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Evaluation and Verification of Advanced Methods to Assess Multiple-Site Damage of Aircraft Structure

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16. Abstract <p>Widespread fatigue damage (WFD) is a complex phenomenon that is extremely difficult to analyze with standard methods developed from first principles of linear elastic fracture mechanics (LEFM). Because of the limited applications of LEFM, more advanced methods have been explored and developed over the past decade. These include analytical tools to determine parameters governing the onset and growth of cracks and elastic-plastic fracture criterion for residual strength determinations. The tools also include the finite element alternating method (FEAM); a computationally efficient yet rigorous approach to calculate two- and three-dimensional stress-intensity factor solutions governing crack growth; FASTRAN, a fatigue crack growth analysis program using a crack-closure model; and STAGS, an advanced finite element program implemented with fracture mechanics and stable-tearing analysis capabilities for generalized shell structures. The elastic-plastic failure criterion includes the plastic zone touch, crack tip opening angle (CTOA), and the T*-integral. These computational tools must be verified and validated using experimental data to ensure successful transfer of useable and accurate technology to the industry.</p> <p>The objectives of this study were to (1) take existing analysis tools and establish the processes required of using them as engineering tools to determine the effects of multiple-site damage (MSD) on the residual strength of representative aircraft structures and (2) investigate small crack behavior to better understand the formation of MSD in structures. These tools and criteria were used and verified in this study to analyze portions of the multiple-site crack initiation, growth, linkup, and catastrophic fracture process. For the development of MSD, the fundamental phenomenon of fatigue crack initiation in material and the rate of growth were addressed experimentally at the coupon level and analytically using FASTRAN. The initiation of MSD was addressed at the component level on flat panels that were representative of typical fuselage lap splices. Experimentally generated data and a closure-based crack growth code were used to develop equivalent initial flaw sizes. Residual strength analysis used the application of the T*-integral, CTOA, and plastic zone linkup criteria to predict the linkup and fast fracture of MSD in curved panels and aft pressure bulkhead. A procedure was developed that used elastic-plastic finite element analyses with STAGS and CTOA to perform the stable tearing and unstable fracture of MSD in aircraft structures.</p> <p>From extensive experimental work, test data was generated and used for correlation and validation and verification of the various methodologies and criteria.</p>					
17. Key Words Widespread fatigue damage; Multiple site damage; Stress-intensity factor, (SIF); Crack tip opening angle, Plastic zone linkup, (PZL); Plastic zone touch, (PZT); T*-Integral; Crack initiation; Crack growth rate; Equivalent initial flaw size, (EIFS); Residual strength; Fracture mechanics			18. Distribution Statement This document is available to the public through the National Technical Information Service (NTIS) Springfield, Virginia 22161.		
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VOLUME 2

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LIST OF ACRONYMS AND SYMBOLS

AAWG	Airworthiness Assurance Working Group
AFRL	Air Force Research Laboratory
AGARD	Aerospace Research and Development
BES	Boundary element system
BIAM	Beijing Institute of Aeronautical Materials
CAAC	Civil Aviation Authority of China
CTOA	Crack tip opening angle
C(T)	Compact Tension
DOF	Degree of Freedom
EIFS	Equivalent initial flaw size
EO	Engineering order
EPFEAM	Elastic-plastic finite element alternating method
FAA	Federal Aviation Administration
FASTER	Full-Scale Aircraft Structural Test Evaluation and Research
FEA	Finite element analysis
FEAM	Finite element alternating method
GUI	Graphical user interface
LaRC	Langley Research Center
LEFM	Linear elastic fracture mechanics
LVDT	Linear Variable Differential Transformer
MPC	Multiple-point constraint
MSD	Multiple-site damage
NAARP	National Aging Aircraft Research Program
NASA	National Aeronautics and Space Administration
NDI	Nondestructive inspection
NIST	National Institute of Standards and Technology
OEM	Original equipment manufacturer
POD	Probability of detection
PZL	Plastic zone linkup
SCG	Small crack growth
SEM	Scanning Electronic Microscope
SENT	Single-edge notch under tension
SIF	Stress-intensity factor
TCL	Tool command language
USAF	United States Air Force
WFD	Widespread fatigue damage
WPAFB	Wright-Patterson Air Force Base

α	Constraint factor
K	Stress-intensity factor
a_1/L	Dimensionless parameter
a_{eff}	Effective crack length
β_1	Total geometry correction factor for the lead crack
β_2	Total geometry correction factor for the MSD crack
β_u	Correction factors
c/a	Crack depth to crack length
da/dN	Crack growth rate
ΔK	Crack tip stress-intensity factor range
ΔK	Stress-intensity factor range
ΔK_{eff}	Effective stress-intensity factor range
F_{TU}	The material allowable
F_{TY}	Yield strength
F_{TY}	Material yield strength
Γ_ε	Crack tip contour
$M(T)$	Middle-crack tension
R -ratio	Stress ratio (minimum stress/maximum stress)
T^*_R	T^* -integral resistance
Ψ_c	Critical COTA
Ψ_c	Critical tearing angle

EXECUTIVE SUMMARY

This report summarizes the results from a study performed by The Boeing Company, Huntington Beach, CA, during August 1996 to February 2003. This study was undertaken to achieve two goals: (1) take existing analysis tools developed under government-funded research and establish the processes required to use them as engineering tools to determine the effects of multiple-site damage (MSD) on the residual strength in the representative aircraft structures and (2) investigate small crack behavior to better understand the formation of MSD in a structure.

