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DOT/FAA ND-94/3 c. 2 Kim, Yeon-Myung Vertiport capacity - analysis methods
Dear Colleague:

Enclosed is a copy of the report FAA/ND-95/3, Vertiport Capacity - Analysis Methods.

This report presents a methodology for analyzing vertiport capacity and delays. Based on available information, deterministic models were developed to estimate vertiport capacity, analytic queuing models were developed to estimate the delays at vertiports during steady demand periods, and simulation models were developed to estimate delays during peak periods when traffic exceeds capacity.

Results show that vertiport capacity is more likely to be limited by airspace separations or gate availability than by touchdown and liftoff area (TLOF) occupancy times. Equations are provided for estimating relations among aircraft arrival rates, required number of gates, and gate occupancy times. These can be used to determine the required number of TLOF's or gates in particular situations.

The civil tiltrotor (CTR) can take the rotorcraft industry into a dramatically different market niche. Analysis continues to show that a vertiport designed to support scheduled passenger operations will be more than just a big heliport. Such a vertiport will have characteristics of both airports and heliports. However, the vertiport will be a new and different type of landing facility in many ways that have yet to be completely resolved and understood. A paradigm shift in thinking will be required to bring this about.

This effort is one of a variety being conducted to enable the FAA and other organizations to plan for the infrastructure needs of CTR aircraft.

Eileen R. Verna
Acting Manager, General Aviation and Vertical Flight Program Office
Technical Report Documentation Page

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<th>5. Report Date</th>
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<th>7. Author</th>
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<td>This report presents a methodology for analyzing vertiport capacity and delays. Based on available information, deterministic models were developed to estimate vertiport capacity, analytic queuing models were developed to estimate the delays at vertiports during steady demand periods, and simulation models were developed to estimate delays during peak periods when traffic exceeds capacity. A practical capacity, defined as that traffic volume at which average delays are four minutes per operation, can be determined from a volume versus delay curve. Results show that vertiport capacity is more likely to be limited by airspace separations or gate availability than by touchdown and liftoff area (TLOF) occupancy times. Equations are provided for estimating relations among aircraft arrival rates, required number of gates, and gate occupancy times. These may be used to determine the required number of TLOF's or gates in particular situations.</td>
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We gratefully acknowledge the assistance and cooperation of the FAA, NASA and U.S. Air Force in performing this work. Many individuals from various FAA and NASA offices, along with the pilots from the U.S. Air Force contributed to this project. We want to thank Michael Zywkarte of NYMA incorporated, Robert D. Smith and Kenneth W. Peppard of the Federal Aviation Administration, and Brian Sawyer of SAIC for their guidance and support throughout the project and for their reviews of this report. We wish to thank USAF Lt. Col. Joe Arvai, Bill Decker and Loran Haworth from NASA Ames Center and Patti Moss from the New York Terminal Radar Approach Control Center, whose support and comments were very useful in identifying problems and in making assumptions and numerical estimates. Finally, we wish to thank William Reid of Value Systems Services for his support on this project.
The General Aviation and Vertical Flight Program Office of the Federal Aviation Administration (FAA) has undertaken a series of studies on the planning and design of vertiports and other issues related to civil tiltrotor operations and facilities. Some practical methods for estimating vertiport capacity and delays were urgently needed. Thus, the emphasis was on methods that could be developed relatively quickly, based on information that could be obtained in this time frame.

The objectives of this study were to (1) identify and quantify the factors affecting vertiport capacity and delay, (2) develop methods for estimating vertiport capacity and delays, and (3) document these methods to show how they can be applied to estimate vertiport capacity and delay.

In order to identify and quantify the factors affecting vertiport capacity and delay, previous studies were reviewed and the survey information from several pilots and air traffic controllers was evaluated. After considering various technical approaches, a methodology for estimating vertiport capacity and delays was proposed that included deterministic (i.e. non-probabilistic) models, probabilistic queuing models, and simulation models. Deterministic models were found to be most suitable for estimating the capacity of three vertiport subsystems: terminal airspace, touchdown lift-off surfaces (TLOFs), and gates. It was assumed that the smallest capacity found among these subsystems was the total system capacity. The absolute capacities of vertiports and their subsystems were estimated based on feasible intervals between arrivals or occupancy times. For example, the terminal airspace capacity was mainly determined by the minimum separation between aircraft in the air and their speed. TLOF capacity was mainly determined by TLOF ground occupancy time and in-trail separation in the approach path. Thus, the TLOF capacity was the minimum value of two capacities, i.e. TLOF ground capacity and TLOF approach path capacity. Since the taxiway capacity is usually not a limiting component for conventional airports, it was assumed that the taxiway capacity for vertiports would not be critical either. Gate capacity was defined as the maximum number of flights that a given number of gates can handle during a specific time interval.
Probabilistic queuing models were found to be most suitable for estimating delays under steady-state conditions. They were used to analyze the performance of vertiports with different numbers of TLOFs and gates, different arrival/departure patterns, and operational procedures. For modelling purposes, the gate or TLOF occupancy time was treated as a service time, and headways were treated as interarrival times.

A simulation approach was considered appropriate for estimating delays due to temporary peaks when traffic exceeds capacity. Simulation models were developed to deal with transient demand conditions (e.g. effects of peak period duration and various volume capacity levels). Our basic approach included several steps. First, a deterministic analytic model was developed to calculate delay that was expressed as a function of (a) peak period duration, (b) peak period arrival rate, (c) off-peak period arrival rate, (d) service rate, (e) utilization factor during off-peak period, and (f) utilization factor during peak periods. Secondly, the behavior of the probabilistic system was simulated. Thirdly, a "stochastic adjustment factor", which was the ratio of simulated delay to deterministic delay was computed. These simulation models were developed for various numbers of TLOFs or gates, and for different interarrival and service time distributions.

