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RELIABILITY OF NAVIGATION SYSTEMS

FINAL REPORT

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16. Abstract

This report examines reliability as it relates to navigation systems in the National Airspace System. Reliability measures are defined and are applied to VOR, Omega, LORAN-C, and GPS. The reliability equations and the system descriptions are basic in nature and the fundamental concepts are illustrated by calculations.

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SUMMARY

This report examines reliability as it relates to navigation systems in the National Airspace System. Reliability measures are defined and are applied to VOR, Omega, LORAN-C, and GPS navigation systems. The reliability equations and the system descriptions presented are basic in nature, intended only to emphasize the concepts rather than to provide a rigorous analysis.

It is emphasized that although reliability is an important element of a system evaluation, it is not a sufficient measure for determining user acceptability. Other technical considerations as well as economic and institutional considerations must also be addressed in a comprehensive system evaluation. The intent of this report, however, is to focus only on system reliability.

Although this report has defined various measures of reliability, a standard for application of these measures does not exist. It is through engineering judgment, therefore, that decisions are being made by the aviation community as to the acceptability or nonacceptability of system or unit reliability.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

The FAA is evaluating various navigation systems for use after 1995. The evaluation depends upon technical, economic, and institutional criteria. Included among the technical criteria are accuracy, coverage, reliability, and integrity. Although a significant amount of work has been done in this evaluation process, reliability has not been sufficiently addressed. This project focuses on the reliability issue in support of the overall navigation system evaluation. The navigation systems included in this study are VOR, Omega, LORAN-C, and GPS.

In general, the term "reliable" is used to indicate that a system performs an intended task without failure for a given period of time. To be useful, the meaning must be precisely defined. For this project, the following definition is used:

> "Reliability is the probability that a system will perform satisfactorily for at least a given period of time when used under stated conditions" (Reference 1).

Application of the reliability definition also requires definitions of the system used, period of time in use, operating conditions, and what is meant by satisfactory performance. The definitions are flexible, being dependent upon the application. In this report, examples are given to illustrate the differences in application.

The evaluation of various navigation systems requires comparative assessments, adding another dimension of complexity to the performance of a reliability analysis. Different navigation systems do not generally provide the same coverage and depend upon dissimilar transmitting networks. Therefore, a common navigation mission for use as a reference is difficult to construct. For example, a transoceanic mission is not a suitable basis for comparing navigation system reliability since VOR does not provide oceanic coverage. Another variation is coverage at different flight altitudes. VOR depends upon the altitude of the user relative to the transmitting facility, whereas coverage of many other systems does not change with altitude. Also, a single VOR station provides adequate guidance for navigation along a short path, but multiple Omega, LORAN-C, and GPS stations are needed.

1.2 PURPOSE

The purpose of this project is to develop reliability measures applicable to navigation systems. A variety of reliability measures are defined in the report and calculations are provided to illustrate fundamental concepts.

1.3 SCOPE

FAA's long-range goal with respect to navigation system reliability is to establish a standard method for determining navigation system reliability that will permit direct comparisons to be made between different systems. This project is an initial step toward that goal. The project was limited to describing reliability fundamentals and to developing a potential approach for evaluating navigation system reliability.

1.4 TECHNICAL APPROACH

This study provides sample calculations as a means of explaining fundamental concepts of reliability as they relate to navigation systems. The calculations use data that reflect actual operating conditions within the United States. In some cases, the calculations are simplified to reduce the complexity of the presentation. When simplifications are made, the effect on the accuracy of the calculations is discussed. In general, the calculations are close approximations and can be considered engineering estimates.

1.5 REPORT ORGANIZATION

Chapter Two presents the reliability definitions developed for this report and the assumptions made in developing the material.

Chapter Three presents the development of reliability measures for application to various navigation systems.

Chapter Four states the conclusions of the report.

All references are listed in Appendix A. The references are numbered as they appear in the text.

Appendix B provides an explanation of the reliability modeling methodology used in this report.

CHAPTER TWO

DEFINITIONS AND ASSUMPTIONS

2.1 DEFINITIONS

The following definitions have been developed for the various categories of reliability associated with navigation systems.

2.1.1 Reliability

> "Reliability is the probability that a system will perform satisfactorily for at least a given period of time when used under stated conditions" (Reference 1).

The reliability definition has several essential elements: probability, system, time, stated conditions, and satisfactory performance. Each of the elements must be described in order to perform a reliability calculation. Any variations in the elements as applied in the respective calculations must be taken into account since all of the elements affect the reliability results.

2.1.2 Module Reliability

The term "module reliability" is used to characterize the reliability of a system module that is identifiable as a subsystem. The module may be a part of a ground station, an airborne station, or other major system.

