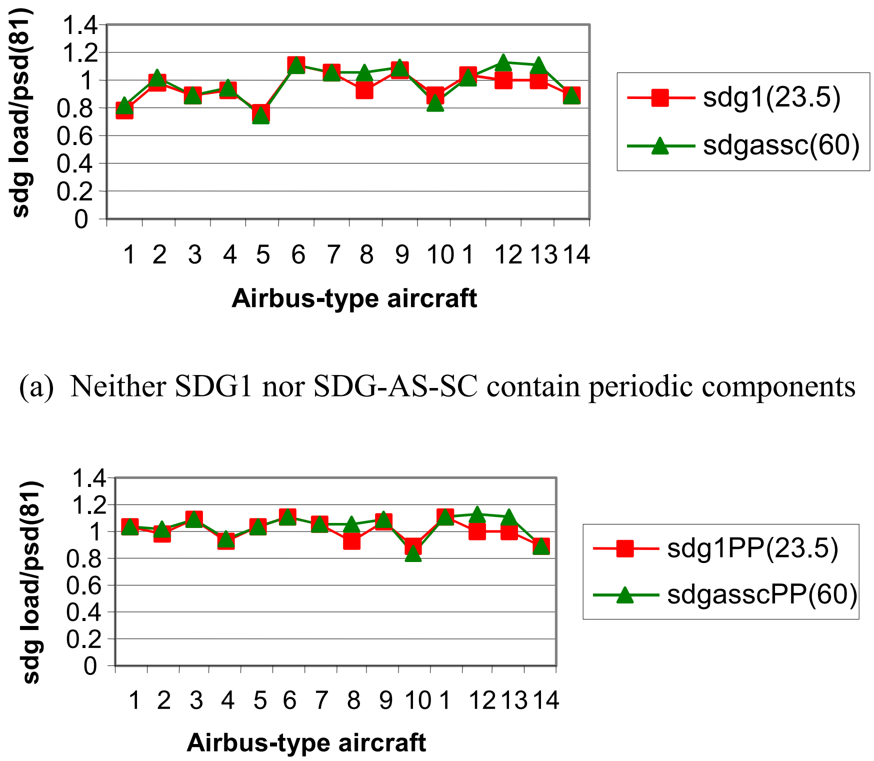
For Airbus-type aircraft, two different configurations and flight conditions, referred to as Aircraft A and Aircraft B, have been studied. Aircraft A represents a wide-bodied airbus flying at a cruise speed of 853 f/s true airspeed (TAS). Aircraft B represents a (different) wide-bodied airbus flying at a cruise speed of 476 f/s TAS. The response quantities evaluated are numbered, for reference in the figures, as follows:

Vertical gust response

1 Engine lateral acceleration (aircraft A)
2 Wing bending moment (aircraft A)
3 Wing torque (aircraft A)
4 Normal acceleration at aircraft cg (aircraft A)
5 Engine lateral acceleration (aircraft B)
6 Inboard wing bending moment (aircraft B)
7 Outboard wing torque (aircraft B)
8 Inboard wing torque (aircraft B)
9 Horizontal tail bending (aircraft B)
10 Horizontal tail torque (aircraft B)
11 Front fuselage bending moment (aircraft B)
12 Rear fuselage bending moment (aircraft B)
13 Fuselage acceleration, location unspecified (aircraft B)
14 Elevator angle (aircraft B)

Figure 1 shows the associated design loads, for the above 14 response quantities, for the two SDG models, normalized in each case with respect to the corresponding design load calculated by means of the PSD method. The design gust intensities used in this comparison are U_0 = 23.5 (ft. sec. units) for SDG1, U_{ref} = 60 f/s for SDG-AS-SC, and U_σ = 81 f/s for the PSD model. These values have been shown in previous work to give, for the response to vertical gusts of the Airbus-type aircraft, average load ratios close to unity.

In figure 1(a), neither the SDG1 nor the SDG-AS-SC gust models incorporate the periodic gust components, introduced recently on the basis of an analysis of severe turbulence records (section 7.3). In figure 1(b), the periodic gust components are incorporated into both the SDG-AS-SC and SDG1 gust models (following their specification in reference 75).



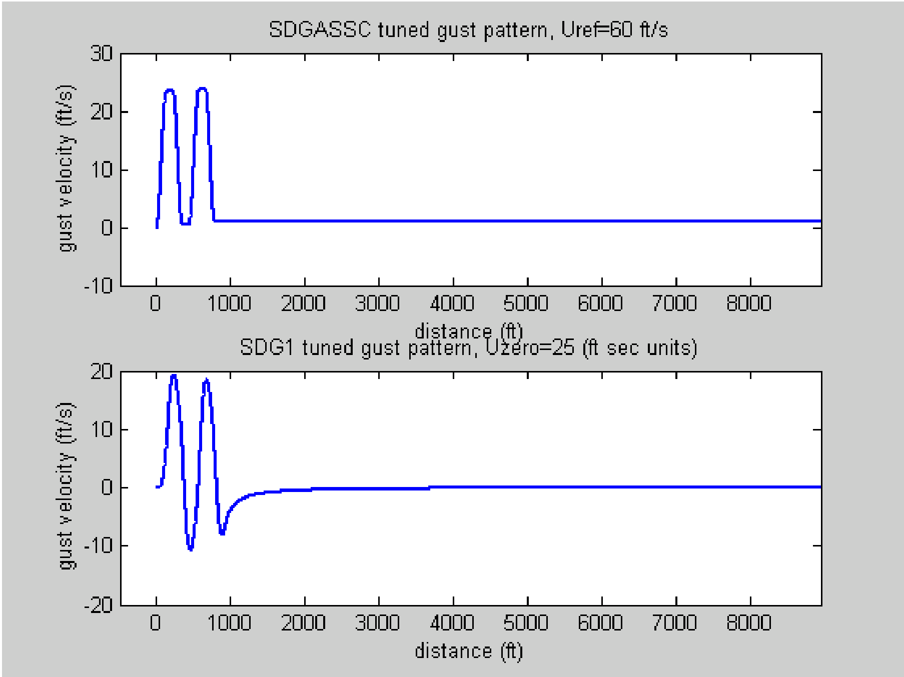
(a) Neither SDG1 nor SDG-AS-SC contain periodic components

(b) SDG1 and SDG-AS-SC both contain periodic components

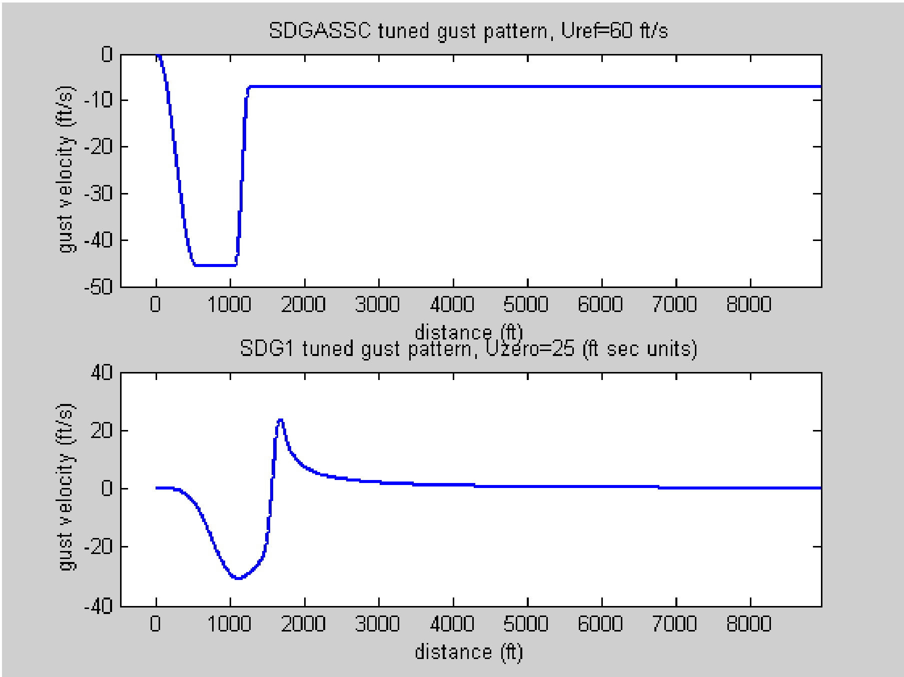
Note: Responses 1 to 4: Aircraft A; responses 5 to 14: Aircraft B; see section 7.4 to identify individual response quantities and gust intensities.

FIGURE 1. AIRBUS-TYPE AIRCRAFT—DESIGN LOADS FOR TWO SDG MODELS OF VERTICAL GUSTS NORMALIZED WITH RESPECT TO PSD DESIGN LOAD BASED ON A-BAR

Figure 1(a) demonstrates good consistency between the more mathematically comprehensive model SDG1 and the simplified model SDG-AS-SC, both following very similar trends with respect to the PSD design loads. In particular, the SDG loads predicted for aircraft A, cases 1 to 4, differ by less than 5 percent. The associated tuned input gust patterns are illustrated in figure 2. These show, for cases 1 to 4, a one-to-one correspondence between the individual ramp gust components that make up the tuned gust patterns of the respective SDG gust models. At the same time, they illustrate characteristic differences associated with the fact that the elementary gust component of the SDG1 model is a ramp-with-decay, whereas the SDG-AS-SC model is a ramp-hold. One consequence is that, whereas in the SDG1 model, the gust velocity always decays eventually to zero, this is not the case for SDG-AS-SC. It may be noted that case 3 (figure 2(c)) is an example in which both gust patterns comprise two staircases (up-up then down-down). Cases 2 and 4 (figures 2(b) and 2(d)) both show the characteristic occurrence of a longer gradient distance ramp, which excites the rigid-body aircraft response, followed at a later time by a shorter ramp, which excites the elastic response such that maximum resonance between the rigid-body and elastic responses occurs.

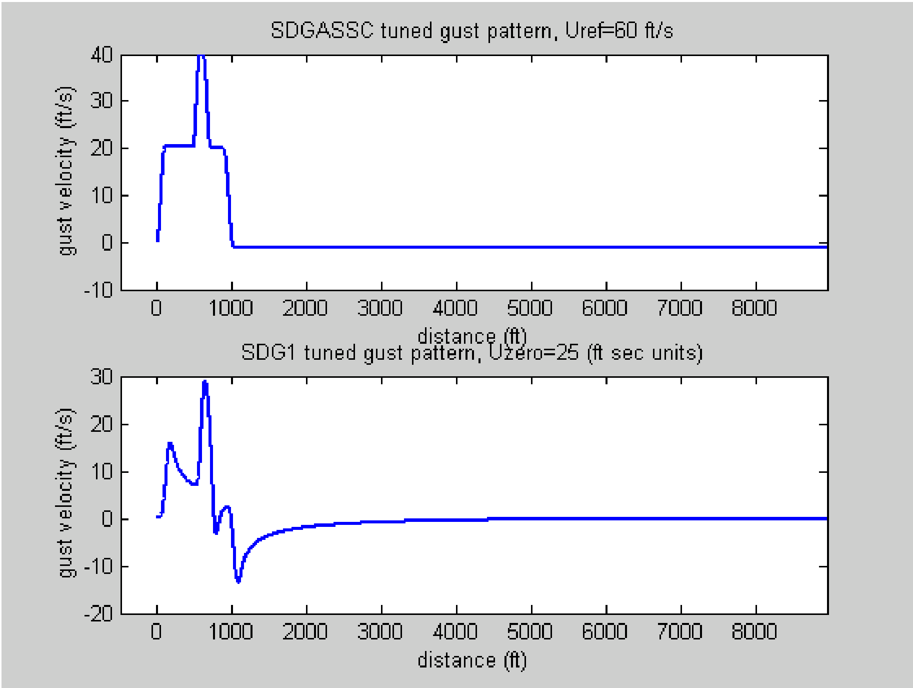


(a) Case 1 (figure 1(a))

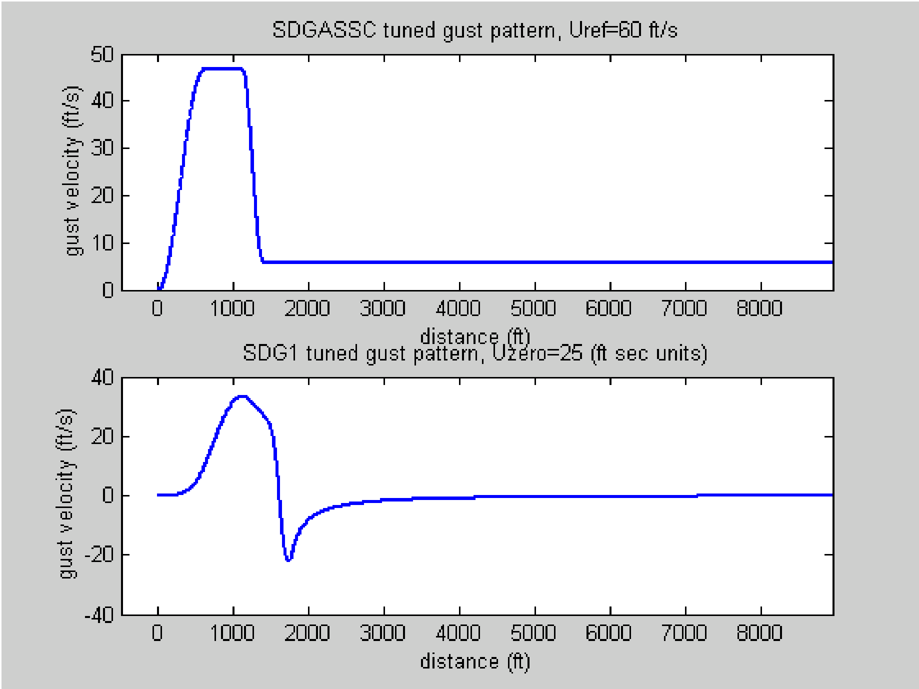


(b) Case 2 (figure 1(a))

FIGURE 2. COMPARISON OF SDG-AS-SC AND SDG1 TUNED GUST SHAPES
(Neither SDG1 nor SDG-AS-SC contain periodic gust components.)



(c) Case 3 (figure 1(a))



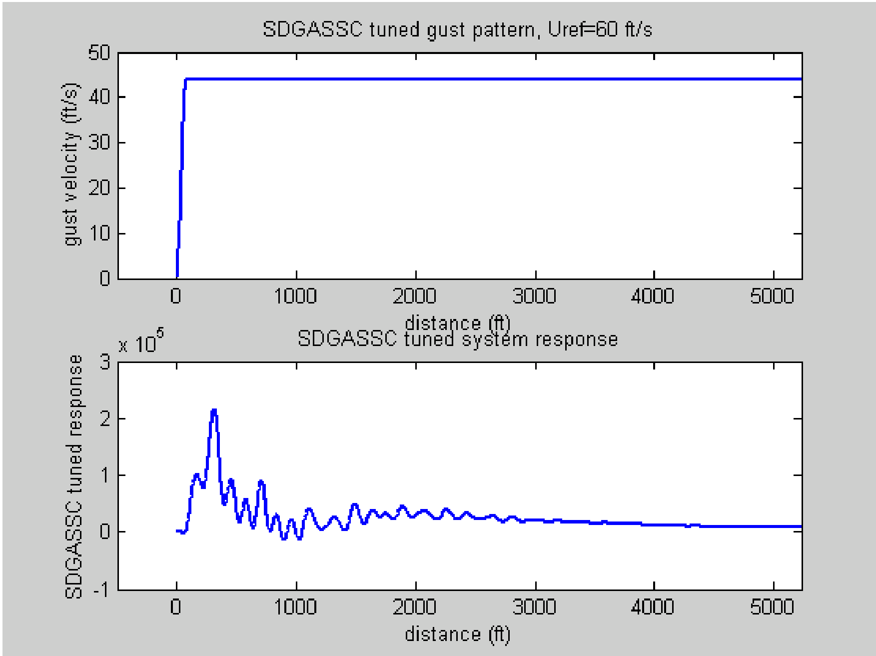
(d) Case 4 (figure 1(a))

FIGURE 2. COMPARISON OF SDG-AS-SC AND SDG1 TUNED GUST SHAPES (Continued) (Neither SDG1 nor SDG-AS-SC contain periodic gust components.)

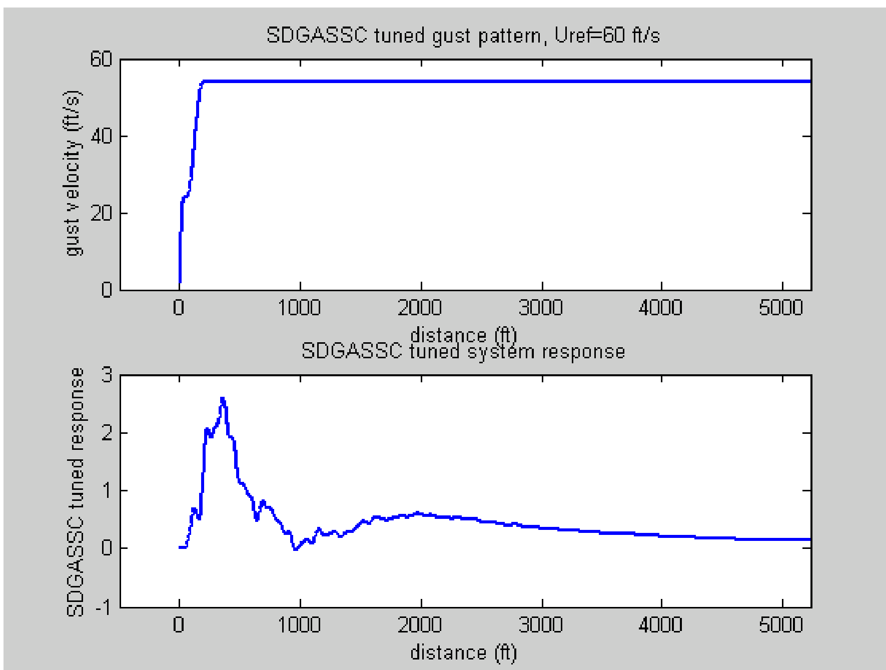
Returning to figure 1(a), response quantity 8 is an instance in which the SDG-AS-SC load is approximately 14 percent greater than the SDG1 load. This is a good example of unavoidable differences that occur between the loads predicted by the two models because of the difference between the ramp decay elementary gust component of the SDG1 model and the ramp-hold component of SDG-AS-SC. As illustrated in figure 3(a), which shows the SDG-AS-SC tuned input and tuned response for response quantity 8, the response comprises a combination of rigid-body and elastic modes, with the peak in response occurring some time after the end of the input ramp. In this situation, the constant hold component of the ramp-hold gust pattern is causing significant excitation of the rigid-body response. When excited by a similar ramp up-gust, but followed by the immediate decay characteristic of the SDG1 gust model, the magnitude of the peak response is significantly reduced. In such situations, the SDG-AS-SC gust model will tend to give conservative results (despite the fact that the P_1 complexity factor has been reduced to 0.92 in SDG-AS-SC in order to reduce this conservatism). SDG-AS-SC is similarly conservative with respect to SDG1 in the cases of response quantities 12 and 13 (figure 1(a)). Figure 3(b) shows the tuned inputs and responses for case 13. As in case 8, (figure 3(a)), the conservatism is associated with excitation of the rigid-body response by the hold component of the input gust pattern, in this case a two-ramp staircase. In contrast, figure 4 illustrates a situation (case 11 of figure 1(a)) in which the SDG-AS-SC and SDG1 design loads are in good agreement despite the fact that the tuned input gust patterns are somewhat different.

Case 10 (figure 1(a)) is an instance in which both SDG models predict significantly lower response amplitudes than the PSD model. The reason for this lies in the fact that, for each SDG model, the tuned gust length is of the order of 800 ft and, as will be argued in section 7.5, for responses associated with gradient distances of this large magnitude the PSD model tends to be overly conservative. Other examples exhibiting the same phenomenon are discussed in section 7.4.2.