The main results obtained with available data and our proposed methods are summarized below:

- The terminal airspace capacity for one approach path ranges from 16 to 24 flights/hour with the minimum in-trail separation distance (from 3 nautical miles to 6 nautical miles).
- TLOF capacity is the same as terminal airspace capacity, since its capacity is affected by terminal airspace capacity. It varies from 16 operations/hour (pessimistically) to 24 operations/hour (optimistically).
- Gate capacity depends on the gate utilization factor and gate occupancy time, as shown in Table ES-1.
- The vertiport capacity is determined by the minimum capacity among the capacities of subsystems in series, as shown in Figure 3. Preliminary results show that the airspace capacity or gate capacity are more likely to be critical than TLOF capacity.
- From the relations among terminal airspace, TLOFs and gates (Equation 4.1), we can
estimate the required number of gates, as shown in Table ES-1, regarding to various gate occupancy times, arrival rates and utilization factors. The utilization factor is defined as the ratio between arrival rate and service rate (per time unit). Typically, the gate utilization factor at conventional airports varies between 0.5 and 0.8. These utilization factors (0.5 and 0.8) are reflected in Table ES-1.

**Table ES-1. Estimation of Number of Gates Required for Various occupancy Times and Utilization Factors**

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† Required number of gates with reserve factor (\( G^* = \lceil G + \sqrt{G} \rceil \)), \( \lceil \cdot \rceil \) means rounded up value, \( G \) = number of gates required without reserve factor.

‡ This utilization factor is used for peak period.

- Delays due to peak period overflows were analyzed by combining a deterministic analytic model with simulation results.
- Gate capacity will be dramatically reduced if CTRs cannot operate independently (simultaneously) at adjacent gates. At vertiports where both capacity and land costs are important issues, the use of "jetways", or some similar structures for passenger loading/unloading, would probably be the most cost-effective solution.
As in previous capacity analysis methods for conventional airports, this study focuses on aircraft rather than passengers. To translate aircraft capacities and delays into corresponding passenger numbers, we can multiply aircraft by their seat capacities (approximately 40 for early CTR's) and load factors (approximately 65%, based on conventional airline operations). Further studies may consider the mix of aircraft sizes and variations in load factors for various periods and situations.

The methodology and results should be useful in evaluating the commercial feasibility of particular vertiports and the effects of vertiports on a larger air transportation system. To improve the reliability and precision of models for vertiport capacity and delay, the following additional research tasks are recommended:

- Development of relations between aircraft and passenger capacity.
- Optimization of gate configuration and gate sizing.
- Sensitivity analysis on gate separation (as indicated in Ref. No. 6) with respect to:
  a) small separations with dependent operations at adjacent gates,
  b) large separations with independent operations at adjacent gates,
  c) use of "jetways" allowing small separations with independent operations at adjacent gates.
- Estimation of terminal airspace capacity and delay for mixed operations of CTR and conventional aircraft.
- Incremental analysis of the costs and benefits of additional gates.
- Sensitivity analysis on TLOF occupancy time and number of CTR operations with respect to different taxiing disciplines, including conventional and hover taxiing.
TABLE OF CONTENTS

1. INTRODUCTION 1
   1.1 Problem Statement 1
   1.2 Objectives 1
   1.3 Scope 2
   1.4 Audience 2
   1.5 Terminology 2
   1.6 Report Organization 4

2. LITERATURE REVIEW 5
   2.1 Information Collection and Review 5
     2.1.1 Specific Findings 5
   2.2 Identification and Quantification of Factors Affecting Vertiport Capacity 9
     2.2.1 Vertiport Components 9
     2.2.2 Operation Procedures 10
     2.2.3 Air Traffic Control Factors 11
     2.2.4 Environmental Restrictions 12
     2.2.5 Demand Pattern 12
     2.2.6 Other Factors 13

3. METHODOLOGY FOR ESTIMATING VERTIPORT CAPACITY AND DELAYS 15
   3.1 Deterministic Analytic Method 15
       3.1.1 Terminal Airspace Capacity 15
       3.1.2 TLOF Capacity 16
       3.1.3 Taxiway Capacity 19
       3.1.4 Gate Capacity 20
3.2 Probabilistic Queuing Analysis
   3.2.1 Single Server Queuing Model
   3.2.2 Multiple Server Queuing Model

3.3 Simulation Analysis
   3.3.1 Deterministic Delay Due to Peak Period Overflow
   3.3.2 Stochastic Delay Due to Peak Period Overflow
   3.3.3 Average Delay of Multiple Parallel Servers System

4. CAPACITY AND DELAY DETERMINATION
   4.1 System Capacity Estimation
      4.1.1 Terminal Airspace Capacity
      4.1.2 TLOF Capacity
      4.1.3 Gate Capacity
      4.1.4 Relations among Airspace Capacity, TLOF Capacity and Gate Capacity

   4.2 Estimation of Delay
      4.2.1 Delay at a Single Server Queuing System
      4.2.2 Delay at a Queuing System with Multiple Parallel Servers

   4.3 Simulation Results
      4.3.1 Total Delay of Deterministic and Stochastic Traffic Flow
      4.3.2 Estimation of Stochastic Adjustment Factors
      4.3.3 Estimation of Average Delay for Multiple Server System

