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ADVANCED PAVEMENT DESIGN: FINITE ELEMENT MODELING FOR RIGID PAVEMENT JOINTS, REPORT III: MODEL SIMPLIFICATION AND APPLICATION

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A rational, three-dimensional (3D) finite element technique was applied to model the structural response of the jointed concrete airport pavement system. Modeling features include explicit 3D modeling of the slab continua, load transfer capability at the joint, explicit 3D modeling of the base course continua, load transfer capability across the cracks in the base course, and contact interaction between the slabs and base course. The results of these models were simplified for incorporation into concrete pavement design applications. A set of joint response algorithms was developed that provides a complete system for relating commonly used deflection- and stress-based metrics of joint response to a dimensionless quantity that can be used to establish joint properties for finite element calculations. They may be readily implemented in personal computer spreadsheets or calculated by hand using an electronic calculator. Multivariate statistical analysis techniques were applied to increase understanding of the contribution of the input variables to variability of load transfer estimates and the sensitivity of the model to changes in input variables.
PREFACE

This research reported was sponsored by the U.S. Department of Transportation, Federal Aviation Administration (FAA), Airport Technology Branch, under Interagency Agreement DTFA03-94-X-00010 by the Airfields and Pavements Division (APD), Geotechnical Laboratory (GL), and U.S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Mississippi. Dr. Xiaogong Lee, Airport Technology R&D Branch, FAA, was the technical monitor. Dr. Satish Agrawal is the manager of the Airport Technology R&D Branch, FAA.

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TABLE OF CONTENTS

EXECUTIVE SUMMARY ix

1. INTRODUCTION 1-1
   1.1 Background 1-1
   1.2 Objective 1-2
   1.3 Scope 1-2

2. MODEL SIMPLIFICATION 2-1
   2.1 Introduction 2-1
   2.2 Joint Response Algorithms 2-2
   2.3 Doweled-Joint Response 2-4
   2.4 Implementation for Evaluation of Joints 2-8
   2.5 Implementation for Design of Doweled Joints 2-9
   2.6 Uses and Limitations 2-10

3. MULTIVARIATE ANALYSES 3-1
   3.1 Introduction 3-1
   3.2 Sensitivity of Model to Changes in Input Variables 3-1

4. CONCLUSIONS 4-1

5. BIBLIOGRAPHY 5-1
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Relationship Between $LTE_{\delta}$ and $f$</td>
<td>2-3</td>
</tr>
<tr>
<td>2-2</td>
<td>Relationship Between $LTE_{\delta}$, $LTE_{\sigma}$, and $\varepsilon/\ell$</td>
<td>2-4</td>
</tr>
<tr>
<td>2-3</td>
<td>Friberg’s (1940) Analysis of Dowel Bar Support</td>
<td>2-5</td>
</tr>
<tr>
<td>2-4</td>
<td>Dowel Design Spreadsheet</td>
<td>2-9</td>
</tr>
<tr>
<td>3-1</td>
<td>Histogram for Load Transfer Obtained From the Sensitivity Analysis</td>
<td>3-2</td>
</tr>
<tr>
<td>3-2</td>
<td>Sensitivity of Load Transfer to Concrete Slab Thickness</td>
<td>3-3</td>
</tr>
<tr>
<td>3-3</td>
<td>Sensitivity of Load Transfer to Modulus of Dowel Support</td>
<td>3-4</td>
</tr>
<tr>
<td>3-4</td>
<td>Sensitivity of Load Transfer to Modulus of Base Reaction</td>
<td>3-4</td>
</tr>
<tr>
<td>3-5</td>
<td>Sensitivity of Load Transfer to Dowel Spacing</td>
<td>3-5</td>
</tr>
<tr>
<td>3-6</td>
<td>Sensitivity of Load Transfer to Joint Opening</td>
<td>3-5</td>
</tr>
<tr>
<td>3-7</td>
<td>Sensitivity of Load Transfer to Concrete Chord Modulus</td>
<td>3-6</td>
</tr>
<tr>
<td>Table</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>3-1</td>
<td>Magnitudes of Independent Variables for Sensitivity Analysis</td>
<td>3-1</td>
</tr>
<tr>
<td>3-2</td>
<td>Descriptive Statistics for Load Transfer (%) From Sensitivity Analysis</td>
<td>3-2</td>
</tr>
<tr>
<td>3-3</td>
<td>Partial Correlation Coefficients Between Model Input Variables and Load Transfer</td>
<td>3-6</td>
</tr>
</tbody>
</table>
LIST OF ACRONYMS

<table>
<thead>
<tr>
<th>2D</th>
<th>Two-dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Administration</td>
</tr>
<tr>
<td>FWD</td>
<td>Falling weight deflectometer</td>
</tr>
<tr>
<td>DCI</td>
<td>Dowel-concrete interaction</td>
</tr>
</tbody>
</table>
EXECUTIVE SUMMARY

A rational, three-dimensional (3D) finite element technique was applied to model the structural response of the jointed concrete airport pavement system. Model features include explicit 3D modeling of the slab continua, load transfer capability at the joint (modeled as springs between the slabs), explicit 3D modeling of the base course continua, load transfer capability across the cracks in the base course (again, modeled by springs across the crack), and contact interaction between the slabs and base course. The contact interaction model allows gaps to open between the slab and base. Furthermore, where the slabs and base are in contact, transfer of shear stresses across the interface via friction is modeled.