2.1.3 Receiver Reliability

Receiver reliability is that associated with the airborne navigation equipment. The equipment configuration may incorporate single, dual, or triple redundancy.

2.1.4 Station Reliability

The reliability of a single transmitting facility is referred to as station reliability. In many cases, a single station is insufficient for navigation.

2.1.5 Signal Reliability

Signal reliability is the reliability associated with a navigation system's capability to provide suitable signals in a specified coverage area. It is a function of the station reliabilities of the associated transmitting facilities.

2.1.6 Operational Reliability

The reliability resulting from combining the signal and receiver reliabilities in a specified area of operation is referred to as operational reliability.

2.1.7 Flight Path Reliability

Flight path reliability is the operational reliability over a given flight path, which usually traverses several coverage areas.

2.1.8 Navigation System Reliability

Navigation system reliability is a composite representation of signal reliability and receiver reliability based on all possible areas of operation. Navigation system reliability is the sum of the operational reliabilities of all coverage areas of interest.

2.1.9 System Effectiveness

System effectiveness is the probability that the system can successfully meet an operational demand within a given time period when operated under specified conditions. The major difference between system effectiveness and system reliability is the concept of operational demand. Operational demand is a measure of system usage based on the number and geographical distribution of users.

2.2 ASSUMPTIONS

2.2.1 Probability of Success

The probability of success (satisfactory performance) is frequently used as a reliability measure. However, it is important to note that the probability is highly dependent on the combination of events that define the scenario or mission. For this report, the term probability of success will be a measure of reliability as applied to specific operational scenarios.

2.2.2 MTBF

The mean time between failures (MTBF) values used in this report are taken from the references. They are assumed to be based on continuous unit operating time and not to include nonoperating periods such as standby time or service time.

2.2.3 Navigation Receivers

For this study, navigation receivers are characterized by two MTBF values and by two common installation configurations, single and dual. The reliability of navigation receivers varies with type of system (VOR, LORAN-C, Omega, GPS, etc.), unit model, manufacturer, type of use, and age. Determination of MTBF values for all receivers of interest is beyond the scope of this effort. The receiver MTBF values and configurations chosen represent reasonable estimates of actual performance with current receivers (Reference 2).

2.2.4 DMEs and TACANs

This report does not present reliability calculations for DMEs or TACANs. DMEs and TACANs are normally used with VORs to provide distance information and can also permit navigation without VORs. The coverage, MTBF, and, therefore, reliability of DMEs and TACANs are similar to those of VORs (Reference 3).

2.2.5 Reduced Capability Modes

Many VOR/DME-based RNAV receivers use both VOR and DME signals. The reliability of such receivers would be dependent on the reliability of the VOR and DME stations being simultaneously used. If any of the stations failed, the receiver could possibly navigate in a reduced capability mode such as VOR navigation without DME. Subtleties such as reduced capability are not considered in this report, but are subject to the same techniques shown.

2.2.6 Procedures for Failure Recovery

This report does not consider the procedures to be taken to recover from a navigation failure. They are not part of reliability, but they are an important consideration in system design and operation. In some cases the procedure may be elementary. For example, if a redundant receiver fails, no action other than switching to an operable receiver will be immediately necessary. However, if the failure occurs in a required ground station, continued use of the navigation system may be impossible. Although the scope of this study does not permit investigation of all possible failure modes, it should be noted that all failures are not of equal consequence. For example, if a VOR ground station fails, the service area affected is relatively small compared to the area affected by the failure of a wide area navigation system such as GPS. Time is another example of different consequences. A failure may be as short as a few seconds or in excess of several days.

2.2.7 Operational Time Period

As stated in Section 2.1.1, the application of the reliability definition requires that a time period of operation be specified. For most of the calculations in this report an operational time period of 1 hour is used. A 1-hour period is appropriate because a majority of flights are on the order of 1 to 2 hours duration (Reference 4).

2.2.8 Maintenance

For the reliability values presented, it is assumed that a unit that fails is not repaired or returned to service during the time period of interest. This assumption becomes invalid over longer periods of time. In order to consider maintenance effects on reliability, detailed data on equipment repair time would be necessary, which is beyond the scope of this report.

2.2.9 Equipment Configuration

For this report, avionic units are considered to operate independently of each other. As units become integrated, interdependencies can affect reliability. For example, airborne navigation systems are being developed that use numerous, and dissimilar, navigation sensors, controls, and displays. The numerous sensors, controls, and displays provide redundancy and, if they are dissimilar, a means of checking system performance. However, the dependence between system elements must be carefully analyzed to prevent a "fault" in the system from propagating through the system elements. The reliability of an integrated system depends upon the reliability of each element, the element interdependencies, and the failure modes.