5. CONCLUSIONS AND RECOMMENDATIONS
   5.1 Conclusions
5.2 Recommendations  

<table>
<thead>
<tr>
<th>REFERENCES</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>APPENDIX A.</td>
<td>ACRONYMS</td>
</tr>
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# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CTR Approach Path Profile</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>Typical Vertiport Layout</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Capacity Components of Vertiport Airside</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>Flow Chart of Simulation Model</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>Total Deterministic Delay Due to Peak Period Overflows</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>Effect of Gate Numbers and Occupancy Time on Gate Capacity (0.8 Utilization)</td>
<td>37</td>
</tr>
<tr>
<td>7</td>
<td>Average Waiting Time for a Single-Server Queuing System</td>
<td>41</td>
</tr>
<tr>
<td>8</td>
<td>Idle System Probability ($P_0$)</td>
<td>43</td>
</tr>
<tr>
<td>9</td>
<td>Average Waiting Time for Queuing System with Multiple Parallel Servers</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>Deterministic Delay for Peak Period Overflows</td>
<td>46</td>
</tr>
<tr>
<td>11</td>
<td>Deterministic and Stochastic Delays Due to Peak Overflows for M/M/k Case</td>
<td>48</td>
</tr>
<tr>
<td>12</td>
<td>Deterministic and Stochastic Delays Due to Peak Overflows for M/N/k Case</td>
<td>49</td>
</tr>
<tr>
<td>13</td>
<td>Stochastic Adjustment Factor for M/M/k Case</td>
<td>51</td>
</tr>
<tr>
<td>14</td>
<td>Stochastic Adjustment Factor for M/N/k Case</td>
<td>52</td>
</tr>
<tr>
<td>15</td>
<td>Average Delay for Multi-Server Systems (M/N/k and M/M/k Cases)</td>
<td>55</td>
</tr>
<tr>
<td>16</td>
<td>Average Delay for Multi-Server Systems (D/N/k and D/M/k Cases)</td>
<td>56</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1. Civil Tiltrotor Descent Phase of Altitude, Airspeed and Time 7
Table 2. Average Waiting Time and $P_0$ for M/M/k Case 26
Table 3. Comparison of Average Waiting Time in the Queue
   Between Theory and Simulation Results (M/M/k) 32
Table 4. Gate Capacity (CTRs/hour) 36
Table 5. Required Number of Gates 39
Table 6. Deterministic Delay Due to Peak Period Overflows 45
Table 7. Stochastic Delays for Peak Period Overflows 47
Table 8. Stochastic Adjustment Factor ($F_s = D_s/D_o$) 50
Table 9. Average Delay in Multiple Server Systems for M/M/k and M/N/k Cases 53
Table 10. Average Delay in Multiple Server Systems for D/M/k and D/N/k Cases 54
1. INTRODUCTION

1.1 Problem Statement

Interest in vertiports is due to recent advances in tiltrotor technology, increasing congestion expected at existing airports, difficulties in providing new runway capacities and difficulty in finding acceptable sites for new air carrier airports.

Recently, there has been considerable research and development on Civil Tiltrotor (CTR) aircraft. The Federal Aviation Administration (FAA) has initiated several studies on CTR operations and has awarded numerous Airport Improvement Grants (AIP) to states and local governments to assess vertiport feasibility (4, 7, 16, 26, 27). The Port Authority of New York & New Jersey (PANYNJ) has commissioned several feasibility studies under AIP Grants on Civil Tiltrotor Service in the New York metropolitan area (16, 28). The FAA has also distributed Advisory Circular 150/5390-3 for vertiport design. This circular provides guidance for the planners and communities interested in developing a civil vertiport. Vertiports can be located in urban areas, in suburban areas, and at major hub airports.

This report documents methods for estimating vertiport capacities and delays.

1.2 Objectives

The objectives of this study are to:

1) Identify and quantify the factors affecting vertiport capacity and delay.
2) Develop methods for estimating vertiport capacity and delays.
3) Document these methods and show how they can be applied to estimate vertiport capacity and delay.
1.3 Scope

When this study was initiated, some practical methods for estimating vertiport capacity and delays were needed urgently, i.e. within two months. Hence, the emphasis was on methods that could be developed relatively quickly, based on information that could be obtained in this time frame. Literature from previous studies was reviewed and the survey returns from several pilots and air traffic controllers were evaluated. After considering various technical approaches, deterministic models were found to be most suitable for estimating the capacity of various vertiport components. Probabilistic queuing models were found to be most suitable for estimating delays under steady-state conditions and a simulation approach was deemed appropriate for estimating delays due to temporary peaks when traffic exceeds capacity. This report documents these methods and their results.

1.4 Audience

This report has been written for the FAA General Aviation and Vertical Flight Program Office (AND-610), vertiport planners and designers, urban planners, and other decisionmakers in aviation. Readers should be familiar with the basic terminology used in deterministic models, probabilistic queuing models and simulations. Readers unfamiliar with such terminology are advised to read Section 1.5.

1.5 Terminology

This section defines the terms used in this report that are not very common. These terms include:

*Capacity:* The maximum number of aircraft operations (i.e., arrivals and departures) per unit of time, (typically in operations per hour) that can be accommodated under specified operating conditions.

*CTR:* Civil tiltrotor aircraft capable of vertical takeoffs and landings.
Delay: The time difference between scheduled and actual events, such as aircraft arrivals. Congestion delays or queuing delays are excess service times (above normal service times, which occur at near zero traffic volumes). Delays depend on capacity and the magnitude and fluctuation in demand.

Deterministic: Predictable, i.e. not random.

Headway: Time interval (in minutes and seconds) between two consecutive aircraft.

In-trail Separation: The distance interval (in nautical miles) between two aircraft. It depends on weather conditions (Visual Meteorological Conditions (VMC) or Instrument Meteorological Conditions (IMC)), types of leading and following aircraft (small - heavy, heavy - large, etc), type of overflown surface (continent or ocean) or type of the navigational aid in use.