The results of these models were simplified for incorporation into concrete pavement design applications. A set of joint response algorithms was developed that provided a complete system for relating commonly used deflection-based and stress-based metrics of joint response to a dimensionless quantity that can be used to establish joint properties for finite element calculations. The functional forms of these regression algorithms were arbitrary from an engineering viewpoint and were selected from a large number of investigated choices. They may be readily implemented in personal computer spreadsheets or calculated by hand using an electronic calculator. Multivariate statistical analysis techniques were applied to increase understanding of the contribution of the input variables to variability of load transfer estimates and the sensitivity of the model to changes in input variables.
1. INTRODUCTION.

1.1 BACKGROUND.

The Westergaard idealization has been the basis for the Federal Aviation Administration’s (FAA) concrete pavement structural design procedure. In 1926, Westergaard developed a method for computing the response of rigid pavement slabs-on-grade subjected to wheel loads by modeling the pavement as a thin, infinite or semi-infinite plate resting on a bed of springs (Westergaard 1926). Although available Westergaard solutions have been extensively used, they are limited by two significant shortcomings: (a) only a single infinite slab panel is modeled in the analysis; therefore, load transfer at joints is not accounted for, and (b) the layered nature of the pavement foundation is not explicitly reflected in the Winkler foundation model. To account for the increased capacity of the foundation caused by a stabilized layer, the modulus of subgrade reaction is increased in the Westergaard model. This approach, in which the “top-of-the-base” modulus is determined empirically, is required by the assumptions of the Westergaard theory.

Multilayered, linear elastic models, as used in the new FAA design method (commonly referred to as LEDFAA) released in 1995 (FAA Advisory Circular 150/5320-16), consider the complete layered system in the vertical direction, thereby addressing the second limitation of the Westergaard theory. In the horizontal direction, however, the layers are assumed to be infinitely long with no discontinuities such as edges or joints. Consequently, the load transfer limitation remains.

Translational and rotational springs and beam elements are used by some two-dimensional (2D) finite element programs to model load transfer at a joint. The slabs and base course layers are modeled as thin plates that may be fully bonded or fully debonded. If the slab and base course are considered to be bonded (full strain compatibility between slab and base), transformed section concepts are used to formulate a plate element with an equivalent composite plate stiffness.

In a previous report in this series, Hammons (1997) identified several features of an improved concrete pavement analytical model, which address some of the deficiencies of current analysis techniques. All members of the concrete pavement system including slabs, joints, base, and subgrade were modeled as linear elastic components. The primary findings of the previous research include the following:

• The concept of the composite or top of the base modulus of subgrade reaction ignores the composite action of the slab-stabilized base structural system. This concept should be abandoned in favor of a more realistic model that explicitly includes the structural benefits of the stabilized base.

• Experimental evidence suggests that the joint efficiency depends upon the presence and condition of a stabilized base. The presence of cracking in the base and the degree of bonding between the slabs and stabilized base course influences the structural capacity and load transfer capability of the rigid pavement structure.
A comprehensive three-dimensional (3D) finite element modeling technique provides a more rigorous approach to modeling the structural response of the jointed rigid airport pavement system. Modeling features that are required include explicit 3D modeling of the slab continua, load transfer capability at the joint (modeled as springs between the slabs), explicit 3D modeling of the base course continua, aggregate interlock capability across the cracks in the base course (again, modeled by springs across the crack), and contact interaction between the slabs and base course. The contact interaction model feature must allow gaps to open between the slab and base. Furthermore, where the slabs and base are in contact, transfer of shear stresses across the interface via friction should be modeled.

The incremental finite element analysis procedure used to solve the contact interaction problem can be computationally intensive. In the event that solution times and memory requirements are greater than the available computer resources, the slabs can be modeled by thick plate or shell elements with little sacrifice in accuracy. However, the capability to predict load transfer across cracks in the base course and debonding between the slabs and base are essential and must be retained.

1.2 OBJECTIVE.

The objective of this research was to develop a 3D finite element model of the concrete pavement slab-joint-foundation system that can be implemented in the advanced pavement design concepts currently under development by the FAA. The basic criteria for this model development were (a) soundness of the theory and (b) precision of the model consistent with the requirements of the improved FAA pavement design methodology. The model developed will serve as an analytical stepping stone to increased understanding of the behavior of concrete pavement systems. By judiciously applying this increased understanding of behavior, improved design criteria can be developed resulting in enhanced concrete pavement performance.