In figure 1(b), the periodic gust components are incorporated into the SDG-AS-SC gust model (following its specification in reference 75). SDG1 has also been supplemented by these components. Comparison with figure 1(a) shows that the inclusion of these periodic components has the effect, for the response to vertical gusts of the Airbus-type aircraft, of reducing some of the disparities between the design loads predicted by the SDG and PSD gust models. This is particularly evident in cases 1 and 5, both being examples of lightly damped engine acceleration responses. In both cases, the SDG and PSD design loads are now within 3 percent agreement.



(a) Case 8 (figure 1(a))



(b) Case 13 (figure 1(a))

Note: (b) is an example of an SDG-AS-SC staircase gust pattern.

FIGURE 3. SDG-AS-SC TUNED INPUT AND TUNED RESPONSE, ILLUSTRATING CASES WHERE THE PEAK RESPONSE IS DELAYED BEYOND THE END OF AN INPUT RAMP-HOLD GUST

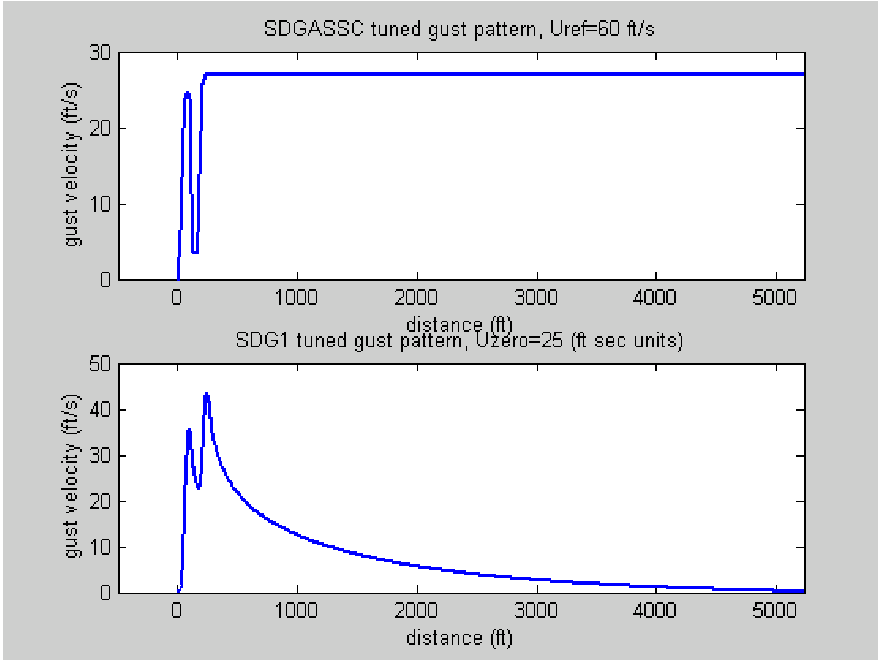


FIGURE 4. COMPARISON OF SDG-AS-SC AND SDG1 TUNED GUST SHAPES, ILLUSTRATING AN SDG1 STAIRCASE GUST PATTERN
(Case 11, figure 1(a))

7.4.2 Turboprop-Type Aircraft.

For the turboprop-type aircraft, responses to both vertical and lateral gusts have been studied for a configuration flying at a speed of 472 f/s TAS. For this aircraft, the response quantities evaluated are coded, for reference in the figures, as follows.

Vertical gust response

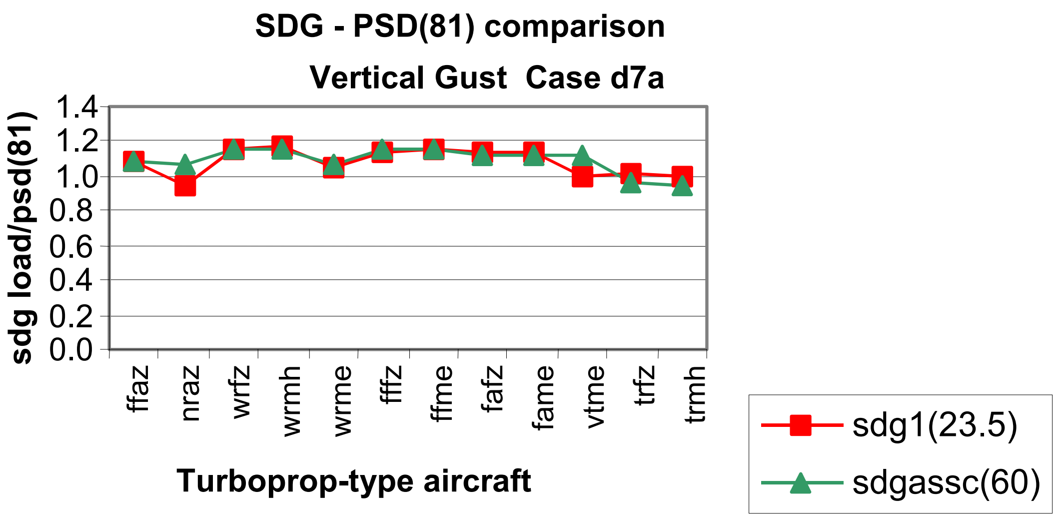
ffaz	Normal acceleration at aircraft cg
nraz	Normal acceleration at power plant cg
wrfz	Wing root vertical shear
wrmh	Wing root vertical bending moment
wrme	Wing root torque
fffz	Front fuselage root vertical shear
ffme	Front fuselage root vertical bending moment
fafz	Aft fuselage root vertical shear
fame	Aft fuselage root vertical bending moment
vtme	Vertical tail root pitching moment
trfz	Tailplane root vertical shear
trmh	Tailplane root vertical bending moment

Lateral gust response

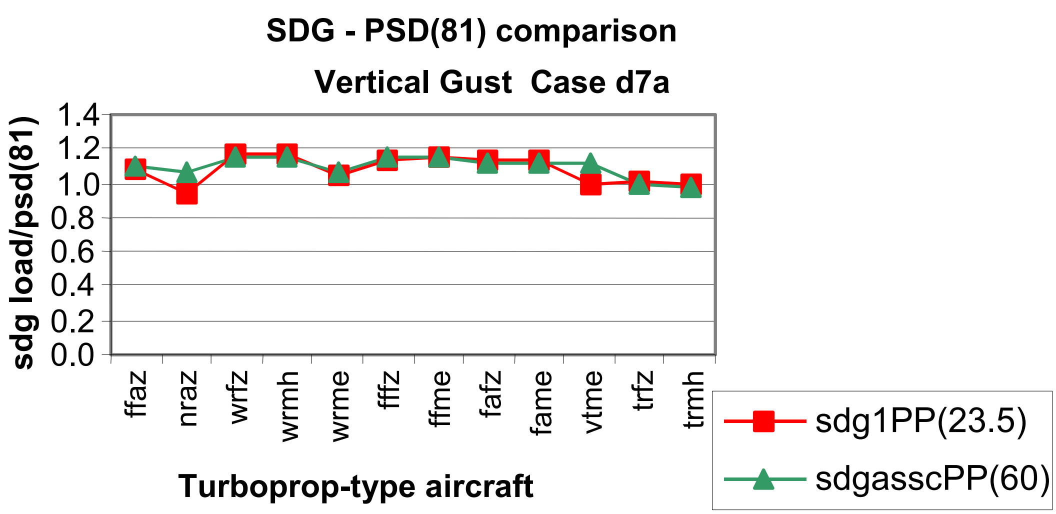
ffay	Lateral acceleration at aircraft cg
nray	Lateral acceleration at power-plant cg
fffy	Front fuselage root lateral shear
ffms	Front fuselage root lateral bending moment
ffmh	Front fuselage root torque
fafy	Aft fuselage root lateral shear
fams	Aft fuselage root lateral bending moment
famh	Aft fuselage root torque
vtfy	Vertical tail root lateral shear
vtmh	Vertical tail root lateral bending moment
vtms	Vertical tail root torque
trmh	Vertical tail tip rolling moment

Figures 5 and 6 illustrate, for the above response quantities in one particular flight condition, the SDG1 and SDG-AS-SC design loads, normalized with the PSD design loads exactly as in figure 1. In figures 5(a) and 6(a), neither the SDG-AS-SC nor the SDG1 gust model contain the periodic gust components (section 7.3). In figures 5(b) and 6(b), periodic components are incorporated into both the SDG-AS-SC and SDG1 gust models, following their specification in appendix C.

A comparison of figures 5(a) and 5(b) shows that the incorporation of the periodic components has had very little effect on these responses to vertical gusts. In all cases, figure 5, the only significant differences between the SDG-AS-SC and SDG1 models occur such that the former model is relatively conservative: an acceptable result because SDG-AS-SC is the relatively simplified model.



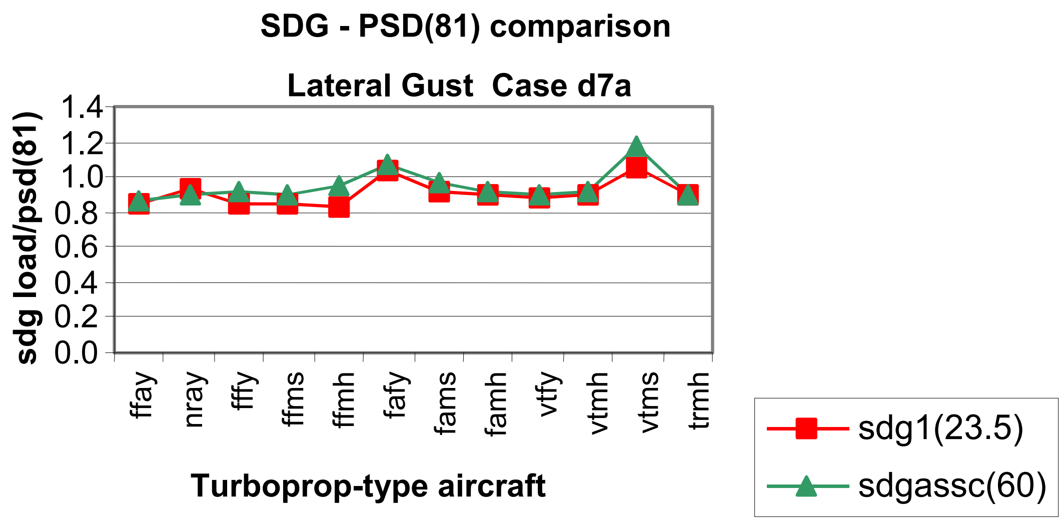
(a) Neither SDG1 nor SDG-AS-SC contain periodic gust components



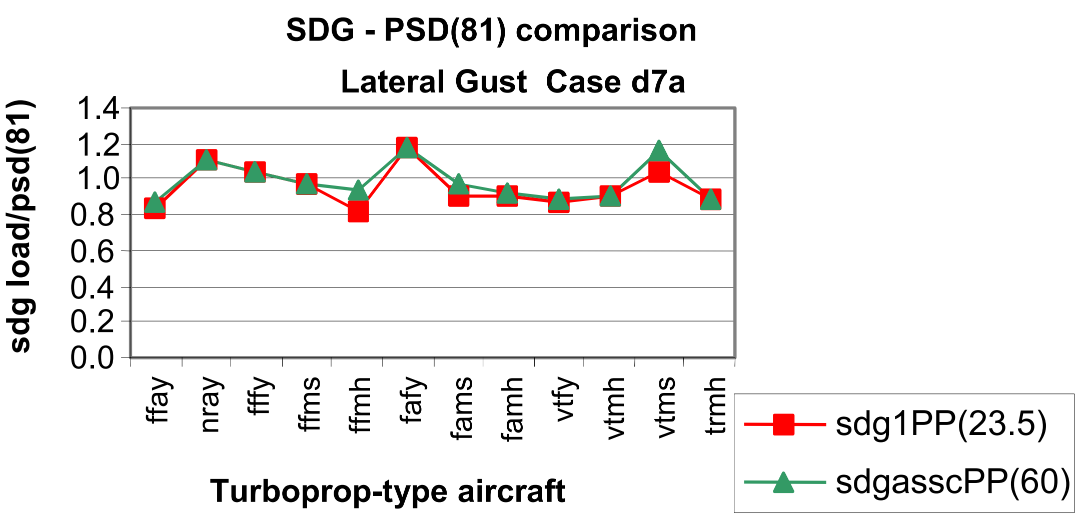
(b) SDG1 and SDG-AS-SC both contain periodic gust components

Note: See section 7.4.2 to identify individual response quantities and gust intensities.

FIGURE 5. TURBOPROP-TYPE AIRCRAFT—DESIGN LOADS FOR TWO SDG MODELS OF VERTICAL GUSTS NORMALIZED WITH RESPECT TO PSD DESIGN LOAD BASED ON A-BAR



(a) Neither SDG1 nor SDG-AS-SC contain periodic gust components



(b) SDG1 and SDG-AS-SC both contain periodic gust components

Note: See section 7.4.2) to identify individual response quantities and gust intensities.

FIGURE 6. TURBOPROP-TYPE AIRCRAFT—DESIGN LOADS FOR TWO SDG MODELS OF LATERAL GUSTS NORMALIZED WITH RESPECT TO PSD DESIGN LOAD BASED ON A-BAR

7.5 TRENDS IN SDG AND PSD LOAD RATIOS.

7.5.1 Comparison of Responses to Lateral and Vertical Gusts.

It can be seen from figure 5(b) that for the turboprop-type aircraft, both SDG models consistently predict higher design loads than the PSD model for responses to vertical gusts (the normalized values are greater than, or approximately equal to, unity). In contrast, the corresponding figure for responses to lateral gusts, figure 6(b), shows, with only two exceptions, the SDG design loads for the aft fuselage and vertical tail to be less than the PSD design loads, with the two SDG models being in good agreement (the differences being again in the sense that SDG-AS-SC is relatively conservative).

While the above results refer to only one aircraft configuration and flight condition, they indicate that, for this type of turboprop aircraft, if the design intensity U_{ref} for an SDG gust criterion (appendix C) were to be specified such that the average ratio of SDG to PSD design loads was equal to unity for vertical gusts, then the corresponding average for the responses at the aft fuselage and vertical tail to lateral gusts would be less than unity by between 10 and 20 percent. A design based on the SDG model that took into account this predicted load reduction could result in a considerable saving of weight. Alternatively, if the design intensity U_{ref} for the SDG gust criterion were to be chosen such that the average ratio of SDG to PSD design loads, including both vertical and lateral gust cases, was equal to unity, then the adoption of an SDG criterion would result in a redistribution of aircraft weight (see item 4 of appendix E). By putting the strength where it is most required, a consequence could be an associated increase in safety margins. These conclusions confirm similar conclusions reached previously by Glaser in reference 95.

As the above observation, i.e., the SDG and PSD ratio for design loads is significantly smaller for lateral gust responses (such as aft fuselage and fin bending moments) than for vertical gust responses (such as wing bending, shear, and torque), is likely to have more general applicability, a study has been made of the underlying causes. This has been done by examining the consequences of making systematic changes to the numerical parameters in the SDG-AS-SC gust model, and the following conclusions have been drawn.

The differences between the SDG and PSD load ratios for vertical and lateral responses arise from two sources. First, the value $k = 1/6$ assumed in the SDG model to be the scaling exponent relating the amplitude of extreme gusts to gust gradient distance. This contrasts with a value of one-third, which relates to the measured scaling of the rms of the fluctuations and also determines the scaling of extreme gusts in the PSD method. The $k = 1/6$ scaling in the SDG model arises (section 4.5) from the measured strengths of the tails of the probability distribution of velocity increments (ramp gust amplitudes) being stronger at the shorter gradient distances. Such distributions contrast with the distributions assumed in the PSD model, based on Gaussian assumptions, which have the same shape at all gradient distances. A consequence is that the SDG and PSD load ratios in response to vertical gusts at the shorter gradient distances that are relevant to loads, such as wing bending, tend to be greater than the SDG and PSD load ratios in response to lateral gusts at the longer gradient distances, associated with loads such as fin bending. In the latter case, on the basis of the shapes of the measured probability distributions of velocity increments, the PSD model tends to be overly conservative.

The second source of the differences between the SDG and PSD load ratios for vertical and lateral responses lies in the numerical values of the complexity factors (p-factors) in the SDG model. As summarized in section 5.4, these reflect the non-Gaussian characteristic that severe and extreme turbulence occurs in relatively short-duration bursts. When this is taken into account, in the SDG model, through the introduction of appropriate complexity factors, the magnitudes predicted by the PSD method for lateral loads such as fin bending, which are related to a relatively lightly damped mode of response (Dutch Roll) and build up over a correspondingly extended sequence of cycles, are significantly reduced.

7.5.2 Comparison of Responses of Different Aircraft to Vertical Gusts.

The difference between the value k = 1/6, assumed in the SDG model to be the scaling exponent relating the amplitude of extreme gusts to gust gradient distance, and the value of 1/3, which determines the analogous scaling in the PSD method, produces a trend in the SDG. The PSD load ratios in response to vertical gusts when different aircraft, or different flight conditions, are compared. This is apparent in the inboard wing bending moments for Airbus types A and B (see responses 2 and 6 in figure 1(a)) and the turboprop-type aircraft (see response wrmh in figures 5(a) and 5(b)). For these three cases, the SDG and PSD load ratios for wing bending moments are 1.00, 1.10, and 1.16. This trend of increasing load ratios can be explained in terms of an associated trend in which the respective gradient distances of the tuned ramp gusts, which dominate these responses decrease, being approximately 500, 160, and 100 ft. Relative to the loads predicted by the PSD method, the SDG loads are increased at the shorter gradient distances because of an associated increase in the strengths of the tails of the probability distribution of gust amplitude (section 4.5).

8. ADVANTAGES OF THE SDG MODEL IN RELATION TO TURBULENCE MODELS IN EXISTING REQUIREMENTS.

In a review of alternative theoretical models of the turbulent atmosphere, developed for the prediction of the response of aircraft, Etkin reference [2] pointed out that such models have to accommodate those events that are perceived as discrete, and usually described as gusts, as well as the phenomenon described as continuous turbulence, even though some discrete gusts are actually rare excursions of a continuous process. This dichotomy is reflected in current airworthiness requirements for aircraft structural loads (Federal Aviation Regulations (FAR), Part 25, Airworthiness Standards: Transport Category Airplanes U.S. Dept. of Transportation, Federal Aviation Administration), which cater independently for continuous turbulence and isolated discrete gusts. As a basis for reviewing the relative advantages of the SDG model, some limitations of the turbulence and gust models in these existing requirements should be outlined.

8.1 LIMITATIONS OF TURBULENCE MODELS IN EXISTING REQUIREMENTS.

8.1.1 The PSD Model.

The traditional statistical approach to the prediction of aircraft dynamic loads in atmospheric turbulence, the PSD method, is based mathematically upon the simplifying assumption that, at least over patches of limited extent, the turbulence can be represented as a stationary Gaussian process. While it is now accepted that turbulence velocity is, in fact, a non-Gaussian process, the method continues to be used, in particular for relating the loads on one aircraft to the loads on another with differing dynamic response and for relating the loads at different stations on the same aircraft. The limitations of the PSD model are, therefore, related to its neglect of non-Gaussian characteristics, which are important features of measured turbulence structure [79, 80, and 81] and have a significant influence on related aircraft loads.

The non-Gaussian structure of measured turbulence-velocity records influences aircraft response predictions in three ways. First, turbulence-velocity increments have strong-tailed probability distributions of broadly exponential, rather than Gaussian, form [80]. The tails of these exponential-type distributions stand out beyond the tails of a Gaussian distribution, resulting in much higher predicted probabilities of events at, for example, five or six times the rms amplitude. However, this does not in itself necessarily produce errors in the PSD method of loads prediction, as exponential tails can be reproduced by the expedient of modelling turbulence as a sequence of Gaussian patches whose rms itself follows a specified distribution [3]. In the past, this result has sometimes been quoted as the means by which the PSD method can deal with the non-Gaussian structure of turbulence. So far as producing exponential tails on the distributions is concerned, this is certainly the case. But this is the least important of the influences of non-Gaussian structure on aircraft response predictions and of the associated errors introduced by the basic assumptions of the PSD method. For the replacement of a single Gaussian patch of specified rms with a sequence of Gaussian patches having a specified distribution of rms's has no effect whatever on the ratios of predicted loads on two aircraft having differing dynamic characteristics. In contrast, the non-Gaussian properties of turbulence structure summarized below do influence such load ratios and failure to account for them is the primary limitation of the PSD method.