Kendall Notation: General notation used to describe a queuing system in the form a/b/c/d/e where:

(a) describes the type of arrival process (interarrival time distribution)
(b) describes service time distribution
(c) describes the number of servers
(d) describes the queue storage capacity
(e) describes the queue discipline

Common distributions listed in (a) or (b) include exponential (M), general (G) and deterministic (D). Common queue service disciplines listed in (e) include FIFO (First-in First-out), LIFO (Last-in First-out) and SIRO (=Service in Random Order).

Platoon: A group of aircraft with similar characteristics.

Probabilistic: Influenced by random variables with specified probability distributions.
**Queuing System:** A system in which demand may temporarily exceed capacity. When the arrival rate exceeds the service rate the excess arrivals wait in a queue. Usually, the arrival process is described in terms of the probability distribution of the interarrival times of customers (aircraft) and it is usually assumed that these interarrival times are independent, identically distributed random variables. The distribution of interarrival and service times, the queue discipline, queue storage capacity and number of servers are among the important characteristics of queuing systems.

**Stochastic:** Influenced by random variables with specified probability distributions.

**TLOF:** Touchdown lift-off surface (hard or paved) capable of supporting the heaviest tiltrotor that is expected to operate at the vertiport.

**Transient Demand:** Demand that is not deterministic, but fluctuates over time.

**Utilization Factor:** A nondimensional value that defines the ratio between arrival rate and service rate (per time unit). Typically, the gate utilization factor at conventional airports varies between 0.5 and 0.8. This utilization factor accounts for demand variability and for the time required to maneuver aircraft in and out of gates.

**1.6 Report Organization**

The report is organized into five sections. Section 1 provides the context for *Vertiport Capacity Analysis Methods*, including problem statement, objectives, intended audience and definition of terms. Section 2 reviews the literature and discusses various factors affecting vertiport capacity. Section 3 presents several methodologies for estimating vertiport capacity and delays. Capacity and delay determinations are presented in Section 4, while needs for further research, summary and conclusions are presented in Section 5.
2. LITERATURE REVIEW

In order to identify and quantify the factors affecting vertiport capacity and delay, an extensive literature review was conducted and the information collected from different sources was carefully analyzed.

While reviewing the literature and gathering information, considerable inconsistency was found in sizing the various elements of a vertiport. To some extent, this problem arises because CTR aircraft are still in the developmental stage and many of the performance characteristics are changing as the aircraft design evolves. Some of the information on CTR performance characteristics is still considered sensitive, and therefore is quite difficult to obtain.

2.1 Information Collection and Review

In addition to reviewing the written documents listed in the reference section, several pilots, aerospace engineers and air traffic controllers were interviewed. Their suggestions were compared with the relevant information found in published sources and their expertise was used to clarify operational procedures and to help estimate some numerical factors.

2.1.1 Specific findings to date include the following:

a) A comprehensive set of vertiport requirements (vertiport sizing, lighting, navaids, vertiport capacity and delay issues, etc.) has not been defined (5, 12, 27).

b) Existing taxiway, parking, and terminal vertiport areas models are quite preliminary, and are subject to change (27). The reasons given are that the CTR aircraft is still in its developmental stage.

c) The relations among various elements that could affect vertiport capacity and delay, such as headway, touchdown and liftoff area (TLOF) (i.e. rollway) occupancy time and taxi time, have not been modelled in a mathematically comprehensive way.

d) Operations research methodologies or optimization models for vertiport capacity and delay were not found in the reviewed literature.

e) The Microwave Landing System (MLS) has been considered as the primary future
navigational aid for landings (16, 28). The impacts of Global Positioning System (GPS), Automatic Dependent Surveillance (ADS), Data Link, Precision Runway Monitor (PRM) and other new technologies on CTR operations near and at vertiports have not been fully considered. However, this work is in process. This technology is important for determining the level of automation needed and in-trail separation (headway) on the final approach.

g) Information on minimum TLOF occupancy time was not available in the literature but was obtained during interviews. This information is needed for estimating the relative effect of TLOFs and gates on vertiport capacity.

h) Information on in-trail separation requirements for CTRs was not standardized. (Separations between 3 and 6 miles have been previously considered.)

i) No standardized requirements for TLOF clearance were found. The TLOF clearance was another vertiport element important in determining TLOF throughput (i.e., maximum arrival rate).

j) The effects of headways for CTRs were calculated at a "macro" level, with respect to the number of CTRs required (for example) in the Northeast Corridor, and sensitivity analysis for 30 - 60 minutes headways was used to estimate numbers of CTRs required. No sensitivity analysis for gate requirements with respect to changes in headway was found (8).

k) The preliminary information on flight profile characteristics (i.e. vertical and horizontal speed, conversion, etc) and airside requirements (i.e. approach angle, decision height, etc) based on CTR 2000 aircraft characteristics was obtained from the Civil Tiltrotor Aircraft Performance (6), and was used in this report (Table 1 and Figure 1). This information was compared with information obtained from an interview with Lt. Col. Joe Arvai and differences were noted Section 4.1.1.
### Table 1. Civil Tiltrotor Descent Phase of Altitude, Airspeed and Time