1.3 SCOPE.

This research effort was initiated in June 1994 under Interagency Agreement DTFA03-94-X-00010 between the FAA and the Airfields and Pavements Division of the U.S. Army Corps of Engineers Waterways Experiment Station. The provisions of Task 1, “Modeling of Rigid Pavement Joints for Advanced Pavement Design,” of this interagency agreement included six subtasks:

- Subtask 1.1: Review and Evaluation of Existing Joint Models
- Subtask 1.2: Perform a Response and Sensitivity Analysis of Rigid Pavement Systems
- Subtask 1.3: Develop a General 3D Analytical Model
- Subtask 1.4: Perform Laboratory-Scale Testing
- Subtask 1.5: Model Application
- Subtask 1.6: Model Simplification for Implementation into FAA Design Procedures

Two previous reports have been published as a part of this research effort. The first report (Hammons and Ioannides 1996), which cover subtask 1.1, presents a detailed review and evaluation of existing joint models, an analysis of experimental data on small-scale model data generated by the
Corps of Engineers in the 1950s, and a new Westergaard-type, closed-form solution for load transfer at rigid pavement joints.

The second report in this series (Hammons, 1997) described the results from subtasks 1.2, 1.3, and 1.4 as previously outlined. A comprehensive finite element response and sensitivity study for rigid pavement single- and multiple-slab models founded on a Winkler subgrade using the finite element code ABAQUS were described. A series of laboratory-scale experiments on jointed rigid pavement models was conducted, and the data from the experiments were analyzed. Finally, a 3D finite element analysis system was developed that includes the influence of the base course on the structural capacity of the rigid pavement slab-joint-foundation system.

This report describes the final two research subtasks previously outlined. In section 2 of this report simplification of the model for inclusion in the FAA’s advanced structural models is presented. A mechanistic method for the analysis of concrete pavement joints is presented. This method is then implemented for mechanistic design of doweled joints. Section 3 contains multivariate analyses conducted to increase understanding of the proposed method.
2. MODEL SIMPLIFICATION.

2.1 INTRODUCTION.

By their nature, joints are structurally weakening components of the concrete pavement system. Thus, the response and effectiveness of joints are primary concerns in concrete pavement analysis and design. The concept of load transfer is fundamental to the FAA concrete pavement design criteria. The philosophical basis of load transfer is very simple: the maximum stresses and deflections in a loaded slab can be reduced if a portion of the applied load is transferred to an adjacent slab. If a joint is capable of transferring load, statics dictates that the portion of the applied load supported by the loaded slab, $P_L$, and the complementary load supported by the adjacent slab, $P_U$, commonly referred to as the unloaded slab, must sum to the total applied wheel load $P$, i.e.,

$$P_L + P_U = P$$  \hspace{1cm} (2-1)

Load may be transferred across a joint by shear and/or bending moment. However, it has been commonly argued that load transfer is primarily by shear (American Concrete Institute 1956, Huang 1978). If the slabs on both sides of the joint are identical in thickness and elastic properties, and if a pure shear load transfer mechanism is assumed, the maximum bending stress responses near the edge of the slabs are related by

$$\sigma_L + \sigma_U = \sigma_f$$  \hspace{1cm} (2-2)

where $\sigma_L$ is the maximum bending stress in the loaded slab, $\sigma_U$ is the maximum bending stress in the adjacent unloaded slab, and $\sigma_f$ is the maximum bending stress for the corresponding free edge condition (no load transfer). Similarly, maximum slab deflections near the edge are related by

$$\Delta_L + \Delta_U = \Delta_f$$  \hspace{1cm} (2-3)

where $\Delta_L$ is the maximum edge deflection of the loaded slab, $\Delta_U$ is the maximum edge deflection of the adjacent unloaded slab, and $\Delta_f$ is the corresponding maximum free edge deflection (no load transfer).

Deflection load transfer efficiency ($LTE_{\delta}$) is defined as the ratio of the deflection of the unloaded slab to the deflection of the loaded slab, as follows:

$$LTE_{\delta} = \frac{\Delta_U}{\Delta_L}$$  \hspace{1cm} (2-4)

Similarly, stress load transfer efficiency ($LTE_{\sigma}$) is defined as the ratio of the edge stress in the unloaded slab to edge stress in the loaded slab, as follows:

$$LTE_{\sigma} = \frac{\sigma_U}{\sigma_L}$$  \hspace{1cm} (2-5)
Load transfer \((LT)\) in the FAA rigid pavement design procedure is defined as that portion of the free edge stress that is carried by the adjacent unloaded slab:

\[
LT = \left( \frac{\sigma_U}{\sigma_f} \right) = \left( \frac{1}{1 + \frac{\sigma_L}{\sigma_f}} \right) \tag{2-6}
\]

All three load transfer measures are commonly expressed as percentages. It should be noted from the above equations that the ranges of \(LTE_\delta\) and \(LTE_\sigma\) are from 0 to 100 percent, while the range of \(LT\) is from 0 to 50 percent. Equation 2-6 can be related to equation 2-5 as follows:

\[
LT = \frac{LTE_\sigma}{1 + LTE_\sigma} \tag{2-7}
\]

The FAA design criteria prescribe \(LT = 25\) percent, effectively reducing the design stress and allowing a reduced slab thickness. This accepted value is primarily based upon observations from experimental pavements trafficked from the mid-1940s to the mid-1950s (Ahlvin, 1991). If the load transfer requirement is violated through a degradation of the joint system, the pavement life can be significantly reduced.