CHAPTER THREE

APPLICATION OF RELIABILITY MEASURES

3.1 INTRODUCTION

This chapter illustrates the various definitions given in Chapter Two by calculating the reliability measures for various navigation systems. The calculations are based on systems presently in use or proposed for future use in the domestic U.S. National Airspace System (NAS). The system configurations used are considered typical but do not represent all possibilities. The construction of the models used in this chapter is explained in Appendix B. For convenience, all values are rounded to six decimal places.

3.2 MODULE RELIABILITY

The term "module" is used to indicate an element of a larger assembly. For example, the module could be an electronic component, an electronic board, a line replaceable unit, or an entire station in a chain. Each system level has a reliability that is dependent upon the reliabilities of its defined modules.

We can use an example MTBF of 1000 hours to illustrate the probability of success, P_S, for a single module. The probability that the module will not fail during a 1-hour period is

$$P_S = e^{-t/MTBF} = e^{-1/1000} = 0.999000$$

The probability calculated does not predict when failure will occur. The module may fail after one minute of operation or after one year. The calculation also assumes that the module is either operating satisfactorily or is totally failed (i.e., no degraded state exists). As the time interval is increased, the module is less likely to operate satisfactorily and more likely to fail. (The probability of failure is one minus reliability). The probabilities for longer time intervals are shown in Table 3-1.

The probability values of Table 3-1 could be used to estimate the number of modules required to provide continuous operation during the

Table 3-1. PROBABILITIES REPRESENTING MODULE RELIABILITY FOR AN MTBF OF 1,000 HOURS		
t (Hours)	Probability of Success P_S	Probability of Failure $1-P_S$
1	0.999000	0.000100
10	0.990050	0.009950
100	0.904837	0.095163
1000	0.367879	0.632121
8760*	0.000157	0.999843

*An operating time of 8760 hours is equivalent to 1 year of continuous operation.

year. Further, such calculations can be used to decide how to design systems. For example, the module could be improved to a 2000-hour MTBF or two modules could be installed in a redundant configuration.

Table 3-2 compares the probability of success for a single module of 1000 hours MTBF, a 2000-hour-MTBF module, and a redundant configuration of two 1000-hour MTBF modules. For a redundant pair composed of two identical modules, the probability of success is determined by the probability of either module being operational. The resulting equation is

$$P_S = 2P - P^2$$

where

P is the probability of success for a single module.

The probability values for time periods greater than 10 hours are presented to illustrate the mathematical trend of the equations. Since flights rarely last over 10 hours, the P_S values in Tables 3-1 and 3-2 for the 100, 1,000 and 8,760 hour periods are not applicable to flight-related missions.

3.3 RECEIVER RELIABILITY

The reliability of navigation receivers varies with type (VOR, LORAN-C, Omega, etc.), unit model, manufacturers, usage, and age (see Section 2.2.3). In calculating the receiver probability of success, two typical values for receiver MTBF are used in this report (MTBF values

Table 3-2. PROBABILITIES REPRESENTING MODULE RELIABILITY FOR SINGLE AND REDUNDANT CONFIGURATIONS			
t (Hours)	P_s MTBF = 1000	P_s MTBF = 2000	P_s Redundant MTBF = 1000
1	0.999000	0.999500	0.999999
10	0.990050	0.990512	0.999901
100	0.904837	0.951229	0.990944
1000	0.367879	0.606531	0.600424
8760	0.000157	0.012525	0.000314

taken from Reference 2). Receivers are usually installed in either a dual or single configuration. (For the reliability modeling in this report, the receivers in all configurations are assumed to be in operation rather than some operational and some in standby.) For simplicity, the designators SL, DL, SH, and DH are used in this report to identify the receiver configurations, where

SL = single unit with low MTBF (500 hours)

DL = dual redundancy with low MTBF (500 hours)

SH = single unit with high MTBF (2000 hours)

DH = dual redundancy with high MTBF (2000 hours)

If the probability of success for each receiver configuration is calculated over various periods of time, the values of Table 3-3 result. The calculations are based on operating time. Receivers may be operated only a few hundred hours a year or may be used several thousand hours in a year.

3.4 VOR RELIABILITY

The solid-state VOR stations being installed have a specified MTBF of 10,000 hours. Existing tube-type VOR stations achieve an MTBF on the order of 3400 hours (Reference 3).

3.4.1 Station Reliability

The station reliability for various operating periods is shown for the old and new stations in Table 3-4.