<table>
<thead>
<tr>
<th>Distance from Touchdown (nm)</th>
<th>Altitude (ft)</th>
<th>Airspeed (ktas)</th>
<th>Time from Touchdown (min)</th>
<th>Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Touchdown Landing Decision Height</td>
</tr>
<tr>
<td>0.07</td>
<td>75</td>
<td>25</td>
<td>0.2</td>
<td>Segmented approach max. R/D = 800 fpm (9 degree max. glide slope)</td>
</tr>
<tr>
<td>0.45</td>
<td>441</td>
<td>55</td>
<td>0.6</td>
<td>1 min. partial conversion</td>
</tr>
<tr>
<td>1.07</td>
<td>908</td>
<td>70</td>
<td>1.2</td>
<td>0.1 g decel. const. alt.</td>
</tr>
<tr>
<td>1.57</td>
<td>1227</td>
<td>80</td>
<td>1.6</td>
<td>Avg. 1000 fpm descent @250 ktas</td>
</tr>
<tr>
<td>2.32</td>
<td>1546</td>
<td>100</td>
<td>2.1</td>
<td>Avg. 2000 fpm descent @ ft. idle</td>
</tr>
<tr>
<td>3.05</td>
<td>1780</td>
<td>120</td>
<td>2.5</td>
<td>Cruise</td>
</tr>
<tr>
<td>5.55</td>
<td>2000</td>
<td>180</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>8.01</td>
<td>2000</td>
<td>257</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>23.0</td>
<td>5300</td>
<td>273</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td>45.2</td>
<td>10000</td>
<td>291</td>
<td>12.3</td>
<td></td>
</tr>
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<td>51.2</td>
<td>12129</td>
<td>321</td>
<td>13.5</td>
<td></td>
</tr>
<tr>
<td>57.2</td>
<td>14257</td>
<td>350</td>
<td>14.5</td>
<td></td>
</tr>
<tr>
<td>87.2</td>
<td>24900</td>
<td>360</td>
<td>19.6</td>
<td></td>
</tr>
<tr>
<td>87.5</td>
<td>25000</td>
<td>320</td>
<td>19.7</td>
<td></td>
</tr>
<tr>
<td>90.0</td>
<td>25000</td>
<td>320</td>
<td>20.1</td>
<td></td>
</tr>
</tbody>
</table>

† Table was developed by the Boeing Defense Space Group.
2.2 Identification and Quantification of Factors Affecting Vertiport Capacity

Vertiports are defined as "facilities providing full support for the takeoff and landing of tiltrotor aircraft (Figure 2). Such facilities would be capable of accommodating the operation of helicopters as well. They may be developed as public-use or private-use facilities. A variety of sites may have potential for development as vertiports, including rooftops, land along waterfronts, space over highways, and unused land at existing airports." (5). Many factors affect vertiport capacity. These factors can be classified into the following groups:

2.2.1 Vertiport Components

Vertiport components include the number of TLOFs, number of gates, taxiways, and terminal airspace. These factors are closely related. Based on Horonjeff's equation (11, page 381), \( G = CT/U \), where, \( G \) = number of gates, \( C \) = maximum volume of aircraft arrivals, in CTR per hour, \( T \) = weighted average gate-occupancy time, \( U \) = gate utilization), the required number of gates is determined by the maximum volume of aircraft arrivals, gate occupancy time and gate utilization.

Figure 2. Typical Vertiport Layout (Source: Reference 27)
The maximum volume of aircraft arrivals is based on the capacity of the terminal airspace and of the TLOF. If the maximum volume of aircraft arrivals or the gate occupancy time increases, the required number of gates may have to be increased. For example, let us assume that $U=0.8$, the headway between operations at TLOF=$120$ seconds, and $T=20$ minutes. The TLOF can be used for $30$ operations/hour (3600sec./120sec.=$30$, 15 landings and 15 takeoffs per hour). The number of gates required for 15 arrivals/hour can be calculated as follows:

$$ G = C^* T / U = (15 * 20/60) / 0.8 = 6.25 \approx 7 \text{ gates.} \quad (2.1) $$

If we add a reserve factor suggested by De Neufville (10), equal to the square root of the original number of gates, the adjusted number of gates required is

$$ G^* = n + \sqrt{n} = 6.25 + \sqrt{6.25} = 8.75 \approx 9 \text{ gates.} \quad (2.2) $$

The spacing between gates, between TLOFs or between gates and TLOFs can also affect the vertiport capacity. The gate capacity may be restricted by constraints, e.g. on land availability or required spacing between adjacent gates.

### 2.2.2 Operation Procedures

These pertain to gate operations, TLOF operations, ratio of arrivals and departures, and mix of aircraft sizes. The gate occupancy time and, hence, gate capacity depend on the gate operation procedures. The mean maneuvering time and the maximum value among four values, durations for activities that are presumed to be concurrent (passenger unloading or loading time, baggage unloading or loading time, inspection and check out, and fueling time) determine the occupancy time. The gate capacity increases as these gate occupancy time components decrease. The occupancy time is formulated as follows:

$$ T_g = F + \text{Max (} T_p, T_b, T_v, T_f \text{)} \quad (2.3) $$
where $T_g$ = gate occupancy time
$F$ = mean fixed or set up time for maneuvering of gate
$T_p$ = mean passenger loading and unloading time
$T_b$ = mean baggage loading and unloading time
$T_i$ = mean inspection and check out time
$T_f$ = mean fueling time

The TLOF occupancy time affects the TLOF capacity. The TLOF occupancy time depends on air traffic control procedures, aircraft ground speeds, taxiing procedures, turn-off speeds, TLOF exits, and the distance between TLOF and gate. The terminal airspace is affected by the ratio of arrivals and departures and the mix of aircraft sizes.