### 2.2 JOINT RESPONSE ALGORITHMS

If the joint between two slabs is assumed to be elastic and continuous and to transfer load by vertical shear only without bending moments, then load transfer across the joint can be represented as a dimensionless joint stiffness, \(f\), in terms of the radius of relative stiffness, \(l\), modulus of subgrade reaction, \(k\), and a joint stiffness parameter, \(q\) (Skarlatos, 1949). The latter represents the force transferred across a unit length of joint per unit differential deflection across the joint as follows:

\[
f = \frac{q}{k\ell} \tag{2-8}
\]

Using this approach, Skarlatos (1949) developed relationships involving integral equations for the maximum stress and deflection on the unloaded side of the joint. Using modern personal computers and powerful mathematical software, Ioannides and Hammons (1996) were able to perform the necessary integrations for square loaded areas of various sizes, \(2\varepsilon \times 2\varepsilon\). Following the same approach as Westergaard, closed-form equations for the maximum deflection and maximum bending stress on the unloaded side of a joint capable of load transfer were developed. Together with Westergaard’s edge loading equations, the relationships developed by Ioannides and Hammons (1996) can be used to investigate the load transfer problem.

The results of the analytical development work were used to develop a series of closed-form relationships convenient for routine engineering calculations. Nonlinear regression was used to develop the following expressions relating \(f\), \(\varepsilon/\ell\), and \(LTE_\delta\).
\[
\log f = \left[ 0.434829 \left( \frac{\varepsilon}{\ell} \right) - 1.23556 \right] \log \left( \frac{I}{LTE_{\delta}} - 1 \right) + 0.295205 \tag{2-9}
\]

or

\[
LTE_{\delta} = \frac{I}{\log^{-1} \left[ \frac{0.214 - 0.183 \left( \frac{\varepsilon}{\ell} \right) - \log f}{1.180} \right] + \log f} \tag{2-10}
\]

A plot of this relationship for selected values of $\varepsilon/\ell$ is shown in figure 2-1.

Likewise, nonlinear regression was used to develop expressions relating $LTE_{\delta}$, $LTE_{\sigma}$, and $\varepsilon/\ell$:

\[
LTE_{\delta} = \frac{\left[ 1206 \left( \frac{\varepsilon}{\ell} \right) + 377 \right] LTE_{\sigma}^2 - 693 \left( \frac{\varepsilon}{\ell} \right) LTE_{\sigma}^3}{\left[ 1 + 689 \left( \frac{\varepsilon}{\ell} \right) LTE_{\sigma} + \left[ 370 - 154 \left( \frac{\varepsilon}{\ell} \right) \right] LTE_{\sigma}^2 \right]} \tag{2-11}
\]

or

\[
LTE_{\sigma} = \frac{\left[ 10.14 \left( \frac{\varepsilon}{\ell} \right) + 4.00 \right] LTE_{\delta} - \left[ 4.3 \left( \frac{\varepsilon}{\ell} \right) + 3.98 \right] LTE_{\delta}^2}{\left[ 21.03 + \left[ 5.74 \left( \frac{\varepsilon}{\ell} \right) - 20.98 \right] LTE_{\delta} \right]} \tag{2-12}
\]

Figure 2-1. Relationship Between $LTE_{\delta}$ and $f$
A graph showing the relationship between $LTE_\delta$, $LTE_{\sigma}$, and $\epsilon/\ell$ is shown in figure 2-2. The following equation has been obtained for $LT$ (percent):

$$LT\ (\%) = \frac{34.3\left(\frac{\epsilon}{\ell}\right) + 14.98}{1 + 0.686\left(\frac{\epsilon}{\ell}\right) - 0.995} \frac{LTE_\delta - 14.835 LTE_\delta^2}{LTE_\delta}$$

(2-13)

Figure 2-2. Relationship Between $LTE_\delta$, $LTE_{\sigma}$, and $\epsilon/\ell$

The joint response algorithms presented in equations 2-9 through 2-13 provide a complete system for relating commonly used deflection- and stress-based metrics of joint response ($LTE_\delta$, $LTE_{\sigma}$, and $LT$) to a dimensionless quantity that can be used to establish joint properties for finite element calculations. The functional forms of these regression algorithms were arbitrary from an engineering viewpoint and were selected from among a large number of choices investigated. They may be readily implemented in personal computer spreadsheets or calculated by hand using an electronic calculator.