Table 3-3. PROBABILITY OF SUCCESS FOR VARIOUS RECEIVER CONFIGURATIONS				
Operating Time (Hours)	Receiver Configuration			
	SL	DL	SH	DH
1	0.998002	0.999996	0.999500	1.000000
10	0.980199	0.999608	0.995012	0.999975
100	0.818731	0.967141	0.951229	0.997621
1000	0.135335	0.252355	0.606531	0.845182
8760	0.000000	0.000000	0.012525	0.024894

Table 3-4. PROBABILITIES REPRESENTING VOR STATION RELIABILITY		
Operating Time (Hours)	Probability of Success	
	P_S (Old)	P_S (New)
1	0.999706	0.999900
10	0.997063	0.999000
100	0.971017	0.990050
1000	0.745189	0.904837
8760	0.076042	0.416445

At this level of modeling, the VOR station reliability is conceptually identical to the module reliability of Section 3.2. In later sections, the reliability models are slightly more complex and treat the VOR as a module within a system.

3.4.2 Signal Reliability

Although station reliability is an important factor in navigation reliability, the user is generally more concerned with signal reliability. If signals are available from an alternative VOR, the user can navigate without the failed VOR. VOR service range depends upon altitude. Thus, at 10,000 feet only one or two VOR stations may be usable, while at 20,000 feet as many as six stations may be usable (Reference 5). Table 3-5

Table 3-5. PROBABILITIES REPRESENTING NAVIGATION SIGNAL RELIABILITY AS A FUNCTION OF VOR STATION REDUNDANCY*

Operating Time (Hours)	P_S (1)	P_S (2)	P_S (3)	P_S (4)	P_S (5)	P_S (6)
	1 Station	2 Stations	3 Stations	4 Stations	5 Stations	6 Stations
1	0.999900	0.999999	1**	1	1	1
10	0.999000	0.999999	1	1	1	1
100	0.990049	0.999900	0.999999	1	1	1
1000	0.904837	0.990944	0.999139	0.999918	1	1
8760	0.416445	0.659463	0.801278	0.884035	0.932328	0.960509

*MTBF = 10,000 hours for each station.
**Probability values greater than 0.999999 are shown as 1.

presents the signal probability of success, P_s, for various levels of signal redundancy. The value shown for P_s is the probability that at least one VOR station will provide a usable signal to the user. (For an RNAV mission, more than one VOR/DME station would be required, which would reduce the probabilities.) The general formula for a redundant configuration is

P_s (Redundant) = $1-(1-P_s)^N$ for redundancy of N stations, with a common P_s for each signal transmitted

3.4.3 Operational Reliability

Since neither receiver nor stations are useful alone, operational reliability combines signal reliability and receiver reliability to provide a more meaningful measure. Table 3-6 presents probability values, P_s, representing VOR operational reliability, for a 1-hour operating period. Note that P_s values for longer time periods (10, 100, 1000, and 8,760 hours) are not used in computing operational reliability. Even if an individual plane flew continuously for 10 hours or longer, it is unlikely that it would remain in the same coverage area upon which the value of P_s is based. Since the probability value of the signal reliability approaches one, it is the receiver reliability that limits the operational reliability.

3.4.4 Flight Path Reliability

Since the service range of a VOR depends upon altitude (Reference 6), the operational reliability over a particular flight path can vary widely. Although the operational reliability of GPS, LORAN-C, and Omega will also vary with flight path, the larger coverage areas of those systems make the effect much less significant. The following sections describe two typical cases: a low-altitude flight and a high-altitude flight.

3.4.4.1 Low-Altitude Flight Path Reliability

For an aircraft flying at an altitude of 10,000 feet above ground level, the standard service range of a VOR station is 40 nautical miles (nm). For this example the aircraft flies an average speed of 100 knots and is equipped with a single, 500 hour MTBF receiver (SL). A hypothetical low-altitude route showing VOR coverage is included in Figure 3-1.

The probability of success for a given flight is composed of the individual probabilities of success for each of the flight path segments:

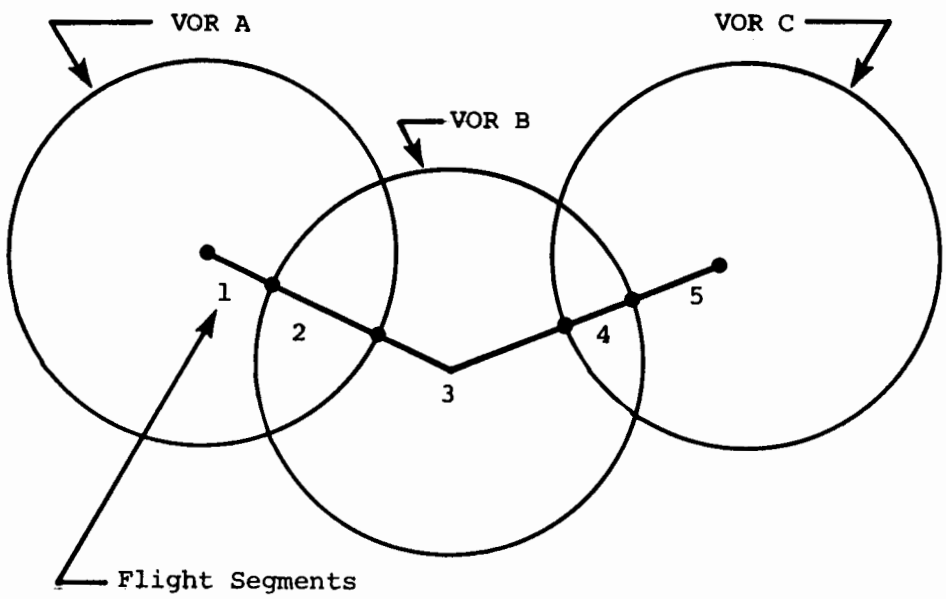
$$P_{Flight} = P_1 \times P_2 \times P_3 \times P_4 \times P_5$$