2.2.3 Air Traffic Control Factors
These factors include separation requirements, dimensions of approach and departure paths (speeds, approach and descent angles), and air traffic services provided. Among the most critical factors affecting airspace capacity are the air traffic control standards. There should be specific minimum vertical, horizontal, and lateral separations for safety. These requirements restrict the airside capacity and TLOF capacity. The required separation is a function of the following variables:

- aircraft wake vortices
- weather conditions (e.g. ceiling and visibility)
- aircraft size
- air traffic services and type of navigation (e.g. radar or non radar separation standards)

The dimensions of approach and departure paths affect the terminal airspace capacity. The duration of transition mode (from rotors-forward to vertical flight or vice versa), speed, and approach and descent angle also affect the capacity. Figure 1 shows the approach path. The TLOF approach path capacity may be increased by sequencing or platoon controls. For example, a departure can be inserted between two arrivals based on separation rules. We can also increase
the capacity by grouping landing and departing aircraft in platoons when there are civil tiltrotor aircraft of several different sizes. This separation in the air rather than headway on the TLOF is more likely to restrict capacity.

2.2.4 Environmental Restrictions

These include weather condition and noise abatement requirements. Environmental factors can reduce the vertiport capacity. The air traffic controllers and pilots will tend to increase separations between aircraft under poor visibility conditions. In addition, under instrument meteorological conditions (IMC) aircraft are required to operate under instrument flight rules (IFR). IFR removes the control flexibility associated with visual flight rules (VFR) and greatly decreases a facility's capacity.

Noise abatement requirements may also reduce vertiport capacity since the noise abatement requirements restrict the vertiport configuration, approach paths, and the times of day when operations may occur.

2.2.5 Demand Patterns

These pertain to hub or non-hub operations, distribution over days, weeks and seasons, and peak hour characteristics. Flight demands vary depending on whether we have hub or non-hub operations and depending on time of day, week or season. The vertiport capacity varies with demand patterns. In hub vertiports, the flights will probably be scheduled in batch operations. These operations will require a high capacity for a short time. For non-hub operations, demand distributions over a day are generally steadier. Therefore, the capacities are different even if the daily demands are the same. When the interarrival time distributions and service time distributions are known, probabilistic queuing models with simple assumptions could be used. For example, if it is assumed that interarrival times and service times for the TLOF are exponentially distributed, the delay at the TLOF can be obtained by applying simple queuing models. From the delay curves (e.g. Figures 7 or 9 in Section 4), we can estimate the practical TLOF capacity. These delay curves can be obtained from queuing models or simulation models.
2.2.6 Other Factors

These include aircraft performance and operating profiles, pilot technique, and proximity to other air traffic. The aircraft performance and operating profiles affect the vertiport capacity. The aircraft maneuverability may affect the gate occupancy time, TLOF occupancy time and ground speed. The TLOF capacity may be affected by pilot skill, since expert pilots may control their aircraft more precisely to arrive at the right time. Also, the vertiport capacity is affected by proximity to other air traffic.

The factors discussed above affect various vertiport components. Some affect the TLOF capacity or terminal airspace capacity and others affect the gate capacity. The vertiport airside may be treated as four subsystems in series: terminal airspace, TLOFs, taxiways, and gates, as shown in Figure 3. The vertiport capacity is then determined by the smallest capacity among these four subsystems.
Figure 3. Components of Vertiport Airside
3. METHODOLOGY FOR ESTIMATING VERTIPORT CAPACITY AND DELAYS

As previously mentioned, the vertiport airside may be treated as four subsystems in series: terminal airspace, TLOFs (i.e., rollways), taxiways and gates. The absolute capacities of vertiports and their subsystems are estimated with deterministic analytic models, based on feasible intervals between arrivals or occupancy times. Probabilistic queuing models are used to estimate delays and the "practical" capacity at which average delays have a certain value (e.g., 4 minutes). A combination of deterministic queuing models and simulation models is used to determine delays during peak periods in which traffic exceeds capacity.

3.1 Deterministic Analytic Method

Analytical models are first developed for estimating the capacity of each of four subsystems.

3.1.1 Terminal Airspace Capacity
The term capacity can be defined as the maximum number of operations (arrivals and departures) per unit of time, (typically in operations per hour) that can be accommodated under specified operating conditions. The terminal airspace capacity is the maximum number of CTR operations that can be accommodated during a specific period. Airspace capacity is determined mainly by the minimum separation distance in the air (approach path) and aircraft speed. This minimum separation distance should be based on safety considerations (including wake vortices behind CTRs) and the performance of the ATC surveillance systems. However, the minimum separation distance for CTRs has not yet been officially determined by the FAA.

We can estimate the terminal airspace capacity for one approach path as follows:

\[ c_a = \frac{1}{E[h_a]} \]  \hspace{1cm} (3.1)

where \( c_a \) = airspace capacity for one approach path (flights/hour)
\[ E[h_a] = \text{the weighted minimum allowable headway between successive aircraft (hours/operation)} \]
\[ = \sum_k \sum_{k'} p_{k'k''} h_{k'k''} \]
\[ p_{k'k''} = \text{fraction of pairs in approach path including CTR types } k' \text{ and } k'' \]
\[ h_{k'k''} = \text{minimum allowable headway between a leading CTR of type } k' \text{ and a trailing CTR of type } k'' \text{ (minutes)} \]

For example, if the weighted minimum allowable headway is 2.5 minutes (=150 seconds), the hourly terminal airspace capacity for one approach path can be estimated as:

\[ c_a = I/E[h_a] = (1/2.5)\times60 = 24 \text{ flights/hour} \tag{3.1.1} \]

This capacity is affected by visibility. Poor visibility decreases the terminal airspace capacity. The terminal airspace capacity is typically lower under IFR conditions than under VFR conditions. With advanced IFR equipment, some mitigation of IFR capacity constraints is possible.