2.3 DOWELED-JOINT RESPONSE.

Friberg (1940) presented an analysis of stresses in doweled joints based upon the work of Timoshenko and Lessels (1925). His analysis was based upon considering the dowel as a semi-infinite beam on a Winkler foundation. His basic relationship for dowel-concrete bearing stress, $\sigma_b$ [FL$^{-2}$] ($F =$ force $L =$ length), was

$$\sigma_b = K \Delta_o$$

(2-14)
where

\[ K = \text{modulus of dowel support [FL}^{-3}] \]
\[ \Delta_0 = \text{deflection of the dowel with respect to the concrete at the face of the joint [L]} \]

Friberg’s (1940) analysis of dowel bar support is shown in figure 2-3. Grinter (1931) reported that the value of \( K \) depended on the modulus of the slab concrete, \( E_c \); the thickness of the slab, \( h \); and the modulus of subgrade reaction, \( k \). Reported values of \( K \) vary greatly. Tabatabaie (1978) reported finding values in the literature from \( 0.08 \times 10^6 \text{ MPa/m (0.3} \times 10^6 \text{ psi/in.) to } 8.6 \times 10^6 \text{ MPa/m (32} \times 10^6 \text{ psi/in.)}. \) The value typically assumed is \( 0.41 \times 10^6 \text{ MPa/m (1.5} \times 10^6 \text{ psi/in.)}. \)

\[ \sigma = K\Delta \]

Figure 2-3. Friberg’s (1940) Analysis of Dowel Bar Support
Friberg’s (1940) relationship for the value of $\Delta_o$ under a dowel bar carrying a transferred load, $P_t$, is

$$
\Delta_o = \frac{P_t}{4 \beta^3 E_d I_d} (2 + \beta \omega) \\
(2-15)
$$

where

$\omega =$ joint opening [L]

$E_d =$ modulus of elasticity of the dowel [FL$^{-2}$]

$I_d =$ moment of inertia of dowel bar [L$^4$]

$\beta =$ relative stiffness of the dowel-concrete system [L$^{-1}$]

Friberg (1940) adopted Timoshenko’s (1925) definition of the relative stiffness of a bar embedded in concrete as

$$
\beta = \sqrt[4]{\frac{K_d}{4 E_d I_d}} \\
(2-16)
$$

where $d$ is the diameter of the dowel bar. The bearing stress on the concrete at the joint face then comes from equation 2-14:

$$
\sigma_b = \frac{KP_t}{4 \beta^3 E_d I_d} (2 + \beta \omega) \\
(2-17)
$$

Interpreting results from analyses made with the finite element code ILLI-SLAB, Ioannides and Korovesis (1992) developed the concept of a dimensionless joint stiffness for the dowelled joint expressed by the ratio $D/skl$ where $D$ is the stiffness of a vertical spring element, $s$ is the dowel spacing, and $l$ is Westergaard’s radius of relative stiffness for the slab-subgrade system, defined as

$$
l = \sqrt[3]{\frac{E_c h^3}{12(l - \nu_c^2)k}} \\
(2-18)
$$

where $E_c$ and $\nu_c$ are Young’s modulus and Poisson’s ratio of concrete, respectively. The value of $D$ reflects the vertical stiffness contributed by the support of the concrete, called the dowel-concrete interaction ($DCI$), and the vertical stiffness offered by beam bending, $C$. These two contributions are summed in a manner analogous to springs in series, leading to

$$
D = \frac{l}{2 DCI} + \frac{l}{12C} \\
(2-19)
$$

Note that, factor 2 in the first term of the numerator of equation 2-19 is needed to account for deformation due to bearing of the dowel bar on both slabs (Brill and Guo, 2000). The value of $DCI$
can be calculated assuming the dowel is a beam on a spring foundation (Friberg analysis) using the following relationship

\[ DCI = \frac{4\beta^4}{(2+\beta\omega)} E_d I_d \]  

(2-20)

Comparing this relationship with equation 2-15 reveals that \( DCI \) is identical with the ratio \( P_t/\Delta_o \) in the Friberg analysis and has dimensions of \([FL^{-1}]\). The term \( C \) in equation 2-19 is defined by the relationship

\[ C = \frac{E_d I_d}{\omega^3(1+\phi)} \]  

(2-21)

where

\[ \phi = \frac{12 E_d I_d}{G_d A_z \omega^2} \]  

(2-22)

\( G_d \) is the shear modulus of the dowel bar, given by

\[ G_d = \frac{E_d}{2(1+\nu_d)} \]  

(2-23)

where \( \nu_d \) is the Poisson’s ratio of the dowel. \( A_z \) is the effective cross-sectional area in shear and is assumed to be 0.9 times the circular area as follows:

\[ A_z = 0.9 \left( \frac{\pi d^2}{4} \right) \]  