The values of P_1 through P_5 are the operational reliabilities of each flight segment based on the time period required to transit the segment. The operational reliabilities are the product of the receiver reliability (P_R) and the signal reliability of the signals being received (e.g., P_A - the probability denoting the signal reliability of VOR A).

Table 3-6. PROBABILITIES REPRESENTING VOR OPERATIONAL RELIABILITY* AS A FUNCTION OF STATION REDUNDANCY

Receiver Configuration (***)	P_S (1)	P_S (2)	P_S (3)	P_S (4)	P_S (5)	P_S (6)
	(0.999000)**	(0.999999)**	(1)**	(1)**	(1)**	(1)**
SL (0.998001)	0.997901	0.998800	0.998001	0.998001	0.998001	0.998001
DL (0.999996)	0.999896	0.999895	0.999996	0.999996	0.999996	0.999996
SH (0.999500)	0.999400	0.999499	0.999500	0.999500	0.999500	0.999500
DH (0.999999)	0.999899	0.999998	0.999999	0.999999	0.999999	0.999999

*One hour operating period.
**Probabilities from Table 3-5 representing signal reliability.
***Probabilities from Table 3-3 representing receiver reliability.



Segment	Time (minutes)	Length (nm)	VORs
1	7.5	12.5	A
2	12.0	20.0	A, B
3	22.5	37.5	B
4	7.5	12.5	B, C
5	10.5	17.5	C

Figure 3-1. HYPOTHETICAL ROUTE FOR FLIGHT PATH RELIABILITY

For segment 1, in which only VOR A is in range:

$$P_1 = P_A \times P_R \qquad (t = 7.5 \text{ minutes})$$

$$P_A = e^{-t/MTBF} = e^{-(7.5/60)/10,000} = 0.999987$$

$$P_R = e^{-t/MTBF} = e^{-(7.5/60)/500} = 0.999750$$

$$P_1 = (0.999987)(0.999750) = 0.999737$$

For the other segments, the operational reliabilities are represented by the following probabilities:

$$P_2 = (P_{A,B})\ (P_R)$$

where $P_{A,B}$ indicates the combination of VOR A and VOR B and is given by $P_{A,B} = 2P-P^2$, for $P = P_A = P_B$

$P_2 = 0.999600$ (t = 12 minutes)

$P_3 = (P_B)(P_R) = (0.999962)(0.999250) = 0.999212$
(t = 22.5 minutes)

$P_4 = (P_{B,C})(P_R) = (0.999999)(0.999750) = 0.999749$
(t = 7.5 minutes)

$P_5 = (P_C)(P_R) = (0.999982)(0.999660) = 0.999642$
(t = 10.5 minutes)

$P_{Flight} = (0.999737)(0.999600)(0.999212)(0.999749)(0.999642)$

$= 0.997941$

3.4.4.2 High-Altitude Flight Path Reliability

If an aircraft flew at a higher altitude than that used for the case example of Section 3.4.4.1, then more VOR signals would be available and the reliability would be increased. Consider an aircraft flying at 20,000 feet or higher at a speed of 570 knots and equipped with dual, high-MTBF VOR receivers (DH). At 20,000 feet and above, six or more VOR stations are available in much of the U.S. (Reference 5). Thus the probability representing flight path reliability at high altitude is equal to the probability shown in Table 3-6 as $P_s(6)$ for a DH configuration, which is 0.999999.

3.5 LORAN-C RELIABILITY

LORAN-C is currently being used in the U.S. as a supplemental navigation system for civil aviation. LORAN-C stations operate in "chains" with a master station and two or more secondary stations. Most receivers in use require the master station signal and at least two secondaries. The individual station coverage ranges from 600 nm to 1,500 nm. At present, LORAN-C chains do not cover the entire U.S., and there is little redundant coverage.