The second, and more significant, aspect of non-Gaussian turbulence, which influences aircraft response predictions, is not that the tails of the distributions of velocity increments are exponential but rather that the strengths of the tails vary with gradient distance, the strength increasing as gradient distance is reduced. Whereas the rms (of the increments) scales approximately as $H^{1/3}$ (one-third power) at the low probability levels associated with limit loads, the amplitudes of the increments scale approximately like $H^{1/6}$ (one-sixth power). As a result, for small values of H, the fluctuations at amplitudes relevant to limit loads become stronger relative to the rms, whereas for large values of H, they become weaker. The neglect of this phenomenon, by the PSD method, influences the ratios of aircraft loads which tune to different values of H. In particular, loads predicted by the PSD method, which assumes that large loads scale in proportion to the rms, tend to be conservative at large gradient distances and unconservative at short gradient distances.

A third aspect of non-Gaussian turbulence that influences aircraft response predictions relates to the fact that the most intense fluctuations tend to occur in short bursts (section 3). In reference 25, for example, the ratios of the magnitudes of the peak responses in the outputs of lightly damped systems and of well-damped systems were evaluated for inputs in the form of moderate turbulence and for inputs in the form of severe turbulence. It was found that this ratio is less in severe turbulence than in moderate turbulence. However, as the power spectral densities were of similar shape in the two cases, the PSD method predicted that this ratio is independent of the severity of the turbulence. The conclusion may be drawn that, in the case of severe turbulence, the fluctuations of highest intensity occurred in relatively short bursts, or short patches, such that the responses did not reach the statistical equilibrium assumed in the PSD approach. As is noted in reference 25, the PSD method has no free parameters to correct for this discrepancy, whereas in the SDG method, it can be accounted for simply by reducing the p-factor, or pattern amplitude factor, p_n associated with the gust pattern tuned to the more lightly damped system.

8.1.2 The Tuned Isolated Gust Model.

To take into account the more extreme, and relatively isolated, atmospheric disturbances, a tuned isolated discrete gust (IDG) model is included in the airworthiness requirements. This does address the problem of representing a very localized fluctuation and takes into account, statistically, some properties of severe-to-extreme turbulence. It is defined by a limited family of velocity profiles, of one-minus-cosine form, incorporating prescribed changes in gust velocity that are proportional to gradient distance to the power one-sixth. The assessment of an aircraft load quantity involves finding the maximum response to this prescribed family of discrete gust profiles. While the one-sixth scaling law in this gust model is consistent with measurements made of severe turbulence [80] its use of just a single shape of gust profile severely limits its ability to take into account, realistically, the widely differing aircraft dynamic response characteristics, which have been shown to tune to associated gust patterns of differing shapes.

As with the PSD method, discussed in section 8.1.1, the significance of this deficiency is most acute when the ratios of design loads on two aircraft, or two different response quantities on the same aircraft, are compared. In reference 75, examples are presented of ways in which this restriction to just a single shape of gust profile can underestimate response magnitudes for some load quantities while overestimating response magnitudes for others.

8.2 RELATIVE ADVANTAGES OF SDG MODEL.

It was to meet the above deficiencies in the existing requirements that the SDG method was developed. In the following sections, some specific advantages claimed for the SDG model are summarized.

8.2.1 Realistic Relationship Between Gust Amplitude and Gradient Distance.

Unlike the PSD method, the SDG representation takes into account the non-Gaussian statistical structure of measured turbulence. A particular non-Gaussian property of measured severe turbulence, incorporated into the SDG model, is that the strengths of the tails of the probability distributions of turbulence-velocity increments increase as the gradient distance is reduced. This effect is quantified in the SDG model by means of the $k = 1/6$ scaling law, which relates the amplitude of extreme gusts to gradient distance and that, as described in section 8.1.1, differs from the $k = 1/3$ law, which applies to the scaling of the rms (of velocity increments).

One example of the practical advantages to be gained from a gust model, which incorporates this non-Gaussian property of severe turbulence, was described in section 7.5.1. Here, it was shown that for a particular class of aircraft, the ratio of lateral gust loads (such as fin bending) to vertical gust loads (such as wing bending) predicted by the SDG method is typically smaller than the corresponding ratio predicted by the PSD method. Thus, the adoption of a gust criterion based on the SDG method (appendix C) would lead to a redistribution of the magnitudes of predicted responses to vertical and lateral gusts, leading to a distribution of aircraft strength and weight more consistent with the measured statistics of severe turbulence.

8.2.2 Incorporates Representation of Short-Duration Bursts.

The fact that severe-to-extreme turbulence tends to occur in short-duration bursts of non-Gaussian turbulence (section 3) is taken into account in the SDG model through the complexity factors, or p-factors, which define the relative amplitudes of single isolated ramp gusts and of gust patterns comprising extended sequences of ramp gust components [87] occurring at the same level of probability. As is demonstrated explicitly in reference 87, the ratio of these amplitudes measured in severe turbulence differs from the same ratio measured for a Gaussian process. Expressed in terms of the complexity factors pn associated, in the SDG model, with compound gust patterns comprising n ramp gust components, for $n > 2$, the factors pn applicable to severe turbulence are less than the corresponding factors applicable to moderate turbulence or to a Gaussian process.

As discussed in section 7.5.1, the incorporation of this measured property of severe turbulence into the design process could lead to a redistribution of the relative magnitudes of predicted aircraft loads associated with lightly damped and well-damped response modes.

8.2.3 Single Requirement Instead of Two.

The replacement in the airworthiness requirements of two separate turbulence models (section 8.1) by a single unified representation of atmospheric gusts, as in the proposed gust Criterion (appendix C and reference 75), would result in a simplified implementation and documentation of the certification process and an associated overall reduction in workload for both the aircraft manufacturer and the certification authority.

Another advantage of such unification lies in the reduction of potential wasted time, particularly in the early stages of a new design when the loads department of a manufacturing company has to give the stress office a set of loads for some chosen flight condition(s). Because the design is at this stage only provisional, there can be uncertainty as to which of the two gust models in the requirements will be critical. Early in a project, in order to meet this demand and subject to a demanding schedule, prior experience has to be used to select only a limited set of cases which are expected to be critical. This might entail, for example, only performing a PSD analysis and leaving the tuned IDG analysis for a later time (or vice versa). One advantage of designing to a single unified turbulence requirement would be in the reduced risk of missing a significant case early in the program and the subsequent disruption caused by a late loads increase.

Finally, the replacement of the two gust and turbulence models in the existing airworthiness requirements by a single SDG model has the advantage of providing an unambiguous criterion for the design and optimization of control systems for gust alleviation. In this context, the problems raised by the limitations of the present gust requirements were highlighted by H.P.Y. Hitch in a paper on gust alleviation systems delivered to the Royal Aeronautical Society [7] in 1979. To quote from this paper: "… any gust load alleviation system must alleviate appropriately real gust turbulence and not just the idealized gust/turbulence of current regulations, which admittedly have produced satisfactory strengths of civil aircraft over the past 30 years and more." Referring to the "still deep-rooted divisions of opinion on the merits of these two procedures" in the existing requirements, Hitch [7] goes on "The IG (isolated gust) description seems to represent rather better, goes the argument, the conditions of the extreme event which is presumed part of some special deterministic process such as a convective storm.

The PSD description seems to represent rather better the conditions in which these extreme events are embedded rather than the events themselves. There are genuine objections to each and there is a need of a representation which embraces them both as special cases…" Hitch [7] proceeds to give specific examples to illustrate the difficulty of designing a system for which the predicted benefits, as evaluated by the two turbulence models in the existing requirements, are consistent. Hitch concludes that 'These matters are not academic for … it is conceivable that a gust alleviation system designed to a PSD description would not be satisfactory if subjected to an isolated gust and vice-versa. A unifying theory - as for example SDG – would resolve such matters."

These concerns were expressed in a lecture concerned with structural load alleviation systems. However, it may be noted that they are equally valid in the context of systems designed to alleviate problems of passenger and crew injury due to unexpected encounters with severe gusts in clear air.

<u>8.2.4 Data Analysis Tools for Severe Turbulence Records</u>.

The principal hypothesis underlying the SDG method is that the most fundamental properties of atmospheric turbulence, having relevance to aircraft design engineers, are quantities related to the statistics of turbulence-velocity increments (i.e., ramp gusts). Of particular importance are parameters that quantify how these increments vary in amplitude as the scale (gradient distance) is varied and parameters that quantify how sequences of increments cluster. The central roles played by these parameters, in influencing aircraft loads, were demonstrated in documented studies by BAe [25]. In consequence, associated data analysis tools and algorithms have been developed to extract these parameters (which cannot be inferred from the PSD) from measured data. These SDG-based algorithms are as follows.

1. Turbulence smoothing and differencing filters to detect velocity increments and software designed to extract scaling (k) parameters from the filter outputs

2. Combinations of the above filters to detect sequences of increments and software designed to extract cluster parameters (p-factors)

3. A phase randomization algorithm, which produces from measured turbulence records related Gaussian signals having identical power spectra but Gaussian statistics.

Application of algorithms of types 1. and 2. above to both a turbulence record and to the associated Gaussian signal 3. allows results of the data analysis, which depend only upon the power spectrum to be separated from results that reflect the non-Gaussian turbulence structure (technically it allows the second-order statistics and the higher-order statistics to be measured independently).

Illustrations of the most recent use of these algorithms are presented in references 87 and 80.

As airlines are increasingly flying aircraft on routine operational flights that carry digital flight-data recorders from whose outputs wind components can be reconstructed [20 and 76], encounters with severe turbulence that cause crew and passenger injury are likely to become the major source of data relating to the nature of atmospheric turbulence. As a result, there is an increasing need for appropriate data analysis tools that extract the maximum relevant information from these records in a form applicable to the calculation of associated aircraft dynamic response, particularly for the purpose of designing control systems that alleviate the gust-induced accelerations.

One way to use high-quality measured turbulence records is to store them in the form of a library for use in the numerical simulation, and possibly optimization, of system response. This has the disadvantage that, because of the non-Gaussian nature of the records, in order to provide sufficient confidence in the generality of the results, it is necessary to run a very large number of sample inputs in the simulation of each configuration (a much larger number than would be necessary for a Gaussian process, because of the rarity of the more extreme events). While a particular system may exhibit apparent advantages in the form of reductions in aircraft response with respect to the samples in the library, there remains the question as to whether equiprobable samples of severe turbulence could cause larger response amplitudes or result in different predicted trends.

To remove this disadvantage, for the purpose of assessing system benefits, rather than use a collection of turbulence records directly in the simulation process, the records of turbulence velocity may be subjected to the data analysis tools outlined above, in order to confirm the validity of an appropriate statistical model and to determine the numerical values of model parameters. The design and assessment of the various systems can then be based on evaluating the benefits of the systems with respect to the calibrated model. In effect, the use of a turbulence model provides a means for extrapolating from (or interpolating between) a given set of records to the family of potential records from which this particular set has been randomly selected. In the application of the model to system assessment, rather than simulation using measured time histories, a systematic evaluation of critical cases may be made, using standard SDG response evaluation procedures (section 6). Of course, increased confidence may be provided by evaluating, as a supplementary procedure, the system response to a limited number of samples of the original recorded turbulence, but this by itself cannot reliably form the basis of the demonstration of system benefits in statistical terms.

It is a major limitation of the PSD and tuned isolated discrete gust models of turbulence in the current airworthiness requirements that neither has the capability of representing the most useful information to be derived from flight records relating to severe turbulence incidents. This information, contained in measured flight profiles, does not lie in the PSD of the records but in the phase components of the Fourier representation (figure E-1 of appendix E). Furthermore, measured wind profiles can only be related in a relatively crude manner to the restricted shape of gust profile in the tuned isolated discrete gust model.

In contrast, data analysis procedures do exist, as outlined above, for deriving SDG parameters from measured records of severe turbulence, including isolated gusts, whether derived from research aircraft or from routine operational flight data.

8.2.5 Assessment of Aircraft With Nonlinear Systems.

As this report is concerned specifically with the linear SDG method, here one simply makes reference to this important advantage of the SDG method. For a nonlinear aircraft, or automatic control system, the implementation of the proposed draft criterion based on the SDG method [75] requires a search to be performed in the time-plane, for each load condition, to find from the prescribed family of gust patterns that pattern which causes maximum response. Research presented at the Gust Specialists Meetings has pointed out the way in which the criterion could be met, in this case by a computational search for the critical gust pattern, based on currently available genetic algorithms.

9. CONCLUSIONS.

A documentation of the statistical discrete gust (SDG) method has been presented herein, including a review of the historical background and a comprehensive list of references, which contains previously unpublished internal reports that are now to be made available on a CD-Rom disc.

Previously unpublished analysis of severe turbulence records, in support of numerical parameters in the SDG gust model, has also been provided on the CD-Rom disc in the form of supplementary reports [87 and 80]. Also provided on the disc is a report [75] containing details of the SDG-AS-SC gust model. Matlab software to implement this model on a PC is to be made available from Stirling Dynamics Ltd. on request.

The SDG method for predicting aircraft loads in severe-to-extreme atmospheric turbulence may be contrasted with the two gust and turbulence models in the current airworthiness requirements for aircraft structural loads (FAR, Part 25, Airworthiness Standards: Transport Category Airplanes'. U.S. Dept. of Transportation, Federal Aviation Administration), which cater independently for tuned isolated discrete gust (IDG) and continuous turbulence. On the one hand, it can be interpreted as generalizing the existing IDG model to take into account for tuning to gust patterns of different shapes. On the other, it is expressed in a statistical format that parallels that of the existing power spectra density (PSD) method for flight in continuous turbulence, being applicable in both Mission Analysis and Design Envelope forms. However, it differs from the PSD method in that the analytical foundations of the PSD method for the calculation of critical loads implicitly assume phase correlations to be purely random, through Gaussian process assumptions, the SDG representation takes into account the phase correlations in measured severe turbulence, which result in the associated statistics being highly non-Gaussian.

Two major consequences of the phase correlations in non-Gaussian turbulence are:

1. The strengths of the tails of the probability distributions of velocity increments (ramp gust amplitudes) vary with gradient distance, the strength increasing as gradient distance is reduced. This phenomenon is accounted for in the SDG model in that extreme gust amplitudes follow the $H^{1/6}$ law, whereas the PSD model assumes that the larger gust amplitudes follow the same scaling law as the root mean square (rms), i.e., the $H^{1/3}$ law. Compared with the PSD model, the ratio of the amplitudes of short gusts to the

amplitudes of long gusts, for a prescribed level of probability, is thus increased in the SDG model of extreme turbulence.

2. The most intense gusts in severe-to-extreme turbulence tend to occur in short bursts. This is accounted for in the SDG method by complexity factors, which are chosen to reflect the fact that more complex patterns, comprising extended sequences of ramp components, are relatively less probable than would be the case in a random phase process, i.e., in the PSD model.

In consequence 1., the ratio of (SDG design loads) to (PSD design loads) tends to be greater for response quantities that tune to relatively short gusts than for response quantities that tune to longer gusts, i.e., for higher-frequency response quantities than for lower-frequency response quantities. In consequence 2., the ratio is also greater for more highly damped response quantities, i.e., for response quantities that tune to relatively elementary gust patterns (one or two ramps), than for quantities that tune to more extended complex patterns. A particular consequence, which reflects both properties of 1. and 2., is that the ratio of SDG design loads to PSD design loads will, in general, tend to be greater for wing responses, such as wing root bending loads, than for tail responses, such as rear fuselage and fin bending loads. These trends are consistent with trends already well-known in the IDG design loads and PSD design loads ratio. It has been pointed out that if SDG and PSD loads criteria were to be matched for the case of wing response to vertical gusts, then an SDG criterion (appendix C) would tend to reduce the predicted tail response to lateral gusts, with possible scope for associated weight reduction. Conversely, on the basis of properties 1. and 2., a redistribution of existing weight to meet an SDG criterion (appendix C) would result in increased safety margins.

The advantages for the SDG method are summarized as follows:

1. Loads predicted by the SDG method are in greater conformity with measured properties of severe-to-extreme turbulence in the real atmosphere. This applies in particular to the ratios of vertical responses, such as wing root bending loads, to lateral responses, such as rear fuselage and fin bending loads. It is possible that advantage could be taken of these improved predictions to save weight or, alternatively, to increase safety margins.

2. The fact that predicted load ratios are in greater conformity with measured properties of the real atmosphere also means that it provides the most realistic gust model appropriate for the design and assessment of the benefits of gust load alleviation systems. In this case, the relevant ratios relate the loads with and without the system active.

3. On the same basis, it provides the most realistic gust model appropriate for the design of cabin motion response control systems, addressing passenger and cabin crew injury in encounters with severe turbulence.

4. An airworthiness requirement for structural loads based on the SDG method would require only one criterion, in place of the two existing criteria, with resulting savings in the amount of necessary documentation, workload, and design cycle time. A related advantage to the manufacturer of designing to a single unified turbulence requirement

may be the reduced risk of missing a significant case early in the program , when an initial stress analysis is performed by some manufacturers before a full comparison of the loads predicted by the two existing criteria has been completed.

5. It provides appropriate data analysis tools for extracting the relevant information from records of severe turbulence incidents, including encounters with isolated gusts, whether derived from research aircraft or from routine operational flight data recorders.

6. It provides a realistic, time-domain, gust model for the loads analysis of aircraft with highly nonlinear systems.

7. Accepted gust criteria having a common basis with the SDG method already exist in the current airworthiness requirements in the form of the $H^{1/6}$ scaling law in the tuned IDG requirement and the 0.85 reduction factor to represent non-Gaussian effects in the recent multiaxis requirement [96].

Two recent implementations of the SDG method for application to severe-to-extreme turbulence, the SDG1 and SDG-AS-SC models, have been described and compared in terms of their load predictions and associated input gust patterns. Of these, SDG1 is the more comprehensive analytical version having a minimum of simplifying assumptions and SDG-AS-SC is a simplified version, now proposed [75] as the basis for a viable design criterion (appendix C).

Limitations in the current implementations of the SDG method for loads prediction, as for current design criteria, still exist in regard to the representation of particular fluid-mechanical phenomena, including vortex cores and wave-like periodic structures (possibly vortex-induced). These limitations are related to the present lack of empirical evidence in sufficient quantity, and of appropriate form, to derive the required statistical parameters.

The possible future incorporation of explicit vortex-core patterns is discussed in reference 80. There, it is pointed out that the inclusion of a vortex-core representation in the SDG model was already proposed in 1976, and a possible profile shape to represent the velocity distribution for a vortex cross section was described. The position regarding the statistical analysis of empirical data representative of severe-to-extreme gusts has not, in fact, changed significantly over the intervening years and the above proposal still stands. The validation of the vortex profile shape, and the acquisition of related statistics in terms of amplitude distribution and scaling laws for velocity increments, still remains a challenge for future work.

The current implementations of the SDG method are also limited in their representation of wave-like periodic structures in turbulence. Recent analysis of measured severe turbulence data, which highlights the existence of such structures over a wide range of wavelengths, is described in reference 87. However, until such time as the results are substantiated by the analysis of data from a wider database, it has been decided to limit their inclusion in the current SDG models to gradient distances having an upper limit of 350 ft. While this is a somewhat arbitrary upper limit for a sharp cutoff, it is comparable with the analogous cutoff of the 1-cosine profile in the existing tuned IDG requirement, which already has wide acceptance.

10. REFERENCES.

The following reference codes [B-01], …, [E-01], …, etc., identify the locations of reports on a CD-Rom disc, now available for unlimited circulation. The prefix B refers to internal Royal Aircraft Establishment (UK) Reports and Memoranda and the prefix E refers to reports prepared for Gust Specialist Meetings convened by the FAA.

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APPENDIX A—SUMMARY OF THE SDG THEORY AS PRESENTED IN 1980
(Adapted from reference 38)

The gust model is expressed as a function of distance in space; conversion to time as independent variable, for aircraft response assessment, may be made in the usual way using the frozen field hypothesis.