WFD is a complex phenomenon that is extremely difficult to analyze with standard methods developed from first principles of linear elastic fracture mechanics (LEFM). Because of the limited applications of LEFM, more advanced methods have been explored and developed over the past decade with the support and sponsorship of the Federal Aviation Administration (FAA) and the National Aeronautics and Space Administration. This includes analytical tools to determine parameters governing the onset and growth of cracks and elastic-plastic fracture criterion for residual strength determinations. The tools include the finite element alternating method (FEAM); a computationally efficient yet rigorous approach to calculate two- and three-dimensional stress-intensity factor (SIF) solutions governing crack growth; FASTRAN, a fatigue crack growth analysis program using a crack-closure model; and STAGS, an advanced finite element program implemented with fracture mechanics and stable-tearing analysis capabilities for generalized shell structures. The elastic-plastic failure criterion include the plastic zone touch (PZT), crack tip opening angle (CTOA), and the T^* -integral.

These tools and criteria were used and verified in this program to analyze portions of the multiple-site crack initiation, growth, linkup, and catastrophic fracture process. For the development of MSD, the fundamental phenomenon of fatigue crack initiation in material and the rate of growth were addressed at the coupon level experimentally and analytically using FASTRAN. The initiation of MSD was dealt with at the component level on flat panels that were representative of typical fuselage lap splices. Experimentally generated data and a closure-based crack growth code were used to develop equivalent initial flaw sizes (EIFS). For residual strength analysis, the approach taken was to apply the T^* -integral, CTOA, and plastic zone linkup (PZL) criteria to predict the linkup and fast fracture of MSD. A procedure was developed that used elastic-plastic finite element analyses using STAGS and CTOA to perform the stable tearing and unstable fracture of MSD in aircraft structures.

Computational tools must be verified and validated using experimental data to ensure successful transfer of useable and accurate technology to industry. Extensive experimental work was carried out to generate test data for correlation and validation of the various methodologies and criteria. Testing was conducted collaboratively by five organizations: (1) The Boeing Company, Long Beach, CA; (2) United States Air Force (USAF), Air Force Research Laboratory (AFRL) at Wright-Patterson Air Force Base; (3) Beijing Institute of Aeronautical Materials (BIAM); (4) Civil Aviation Authority of China (CAAC); and (5) the FAA William J. Hughes Technical Center.

This project was divided into six major tasks to analyze portions of the multiple-site crack initiation, growth, linkup, and catastrophic fracture process.

1. Crack Initiation. Cracks were initiated and developed from edge-notched coupons and analyzed using FASTRAN. Testing was conducted by BIAM and CAAC under several fatigue loading conditions, including constant-amplitude and spectrum loading. Good correlation between analysis and experimental data was demonstrated under constant-amplitude loading for crack sizes larger than 0.005 inch. However, under spectrum loading, there was poor correlation.
2. Equivalent Initial Flaw Size. A semiempirical procedure was developed for the determination of EIFS using FASTRAN. Tests were conducted by AFRL using large flat panels with four joint configurations representative of aircraft joint construction. The attempts to develop comprehensive SIF solutions by compounding or superpositioning for the individual effects derived from first principles were not effective. An iterative method was developed to empirically account for these factors for the prediction to match the experimental results. In this way, the EIFS was determined to fall between 0.0001 to 0.0015 inch for the four types of splice joints. However, whether the results can be applied to other structures under different loading conditions or using different crack growth models requires additional study.
3. Small Crack Growth. Small crack growth data was generated in pin-loaded specimens and analyzed using FASTRAN. Testing was conducted by AFRL under various load transfer conditions. In general, good correlation was obtained between test and analysis for open-hole specimens under constant-amplitude loads. However, analysis predicted 20 to 30 percent faster crack growth rate for the pin-loaded specimens.
4. MSD in Flat Panels. The CTOA, T*-integral, and PZL criteria were used to analyze flat panels with MSD. Tests were conducted by AFRL using large flat panels with four joint configurations representative of aircraft joint construction. The PZL criterion provides a quick and simple way for residual strength estimations. T*-integral, together with FEAM, was able to predict the stable tearing of MSD cracks in a flat spliced panel. Using STAGS code, the CTOA criterion was able to predict the residual strengths of MSD in splice joints. Using these criteria, predictions of the residual strength were within 8 percent.
5. MSD in Curved Panels. The CTOA criterion was used to analyze curved panels with MSD. Tests were conducted by the FAA William J. Hughes Technical Center using the Full-Scale Aircraft Structural Test Evaluation and Research facility. CTOA predictions agree well with the curved panel test results, within 5 percent.
6. MSD in Aft Pressure Bulkhead. The CTOA criterion was used to analyze an aft pressure bulkhead with MSD. Tests were conducted by the AFRL. CTOA predictions agree well with the curved panel test results, within 5 percent.

In summary, this project demonstrated a successful transfer of technology developed from basic research to real-work applications. Using this technology, a methodology to assess the development of MSD and its effect on the residual strength of aircraft structure was developed. The three major components of the methodology are crack initiation, crack growth and linkup,

and residual strength. The crack initiation methodology used experimentally generated EIFS data and an analytical closure model to determine initial flaw sizes and distribution for multiple-site cracking. The CTOA, T*-integral, and PZT criteria were used to predict crack growth and linkup. Elastic-plastic finite element analyses were used with the CTOA to determine the residual strength of an aircraft structure containing a long lead crack in the presence of MSD. The methodologies were verified through a comprehensive test program.

APPENDIX A—CRITERIA

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