3.5.1 Station Reliability

Solid-state LORAN-C stations currently achieve an MTBF of approximately 465 hours (Reference 7). The probability representing station reliability for 1 hour of operation is

$$P_{st}(t = 1 \text{ hour}) = e^{-1/465} = 0.997851$$

3.5.2 Signal Reliability

The typical station configuration for LORAN-C signal reliability is 1 master station (with a reliability represented by the probability P_m) with 2 secondaries (P_1 and P_2). The composite signal probability is therefore

$$P_s = P_m \times P_1 \times P_2$$

If all 3 stations have an MTBF equal to 465 hours, the probability representing a composite signal reliability for a 1-hour period is the product of the individual station probabilities, (P_{ST}) or:

$$P_s = P_{ST}(M) \times P_{ST}(1) \times P_{ST}(2)$$

$$= (e^{-t/MTBF})3$$

where

$$P_{ST}(M) = P_{ST}(1) = P_{ST}(2)$$

$$P_s(t=1) = (e^{-1/465})^3 = 0.993569$$

If 3 secondary stations are available, the chain will have some redundancy although the master station is still essential. The signal reliability for 1 master and 2 of 3 available secondaries is represented by the following probabilities:

$$P_s = P_m \times (P_{2,3})$$

where

$P_{2,3}$ is the probability that any 2 of the 3 stations are operating.

$$P_{2,3} = 3P^2 - 2P^3$$

where $P = P_1 = P_2 = P_3$

$$P_s = P_m(3P^2 - 2P^3)$$

For t = 1 hr:

$$P_s = (0.997851)\ (2.987124-1.987138)$$

$$= 0.997836$$

3.5.3 Operational Reliability

The operational reliability of LORAN-C can be calculated by using the probability values calculated in Section 3.5.2 for the signal and those in Section 3.3 for the receiver. The results for t = 1 hour are shown in Table 3-7.

Table 3-7. PROBABILITIES REPRESENTING OPERATIONAL RELIABILITY* OF LORAN-C AS A FUNCTION OF STATION REDUNDANCY		
Receiver Configuration	Station Configuration	
	Master With 2 Secondaries (P_s = 0.993569)**	Master With 3 Secondaries (P_s = 0.997836)**
SL (0.998001)#	0.991582	0.995841
DL (0.999996)#	0.993565	0.997832
SH (0.999500)#	0.993072	0.997337
DH (0.999999)#	0.993568	0.997835

*One hour operating period.
**Probabilities from Section 3.5.2 representing signal reliability.
#Probabilities from Table 3-3 representing receiver reliability.

3.6 OMEGA RELIABILITY

Navigation with Omega generally requires the use of 3 stations. Individual station range is approximately 5,000 nautical miles. Because of the long range, eight stations provide nearly world-wide coverage. The geographic distribution of transmitters does not provide signal redundancy in many areas. Most receivers also utilize signals from VLF communication stations to supplement Omega signal. In order to assess the reliability of the Omega navigation system as it is officially defined (i.e., eight transmitters), the supplemental use of VLF communication stations is not included in the following calculations.

3.6.1 Station Reliability

Individual Omega stations have an MTBF of 216 hours (Reference 8). The station reliability for a 1-hour period is represented by the following probability:

$$P_{st} = e^{-t/MTBF} = e^{-1/216} = 0.995381$$

3.6.2 Signal Reliability

The signal reliability of Omega over a 1-hour period and assuming as MTBF of 216 hours is represented by the following probabilities:

where

$$P = P_1 = P_2 = P_3$$

$$P_s = P_1 \times P_2 \times P_3$$

$$= P^3$$

$$= (0.995381)^3$$

$$= 0.986207$$

If an additional Omega station provides redundancy, then

where

$P_{3,4}$ is the probability that any 3 of the 4 stations are operating,

$$P_s = P_{3,4}$$

$$= 4P^3 - 3P^4 \quad \text{for } P = P_1 = P_2 = P_3 = P_4$$

$$= 4(0.995381)^3 - 3(0.995381)^4$$

$$= 0.999872$$

3.6.3 Operational Reliability

Using the probability values given in Section 3.6.2 for the signal reliability and those from Section 3.3 representing receiver reliability, the operational reliability of Omega for a 1-hour duration is represented by the probability values given in Table 3-8.

3.7 GPS RELIABILITY

GPS navigation signals are transmitted by satellites rather than by ground stations. Since the satellites are in constant movement, the coverage provided is in constant change. The dynamic coverage effect can be ignored for the following calculations with little loss of accuracy because of the short time period of interest.