(2-24)

The contributions of Friberg (1940), Skarlatos (1949), Ioannides and Korovesis (1990), and Ioannides and Hammons (1996) can be synthesized to extend the joint response algorithms presented in section 2.2 to doweled joints by making the direct substitution

\[ q = \frac{D}{s} \]  

(2-25)

into equation 2-8. Therefore, the dimensionless joint stiffness ratio for a doweled joint then becomes

\[ f = \frac{D}{sk\ell} \]  

(2-26)

and equations 2-9 and following can be applied directly.
2.4 IMPLEMENTATION FOR EVALUATION OF JOINTS.

Nondestructive deflection testing of pavements using falling weight deflectometer (FWD) devices has gained considerable popularity in the last 15 years. The most common application of such testing is in the backcalculation of layer moduli for the assessment of the structural condition of existing pavements and for the design of overlays. A closed-form backcalculation procedure developed using the principles of dimensional analysis (Ioannides, Barenberg, and Lary, 1989; Ioannides, 1990) has been extensively used in the estimation of in situ structural stiffness parameters of concrete pavements. Midslab deflection testing is used for this purpose. The procedure typically leads to an estimate of \( \ell \) as well as the effective \( k \) and the slab’s Young’s modulus, \( E_c \). Through further application of the principles of dimensional analysis, the procedure was extended to the interpretation of edge deflection testing yielding estimates of the load transfer effectiveness of concrete pavement joints (Ioannides and Korovesis, 1990). The following is a brief outline of the combined midslab-edge evaluation procedure (Ioannides, Alexander, Hammons, and Davis, 1996):

a. With the FWD at a midslab location, drop the weight, \( P \), and measure deflections \( D_0, D_1, D_2, \) and \( D_3 \) at 0, 305, 610, and 915 mm (0, 12, 24, and 36 in.) from the center of the loading plate.

b. Calculate the \( AREA \) (in inches) of the deflection basin, using:

\[
AREA \text{ (in.)} = \frac{6}{D_0} \left[ D_0 + 2(D_1 + D_2 + D_3) \right]
\]  

(2-27)

c. Backcalculate the radius of relative stiffness, \( \ell \), of the pavement system using the appropriate relationship between this parameter and \( AREA \), pertaining to the particular plate-sensor arrangement considered. For the 300-mm-diameter FWD plate and the four-sensor arrangement noted in a., the following regression formula can be employed, leading to an estimate of \( \ell \) in inches:

\[
\ell \text{ (in.)} = \left( \ln \left( \frac{36 \times AREA}{2673.64} \right) \right)^{4.75} \times \frac{1}{2.93}
\]  

(2-28)

d. Estimate the load size ratio \( (a/\ell) \) for this midslab location, where \( a \) is the radius of the FWD plate (150 mm (5.9055 in.)).

e. Move the FWD to an edge location; drop the weight, \( P \); and measure the deflections at conjugate points on the loaded and unloaded sides of the joint, \( \Delta_L \) and \( \Delta_U \), respectively.

f. Calculate the deflection load transfer efficiency of the joint, \( LTE_\delta \) (equation 2-4)
g. Using the values of \( LTE_\delta \) and of \((a/\ell)\) determined in this fashion, backcalculate the corresponding value of the stress load transfer efficiency, \( LTE_\delta \) (equation 2-12) and load transfer, \( LT \) (equation 2-13). Implicit in this determination is the conservative assumption that the midslab \( \ell \) value is valid at the edge, as well. If the total loaded area remains constant, the error associated with substituting a square loaded area (described by the length of the side, \( 2\varepsilon \)) for a circular loaded area (described by the radius, \( a \)) is negligible.

h. Using equation 2-9, estimate the value of the dimensionless joint stiffness, \( f \), and other joint parameters, including the modulus of dowel support, \( K \).

2.5 IMPLEMENTATION FOR DESIGN OF DOWELED JOINTS.

The relationships presented in this chapter may be implemented to develop a mechanistic design methodology for doweled joints in concrete pavements. A spreadsheet was developed that allows a designer to interactively select the required dowel diameter and spacing to yield a desired value of load transfer.

Figure 2-4 shows the layout of the spreadsheet, which has been designed such that any consistent set of units may be used. In the upper left-hand block of the spreadsheet, the user is prompted to enter the thickness of the pavement slab, the modulus of subgrade reaction, the radius of the loaded area, and a trial dowel diameter. Suggested values for dowel diameter follow the FAA criteria (FAA, 1995) and range from 19 mm (3/4 in.) to 51 mm (2 in.) in increments of approximately 6 mm (1/4 in.).