Gust patterns are built up using discrete ramp components. The amplitudes w_j and gradient distances H_j of the individual ramp components in a pattern are related to an intensity parameter u_0 by the equations

$$\frac{w_1}{H_1^k} = \frac{w_2}{H_2^k} = \ldots = p_i u_0 \qquad \text{(A-1)}$$
$$H_j \le L$$

where the scaling exponent k is a constant in the range $0 < k < 1$, p_i is a 'complexity factor' that depends on the number i of components in the pattern and L is the turbulence scale length. $p_1 = 1$ and the p_i decrease monotonically with i : numerical values are discussed in reference B-22, section 2.2]. In addition, we introduce the vector 'configuration' variable

$$H = \{H_1, H_2, \ldots, H_{S1}, H_{S2}, \ldots\}, \qquad \text{(A-2)}$$

where the H_{Sj} define the relative positions of the components. In the case of a gust pair, one such additional variable is required and defines the 'spacing'. A particular pattern, denoted by $\{H, u_0\}$, is then completely defined by equations (A-1) and (A-2).

For an aircraft response variable y we denote by $y(H,u_0)$ the magnitude of the largest peak in the response to the pattern $\{H,u_0\}$ and introduce the scaled peak-response amplitude

$$\gamma_y(H,u_0) = y(H,u_0)/u_0 \,. \qquad \text{(A-3)}$$

In the case of a linear system, $\gamma_y(H,u_0)$ becomes independent of u_0 and we write

$$\gamma_y(H) = \gamma_y(H,u_0), \qquad \text{(A-4)}$$

Subject to the constraint imposed by combining equation (A-1) with a bound on u_0 it is possible to find a tuning condition, or 'worst case', at which a maximum in $\gamma_y(H,u_0)$ (nonlinear system) or $\gamma_y(H)$ (linear system) occurs. In most applications this is a stationary maximum but in some cases it occurs on a boundary $H_j = L$. We denote the maximum value by

$$\overline{\gamma_y}(u_0) = \gamma_y\{\overline{H}(u_0), u_0\} \quad \text{(nonlinear system)} \qquad \text{(A-5)}$$

or

$$\overline{\gamma_y} = \gamma_y\{\overline{H}\} \quad \text{(linear system)} \tag{A-6}$$

and the associated 'tuned' pattern configuration by $\overline{H}(u_0)$ or $\overline{H}$. In addition a 'sensitivity' term λ is defined, which is a measure of the sharpness or curvature of $\gamma_y(H, u_0)$ or $\gamma_y(H)$ at the tuning condition. For many practical applications the variations in λ are relatively insignificant and a constant value λ = 0.2 (a typical average value) may be assumed.

In a deterministic assessment of system response the performance measures:

$\overline{\gamma_y}, \overline{H}$ (1inear system)

$\overline{\gamma_y}(u_0), \overline{H}(u_0)$ (nonlinear system)

are employed. For an aircraft airworthiness requirement these measures would be related to a prescribed 'design' value $u_0 = U_0$ through equation (A-3). Such a requirement would be referred to as a 'Design Envelope' requirement.

Alternatively, an explicit statistical interpretation of the same performance measures exists, based on the rate of occurrence (per unit distance flown) N_y of response peaks greater than magnitude y, when the input is modeled by means of a prescribed statistical aggregate of gust patterns. An airworthiness requirement in this form is referred to as a 'Mission Analysis' requirement.

In the case of a linear system, N_y is given by

$$N_y = (\alpha / \lambda \overline{\overline{H}}) \exp\{-y / \beta \overline{\gamma_y}\}, \tag{A-7}$$

where α and β are parameters characterizing the turbulence environment, and $\overline{\overline{H}}$ is a single scalar length characterizing the scale of the pattern $\overline{H}$ (evaluated as a weighted average of the components in equation (A-2)).

In the case of a nonlinear system it is convenient to regard u_0 as a parametric variable with which to relate N_y and y implicitly. From equation (A-3) it follows that, at the tuning condition, the corresponding value of y is given by

$$y = u_0 \overline{\gamma_y}(u_0). \tag{A-8}$$

In addition N_y is given by

$$N_y = \{\alpha / \lambda \overline{\overline{H}}(u_0)\} \exp\{-u_0 / \beta\} \tag{A-9}$$

(compare equation (A-7)) where $\overline{\overline{H}}(u_0)$ is a length characterizing the scale of the tuned pattern $\overline{H}(u_0)$. From equations (A-8) and (A-9) we may thus obtain a functional relationship between N_y and y with u_0 as a parametric variable.

APPENDIX B—RESPONSE TO GUST PATTERN AS LINEAR SUPERPOSITION OF RESPONSES TO A SET OF INDIVIDUAL RAMPS
(Adapted from reference 26)

In cases where the response of an aircraft is defined by linear differential equations, the principle of superposition may be employed to synthesize the maximum response to higher-order gust patterns from the responses to isolated ramp gusts, thus avoiding the need for an explicit multi-dimensional search for the critical or worst case.

The basis for this simplification may be described as follows. Suppose that the aircraft response (for example a shear or bending moment) is dominated by a relatively lightly damped mode so that the transient response to a single ramp gust takes both positive and negative (incremental) values of significant amplitude, as sketched in figure B-1. By means of a single-dimensional search over gust length H and accounting for the scaling law that prescribes the "family" of ramp inputs, the magnitude of the largest initial response peak, say $\bar{\gamma}_I\left(\bar{H}_I\right)$, and the magnitude of the largest second (or "overswing") peak, say $\bar{\gamma}_{II}\left(\bar{H}_{II}\right)$ are found. This idea is sketched in figure B-2. Note that in general the tuned-gust lengths $\bar{H}_I$ and $\bar{H}_{II}$ for the maximum peak response amplitudes are not equal.

A single excitation profile that combines these maximum response cases (first and second peaks) can be constructed by placing two ramps in sequence, separated by a ramp spacing H_s, as sketched in figure B-3. The response to the second ramp is, of course, superposed with the response to the first ramp. If the spacing is chosen appropriately, the maximum overswing response peak from the first ramp will coincide with the maximum first peak response from the second (in this example, negative) ramp. This response case is essentially a resonance (i.e. response to a tuned excitation). Hence if the maximum response is all that is required, the spacing need not be determined explicitly and the overall magnitude of the maximum response is then

$$\bar{\gamma} = \bar{\gamma}_I\left(\bar{H}_I\right) + \bar{\gamma}_{II}\left(\bar{H}_{II}\right) \tag{B-1}$$

It should be emphasized that during the one-dimensional search procedure with variable H, attention was given to sequential or successive peaks in the *response* to a *single* ramp gust. For the case of the tuned pattern, comprising a *pair* of ramp gusts, the individual ramp response peaks of magnitudes $\bar{\gamma}_I\left(\bar{H}_I\right)$ and $\bar{\gamma}_{II}\left(\bar{H}_{II}\right)$ are superposed and the associated ramp gusts become sequential.

The above procedure has been described for gust patterns comprising just a pair of ramp components. By taking account of further successive peaks in the response to a single ramp gust the method extends simply to higher-order gust patterns.

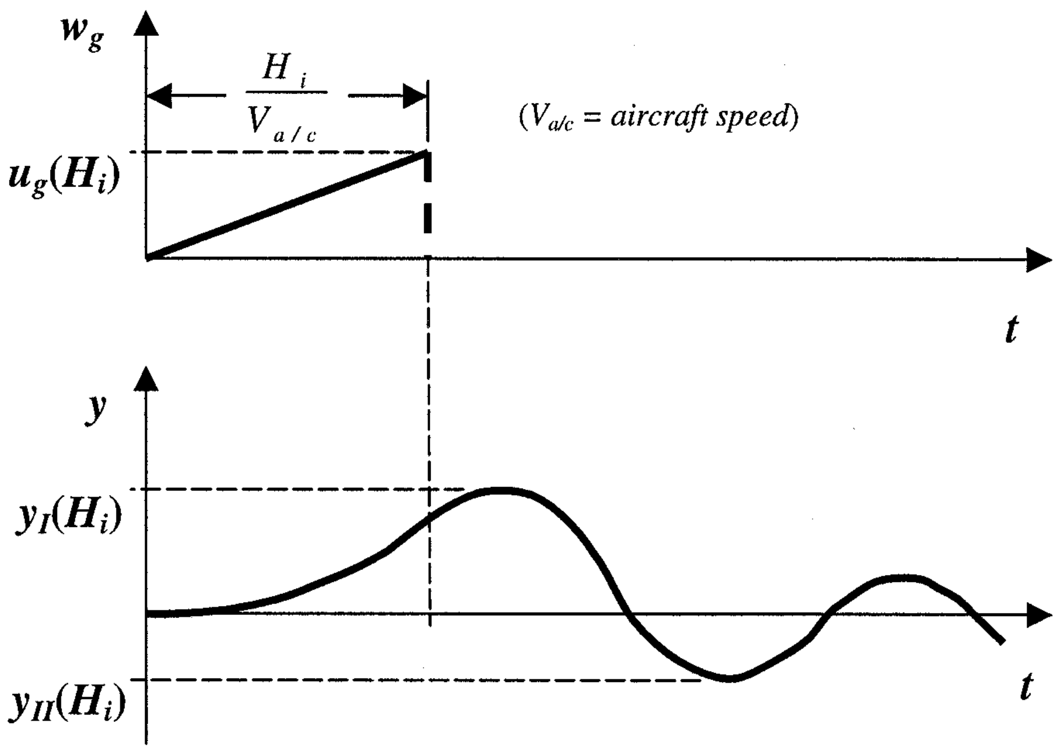


FIGURE B-1. RESPONSE OF A LINEAR SYSTEM TO A RAMP INPUT

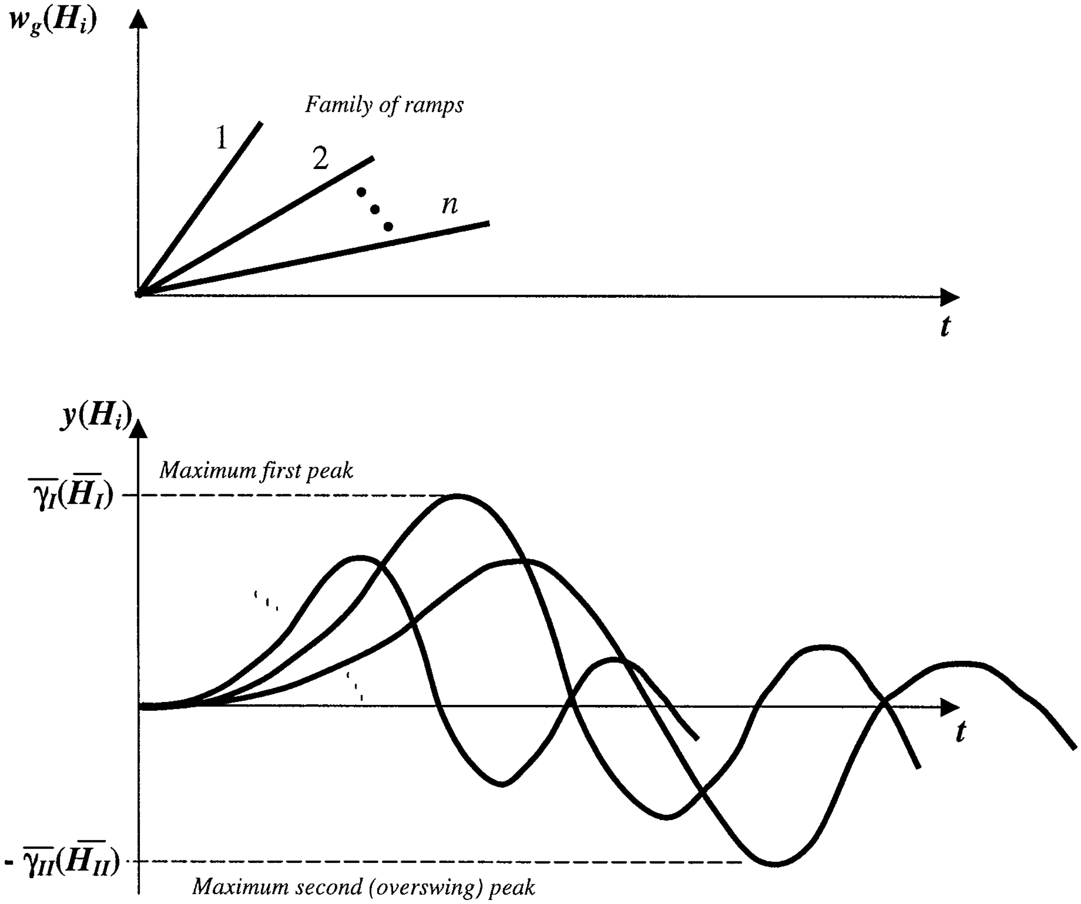


FIGURE B-2. RESPONSES TO FAMILY OF RAMP INPUTS

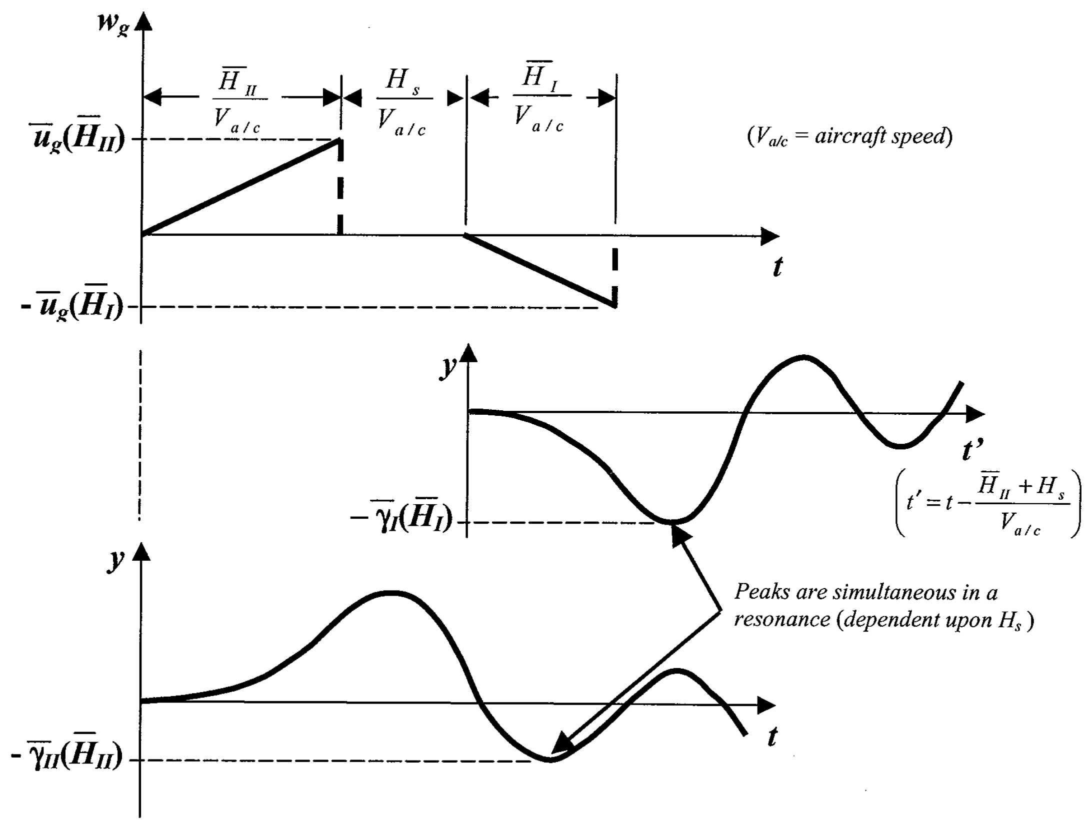


FIGURE B-3. RESONANT COMBINATION OF MAXIMUM FIRST AND SECOND PEAKS FOR A SEQUENCE OF RAMP INPUTS

APPENDIX C—PROPOSED DRAFT GUST CRITERION BASED ON THE SDG METHOD

This proposed criterion replaces subparagraphs (a)(1) through (a)(4) and eliminates paragraph (b) Continuous Gust Design Criteria and its associated appendix G.

§ 25.341 Gust and turbulence loads

(a) Discrete Gust Design Criterion. The airplane is assumed to be subjected to symmetrical vertical and lateral gusts in level flight. Limit gust loads must be determined in accordance with the following provisions:

(1) Loads on each part of the structure must be determined by dynamic analysis. The analysis must take into account unsteady aerodynamic characteristics, control-system dynamics and all significant structural degrees of freedom including rigid body motions. The analysis must also take into account in a realistic or conservative manner any nonlinear structural, aerodynamic or automatic control system characteristics affecting load responses.

(2) The design incremental load response $\bar{\gamma}$ for each load quantity is given by the maximum incremental load that can occur when the input comprises a sequence of up to four non-overlapping ramp-hold gust components having arbitrary spacing, whose gradient distances are varied independently, or a 'periodic' pattern of two, four, eight or sixteen ramps of alternating sign having equal gradient distances and zero spacing. In each case the amplitudes are constrained, as prescribed below, such that the resulting gust patterns have equal probability of occurrence. Both positive (up) and negative (down) ramp-hold gust components are to be considered, but no more than two sequential ramps are to be in the same direction. A pattern of sequential ramps in the same direction will be referred to as a 'staircase'.

The shape and amplitude of each individual ramp-hold gust is given by:

$U = 0.5 * P_k * U_{ds} [1-\cos(\pi s/H)]$ for $0 \leq s \leq H$
$U = P_k * U_{ds}$ for $s > H$

where-

s = distance penetrated into the ramp-hold gust (feet);

U_{ds} = the design gust velocity in equivalent airspeed, as specified in subparagraph (4) of this paragraph;

H = the gust gradient distance (feet) which is the distance parallel to the airplane's flight path for each gust to reach its maximum velocity, taking values in the range from 30 ft. to 2500 ft.;

k (= 1, 2, 3, or 4) is the number of ramp-hold gusts in the sequence; and

P_1, P_2, P_3 and P_4 are complexity factors, included to ensure equal probability of sequences comprising different numbers of ramp components, which take the values 0.92, 0.705, 0.523 and 0.455, respectively.

In the cases of two-, four-, eight- and sixteen-ramp 'periodic' gust patterns the corresponding complexity factors, denoted by P_{2p}, P_{4p}, P_{8p} and P_{16p}, take the respective values 0.9, 0.6, 0.38 and 0.28. The range of gradient distances for these periodic gust patterns is 30 ft. to 350 ft.. (Note that in the above nomenclature for the complexity factors of periodic patterns, the numerical suffix refers to the number of ramp components in the waveform).

For gust patterns that contain a staircase, the following staircase reduction factor, SRF, should be applied to the complete sequence and hence also to the associated load:

(i) for a sequence that comprises just two components (that form a staircase)

$$SRF = 0.893. SSP^{2}$$

(ii) for a sequence comprising three or four components which contains a single two-ramp staircase

$$SRF = SSP^{2}$$

(iii) for a four-component sequence comprising two two-ramp staircases, SSP takes the smaller of the values calculated for each staircase and

$$SRF = 0.925. SSP^{2}$$

where SSP is the staircase scaling parameter, defined to be the minimum value of

$$(H_1 + \Delta H + H_2)^{1/6} / [0.63 \{(H_1)^{1/6} + (H_2)^{1/6}\}] \quad \text{and} \quad 1,$$

and where H_1 and H_2 are the gradient distances of the two ramp gusts forming the staircase and ΔH is the distance between the end of the first ramp and the beginning of the second.