3.7.1 Station Reliability

The operational lifetime of a GPS satellite has been estimated to be at least 7.5 years. Within that lifetime, however, transmitting elements

Table 3-8. PROBABILITIES REPRESENTING OPERATIONAL RELIABILITY* OF OMEGA AS A FUNCTION OF STATION REDUNDANCY

Receiver Configuration	Omega Station Configuration	
	3 Stations (P_s = 0.986207)**	4 Stations (P_s = 0.999872)**
SL (0.998001)#	0.984235	0.997873
DL (0.999996)#	0.986203	0.999868
SH (0.999500)#	0.985713	0.999372
DH (0.999999)#	0.986206	0.999871

*One hour operating period.
**Probabilities from Section 3.6.2 representing signal reliability.
#Probabilities from Table 3-3 representing receiver reliability.

can fail and cause signal outages. Some transmitting elements are redundantly configured and can be switched at a command from the control station. Therefore, temporary interruptions will occur during the switching process. An MTBF of 7,300 hours has been estimated for GPS satellites (Reference 9) to represent the frequency of these service interruptions. If an MTBF of 7,300 hours for a GPS satellite is assumed, the station reliability for a 1-hour period is represented by the following probability:

$$P_{st} = e^{-t/MTBF} = e^{-1/7300} = 0.999863$$

3.7.2 Signal Reliability

Although GPS is still under development, the typical receiver is expected to require a minimum of four satellites. The signal reliability for four satellites (with an MTBF of 7300 hours) over a 1-hour period is represented as

where

$$P = P_1 = P_2 = P_3 = P_4$$

$$P_s = P_1 \times P_2 \times P_3 \times P_4$$

$$= P^4$$

$$= (0.999863)^4 = 0.999452$$

Redundant satellites are operationally desirable and the present constellation does provide five satellites for significant periods. For five satellites (with an MTBF of 7300 hours and where $P_{4,5}$ is the probability that any 4 of 5 satellites are operating), the signal reliability for a 1-hour time period is

where

$$P = P_1 = P_2 = P_3 = P_4 = P_5$$

$$P_s = P_{4,5}$$

$$= 5P^4 - 4P^5$$

$$= 5\ (0.99863^4) - 4(0.99863^5)$$

$$= 0.999997$$

3.7.3 Operational Reliability

Using the probability values from Section 3.7.2 representing signal reliability and those representing receiver reliability from Section 3.3, the operational reliability for GPS for a 1-hour time period is given by the probability values in Table 3-9.

Table 3-9. PROBABILITIES REPRESENTING OPERATIONAL RELIABILITY* FOR GPS AS A FUNCTION OF SATELLITE REDUNDANCY

Receiver Configuration	Station (Satellite) Configuration	
	4 Satellites (P_s = 0.999452)**	5 Satellites (P_s = 0.999997)**
SL (0.998001)#	0.997454	0.997998
DL (0.999996)#	0.999448	0.999993
SH (0.999500)#	0.998952	0.999497
DH (0.999999)#	0.999451	0.999996

*One hour operating period.
**Probabilities from Section 3.7.2 representing signal reliability.
#Probabilities from Table 3-3 representing receiver reliability.

3.8 NAVIGATION SYSTEM RELIABILITY

As stated in the definition (Section 2.1.8), navigation system reliability is the sum of the operational reliabilities for all coverage areas. Modeling the coverage areas is difficult because of the wide range of area sizes and signal redundancy for each system of interest. Therefore, the calculation of navigation system reliability has not been attempted in this report.

3.9 SIGNIFICANCE OF PROBABILITY VALUES

The representations of reliability presented as probabilities in previous sections do not clearly characterize the significance of variations in probability values. The difference between a probability value of 0.999998 and a value of 0.999999 is so small as to appear insignificant. To be meaningful, the probability variations should be related to actual traffic statistics.

One potential method of interpreting the significance of probability values is to base their calculation on aircraft operating hours. For 1979, approximately 48 million operating hours were logged by aircraft operating in the domestic U.S. (Reference 5). Calculating the number of flights interrupted by navigation failures depends on the characteristics of the flight as well as the characteristics of the navigation system. Flight parameters needed are the altitude, length in miles, and duration in hours. System parameters needed include the maintenance and service requirements for the navigation systems in use. Based on the operating parameters, scenarios could be generated to relate the effect of a failure (represented by probability values) to the number of affected flights. The development of a navigation interruptions measure could be the subject of additional study.

CHAPTER FOUR

CONCLUSIONS

The reliability of a navigation system cannot be represented by a single value that is applicable to all situations. Rather, the situation of interest must be carefully defined and the relevant probabilities calculated. The probabilities developed in this study for various reliability measures are based on selected operating scenarios that were judged typical of the respective navigation systems. The probabilities representing station, signal, receiver, and operational reliability measures are presented in Table 4-1.