![Figure 2-4. Dowel Design Spreadsheet](image-url)
In the center left-hand block of the spreadsheet, commonly assumed structural parameters are listed pertaining to pavement and dowels. These values may be changed at the designer’s discretion. The elastic properties of the Portland cement concrete are often set at conservative values for engineering design. The elastic properties of the dowel are values commonly assumed for steel. The default value for the modulus of dowel reaction is the commonly assumed value, although wide variations of this value have been reported in the literature. The three default values for dowel spacings are the three values recommended by the FAA (1995). The user has the option to use other values of dowel spacing at his or her discretion.

The lower left-hand block contains a list of useful parameters which have been calculated from the required and optional input. These spreadsheet cells have been protected so that the user may not alter or override the formulae with user input values.

The graph on the right-hand side of the spreadsheet shows the calculated value of load transfer (based upon the user’s input) as a function of joint opening and dowel spacing for the trial value of dowel diameter. The plots in the graph are automatically updated as the user interacts with the spreadsheet by providing the required and optional input values. By observing the changes in the graph, the user can interactively select a combination of dowel diameter and dowel spacing to achieve a desired load transfer.

2.6 USES AND LIMITATIONS.

The methods provided in this chapter represent a rational, mechanistic method for analysis, design, and evaluation of joints in concrete pavements and constitute a considerable improvement over previous empirical methods. When coupled with the 3D finite element method, they provide a powerful tool for analyzing the response of jointed concrete pavements subjected to vehicle loads. For both the aggregate interlock joint and the doweled joint, the magnitude of \( q \) can be directly used to set the stiffness of linear springs across a joint in a 2D or 3D finite element model. The joint stiffness to yield a particular value of load transfer can be estimated or may be backcalculated from field experiments.

For slabs on a bed-of-springs foundation, application of these developments should lead to accurate predictions of joint response. However, the analyst should be cognizant of the fact that the algorithms set forth in this section are limited by the assumptions of the Westergaard theory, and as such, fail to take into account the influence of the structural capacity of the base course, the presence of cracking in the base course, contact interaction between the base and slab, or other nonlinear effects. However, the weaknesses of these joint response algorithms play into the strengths of the 3D finite element method as described by Hammons (1997). Also, environmental effects are not explicitly included in the model; however, joint opening is a direct input into the doweled-joint models and its influence on doweled joints can be studied.
3. MULTIVARIATE ANALYSES.

3.1 INTRODUCTION.

This chapter describes the results of several statistical analyses that were performed to better understand the load transfer model. These analyses included three phases:

- Evaluation of the sensitivity of the model to broad changes in input variables.
- Estimation of modulus of dowel support using field data.
- Evaluation of the contribution of each input variable to the variability of load transfer estimates, for given field situations.

3.2 SENSITIVITY OF MODEL TO CHANGES IN INPUT VARIABLES.

The purpose of this section is to examine the sensitivity of load transfer, as calculated by the model, to changes in the independent variables. This study provides the pavement analyst with a relative ranking of the input variables in terms of their potential for influencing load transfer, as calculated by the model.

The independent variables included in this analysis were concrete slab thickness \((h)\), modulus of base course reaction \((k)\), static modulus of elasticity for concrete \((E_c)\), modulus of dowel support \((K)\), dowel spacing \((s)\), and joint opening width \((\omega)\). To examine sensitivity, a realistic range of magnitude for each value was established. The term “realistic” here implies that the range includes most values that would be encountered in pavement analysis and design. For each variable, two values defined the realistic range and a third value was selected at their midpoint, as shown in table 3-1. For this analysis, the input variables were considered to influence load transfer independently. This assumption will be examined for appropriateness in later analyses. During this sensitivity study, the following model inputs were assumed to be constant: size of loaded area \((\varepsilon = 0.15 \text{ m})\), dowel diameter \((d = 25 \text{ mm})\), Poisson’s ratio for concrete \((v_c = 0.19)\), modulus of elasticity for the steel dowel \((E_d = 200,000 \text{ MPa})\), and Poisson’s ratio for the steel dowel \((v_s = 0.30)\).

Table 3-1. Magnitudes of Independent Variables for Sensitivity Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum</th>
<th>Midpoint</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete slab thickness, mm (in.)</td>
<td>203 (8)</td>
<td>406 (16)</td>
<td>610 (24)</td>
</tr>
<tr>
<td>Modulus of base course reaction, MPa/m (pci)</td>
<td>67.9 (250)</td>
<td>135.7 (500)</td>
<td>203.6 (750)</td>
</tr>
<tr>
<td>Concrete chord modulus, MPa (psi)</td>
<td>27580 (4.0 x 10^6)</td>
<td>34475 (5.0 x 10^6)</td>
<td>41370 (6.0 x 10^6)</td>
</tr>
<tr>
<td>Modulus of dowel support, MPa/m (pci)</td>
<td>300,000 (1.11 x 10^9)</td>
<td>1,800,000 (6.63 x 10^9)</td>
<td>3,300,000 (12.2 x 10^9)</td>
</tr>
<tr>
<td>Dowel spacing, mm (in.)</td>
<td>305 (12)</td>
<td>381 (15)</td>
<td>457 (18)</td>
</tr>
<tr>
<td>Joint opening, mm (in.)</td>
<td>3.18 (0.125)</td>
<td>9.53 (0.375)</td>
<td>15.88 (0.625)</td>
</tr>
</tbody>
</table>
The load transfer model was executed with all combinations of all input variables. This effort provided 729 (= 3⁶) estimates of load transfer, which has units of percent. Descriptive statistics for these estimates are shown in table 3-2 and a histogram of the results is shown in figure 3-1. The ranges of independent variables provided for a wide range of load transfer estimates (from 8.5 to 38 percent). The distribution of load transfer estimates had a slightly positive skew, which is typical for distributions of this pavement characteristic (Hammons, et al., 1995).