(3) For a LINEAR airplane the following simplified analysis may be performed for each load quantity

(i) Perform time-response calculations for a set of SINGLE ramp-hold gusts as defined in sub-paragraph (2) above, with $P_1 = 1$ and with H taking a sufficient number of values in the range from 30 ft. to 2500 ft. to determine from these results the local maxima and minima of load response. These local extreme values may be either stationary values with respect to both time and gust gradient distance H, occurring at values of H within the range 30 ft. < H < 2500 ft., or they may be stationary values with respect to response time occurring on a scale boundary (30 ft. or 2500 ft.). In addition, perform time-response calculations for a set of periodic gust patterns, as defined in sub-paragraph (2) above, with $P_{2p} = P_{4p} = P_{8p} = P_{16p} = 1$ and with H taking a sufficient number of values in the range from 30 ft. to 350 ft. to determine from these results, in each case, the single global absolute maximum of load response with respect to both response time and gust gradient distance, H.

(ii) For each load quantity, select from the local maxima and minima in the responses to single ramp-hold gusts, determined as in paragraph 3(i), the values of load response defined by the following alternating sign procedure:

E_1 has the largest absolute response magnitude,

E_2 has the largest response magnitude with sign opposite to E_1 such that, when the two time histories containing the response values E_1 and E_2 are displaced in time such that these response values occur at the same instant, the associated component ramps are non-overlapping.

E_3 is the largest response value with magnitude less than E_1 and having the same sign as E_1 such that when the three time histories containing the response values E_1, E_2 and E_3 are displaced in time such that these response values occur at the same instant, the associated component ramps are non-overlapping.

E_4 is the largest response value with magnitude less than E_2 and having the same sign as E_2 such that when the four time histories containing the response values E_1, E_2, E_3 and E_4 are displaced in time such that these response values occur at the same instant, the associated component ramps are non-overlapping.

If any of the above local maxima / minima do not exist, the associated E-value is set equal to zero.

In addition, for each load quantity, denote by E_{2p}, E_{4p}, E_{8p} and E_{16p} the single global absolute maximum value of load response to respectively a set of two-, four-, eight- and sixteen-ramp periodic patterns, determined as in paragraph 3(i).

(iii) For each load quantity, combine the four values E_1 to E_4 to give four response values as follows:

$$\gamma_1 = P_1|E_1|$$
$$\gamma_2 = P_2(|E_1| + |E_2|)$$
$$\gamma_3 = P_3 \max((|E_1| + |E_2| + |E_3|) and (|E_1| + |E_2| + |E_4|))$$
$$\gamma_4 = P_4(|E_1| + |E_2| + |E_3| + |E_4|)$$

where P_1, P_2, P_3 and P_4 are the complexity factors whose numerical values are given in sub-paragraph (2). Corresponding to each response value γ_k, for each load quantity, is a 'tuned' gust pattern consisting of k non-overlapping ramp-hold components whose signs are chosen such that the associated stationary response values reinforce one another. In those cases in which the resulting gust pattern contains one or two staircases, as defined in subparagraph (2), the tuned gust pattern and the associated value of γ_k should both be factored by the staircase reduction factor, SRF, defined in subparagraph (2).

Combine the two values E_1 and E_3, and the two values E_2 and E_4, for each load quantity, to give response values as follows:

$$\gamma_{2s1} = P_2(|E_1| + |E_3|)$$
$$\gamma_{2s2} = P_2(|E_2| + |E_4|)$$

where P_2 is the complexity factor whose numerical value is given in subparagraph (2). Corresponding to these response values are two 'tuned' staircase gust patterns each consisting of two non-overlapping ramp-hold components, displaced in time such that their associated response values of maximum amplitude occur at the same instant and act in the same direction. Both the tuned gust pattern and the associated values γ_{2s1} and γ_{2s2} may be further factored by the staircase reduction factor SRF defined in subparagraph (2).

In addition, evaluate the response values:

$$\gamma_{2p} = P_{2p}(E_{2p})$$
$$\gamma_{4p} = P_{4p}(E_{4p})$$
$$\gamma_{8p} = P_{8p}(E_{8p})$$
$$\gamma_{16p} = P_{16p}(E_{16p})$$

where P_{2p}, P_{4p}, P_{8p} and P_{16p} are the complexity factors whose numerical values are given in subparagraph (2).

(iv) The design incremental load response $\bar{\gamma}$ for each load quantity, defined in subparagraph (2), is then given by the overall maximum value of the four quantities γ_k (factored where appropriate by the staircase reduction factor, SRF, defined in subparagraph (2)), the quantities γ_{2s1} and γ_{2s2} each factored by SRF (defined in subparagraph (2)) and the quantities γ_{2p}, γ_{4p}, γ_{8p} and γ_{16p}, all as described in subparagraph (3)(iii).

(4) Corresponding to the design incremental load response $\bar{\gamma}$, defined in subparagraph (2), is a 'tuned' input gust pattern which, in addition to its association with the design incremental load response, can also be used in the calculation of correlated loads.

Design limit loads are defined as follows:

$$P_L = P_{1-g} \pm \bar{\gamma}$$

where P_{1-g} = the steady 1-g load for the condition.

The design gust velocity must be determined from the relation:

$$U_{ds} = U_{ref}*F_g*(H/350)^{1/6}$$

where-

$U_{ref} =$ the reference gust velocity in equivalent airspeed defined in subparagraphs (5)(i) and.(5)(ii)

$F_g =$ the flight profile alleviation factor defined in paragraph (6).

(5) The following reference gust velocities apply;

(i) At airspeeds between VB and VC:

Positive and negative gusts with reference gust velocities of 56.0 ft/sec EAS must be considered at sea level. The reference gust velocity may be reduced linearly from 56 ft/sec EAS at sea level to 44.0 ft/sec EAS at 15,000 ft. The reference gust velocity may be further reduced linearly from 44.0 ft/sec EAS at 15,000 ft to 20.86 ft/sec EAS at 60,000 feet.

(ii) (no change)

(6) (no change) - defines F_g

(7) (no change) – relates to stability augmentation systems

(b) (Reserved)

APPENDIX D—EXPLANATORY INFORMATION RELATING TO THE PROPOSED DRAFT CRITERION

Introductory comment.

With the exception of subparagraph (3), which is applicable only when the aircraft load response is linear, all subparagraphs are applicable to both linear and nonlinear aircraft.

Subparagraph (2)

This subparagraph prescribes numerical values for complexity factors P_1, P_2, P_3 and P_4, applicable to ramp gust patterns with arbitrary spacing between the component ramps. It may be noted that the specified value of P_1 is not equal to unity (1). The reason for this concerns the 'hold' component of the ramp-hold gust profile. The value $P_1 = 1$ is reserved for a reference ramp gust defined to have the *most probable* profile containing a ramp-shaped transition from a minimum to a maximum value. This profile takes the form of a ramp *followed by a decay*. P-factors for arbitrary gust patterns are defined as ratios with respect to this reference case. Following this procedure, the P_1 factor for a ramp-hold gust profile, which is less probable than the ramp followed by decay, is 0.92. It has not been found necessary to include the ramp-with-decay as an explicit component of the gust Criterion. It may be noted that the ramp-hold pattern can excite an aircraft mode containing both rigid-body and flexible components significantly more than the ramp-with-decay, the ramp exciting the flexible component of the response and the 'hold' exciting the rigid-body component.

This subparagraph also prescribes values for factors P_{2p}, P_{4p}, P_{8p} and P_{16p}, applicable to periodic gust patterns. The need for the incorporation into the SDG model of these latter gust components arises from empirical evidence that severe turbulence, in addition to shear-like structures well represented by patterns of ramps, contains periodic components which, at least in some cases, appear to be vortex-induced. The relatively high intensity of these periodic components is reflected in the magnitudes of P_{2p} / P_2 and P_{4p} / P_4.

This subparagraph also includes references to staircase gust patterns, illustrated in figure D-1. It should be noted that figure D-1 is schematic in the sense that all straight-line ramps shown in this figure in fact represent (1-cosine) ramps. In particular, the situation in which the spacing between two successive (1 – cosine) ramps is equal to zero does not lead to them coalescing into a single (1 – cosine) ramp. Cases 1(a) and 2(a) illustrate two staircase gust patterns which differ only in the spacing BC between the two sequential ramps. In Case 1(a), the dashed line AD joins the beginning of the first ramp to the end of the second ramp. In the following it is shown how the probability of this 'virtual ramp' transition from A to D, can be calculated.

In each case, figure D-1, with each staircase configuration, (a), may be associated a reference gust LM, (b), in the form of a single ramp acting over the same overall distance as the staircase, LN = AG, and scaled in amplitude MN such that this reference gust has the same probability of occurrence as that associated with the two-ramp staircase. This requires that the scaled amplitude

$$M1 = MN / (LN)^{1/6} \tag{D-1}$$

of the reference gust (b) be related to the scaled amplitudes

$$M2 = BE / (AE)^{1/6} = DF / (CF)^{1/6} \tag{D-2}$$

of the individual ramp components AB and CD by a two-component staircase complexity factor P_{2s}:

$$M2 = P_{2s}. M1. \tag{D-3}$$

The complexity factor P_{2s} , introduced in (D-3), is the factor applicable to a two-ramp staircase, and thus is taken to be the product of the complexity factor P_2 (= 0.705), applicable to all two-component gust patterns, and a further reduction factor 0.893 applicable to all two-component sequences that form a staircase:

$$P_{2s} = (0.705)\,(0.893) = 0.63. \tag{D-4}$$

Suppose now that the spacing BC between the component ramps of the staircase (a) is increased while the component ramps themselves, and hence also the overall velocity increment DG over the staircase, remain constant. Under this transformation, the values of M2 and hence also M1 also remain constant. Then, as the associated overall length of the staircase AG, and hence also that of the reference ramp LN, are increased then so also is the amplitude MN of the reference ramp, following the one-sixth law, equation (D-1). Thus for sufficiently large spacing BC the velocity increment MN will exceed the velocity increment DG. This condition is illustrated in Case 1. In this situation the velocity transition from A to D has no influence on the probability of the staircase pattern, Case 1(a).

Conversely, as the spacing BC between the component ramps of the staircase is decreased (figure D-1), the amplitude MN of the reference single ramp (b) is also decreased, while the velocity increment DG over the staircase (a) remains constant. It may be verified that for sufficiently small spacing the velocity increment DG will exceed the velocity increment MN. This condition is illustrated in Case 2. In this situation the velocity transition from A to D comes to have a significant influence on the probability of the staircase pattern, Case 2(a). For a staircase comprising two component ramps of equal length, the 'cross-over' condition, where DG = MN, occurs when the spacing BC is approximately twice the length AE = CF of the component ramps.

If we now associate the velocity increment DG over the staircase with that of a 'pseudo-ramp' AD acting over the distance AG, then the condition DG > MN can be seen to violate the prescribed condition that the gust patterns (a) and (b) have the same probability. In effect, the condition DG > MN implies that the pseudo-ramp AD is less probable than the reference gust LM. The incorporation of such a staircase gust pattern in the design requirement would thus be unnecessarily conservative (i.e. require excessive strength).

This violation of the equal-probability condition, which occurs when the spacing BC is sufficiently small that DG > MN, as in Case 2, may be removed by multiplying the overall amplitude of the staircase gust pattern, and hence DG, by a 'staircase scaling parameter'

$$SSP = MN / DG \quad (< 1) \qquad \text{(D-5)}$$

However, even with the above scaling parameter incorporated, the design condition would still be overly conservative. For it can be shown that the probability of the staircase pattern ABCD is in fact LESS than that of the pseudo-ramp AD. For, when the spacing is reduced to zero, the staircase gust pattern, in the form of two consecutive (1-cosine) ramps, is less probable than the single pseudo-ramp AD. (The probability of any pattern joining A to D, in figure D-1(a), can be related to an energy function whose minimum value, corresponding to maximum probability, occurs when the pattern is maximally smooth). Thus, in Case 2, figure D-1, even with the scaling parameter SSP incorporated, pattern (a) still has lower probability than that of the reference ramp (b).

To offset this conservatism:

(i) for any gust pattern containing a staircase, the staircase-reduction factor SRF prescribed in the Criterion is based on SSP^2, which is LESS than SSP when SSP < 1

(ii) for a sequence comprising three or four components, and containing a staircase, the staircase-reduction factor SRF may be applied to the entire sequence and not just to the staircase

(iii) for a two-component sequence that comprises a staircase, SRF is further reduced by a multiplicative factor of 0.893

(iv) for a four-component sequence comprising two staircases (which can only occur in opposite directions) SRF is further reduced by a multiplicative factor of 0.925.

These reduction factors, together with the prescribed dependence of SRF on the *square* of SSP, have been determined as a result of extensive comparisons of aircraft loads resulting from the proposed Criterion, based on ramp-hold discrete gust components, and associated loads derived by an alternative and more mathematically-rigorous method (the SDG1 method) in which the probability of staircase gust patterns is quantified in a less empirical manner.

For implementation purposes, the staircase scaling parameter SSP, (D-5) above, may be expressed purely in terms of distances in the staircase pattern as follows:

$$\begin{aligned} SSP &= MN / DG = MN / (BE + DF) = M1.\,(LN)^{1/6} / \{M2.(AE)^{1/6} + M2.(CF)^{1/6}\} \\ &= \{M1 / M2\}.\,[(AG)^{1/6} / \{(AE)^{1/6}) + (CF)^{1/6}\}] \\ &= \{1 / P_{2s}\}.\,[(AG)^{1/6} / \{(AE)^{1/6}) + (CF)^{1/6}\}]. \qquad \text{(D-6)} \end{aligned}$$

Subparagraph (3)(i)

For linear aircraft response, for each load quantity the first step is to calculate the response to a set of single ramp-hold gusts of different gradient distances. A range of ramp lengths should be covered, with the minimum ramp length being 30 feet and the maximum ramp length being 2500 feet.

To avoid unnecessarily conservative loads, it should be verified that the load-response equations used in the above calculation are such that the response to an input step function tends to zero for large values of time.

An illustration of a typical set of such responses to a set of ramp-hold gusts is shown superposed in figure D-2, where each ramp input has been taken to start at the same instant. In each trace in figure D-2 a sequence of peaks and troughs (maxima and minima) can be identified. Moreover, with each such extreme value, an associated extreme value may be identified in adjacent traces. From the combined information in the full set of response records associated extreme values in the *envelope* can be identified.

Each such extreme value in the envelope can be associated with a particular instant of time and with a particular gradient distance (and hence a particular record). It is simultaneously an extreme value with respect to time and with respect to gradient distance.

An alternative representation of the information in figure D-2 is shown in figure D-3. This figure shows the same set of load responses as in figure D-2 but now the ramp gust inputs have been displaced in time such that, rather than each ramp starting at the same instant, the mid-point of each ramp occurs at the same instant. The stationary extrema in the envelope (extrema simultaneously with respect to time and gradient distance) can now be seen more clearly.

Finally, the time histories in figure D-3 can be used to produce a contour plot, figure D-4, of which the traces in figure D-3 are now horizontal cross sections. This contour plot is obtained by interpolating across a continuum of gradient distances. The extreme values can be associated with the interiors of nested closed loops in the contours. The complete set of such extreme values forms a set of 'candidate' values of which the required extreme values E_1 to E_4, specified in paragraph (3)(ii), form a subset.

Computationally, two alternative methods of locating the extreme values exist. In the first, so called ‘peak tracking’, the extreme values with respect to time are first identified in the responses to each of a set of ramps with different gradient distances. These extreme values are then ‘tracked’ with respect to gradient distance, by associating extrema in records corresponding to ramp inputs having adjacent gradient distances, to find the extreme value with respect to time which is also an extreme value with respect to gradient distance. In the context of figure D-4, this may be interpreted as finding the tops of ridges and bottoms of valleys, in sections at particular gradient distances, and then tracking up the ridge or down the valley to the highest or lowest point respectively.

An alternative computational approach is to perform a direct two-dimensional search of the ‘surface’, of which contours are shown in figure D-4, to identify the locally extreme values.

Subparagraph (3)(ii)

Subparagraph (3)(ii) requires that the four largest of the locally extreme values, E_1 to E_4, from the envelope, described in subparagraph (3)(i), be identified subject a selection process involving alternating sign and subject to a constraint that associated gust patterns comprise non-overlapping ramp components.

In order to implement the constraint that the ramp gusts associated with E_1 to E_4 follow the ASprocedure and be non-overlapping it is convenient to separate the set of 'candidate' extreme values described in paragraph 3(i) into two subsets, corresponding respectively to maxima and minima, with the magnitudes in each set being in order of decreasing absolute magnitude. With each extreme value will be associated a gradient distance (and hence a particular ramp gust) and also a particular instant of time (at which the associated extreme value in the response to that gust occurs).

The greatest extreme value, E_1, which may be a maximum or a minimum, is first selected from the appropriate subset. Next, displacing the ramp gusts in time such that the associated extreme values of load occur at the *same instant*, delete from both subsets of extreme values all those cases whose associated ramp gradient lengths have any overlap with the gradient length of the ramp gust associated with E_1.

Now let E_2 be the largest remaining extreme value with sign opposite to E_1, and hence in the opposite subset, and apply the process of deleting all extreme values whose associated gradient lengths overlap the gradient length of the ramp gust associated with E_2 when the ramp gusts are displaced in time as before.

Next let E_3 be the largest remaining extreme value with the same sign as E_1 and again apply the associated deletion process. Finally let E_4 be the largest remaining extreme value with sign opposite to E_1.

Subparagraph (3)(ii) also requires that, for each load quantity, the single global absolute maximum value of load response to families of periodic gust patterns, denoted by E_{2p}, E_{4p}, E_{8p} and E_{16p} respectively, be found.

Subparagraph (3)(iii)
Subparagraph (3)(iii) requires that the four values E_1, E_2, E_3 and E_4 be combined to evaluate the four quantities 'gamma-k'. The value of k defines the number of elementary ramp components in the associated gust pattern. The associated input gust patterns are found by adding the time histories of the ramp components, with alternate ramps being inverted in sign, or polarity, and displaced relative to one another in time, or position, such that the associated extreme values in the response occur at the same instant. Then, as a consequence of the 'alternating sign' convention, the time histories of response to the time-displaced ramp inputs will now reinforce one another to produce the maximum response to an overall gust pattern synthesized from the chosen component ramps.

It should be noted that, although the selection process involves extreme values of alternating sign, it is not the case that in the resulting overall gust pattern the sequence of component ramps in time will necessarily alternate in sign. In particular, gust patterns comprising three or four ramps may contain two consecutive ramps in the same direction, the two-ramp staircase described in paragraph (2), and a gust pattern containing four ramps may comprise a sequence of two such two-ramp staircases, of opposite polarity.

Subparagraph (3)(iii) also requires that the two values E_1 and E_3 be combined with the complexity factor P_2 to give a response value γ_{2s1}, and the two values E_2 and E_4 be combined with the complexity factor P_2 to give a response value γ_{2s2}, each pair corresponding to a configuration comprising just a two-ramp staircase. The result of incorporating the staircase-reduction factor SRF is that the probabilities of two-ramp staircases become equal to the probabilities of two-ramp up-down patterns having arbitrary separation. It should be noted that, before the application of the factors SRF, γ_{2s1} will always be greater than γ_{2s2}. However, after the application of the factors SRF this will not necessarily be the case.

Finally, subparagraph (3)(iii) also requires that the quantities 'gamma-2p', 'gamma-4p', 'gamma-8p' and 'gamma-16p' be evaluated, associated with the respective periodic gust patterns that maximize the responses.

Subparagraph (4)

In addition to its association with the design incremental load response $\bar{\gamma}$, defined in subparagraph 2, a further application of the tuned gust pattern is in the calculation of correlated loads. For two given load quantities, the tuned gust pattern for the first load quantity is first found, together with the instant in time at which the maximum value of that load occurs (a quantity that is automatically calculated as part of the process of generating the tuned gust pattern). This tuned gust pattern is then used as an input to the second load quantity. The response of the second load quantity occurring at the instant when the first load quantity experiences its maximum value, together with the response of the first load quantity itself at that same instant, provide the required pair of correlated loads.

Subparagraph (5).
Follows closely the format of the existing rule for the tuned isolated discrete gust (1-cosine).

** For a nonlinear aircraft, the implementation of the draft Criterion requires a search to be performed, for each load condition, to find from the prescribed family of gust patterns that pattern which causes maximum response. Research presented at Gust Specialists Meetings has pointed the way in which the Criterion could be met in this case by a computational search for the tuned gust pattern, based on currently available genetic algorithms.