If the number of approach paths is more than one (e.g., \( n \) approach paths) and there is no conflict among those paths, the total airspace capacity \( C_a \) is \( n \) times the capacity of a single approach path:

\[ C_a = nc_a \tag{3.1.2} \]

where \( C_a = \text{total airspace capacity (flights/hour)} \)
\( c_a = \text{airspace capacity for one approach path (flights/hour)} \)
\( n = \text{number of approach path} \)

However, considering the size of a vertiport, it is not certain that it would be possible to develop more than one non-conflicting approach path. This is particularly the case for IFR operations.
3.1.2 TLOF Capacity

The TLOF is a rollway used for CTR aircraft. FAA provides design standards for a TLOF in AC 150/5390-3 (27). The TLOF capacity is mainly determined by TLOF ground occupancy time and in-trail separation in the approach path. The TLOF ground occupancy time is affected by landing speed, TLOF length, exit location, and clearance time for landing (i.e. the time required to turn a CTR clear of the TLOF to allow safe operation of the next CTR). We can estimate the ground occupancy time from the speed-distance-time-acceleration equations (Equations (3.2.1) (3.2.2)), and the following information:

a) Landing speed and turn-off speed at TLOF.
b) TLOF characteristics and dimensions.
c) Clearance time for safe operation of following CTR.

Given the TLOF occupancy time and in-trail separation in approach path, the TLOF capacity can be calculated from the following equation:

\[ C_{\text{TLOF}} = \text{Min} \{ C_{gr}, C_{app} \} \]

\[ C_{\text{TLOF}} = \text{Min} \left\{ \frac{1}{E[TLOF\ occupany\ time]}, \frac{1}{E[h_{app}]} \right\} \]  (3.2)

where \( C_{\text{TLOF}} = \) TLOF capacity (flights/hour)
\( C_{gr} = \) TLOF ground capacity
\( C_{app} = \) TLOF approach path capacity
\( E(X) = \) expected value of \( X \)

The following example illustrates the TLOF capacity estimation. For landings, assume that landing speed=40 knots, exit turn-off speed=10 knots, distance from landing point to exit point=500 ft, and clearance time (for an aircraft to turn out clear of the TLOF) is 5 seconds. Then, the TLOF occupancy time can be obtained from a kinematic equation as follows:
\[2as = v_o^2 - v^2\] (3.2.1)

where
- \(a\) = acceleration (feet/sec\(^2\))
- \(s\) = distance from landing point to exit point (feet)
- \(v_o\) = initial speed (feet/second)
- \(v\) = final speed (feet/second)

We substitute the variable values into Equation (3.2.1):

\[2a(500\text{ft})= (10\times6076/3600)^2 - (40\times6076/3600)^2\]

Solving for the acceleration we obtain:

\[a = -4.27 \text{ ft/sec}^2\]

When the acceleration and the distance between the landing point and the exit point are given, Equation (3.2.2) can provide the CTR’s rolling time on the TLOF:

\[s = v_o t + 0.5at^2\] (3.2.2)

where \(t\) = CTR’s rolling time (seconds)

Solving the quadratic equation (Equation (3.2.2)) for the CTR’s rolling time we obtain:

\[t = 11.8 \text{ seconds} \approx 12 \text{ seconds}\]

Therefore, the TLOF occupancy time is obtained as follows:

\[
\text{TLOF occupancy time} = \text{CTR’s rolling time} + \text{TLOF clearance time}.
\]

\[
= 12 + 5 = 17 \text{ seconds}
\]

If we assume that the obtained value is the mean occupancy time, then the TLOF ground capacity is estimated as follows:

\[C_g = 1/E[\text{TLOF occupancy time}] = (1/17)\times3600 \approx 212 \text{ operations/hour}\]
Next, if we assume that the mean headway for the approach path is 2 minutes, the TLOF approach path capacity can be estimated as follows:

\[
C_{\text{app}} = \frac{1}{E[h_{\text{app}}]} = \frac{1}{2} \times 60 = 30 \text{ operations/hour}
\]

As discussed previously, the TLOF capacity is the minimum value of the above two capacities:

\[
C_{\text{TLOF}} = \text{Min} \{212 \text{ operations/hour}, 30 \text{ operations/hour}\}
\]

\[
= 30 \text{ operations/hour}
\]

The TLOF ground capacity far exceeds the TLOF approach path capacity. If the TLOF is used only for arrivals or only for departures, the TLOF utilization is very low since the arrival rate at the TLOF equals the approach path service rate. However, if we use the TLOF for alternating operations (take-offs between landings), we can increase the TLOF utilization. Very interestingly, the much lower TLOF occupancy time allows the possibility of inserting a take-off between any two landings without affecting landing intervals. Hence, the number of operations per TLOF might be doubled during periods with similar numbers of take-offs and landings by alternating such operations on one TLOF.

TLOF capacity is decreased by poor weather conditions (low visibility, low ceiling, etc.). The procedures for landings and take-offs of CTRs can affect the TLOF capacity. Unfortunately, these procedures have not yet been standardized. The TLOF capacity can be estimated more accurately when detailed information about procedures under different weather conditions becomes available.

3.1.3 Taxiway Capacity

A taxiway or hover taxiway is defined as a paved link connecting a TLOF to a gate used for passenger service, to a maintenance or refueling locations and to aircraft parking positions. In general, for a conventional airport, the capacity of a taxiway system exceeds the capacity of runways and gates. Taxiways may also be provided in parallel with runways to avoid taxiing on
runways and thus reduce runway occupancy time. In this study it is assumed that taxiway capacity will not be the weak link in the overall vertiport capacity and, hence, we do not analyze it.