Table 3-2. Descriptive Statistics for Load Transfer (%) From Sensitivity Analysis

<table>
<thead>
<tr>
<th>Statistic Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>16.49%</td>
</tr>
<tr>
<td>Median</td>
<td>15.68%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>6.60%</td>
</tr>
<tr>
<td>Range</td>
<td>29.6%</td>
</tr>
<tr>
<td>Minimum</td>
<td>4.73%</td>
</tr>
<tr>
<td>Maximum</td>
<td>34.3%</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.415</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.545</td>
</tr>
</tbody>
</table>

Partial correlation coefficients were used to evaluate the relative influences of independent variables on the dependent variable (i.e., load transfer). A partial correlation coefficient describes the strength of a linear relationship between two variables, while adjusting for relationships involving all the other variables (Freund and Wilson, 1993). Partial correlation provides a measure of the portion of the variability in load transfer that is explained by an independent variable after all the other independent variables have been included in the model. A partial correlation coefficient has properties similar to other correlation coefficients: it can assume a value between -1 and +1; a value of 0 indicates no relationship; and the values of -1 and +1 indicate perfect negative and positive linear relationships, respectively.

Figure 3-1. Histogram for Load Transfer Obtained From the Sensitivity Analysis
Relationships between each independent variable and load transfer are shown in figures 3-2 through 3-7. In each figure, a single independent variable is plotted versus load transfer for all 729 cases, as discussed previously. Partial correlation coefficients for the relationships between the independent variables and load transfer are summarized in table 3-3 in the order of decreasing strength. Concrete slab thickness had the second strongest relationship with load transfer. Fortunately, the thickness of concrete slabs can be controlled accurately. Modulus of dowel support had the strongest relationship with load transfer. Little is known about this design parameter because it cannot be measured directly. In an effort to contribute to our knowledge of this design parameter, the next section uses data from the Denver International Airport to estimate some field values for modulus of dowel support. Efforts such as this are needed to assign realistic values to this influential analysis and design parameter.

![Figure 3-2. Sensitivity of Load Transfer to Concrete Slab Thickness](image-url)

**Figure 3-2. Sensitivity of Load Transfer to Concrete Slab Thickness**
Figure 3-3. Sensitivity of Load Transfer to Modulus of Dowel Support

Figure 3-4. Sensitivity of Load Transfer to Modulus of Base Reaction
Figure 3-5. Sensitivity of Load Transfer to Dowel Spacing

Figure 3-6. Sensitivity of Load Transfer to Joint Opening
Figure 3-7. Sensitivity of Load Transfer to Concrete Chord Modulus

Table 3-3. Partial Correlation Coefficients Between Model Input Variables and Load Transfer

<table>
<thead>
<tr>
<th>Input Variable</th>
<th>Partial Correlation Coefficient</th>
<th>P-Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of dowel support, $K$</td>
<td>0.8835</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Concrete slab thickness, $h$</td>
<td>-0.8735</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Modulus of base course reaction, $k$</td>
<td>-0.4168</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Dowel spacing, $s$</td>
<td>-0.2881</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Joint opening, $\omega$</td>
<td>-0.2102</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Concrete chord modulus, $E_c$</td>
<td>-0.1340</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

*P-value indicates the probability of incorrectly stating the corresponding regression coefficient is other than zero.
4. CONCLUSIONS.

A rational, mechanistic method for analysis, design, and evaluation of doweled joints in concrete pavements is presented. The methodology proposed, a considerable improvement over previous empirical methods, is based upon sound analytical principles and is easily implemented in a personal computer spreadsheet. The required inputs to the analytical model are the slab thickness, modulus of subgrade reaction, and the radius of the loaded area. All other model inputs can be set at default values or be modified at the designer’s discretion. Dowel bar diameters and spacings can be interactively modified by the designer to yield a given level of load transfer capability at the joint. Using the developed spreadsheet, a sensitivity study showed that slab thickness and modulus of dowel support had the most influence on load transfer estimates, relative to all input variables considered.
5. BIBLIOGRAPHY.


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