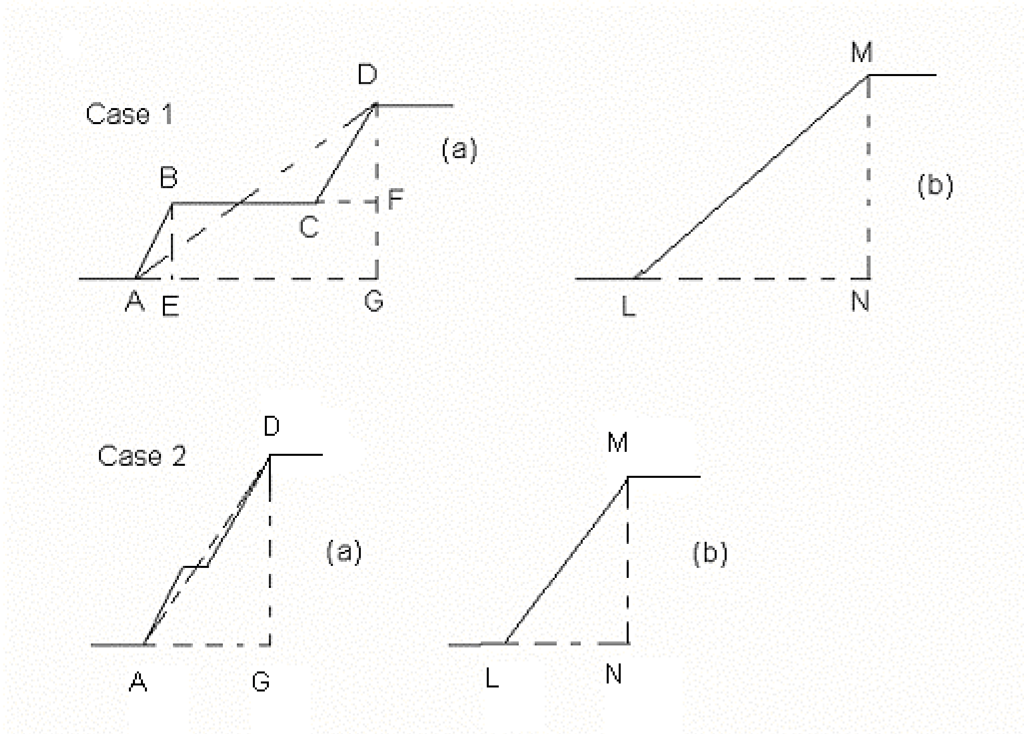


FIGURE D-1. STAIRCASE GUST PATTERNS (SCHEMATIC: ALL STRAIGHT-LINE RAMPS IN FIGURE REPRESENT 'ONE-MINUS-COSINE' RAMP PROFILES)

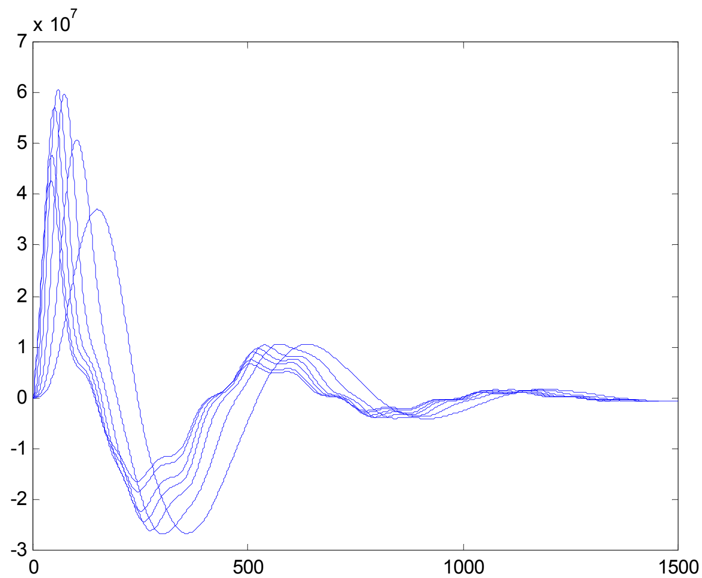


FIGURE D-2. TYPICAL SET OF LOAD RESPONSES TO A SET OF RAMP-HOLD GUST INPUTS, COVERING A RANGE OF GRADIENT DISTANCES, AND ALL STARTING AT THE SAME INITIAL INSTANT (AT SAMPLE NUMBER 1)

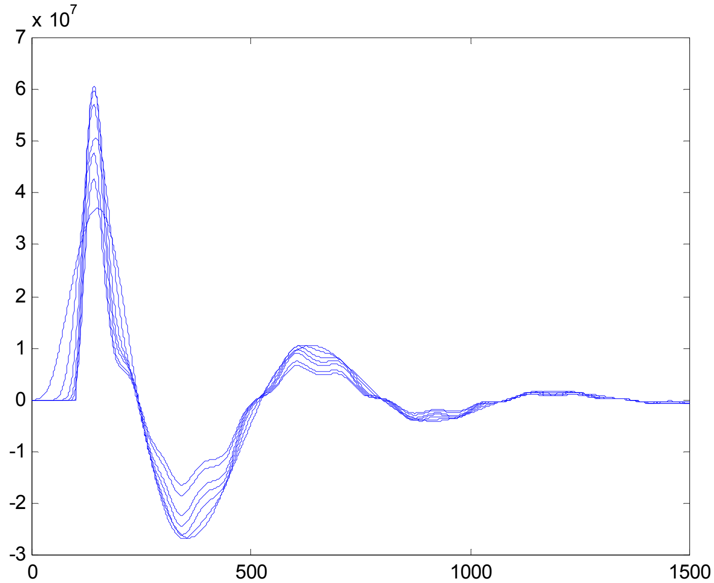


FIGURE D-3. SAME SET OF LOAD RESPONSES AS IN FIGURE D-2 BUT WITH THE RAMP GUST INPUTS DISPLACED IN TIME SUCH THAT THE MID-POINT OF EACH RAMP OCCURS AT THE SAME INSTANT (AT SAMPLE NUMBER 100)

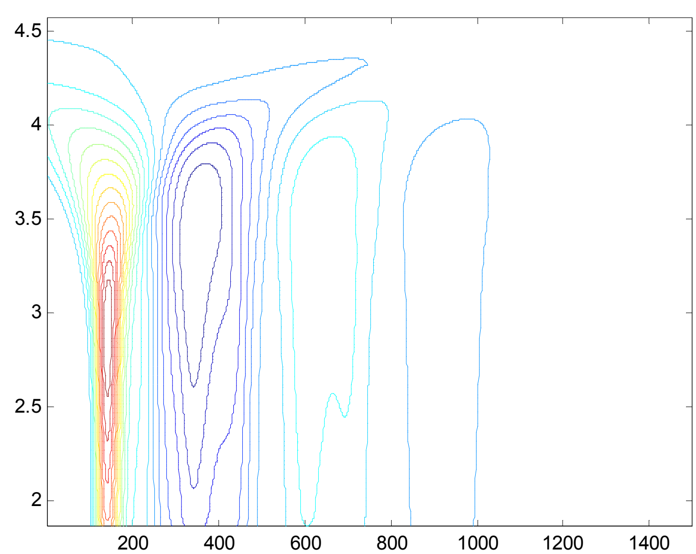


FIGURE D-4. CONTOUR PLOT DERIVED FROM FIGURE D-3 WITH THE VERTICAL AXIS BEING THE LOG OF GRADIENT DISTANCE (TRACES IN FIGURE D-3 ARE HORIZONTAL CROSS SECTIONS HERE)

APPENDIX E—PROPOSAL FOR SDG-BASED AIRWORTHINESS REQUIREMENT MADE IN 1994 [Adapted from reference 66]

The case for a future airworthiness requirement for aircraft limit loads based on the Statistical Discrete Gust (SDG) method.

J G Jones

August 1994

Prepared for Special Gust Specialists Meeting, Hampton, Virginia, September 22, 1994

1. The existing PSD (Power Spectral Density) requirement owes its support largely to the simplicity of its implementation when the aircraft dynamic response can be assumed to be linear. However, it is open to criticism on two major counts:

(i) In terms of a Fourier decomposition, it is representative of the atmosphere only in its Fourier amplitude component. Its Fourier phase component, which is implicitly assumed to be random, is quite unrepresentative of the strong Fourier phase correlations that exist in measured turbulence (figure E-1) and which are associated with the occurrence of relatively sharp ramp-shaped gust components, even in 'continuous' turbulence. These lead to strong (exponential) tails on the amplitude distributions of aircraft response, even for single 'patches' of turbulence. The PSD method of achieving such exponential tails, through the composition of a succession of 'Gaussian patches', is a mathematical expedient which bears no relation to physical reality. Moreover, it has been shown to give quite erroneous results, for example in its predictions of the ratios of the response amplitudes of lightly damped and well-damped modes, when the input is measured turbulence.

(ii) When the aircraft response is nonlinear, even its simplicity of implementation no longer exists. In the absence of applicable analytical tools, resort is usually taken to 'stochastic simulation', using Gaussian patches. However, there is no prospect of reaching widespread agreement as to how an approximate equivalence with the existing requirement for linear aircraft might be achieved.

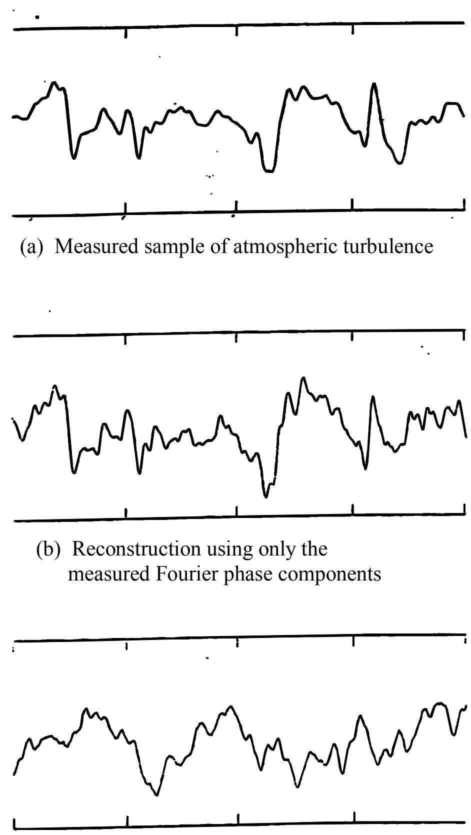
2. The SDG (Statistical Discrete Gust) representation of atmospheric turbulence takes account of the phase correlations in measured turbulence (figure E-1) by modelling explicitly the associated ramp-shaped gust components, and expressing the statistical description of the atmosphere in the form of probability distributions of patterns comprising both single and multiple ramp components. Associated mathematical tools exist both for matching the model to measured turbulence data and for predicting associated aircraft response statistics.

3. The SDG model of turbulence is unique in that it is the only one for which mathematical tools have been developed for adjusting numerical parameters, which specify probabilities, to match data from measured severe gust encounters obtained during routine operational flying (CAADRP).

4. A major conclusion that can be drawn from completed studies of CAADRP data is that the relative probabilities of gust patterns of differing complexity, i.e. number of ramp components, as modeled in the SDG method, differ in recorded turbulence encounters from the relative probabilities that result from the random phase assumption made in the PSD model. To take proper account of these differences would lead to changes in the way in which a given amount of aircraft weight is distributed throughout the structure to minimize the probability of a limit-load encounter.

5. In the formulation of a requirement for flight in continuous turbulence based on the SDG method, outlined in this paper, advantage is taken of the evidence for reduced probabilities of encounters with gust patterns of high complexity to simplify the model by excluding the need to take into account of patterns containing more than eight components. The search procedure proposed for the validation of nonlinear response is thus computationally less expensive than an equivalent implementation of the PSD requirement, for example using the DSP (Deterministic Spectral Procedure).

6. The SDG model offers the advantage of a basis for the future unification of the requirements for flight in continuous turbulence and for encounters with isolated gusts, resulting both in an overall reduction in the amount of computation required to validate an aircraft for flight through atmospheric turbulence and an increase in the degree of realism in the resulting model, achieved by incorporating data from routine operational flying.



(a) Measured sample of atmospheric turbulence

(b) Reconstruction using only the measured Fourier phase components

(c) Reconstruction using only the measured Fourier amplitude components

FIGURE E-1. INFLUENCE OF FOURIER AMPLITUDE AND PHASE ON STRUCTURE OF ATMOSPHERIC TURBULENCE [66]

Reconstruction (b) combines the measured Fourier phase components with theoretical amplitude components corresponding to the von Karman spectral model assumed in the PSD method.

Reconstruction (c) combines the measured Fourier amplitude components with random phase components as assumed in the PSD model.

A comparison of (a) with (b) confirms that the information concerning the existence of sharp ramp-like gradients in the measured turbulence is contained in the phase distribution, a turbulence property that is discarded in the PSD method.

APPENDIX F—REVIEWER COMMENTARY, JOHN GLASER

"DOCUMENTATION OF THE LINEAR STATISTICAL DISCRETE GUST METHOD"

Stirling Dynamics Inc., Report Number SDI-121-TR-1, Issue 1, July 2003, J.G. Jones.

J. G. Jones' documentation report of the linear Statistical Discrete Gust (SDG) method is a significant, thorough dissertation summarising over 30 years of research and development in modelling atmospheric turbulence for aircraft design. It is must reading for anyone responsible for the computation and/or certification of aircraft gust loads. It is must reading for those conducting research in aircraft-related atmospheric processes. And it would be a rewarding read for the technically curious.

The report begins by identifying the limitations of the two gust models currently specified in airworthiness requirements: the tuned Isolated Discrete Gust (IDG) model and the continuous turbulence or Power Spectral Density (PSD) gust model. (These limitations are discussed in considerable detail in Section 8.1.) In a sense, these two models represent opposite ends of the spectrum of gust models. On the one hand, the IDG model, although a valid component of atmospheric turbulence, is too discrete and rarely gives tuned responses involving more than one mode. On the other hand, the PSD gust model is too random because, in addition to random amplitude, it incorrectly assumes random phase and is therefore incapable of representing the (non-Gaussian) high intensity structural bursts that are known to produce the rare but severe-to-extreme gust encounters experienced by aircraft in service. Section 3 provides convincing evidence from both extensive commercial airline flight records (CAADRP) and experimental flight test data from a highly instrumented Gnat trainer, that aircraft structures should be designed for these severe, short-duration bursts of turbulence.

The draft SDG criterion, SDG-AS-SC (Appendix C), proposed by Jones approximates these severe-to-extreme atmospheric bursts of turbulence for a particular parameter and flight condition by a set of non-periodic tuned gust patterns formed by 1, 2, 3 and 4, (1-cos) ramp-hold gust components, each with variable gradient length, H, and corresponding amplitude proportional to $H^{1/6}$. This scaling law is established in Section 4 on the basis of theory and flight test data. For multi-ramp patterns, there is no constraint on the separation between the ramps forming a tuned gust pattern except that the ramps should not overlap. The draft criterion also specifies probability factors on design gust intensity to account for the probability of occurrence or the complexity of each possible tuned gust pattern (i.e. the number of ramps). These factors derive from simulation analysis and commercial flight data. Also identified in severe-to-extreme turbulence and included in the draft criterion are 4 tunable pulse or periodic patterns formed using 2, 4, 8 and 16, $H^{1/6}$ (1-cos) ramp-hold gust components of alternating sign having equal gradient lengths and no separation between the end of one ramp and the beginning of the next. The 2-ramp pulse is simply the gust pattern of the IDG model. The 4, 8 and 16 ramp patterns are particularly effective in exciting modes with very little damping. The probability factors for these pulse/periodic patterns are based on experimental flight test data. The overall maximum load obtained from the two sets of tuned gust patterns, non-periodic and periodic, is taken as the design load for the parameter and flight condition considered. The gust pattern giving maximum load is used to calculate the corresponding time-correlated load distributions for the complete aircraft.

In summary, Jones' documentation report presents compelling evidence that aircraft should be designed for the severe-to-extreme bursts of turbulence actually experienced in service. A design gust criterion has been proposed that simulates, for each load, these turbulence bursts by tuned patterns with gust intensities reflecting their probability of occurrence. This proposed criterion is based on theoretical considerations and is supported by a large body of relevant commercial and experimental flight data. Without doubt, SDG_AS_SC is the most exhaustively researched gust load criterion ever developed!

The present reviewer conducted a reasonably extensive computational evaluation of the proposed linear SDG-AS-SC gust model on a twin turboprop aircraft for 3 significant aircraft configurations at Sea Level and V_c. Bending moment, shear and torsion distributions were determined for the wing due to vertical gusts and for the aft fuselage and fin due to lateral gusts. SDG-AS-SC, IDG and PSD response loads were calculated using MATLAB software provided by Stirling Dynamics Limited.

An indication of the influence that SDG-AS-SC could have on design loads is given by comparing the envelopes of maximum incremental loads obtained for the 3 aircraft configurations considered with those given by IDG and PSD. The design gust intensities U_{ref} = 56 ft/sec EAS and U_σ = 90 ft/sec TAS (ARAC proposed NPRM, in progress) were used for IDG and PSD models respectively. A calibration exercise for wing loads resulted in a corresponding design gust intensity for SDG-AS-SC of U_{rf} = 60 ft/sec EAS. With these design gust values, the maximum load envelopes given by the 3 gust models for wing bending moment, shear and torsion were all in very good agreement, i.e. within 1-2%. However, for the aft fuselage and fin lateral gust loads, the maximum load envelope for PSD was consistently higher than the SDG-AS-SC envelope by 10% to 20% and the maximum load envelope for IDG was consistently lower than the SDG-AS-SC envelope by about the same amount. These results give quantitative support to the long-held views that when IDG and PSD are in agreement for wing loads, the lateral gust loads given by IDG will be unconservative (due to its single mode limitation) and the lateral gust loads given by PSD will be overly conservative (due to its random phase assumption). A detailed explanation for this trend in PSD loads is given in Section 7.5 of the documentation report.

Despite its comprehensive development, based on commercial flight and experimental data, and despite the realistic computational results indicated above, the acceptance of SDG by airworthiness authorities and industry seems lacklustre. The authorities seem to require a significant request for application from industry, and aircraft manufacturers appear to be satisfied with the present IDG and PSD criteria because they have provided "successful designs". This misses the point completely. Based on "successful design", the Pratt formula for turboprop aircraft should be all that is necessary because it had provided successful designs. The point is that aircraft structures should be designed to withstand the gusts to which they are exposed and that information is now available in the form of the proposed SDG-AS-SC criterion! Yes, there will be costs to develop and validate in-house software; and yes, the computer time per load will increase, but labour hours and analysis elapse time should be much reduced because two criteria will be replaced with one. But the most important benefit offered by the SDG formulation is that structural weight would be distributed where it is needed for the gust loads really imposed. It must be emphasised that it is not just the maximum loads that design aircraft structures, it is the combination of maximum loads and their corresponding balanced or time-correlated load distributions (stresses and shear flows) that must be considered, and these corresponding load distributions depend very much on the gust profiles producing the maximum loads.

Those supporting the status quo need to consider, in view of the stringent requirements that severe intermittent gust encounters would impose on control system and actuator design, whether the current gust criteria used to design aircraft structural strength are adequate for designing a gust alleviation system, or for designing a ride control system to address cabin crew and passenger injury.

RECOMMENDATIONS:

While the proposed SDG-AS-SC gust criterion has reached a high level of development, the present reviewer recognises that there may still be room for improvement which he feels can only come from further objective participation by airworthiness authorities and industry. It is therefore recommended that the FAA-hosted ad hoc gust specialists' meetings continue to fulfill its objectives; (a) to reduce the number of gust design criteria, and (b) to recommend a gust design method for advanced aircraft technologies. With respect to SDG, the following activities are suggested:

- To get considered responses from industry and airworthiness authorities to Jones' documentation report,
- To identify and address all direct and indirect issues that industry and airworthiness authorities have that would influence SDG-AS-SC's formulation and acceptance,
- To have industry gain and report on experience in developing in-house SDG-AS-SC code thereby identifying areas in the proposed criterion or in the explanatory information (Appendices C and D of the documentation report) requiring textual clarification.
- To conduct broad in-house computational evaluations of SDG-AS-SC that would include any agreed revisions in the proposed criterion stemming from the above activities.
- To consider model calibration,
- To continue development of the draft criterion and explanatory information with an eye toward the preparation of a draft NPRM/NPA for an SDG-based rule and a draft AC/ACJ encompassing all gust requirements.