Table 4-1. PROBABILITIES REPRESENTING RELIABILITIES FOR VARIOUS NAVIGATION SYSTEMS FOR A 1-HOUR OPERATING PERIOD

Navigation System	Reference Section	Station	Signal	Receiver	Operational
VOR	3.4	0.999900	0.999900*	0.999996	0.999896
LORAN-C	3.5	0.997851	0.993569	0.999996	0.993565
Omega	3.6	0.995381	0.986207	0.999996	0.986203
GPS (4 sat.)	3.7	0.999863	0.999452	0.999996	0.999448
GPS (5 sat.)	3.7	0.999863	0.999997	0.999996	0.999993

*Based on a single station with 10,000-hour MTBF.

As discussed in this report, the reliability of a navigation system depends on the reliability of its subsystems and their configuration. The probabilities for the operational reliability measure were developed as a possible approach to comparing the navigation systems in a manner that would relate to the users of the system. The probabilities for operational reliability shown in Table 4-1 are useful in comparing navigation systems because the values are based on comparable situations and indicate the influence of various navigation subsystems. However, the values shown

should not be interpreted as being conclusive representations of navigation reliability. The scope of this study limited the data on equipment MTBF to those values that were readily available. Some values, such as the MTBF of GPS satellites and of the solid-state VOR stations, are estimates that have not been substantiated by operational experience.

Although this report has defined various measures of reliability, a standard for application of these measures does not exist. It is through engineering judgment, therefore, that decisions are being made by the aviation community as to the acceptability or nonacceptability of system or unit reliability.

APPENDIX A

REFERENCES

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APPENDIX B

CONSTRUCTION OF RELIABILITY MODELS

The models used in this report are based on the steps contained in Appendix A of Mil-HDBK-217C, Reliability Prediction of Electronic Equipment. The steps are:

Step 1 - Define what is required for mission success and translate this into a mission success diagram.

Step 2 - Write the probability of survival (P_S) equation for the system.

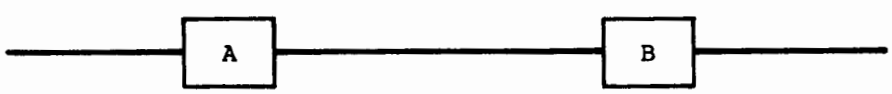
Step 3 - Calculate P_S for each of the equipments in the system.

Step 4 - The probability of survival numbers for the various equipments derived in Step 3 are inserted in the formula derived in Step 2 for the system probability of survival.

Step 5 - A probability of survival curve versus time can be plotted by taking several values of time for mission time and evaluating the probability of system survival by the above procedure for the several values of time chosen.

Step 6 - Additional steps in the analysis will depend upon the decisions that the analysis is intended to optimize.

A simple example of this process is the case of a system consisting of two equipments, A and B. To succeed, both equipments must operate. The success diagram is:



The probability of success equation is:

$P_S = R_A \times P_S$ (if A is operational) + $Q_A \times P_S$ (if A is failed)

where

P_s = probability of mission success (survival)

P_s (if A is operational) = probability of mission success if A is good

P_s (if A is failed) = probability of mission success if A is bad

R_A = reliability of A

Q_A = unreliability of A = $1 - R_A$

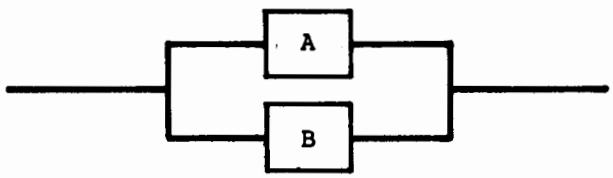
for the previously defined success diagram, the probability of success is:

$$P_s = P_A \times P_B + (1-P_A) \times (0)$$

$$P_s = P_A P_B$$

$$P_s = P_A^2 \quad \text{(if A and B are identical)}$$

If the system consists of two equipments, A and B, either of which can successfully satisfy the mission, then the success diagram is:



The corresponding probability of success equation is:

$$P_s = P_A \times (1) + (1-P_A) \times P_B$$

$$P_s = P_A + P_B - P_A P_B$$

if $P_A = P_B$, then

$$P_s = 2P_A - P_A^2$$

The above examples illustrate the situation of a simple series configuration and a simple parallel configuration. Intuitively, a parallel configuration is more likely to succeed since the probability of both units failing (parallel) is less than the probability of both units operating (series). The parallel configuration with two identical units is often termed redundance.

The probabilities used in the probability-of-success equation result from the assumptions that failures occur randomly in time, that one failure does not influence other failures, and that the failures occur at an average rate which is constant with time. The assumptions are represented by the exponential function:

$$R(t) = e^{-t/m} = e^{-ft}$$

where

R(t) is the reliability for time period t (assuming a failure-free system at t = o)

t is the specified time period

m is the mean-time-between-failures

f is the constant failure rate (m=1/f)

e = 2.71828..., the base of natural logarithms