John Glaser
Principal Engineer, Loads and Dynamics (Retired)
Bombardier Aerospace

APPENDIX G—REVIEWER COMMENTARY, THOMAS A. ZEILER

This document is important in that it brings together in one place much of the vast amount of work done over many years in the development of what is now known as the Statistical Discrete Gust method. It is clear, and has been for quite a long time, that the description of atmospheric turbulence used to calculate aircraft structural loads as a Gaussian random process is not particularly accurate. In this reviewer's opinion, the major obstacle remaining to the fullest development and implementation of this method in practice is in the understanding of the method by the practitioner. At present, there are many remaining questions that this reviewer suspects are more related to lack of understanding by the loads engineering community of the underlying physics of atmospheric turbulence that the SDG method purports to represent, than they are related to any inherent error in the theory itself. Further, it is imperative that the underlying principles be completely understood by the loads engineers so that the method will be used *properly*. A classic example of why this is true came to light during the review meeting held at the FAA Hughs Technical Center in May of 2003. Part of the present documentation was the writing of a sample regulation. The developer of the method incorporated up to four gust ramps in the proposed regulation, thinking that this was all that was needed. It was found that there are some situations, especially in the case of the lightly damped lateral dynamics of wing-mounted engines, in which four ramps are insufficient to capture the pertinent dynamics and their use resulted in the under-prediction of loads. While this shortfall has been addressed, it seems to this reviewer that the number of ramp shapes used is more of a modeling question than it is one of regulation. In this sense, the question of how many ramps are necessary is very similar to the modeling question faced by the loads engineer when the number of structural dynamic modes to be used in dynamic response calculations is being decided. Certainly the number of modes to be used is not regulated by the FAA specifically, and nor *should* it be as that is a number that is very dependent upon the aircraft configuration. It is only required that the loads engineering staff use the number of modes necessary for conservatism (safety) in the loads predictions. Accuracy, while not specifically required, is certainly permitted and is certainly of interest to the manufacturer, especially if greater accuracy should translate into lower predicted loads (and thereby, less weight).

In the course of the preparation, review, and subsequent correspondence related to this document, it has become clear to this reviewer that there are many concepts that need to be elucidated and clarified before the average loads engineer can fully comprehend the SDG method. In contrast, the current assumption that atmospheric turbulence is a Gaussian process, while not physically accurate, does incorporate the characteristic of randomness and is a relatively easy concept to understand and to implement in loads analysis procedures. Linear structural dynamics, linear aerodynamic loading, and Gaussian excitation are a natural blend because of the quadratic nature of both the Gaussian exponent (the energy measure) and the mechanical energy (strain plus kinetic) of a linear structural dynamic system. Thus, once Gaussian statistics are understood (and rather easily at that), knowledge of linear structural dynamics and aeroelasticity are all that is needed for understanding the Power Spectral Density (PSD) based methods. The theory and methods associated with linear structural dynamics and aeroelasticity are, in turn, understandable results from the basic laws of classical mechanics that underlie the education of the typical loads engineer. In particular, the fluid mechanics (aerodynamics) involved are all derivable from the basic laws governing fluid flow: continuity, momentum, energy, and thermodynamic state ("c-m-e-s").

One feature of atmospheric turbulence that is emphasized in the present work is the so-called "correlated phase components" that are related to the gust gradient structure of the turbulence. It would be illustrative to compare the more accurate model of a gust profile (as promoted in this document) with one in which the importance of the gust gradient distance is ignored. Would this be a gust in the form of a step input (the so-called "sharp edged gust")? The importance of the ramp gradient in a ramp excitation is not a difficult concept for the structural dynamicist to understand as this is a standard topic in the basic study of structural dynamics (see Craig's text on structural dynamics, for instance). The difficulty arises when the gust gradient of actual turbulence needs to be characterized. The characterization of the gust gradients appears to involve scaling laws such as those that are an essential part of the SDG method. The scaling laws are a characterization of turbulence, and hence of flow fields, in the atmosphere. If these scaling laws could be related to the basic laws governing fluid mechanics (i.e. "c-m-e-s"), even in a rough but rigorous sense, then what empirical evidence that exists would be more easily understood, and the gradient structure and scaling laws more readily accepted. During discussions between the report author and reviewers that followed the May 2003 meeting at the FAA Hughs Technical Center, it came to the attention of this reviewer that there has indeed been some basic research that has tied scaling laws to basic physical principles governing fluid motion (i.e. the Navier-Stokes rendition of "c.m.e.s."). While this is good news, it still seems to take an expert in the Navier-Stokes equations to fully appreciate the work done to-date. Further, the jury still seems to be out as to the details of appropriate scaling laws as justified by correlations with Navier-Stokes calculations. Nonetheless, there appears to be sufficient similarity between the statistics-based SDG model and the forms being suggested by physics-based research on turbulence that the basic SDG (or an SDG-like) method seems legitimate.

Reiterating, it is the opinion of this reviewer that with the use of an SDG or SDG-like method for performing gust response analyses will come the need for appropriate modeling decisions in representing gust profiles, similar in spirit to the aeroelastic modeling decisions made in representing the aircraft. It seems risky, at the present, for the FAA to specify the gust in detail in a regulation. Rather, if an SDG-like method ever becomes an acceptable means of compliance with airworthiness regulations, the method can be named, but it seems obvious that the details of the implementation (specifically, the number and nature of the ramps used in the gust profile) should be left to the judgement of the analyst in much the same way as aeroelastic modeling details are left to the judgement of the analyst.

However, in order for the modeling of the gust profile to become part of the task of the analyst, then the analyst needs to understand the method. To this end, there are some specific undertakings that this reviewer believes could serve to elucidate the physics of the gust modeling problem, and thereby guide the analyst in the gust modeling task. Following are some suggestions.

Is it possible to construct an example of a random excitation that has some sort of "phase correlation" ? If so, can it then be shown how phase correlation results in non-"Gaussian-ness", and demonstrates how the usual analytical techniques that rely on random excitations of linear systems being Gaussian can no longer be applied.

Can some flow field(s) be derived from "c-m-e-s" that illustrate some or all of the gradient features? Since there are several sources of atmospheric turbulence (flow over or around obstructions; flow in or near convection cells), there are likely to be differing structures of the turbulence. Can it be shown how such flow fields, when randomized in an appropriate fashion, result in a random process possessing the all-important "phase-correlations" in the proposed form?

APPENDIX H—REVIEWER COMMENTARY, P. A. VAN GELDER AND J. P. ROOS

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In the review of the underlying document (Ref. 1) three issues will be addressed:

a) the document itself (including the references), its objectives and realisation,
b) the representation of the atmospheric turbulence in SDG,
c) application of SDG.

a) Review of document

The development of SDG has already started more than 30-35 years ago (see references on the CD-ROM), and since that time many papers, reports and publications have been produced. One of the main objectives of this document was to present the current status of the Linear Statistical Discrete Gust Method (SDG), and to make available the reference material that was un-available until now (mostly RAE/DERA reports with limited distribution).

In this repect the (extensive) report clearly fulfills its purpose, in the sense that it is more or less a road-map, from one document to another, however with many side-roads and detours.

The contents of the CD-ROM (Ref. 2) are plain report-scans, without options for keyword searches and/or hyperlinks and therefor requires a great deal of persistence of the reader to absorb it all.

Three additional reports (Ref. 3 - 5) are part of the documentation as well.

The reader should be aware that the words 'SDG' to denote the method or model is used throughout the years in many different meanings and in different contexts which can be very confusing[1].

When a new method is introduced, the general approach will be to describe the method and all (simplifying) assumptions needed to get a workable/engineering approach. The next step will be to demonstrate/validate the method by showing that (within some boundaries) the method and assumptions are valid. This validation should in this case be based on the proerties of the resulting atmospheric model; the implication for aircraft designloads will be a derived result.

The underlying report does describe the SDG method, not all assumptions (some of which are questionable, see next section) and restrictions are clearly indicated and the final 'proof' is not given.

[1] Eg.: SDG, SDG1, SDG(2), SDG(4), SDG_AS, SDG_00_AS, SDG_AS_SC, SDG_AS_SC_PP, SDG_AS_SC_C9, SDG_AS_SC_C10, etc.

The description of the physical/mathematical background of SDG (multi-fractal) and the theory used for the derivation of the statistical properties and the implementation into the SDG-algorithm (only one scaling law, so mono-fractal) are different.

In the current requirements it is acknowledged, that turbulence may come in different forms, requiring different (simplifying) models.

- The continuous gust or PSD method, based on the energy distribution of amospheric turbulence, determining the steady-state response gust-loads.
- The discrete gust or TDG method, based on short duration peak loads, with a more or less transient behaviour.

In the present report it is assumed that the 'extreme events' found in the limited set of GNAT data will cover all turbulence load cases for all types of aircraft and for all kinds of conditions (see also next section).

The report goes a step too far by implying that the method could be used as a replacement for the current PSD/TDG requirement for airworthiness certification (App. C of Ref. 1, Ref. 5). Applying this proposed requirement directly (so without tuning or tweaking) to a heavy transport aircraft (Ref. 6) contradicts this claim, especially because PSD-critical loads are not covered well with the most recent SDG model (SDG_AS_SC_C9/C10).

b) <u>The representation of the atmospheric turbulence in SDG</u>

The representation of atmospheric turbulence underlying SDG is questionable as it utilises only the 1/6 velocity increment scaling law and one velocity increment probability distribution or pdf for the calculation of loads. Per SDI-121-TR2 the 1/6 scaling law is chosen because it is considered representative for extreme turbulence. The definition of which is usually given in qualitative terms as turbulence in which the aircraft is violently tossed about and is practically impossible to control. Extreme turbulence may cause structural damage, indeed. The problem with using a single scaling law only is that the magnitude of aircraft loads experienced due to turbulence depends on frequency content and not solely on intensity. Thereby, the 1/6 scaling law is shown to be a true scaling law (i.e. scale independent) for a limited range of scales (8 thru 32 m or 26 thru 104 ft) and a limited range of amplitudes only. Now, knowing that intense high-frequency content turbulent bursts occur in the boundary layer shear flows sampled during the GNAT program, it seems only logical to associate the 1/6 scaling law with a range of high-frequencies (smaller scales) rather than a range of low-frequencies (larger scales). In general, small aircraft will experience these bursts as more severe than larger aircraft. Apparently, the choice of velocity increment scaling law and pdf is aircraft size (i.e. flexibility) dependent. A true multifractal reformulation of the SDG method for calculating loads, therefore, seems to be in order to properly take all scaling laws between 1/3 and say 1/6 into account.

The lower limit 1/6 scaling law has been found through analysis of atmospheric data, however, theoretical evidence exists for a range of scaling laws between 1/3 and 1/9 as predicted by the She & Leveque model (Phys. Rev. Lett. 72, p.p. 336, 1994). The latter is known to match anomalous scaling observed in shear flow turbulence data remarkably well and relates the intermittency phenomenon to vortex filaments known to be present in this type of flow. It can also be shown to match the combinations of k and D listed in SDI-121-TR2 reasonably well, which could be seen as an independent verification of the results obtained through SDG analysis of GNAT turbulence data. However, the SDG analysis of atmospheric data is based on the assumption that velocity increment pdf are of the symmetrical stretched exponential type, whilst directly measured distributions of all three velocity components are known to be characterised by a certain amount of skewness due to asymmetric tails. The latter predominantly affects the higher moments[2] and is accounted for in the She & Leveque model because it is related to a log-Poisson cascade process as shown by Dubrulle (Phys. Rev. Lett. 73, p.p. 959, 1994) and She & Waymire (Phys. Rev. Lett. 76, p.p. 262, 1995).

The multifractal model of turbulence is not without controversy as some attribute anomalous scaling entirely to the departure from isotropy in turbulent shear flows. The multifractal interpretation of turbulence and the postulate of local isotropy are tightly bound. If local isotropy does not hold then the small scales cannot be universal as there is direct interaction between the large and the small scales. There is very good evidence for the connection between anomalous scaling and anisotropy in the inertial range. Transverse structure functions have been determined in shear generated turbulence through two point measurements. Odd order transverse structure functions (odd order moments) are found to exist, solely because of the anisotropy of the flow. Experiments even show that their magnitude is of the same order as the even ones and their scaling exponents are such that they increase with increasing order in a way similar to the even ones. This implies that anomalous scaling (i.e. the departure from Kolmogorov scaling) can be accounted for solely in terms of the anisotropy of the flow, because the odd moments (which isotropy requires to be zero) capture the full anomaly. These recent results do not only suggest that the postulate of local isotropy is incorrect, but they also give a strong indication that intermittency and anisotropy result from the same cause; i.e. intense events at the integral scale that directly couple to the small scales. Although early experimental evidence seems to be compelling, much work will be required to base a sound theory on these observations. Warhaft (Proc. Nat. Acad. Sci. USA, Vol. 99, p.p. 2481 – 2486, 2002).

In conclusion, the results of SDG analysis of GNAT turbulence data sampled in the shear dominated atmospheric boundary layer, at altitudes between 30 and 300 meter, and hence the SDG method for calculating aircraft loads cannot be considered universally valid. In other words the present version of SDG strictly applies to boundary layer shear generated turbulence only and cannot be readily applied to turbulence generated through mechanisms like, for instance, Rayleigh-Bernard convection. Moreover, ongoing theoretical work may prove that the multifractal interpretation of turbulence underlying SDG needs adjustment or may lead to entirely new concepts.

c) <u>Application of SDG method</u>

For the development of SDG, use has been made of the so-called A310 model, a representative aircraft model including a fictitious non-linear control system, and the so-called B-model. During the Gust Specialists Meeting John Glaser presented results for a Bombardier Turboprop Aircraft, and Peter van Gelder showed results for a heavy type of aircraft (Airbus A340) (Ref. 6). Especially the determination of lowly-damped engine modes, that are governed by the PSD-requirement, were not captured by far by SDG.

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[2] As SDG focusses on the tails of the velocity increment distribution, skewness becomes a concern.

It should be mentioned that one of the global objectives of SDG is to provide an alternative for the derivation of PSD loads for aircraft equipped with non-linear control systems. This implies that if SDG is not able to capture the PSD designloads for linear aircraft, it will not be able to do so for non-linear cases.

Overall conclusion & major drawbacks

- The report does not give clear indication of the restrictions of the method and algorithms.
- The report gives no proof of underlying assumptions and applied theory with respect to the atmospheric modelling. By 'inverse-engineering' the reader needs to find out for himself whether the assumptions made in the report are valid or not.
- The theory of SDG and the derivation of the statistical properties from the atmosphere are two different things. Here it is assumed that atmospheric turbulence can be described by parameters that have been derived from measurements with a (relatively small) GNAT aircraft at very low altitudes.

References

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5. Jones, J.G.; *The SDG-AS-SC gust model and an associated airworthiness criterion*, SDI-121-TR-4, Issue 1, March 2003. (received as E-16.doc, by E-mail dd. 12-03-2003, final version received as <E16.pdf> by E-mail dd. 18-08-2003)

6. P.A. van Gelder, "*SDG Review Meeting*", presentation at the Gust Specialists ' Meeting in Atlantic City (NJ, USA) 21-22 May 2003

7. J.P. Roos, "*Review of SDG Turbulence Model*", presentation at the Gust Specialists ' Meeting in Atlantic City (NJ, USA) 21-22 May 2003

APPENDIX I—REPLY TO REVIEWER COMMENTARIES, GLYNN JONES

DRAFT 3

REPLY TO REVIEWER COMMENTARIES Glynn Jones

The three reviewer commentaries well illustrate the 'divergent viewpoints regarding the suitability of using the SDG method for the derivation of design gust loads', referred to in the Report Abstract, ranging from highly supportive to dismissive.

John Glaser's contribution is supportive of both the Documentation Report itself and of the SDG-AS-SC gust criterion proposed in the report (Appendix C) as the basis for a single airworthiness requirement to replace the two gust models, tuned isolated discrete gust (IDG or TDG) and power spectral density (PSD), currently specified. As the more contentious issue is the replacement of the PSD model, in the following I confine my comments to implications of this aspect of the proposal.

On the basis of his computational evaluation, Glaser concludes that, assuming his specified values of the design gust intensities U_{rf} (for SDG) and U_{σ} (for PSD), the maximum load envelopes given by these two gust models are in good agreement for wing bending moment, shear and torsion, whereas for the aft fuselage and fin lateral gust loads the maximum load envelope for PSD is consistently higher than the SDG envelope by 10% to 20%. These results are consistent with a broad conclusion reached in the Documentation Report, namely that *when the design gust intensities for these two gust models are matched to give equivalent wing loads* the lateral tail loads specified by PSD will tend to be conservative.

The reason that the relative intensities for the two gust models were chosen in the Report to give approximately equivalent wing loads is that, historically, wing loads have been the more closely related to the data obtained from commercial flight data recorders, and were traditionally taken as the basis of previous calibrations of turbulence models. The initial assessments (around 1970) that, on this basis, the PSD model gave excessive fin loads are described in Section 2.1 of the Documentation Report. It is also described in Section 2.1 of the Documentation Report how a compromise was reached subsequently in which the PSD design intensity was revised such that fin loads were reduced to 'acceptable' values. This was achieved, however, at the expense of reducing the wing design loads. This compromise has persisted to the present day.

A broader interpretation of John Glaser's results, independent of the matching conditions for the design gust intensities, is that (to quote Glaser) 'the most important benefit offered by the SDG formulation is that structural weight would be distributed where it is needed for the gust loads really imposed'.

John Glaser also refers to the stringent requirements that severe intermittent gust encounters impose on control system and actuator design. I would add that related quantities upon which the SDG model is likely to impose stronger requirements than the PSD model in the existing requirements include actuator rates and related local structural loads, which have relatively high frequency content (and thus tune to gusts of short gradient distance).

Tom Zeiler expresses qualified support for the SDG method but also raises issues that he believes need to be resolved. In particular, he expresses the view that the major obstacle remaining to the fullest development and implementation of the method in practice is in the understanding of the method by the practitioner.

A significant aspect of the problem is that of education. As Zeiler points out, the concepts and mathematical tools which provide the foundation of the SDG method differ from the traditional methods, based on linear structural dynamics and aeroelasticity, together with Gaussian statistics, which underpin the power spectral density (PSD) approach and which underlie the education of the typical loads engineer. In contrast, the SDG method (even when applied to linear dynamical systems) is founded upon more mathematically advanced concepts involving asymptotic evaluation of multi-dimensional integrals (generalization of Laplace's asymptotic method to higher dimensions), non-Gaussian statistical distributions (exponential and stretched-exponential distributions) and scaling laws involving fractal dimension. These are not topics currently taught in the engineering departments with which Tom Zeiler is familiar. However, whilst required for a proper understanding of the derivation of the SDG method, familiarity with these topics is certainly not required for its application as an engineering tool.

Tom Zeiler suggests that details of the implementation, specifically the number and nature of the ramps used in the gust profile, should be left to the discretion of the analyst, rather than being specified in the regulations. I cannot believe that the airworthiness authorities could agree with him on this. In its proposed form, the SDG criterion (Appendix C of the Documentation Report) follows the format of the existing requirements in which the gust model is specified in detail in the regulations and the design engineer is required to demonstrate that the resulting aircraft loads lie within an acceptable envelope. Specifically, it follows the format of the existing tuned isolated discrete gust (TDG or IDG) requirement, the difference being that a single gust profile shape is replaces by more general families of gust patterns, each of which has to be taken into account. Whilst superficially more complicated, this process has been demonstrated (by its implementation in the form of Matlab algorithms) to be quite practicable (and fast to execute on an up-to-date personal computer (PC)).

A further issue raised by Zeiler is that of 'phase correlation'. He asks if it is possible to construct an example of a random excitation that exhibits this property and hence to show how phase correlation results in non-'Gaussian-ness'. The only way with which I am familiar that will generate a random process with the type of phase correlation exhibited by samples of turbulence (leading in particular to intermittency) is via a dynamic process which incorporates non-linear interactions between the Fourier modes. This has been achieved in recent times by numerical simulations of turbulence (described for example in Frisch's book[1]). However, an equivalent understanding of phase correlation can be achieved by the much simpler process of starting from a sample of turbulence and demonstrating the manifest changes in statistical properties that results if a related process is generated which has an identical power-spectral density but in which the phase distribution has been randomized. The way in which this procedure converts the characteristic non-Gaussian statistics of turbulence into Gaussian distributions is illustrated graphically in Figure 1 of Ref. 2.

The final issue to be addressed in this 'Reply to Tom Zeiler' concerns the modification of the proposed gust criterion based on the SDG method (Appendix C of the Documentation Report) subsequent to the review meeting held at the FAA Hughes Technical Center in May of 2003. Subsequent to the presentation by Peter van Gelder of results of applying the criterion (in its form as of May 2003) to an A340 Airbus it was apparent that the range of gust patterns included in the criterion at that time was insufficient as a means of providing realistic excitation of the very lightly-damped underwing-engine modes on that configuration. Specifically, the class of 'periodic' gust patterns included in the criterion at that time, which had proved entirely adequate for the purpose of providing realistic excitation of all the aircraft dynamic models to which this author had access, was restricted to patterns comprising only two cycles. However, the possible practical significance of periodic gust patterns containing larger numbers of cycles had already been anticipated in the study of probability parameters of gust patterns in severe turbulence (Ref. 3), performed over the previous year. In this study, periodic gust patterns of up to eight cycles had been shown to exist, at significant amplitudes, in samples of severe turbulence. Of these, the higher-order patterns had been excluded from the proposed criterion as it was believed that they would only be relevant for the excitation of structural modes with unrealistically low damping. Following the inclusion of these higher-order periodic gust patterns in the revised (and current) version of the SDG criterion, the resulting response of the A340 engine modes follows expected trends with respect to the response resulting from the PSD model in the existing requirements. I return to this topic in the 'Reply to Peter van Gelder and Jurjen Roos'.

The more general question raised by this episode, which I suspect underlies Tom Zeiler's reference to it, concerns the extent to which the proposed gust criterion may require continued revision in the light of its application to wider and wider classes of aircraft configurations. To this I respond that in its current form the SDG-based criterion incorporates just those classes of gust patterns which were chosen for statistical investigation in the study described in Ref. 3 and for which measured statistical parameters are now available. Over the entire range of aircraft configurations investigated up to the present time the resulting aircraft loads exhibit consistent trends (not to be confused with agreement) with respect to the design loads resulting from the application of the gust models in the current requirements.

Peter van Gelder and Jurjen Roos are highly critical of the SDG method and of its presentation in the Documentation Report. I address their criticisms in some detail in the following. I identify these criticisms as follows:

1. The report does not give a clear indication of the restrictions within the method and its algorithms, and gives no proof of underlying assumptions with respect to the atmospheric modelling. By 'inverse engineering' the reader needs to find out for himself whether the assumptions made in the report are valid or not.
2. The 'multifractal' SDG statistical model, and the associated theory for the derivation of the statistical properties (of aircraft response), differs from the implementation into the 'SDG-algorithm', which uses only one scaling law and hence is 'monofractal'.
3. The proposed SDG method for predicting aircraft design loads depends entirely on measured low-altitude data, over a limited range of scales, obtained by the 'Gnat' aircraft.
4. Ongoing theoretical work may prove that the multifractal interpretation of turbulence underlying SDG needs adjustment.

5. The aircraft loads predicted by the SDG model do not, in some instances, capture 'by far' the loads resulting from the PSD-requirement.
6. Other issues.

1. Assumptions and restrictions of the SDG method

I agree that I cannot point to a single particular place in the Report where these are summarized concisely. In the following I address that deficiency.

(i) The basic 'multifractal' SDG statistical model is a model of atmospheric turbulence velocity in the 'inertial range' (where the power-spectral-density is proportional approximately to $(\text{frequency})^{-5/3}$).

(ii) Fluctuations in turbulence velocity are represented in terms of sequences of discrete ramp gust elements, representing 'velocity increments', which follow prescribed scaling laws (specifying that the amplitude of a discrete gust of gradient distance H is proportional to H^k, where k is the 'scaling index') and prescribed clustering properties (in which the probability of a gust pattern comprising several ramp components is defined in terms of 'complexity factors' or 'p-factors').

(iii) In the general 'multifractal' formulation (Appendix C of Ref. 2) a probability parameter (mathematically a moment-order parameter) q is introduced such that k and the p-factors are functions of q. Small values of q correspond to fluctuations of low amplitude, at a relatively high probability, whereas large values of q correspond to those high-intensity fluctuations which have low probability but are of the greatest significance for aircraft design loads and turbulence-related 'incidents'.

(iv) In particular, the properties of fluctuations with intensity specified by q = 2 (i.e. defined by second-order statistics) predominantly determine the power-spectral-density of the velocity component. When q = 2, k takes the approximate value 1/3 and the SDG method reduces to an approximate implementation of the PSD method. The gust model SDG2 (Section 2.3 of the Documentation Report) falls into this category.

(v) As q increases, k varies as a decreasing function of q. This function k(q), together with an associated fractal dimension D(q), has been measured in laboratory experiments (Frisch[1]) and in low-altitude flight measurements in severe turbulence by the Gnat aircraft (SDI-121-TR-2)[2].

(vi) For application at the high intensities (large values of q) corresponding to design loads, a value of k = 1/6 has been extrapolated from the measurements made in severe turbulence. p-factors have also been measured in severe turbulence, using data from both experimental flights (Ref. 3) and from 'special events' in commercial airline records (Section 5.1 of the Documentation Report). In the absence of contrary indications, these p-factors have been assumed to apply also at extreme 'design-levels' of turbulence.

(vii) For any specified value of q, and hence prescribed values of k and p-factors, the associated statistical model of turbulence (a 'monofractal' subset of the full multifractal model which covers a range of values of q) is used to determine an associated statistical distribution of aircraft response, given specified aircraft response dynamics. The relationship between the statistical model of the turbulence input and the statistical distribution of aircraft response is determined mathematically by the Laplace asymptotic approximation. By applying this process for a succession of values of q, the statistical distribution of aircraft response corresponding to the full multifractal model may be obtained (Appendix C of Ref. 2).

(viii) The ‘SDG method’ may be interpreted alternatively either as the application of the SDG model to predict aircraft response in statistical terms, as described in (vii) above, or as its application as the basis of a Design Envelope airworthiness criterion / requirement (the ‘SDG-algorithm’, see Item 2, below).

2. The ‘multifractal’ SDG statistical model differs from the implementation into the ‘SDG-algorithm’

van Gelder and Roos comment that the ‘multifractal’ SDG statistical model, and the associated theory for the derivation of the statistical properties (of aircraft response), differ from the implementation in the ‘SDG-algorithm’, which uses only one scaling law and hence is ‘monofractal’. A related criticism is that: ‘The representation of atmospheric turbulence underlying SDG is questionable as it utilises only the 1/6 velocity increment scaling law and only one velocity increment probability distribution or pdf for the calculation of loads’.

These comments fail to take account the relationship that exists between a statistical model of turbulence and a Design Envelope requirement.

The ‘multifractal’ SDG model, specified in Appendix C of Ref. 2, prescribes a probability distribution for turbulence fluctuations covering the full range of amplitudes, or intensities. Fluctuations of small amplitude, and high probability, correspond to small values of the probability parameter q (Item 1(iii)), whereas fluctuations of large amplitude, and correspondingly low probability, correspond to larger values of q. The multifractal model is thus parameterized by a range of values of q, and incorporates a corresponding range of the scaling index k(q) and fractal dimension D(q) (Item 1(v)).

In contrast, the specification of the SDG model in the proposed ‘SDG Criterion’ (Appendix C of the Documentation Report), and the associated ‘SDG-algorithm’, fall into the category of a ‘Design Envelope criterion’, whose purpose is to specify turbulence, and the associated design loads, at just one level of probability (although, in common with the gust models in the existing requirements, this probability and the associated shape of the probability-density-function, or pdf, are not prescribed explicitly in the criterion). In the SDG representation, a single level of probability corresponds to a single value of q, and the scaling index k in the ‘SDG Criterion’ thus takes just a single value, chosen to be equal to 1/6.

3. Sources of information used in support of SDG model

van Gelder and Roos claim that the proposed SDG method for predicting aircraft design loads depends entirely on measured low-altitude data, over a limited range of scales, obtained by the ‘Gnat’ aircraft.

This is not the case. In fact, published evidence consistent with the scaling law ($w \sim H^k$, where $k<1/3$) used in the SDG model goes back to 1970, and involves a variety of sources. This evidence relates to the statistics of turbulence velocity increments in the range of scales where the power-spectral-density of the turbulence velocity is approximately proportional to $(\text{frequency})^{-5/3}$, i.e. the range of scales bounded above by the 'scale length' L as prescribed in the PSD method. This range of scales is the range represented in the SDG model (Item 1(i) above). There are a range of related properties that, particularly since 1970, have been shown to apply to turbulence velocity increments in this 'inertial range'.

(a) The rms (root-mean-square) value of velocity increments (ramp-gust amplitudes) varies with gradient distance H approximately like $H^{1/3}$.
(b) Probability distributions of velocity increments follow approximately a 'stretched exponential' form[1], in which the strength of the non-Gaussian tail increases as the scale H decreases.
(c) A result related to (b) is that the kurtosis of the probability distribution of velocity increments increases as the scale H decreases.
(d) At the lower levels of probability (i.e. higher gust intensities), the amplitudes of fluctuations of given probability vary with gradient distance like H^k, where $k < 1/3$ (compare (a) above). This is necessary for (b) and (c) to apply.
(e) The above scaling index k is a decreasing function k(q) of the probability parameter, or 'moment order', q, where constant values of q correspond to constant values of probability, and increasing values of q correspond to decreasing values of probability (and hence increasing gust intensity).

The earliest reference in the aeronautical literature to the above set of properties of turbulence velocity components in the inertial range, and their relevance to the statistics of aircraft response, is that of Chen[4]. Chen used four independent data sources: High Altitude Clear Air Turbulence (HICAT), Severe Storm Turbulence (SEST), Barbados Oceanographic and Meteorological Experiment (BOMEX), and Wind Tunnel Turbulence (WITT). Particular care was directed to the selection of segments of turbulence that have fairly stationary variance over the entire segment so that non-stationary effects may be neglected. In all four cases the probability distribution of velocity increments showed the characteristic 'stretched exponential' form, with kurtosis significantly greater than that applicable to a Gaussian distribution. Of particular relevance in the present context is the comment by Chen, in discussing the HICAT results, that kurtosis was found to be a decreasing function of H (equivalent to Item (c) above).

A range of subsequent studies, both experimental and theoretical, discussed for example in Ref. 1 (Chapter 8), have shown the above listed properties to be characteristic of turbulence velocity components in the inertial range. In particular, numerical simulations have provided a picture of turbulence-velocity structures, in which the finest scales involve a tangle of vortex filaments, consistent with Items (a) to (e) listed above.

In the context of the above background information, and in particular the HICAT results, the purpose of the analysis of Gnat data was primarily twofold: (i) to provide further confirmation of the above listed properties, particularly Property (e), (ii) to estimate a value of k appropriate for use at the low level of probability associated with design loads.

Both of the above objectives were achieved, as described in Ref. 2. It is true that, due to ground proximity, the range of scales over which the power-spectral-density followed the $(\text{frequency})^{-5/3}$ law was limited above by a scale length L significantly less than the value (2500 ft.) assumed in the PSD model for design loads. However, the particular value of L does not substantially influence the degree of confidence with which the general properties listed above are confirmed by the Gnat data analysis. What is significant is not the particular scale limit but the extent of the range of scales covered by the data analysis. If it were possible to extend this range beyond that covered by the Gnat data analysis, this would have the advantage of allowing the function k(q) to be measured over a wider range of q, and this in turn would allow the extrapolation of k to a specific design-level value to be made with more confidence. Based on the Gnat data, the possible design-level value of k can with confidence be constrained to the range 0.2 to 0.1. The extrapolated value $k = 1/6$ has been chosen as a reasonable compromise.

4. Ongoing theoretical work may prove that the multifractal interpretation of turbulence underlying SDG needs adjustment or may lead to entirely new concepts.

The above is a quote from van Gelder and Roos' commentary. A related comment is: 'the multifractal model of turbulence is not without controversy as some attribute anomalous scaling entirely to the departure from isotropy in turbulent shear flows'.

These comments, that introduce us to the forefront of current turbulence research concern the relationship between anomalous scaling, i.e. values of k such that $k < 1/3$, and the physics of the turbulent flow field. It is true that the original multifractal model of Parisi and Frisch[5] concerned distributions of singularities in homogeneous turbulence whereas more recent work on turbulent shear flows[6] has introduced alternative explanations for anomalous scaling, in which Kolmogorov's concept of an energy cascade is replaced by a model in which large and small scales are directly coupled and the turbulence is anisotropic even at small scales.

However, the 'multifractal SDG model' does not depend on the assumptions made by Parisi and Frisch[5]. Following a probabilistic reformulation of the multifractal model due to Frisch[1], its scaling laws, and associated values of the exponents k and D, are based directly on measured probability distributions for velocity increments and are equally applicable to anisotropic and inhomogeneous shear-flow turbulence. Indeed, in Ref. 2 different exponents are derived for the vertical and lateral components of turbulence velocity. The associated SDG theory for aircraft response shows how such measured probability distributions for velocity increments can be converted to associated distributions for aircraft loads (Appendix C of Ref. 2).

It is, of course, a source of additional confidence when the 'multifractal SDG model' is found to be consistent with some theoretical *physical explanation*. Though maintaining their scepticism regarding the 1/6 scaling law, derived from the fact that the SDG model does not account for velocity increment pdf skewness, van Gelder and Roos state that the model of She and Leveque[7] can be shown to 'match the combinations of k and D listed in SDI-121-TR-2 (Ref. 2) reasonably well, which could be seen as an independent verification of SDG analysis of GNAT turbulence data'.

van Gelder and Roos go on to describe the role of skewness, due to asymmetric tails in measured distributions and in the more recent physical theories, and correctly comment that the SDG multifractal model, in which the fluctuations are represented by symmetric stretched exponential distributions, does not take account of this property. I would simply comment that the SDG multifractal model, as specified in Appendix C of Ref. 2, is consistent with even-order moments in current physical theories and that skewness can be incorporated into SDG if so required, e.g. statistical distributions for up-gusts and down-gusts can be specified separately in regions of overall vertical shear (as described, for example, in Section 4.1 of Ref. 8). However, in line with the turbulence models in the existing airworthiness requirements, this degree of sophistication has not been deemed necessary for the purpose of calculating aircraft structural design loads.

5. The aircraft loads predicted by the SDG model do not, in some instances, capture 'by far' the loads resulting from the PSD-requirement.

To introduce our reply to this comment, we begin with quotations from the standard textbook[9] concerning gust loads on aircraft. Describing how the PSD method depends upon the stationary-Gaussian representation, Hoblit (Ref.–9, Chapter 12.3) remarks that this model should be expected to account *conservatively* for the response of *lightly-damped* modes to discrete gusts. As examples of such lightly-damped aircraft motions Hoblit[9] includes elastic-mode response and the Dutch-roll response to lateral gusts.

Furthermore, Hoblit[9] continues as follows: 'Actually, the stationary-Gaussian model is probably *slightly conservative* for the response of lightly damped modes to continuous turbulence, in that continuous turbulence tends not to be of long enough duration in any one patch for statistical equilibrium to be reached. Also, if indeed the most severe gusts occur essentially as discrete gusts, then the design levels for continuous turbulence, having been set to provide for these discrete gusts, will be unrealistically high for continuous turbulence as such'. These comments have relevance to the quantitative results referred to in the final paragraph of this section.

It is in the light of the above widely accepted views that the comment that heads this section of the Reply should be judged. First, there is the implication that the loads predicted by the SDG model should 'capture' the loads resulting from the PSD-requirement. This completely misses the point that, when calibrated to match (well damped) wing-load data, as was the case for the SDG model employed by van Gelder and Roos, the PSD model tends to be conservative for lightly-damped modes, whereas the SDG model predicts loads that are more realistic (in that its assumptions are in better agreement with measured turbulence data). The result that in the case of lightly-damped modes the SDG loads are less than the PSD loads is thus in line with expectations.

Secondly, the statement that 'the aircraft loads predicted by the SDG model do not, in some instances, capture *'by far'* the loads resulting from the PSD-requirement' is an exaggeration. This comment refers to five cases of loads related to the lightly-damped elastic engine response of the A340 aircraft. Three of these cases tune to the SDG two-cycle (four ramp) periodic waveform, which implies that they are lightly damped, and two cases tune to the eight-cycle (sixteen ramp) waveform, which implies that they are *very* lightly damped. With respect to the PSD loads, the SDG loads are, as an average over these five cases, 5 % less. As an average over the two *very* lightly damped cases, the SDG loads are only 3.7 % less than the PSD loads.

Further information relevant to the interpretation of the above lightly-damped engine-load cases can be obtained from the associated 'tuned' gust time histories (in the case of the PSD method this is given by the matched-filter input). For example, in a typical case of the very-lightly damped response referred to by van Gelder and Roos, the SDG periodic-waveform tuned input consists of just eight cycles covering a total distance of 800 m. In contrast, the PSD load results from a time history of approximately twenty-six cycles, building up over a distance of over 2600 m (approximately 10 sec.). Of the two, the SDG gust profile is much the more realistic as a representation of an encounter with design-level turbulence.

6. Other issues

In their 'Review of the document', van Gelder and Roos comment: 'the reader should be aware that the words 'SDG' to denote the method or model is used throughout the years in many different meanings and in different contexts which can be very confusing'. They then list, in a footnote, ten different SDG 'versions'. To remove the confusion, introduced here by van Gelder and Roos themselves, we should point out that the reader of the Documentation Report will NOT encounter most of the versions that they quote, these being names given to development prototypes subsequently superseded. The only versions of which the reader need be aware are SDG1 and SDG-AS-SC, fully documented in Section 7 of the Documentation Report, and SDG2, whose role is explained in Section 2.3.

Postscript

Despite the unrealistic representation of the atmosphere embodied in the current airworthiness regulations the current view (see below) is that the two gust models in these regulations do provide adequate safety in terms of structural strength. Nevertheless, the SDG method continues to be a candidate as a viable criterion alternative to current practice (see below). Moreover, by providing a more representative / realistic model of the severe patterns of wind fluctuation that can lead to crew and passenger injury, as well as large structural loads, it also has a role to play in the interpretation of such incidents / accidents jointly by the aircraft design engineer and the meteorological specialist.

TO CONCLUDE, THE CURRENT STATUS OF THE METHOD MAY BE SUMMARIZED BY THE FOLLOWING EXTRACT FROM THE MINUTES, ISSUED BY THE FAA, OF THE GUST SPECIALISTS WORKSHOP HELD IN ATLANTIC CITY, MAY 21-22, 2003:

> The FAA is comfortable with the current turbulence design criteria and structural level of safety.
>
> Although a single SDG method may offer some simplification for the applicant, and some useful ability with respect to non-linear systems, it does not appear to offer substantial advantages over-and-above the current combination of PSD and TDG.

However, the SDG method continues to be a candidate as a viable criteria alternative to current practice, and the FAA would be receptive to future representations on the matter from industry or individuals, provided resolution of at least the following:

(1) Traditional levels of safety: the acceptability of reductions in severity of lateral design loads is presently uncertain, and an issue which must be addressed.
(2) Industry acceptance: there should be the prospect of widespread willingness to substitute existing methodologies, not currently in evidence.
(3) Maturity: sufficient additional investigation and analysis to firmly establish the proposed design standard and associated numerical constants, including consideration of altitude effects.

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