3.1.4 Gate Capacity

The gate capacity can be defined as the maximum number of flights that a given number of gates can handle during a specified interval. The gate capacity is the inverse of the weighted average gate occupancy time for all CTRs being served. The gate capacity can be obtained from the following equation:

\[
C_g = \sum_{k} \frac{N_k \cdot U_k}{E[T_k]}
\]

where 
- \(C_g\) = gate capacity (CTR operations/hour)
- \(N_k\) = number of gates that can accommodate CTRs of type \(k\)
- \(U_k\) = gate utilization, or fraction of time that gates are used by CTR of type \(k\)
- \(E[T_k]\) = expected value of gate occupancy time by CTRs of type \(k\).

The gate occupancy time \(T_k\) can be obtained from Equation (2.3) by using service times specific for CTR’s of type \(k\). The passenger loading and unloading time and baggage loading unloading time are functions of the number of passengers.

Gate capacity will be dramatically reduced if CTR’s cannot operate independently at adjacent gates. For example, the use of "jetways" or some similar structure can shield passengers from the rotorwash of a nearby CTR during loading and unloading. In the absence of "jetways", two choices are available. One possibility is to design the vertiport with large separations between gates to allow passenger loading and unloading independently of CTR operations at adjacent gates. A second possibility is to accept that operations at adjacent gates will not be independent and that gate capacity will thus decrease significantly. At vertiports where both
capacity and land costs are important issues, the use of "jetways" or some similar structure may be the most cost effective solution. As a second example, vertiport configuration can also have a significant effect on the interdependence of operations at different gates. Resulting capacity estimates will be discussed in Section 4.1.

3.2 Probabilistic Queuing Analysis

Queuing methods are used in this section to analyze the performance of vertiports with various TLOFs, gates, arrival/departure patterns, and operational procedures. These models are especially useful for estimating delays at vertiports. Queuing models are proposed for systems with a single server or multiple servers in parallel (e.g. gates or TLOFs that operate independently and simultaneously with similar units).

3.2.1 Single Server Queuing Model

A vertiport can be modelled using queuing theory. Several applications of queuing theory have been made for conventional airports (2, 3). Such queuing theory can also be applied for a vertiport. As mentioned earlier, a vertiport may be treated as four subsystems in series. Each subsystem can have a single server (number of TLOFs = 1 or number of gates = 1) or multiple servers in parallel (number of TLOFs or number of gates > 1). Customers can be aircraft and servers can be the approach paths, TLOFs, taxiways and gates. The gate or TLOF occupancy time are treated as the service times and headways are the interarrival times. These service times or interarrival times may have different probability distributions, such as the exponential distribution, normal distribution, uniform distribution, etc. Unfortunately, not enough mathematical queuing models have been developed to date since derivations can be quite difficult. The most common probabilistic queuing model is the M/M/1 model (see Kendall notation in Appendix B). It is based on the following assumptions:

a) The service times are exponentially distributed.

b) The interarrival times are exponentially distributed.

c) The service discipline is First-in First-out (FIFO).

d) The process is in a steady state condition.
c) The utilization factor ($\rho = \frac{\lambda}{\mu}$, ratio of arrival rate and service rate) is less than one. If $\rho > 1$, the queue length will tend to approach infinity.

Based on these assumptions, the mathematical queuing model can be derived (14). From this model, delays at each server can be estimated with one of the following equations:

$$W = \frac{1}{\mu (1-\rho)} = \left( \frac{1}{1-\rho} \right) t_s$$  \hspace{1cm} (3.5)

where

- $W$ = the average waiting time in the system (hours/CTR)
- $\mu$ = average service rate (operations/hour)
- $\lambda$ = average arrival rate (operations/hour)
- $\rho$ = utilization factor = $\frac{\lambda}{\mu}$ (no units)
- $t_s = \frac{1}{\mu}$ = average service time (hours/operation)

For realistic applications, the assumed interarrival time distribution is reasonable because civil airline flights arrive fairly randomly even when scheduled. However, the service time distribution at the TLOF is less likely to be exponential. The following M/G/1 model (see Kendall notation in Section 1.5) assumes a general distribution of service times (14):

$$W = \frac{1}{\mu} + \frac{\lambda^2 \sigma^2 + \rho^2}{2 \lambda (1-\rho)} = \left(1 + \frac{\rho (\sigma^2 + 1)}{2 (1-\rho)} \right) t_s$$  \hspace{1cm} (3.6)

where

- $W$ = the average waiting time in the system (hours/user)
- $\mu$ = average service rate (operations/hour)
- $\lambda = \mu / t_s$ = average arrival rate (arrivals/hour)
- $\rho$ = the utilization factor = $\frac{\lambda}{\mu}$ (no units)
- $\sigma$ = standard deviation of service time (hours/user)
- $t_s = \frac{1}{\mu}$ = mean service time (hours/user)
\[ c = \sigma/t = \text{coefficient of variation (no units)} \]

This model may be used with any empirically observed distributions whose means and variances can be computed.

For deterministic service times, we can obtain the M/D/1 model (see Kendall notation in Section 1.5) by substituting \( \sigma^2 = 0 \) and \( \rho = \lambda/\mu \) into first equation of Equation (3.6).

\[
W = \frac{2 - \rho}{2 \mu (1 - \rho)} = \left( \frac{2 - \rho}{2 (1 - \rho)} \right) \tau_s
\]

(3.7)

For a more general form, we relax assumptions a and b, (i.e., exponential interarrival and service times) in the previous conditions for the M/M/1 model. This G/G/1 queuing model was developed by Dai (9) from Marshall’s formula (20):

\[
W = \frac{\sigma_A^2 + 2 \sigma_s^2 - \sigma_D^2}{2 \tau_A (1 - \rho)} + \frac{1}{\mu}
\]

(3.8)

where \( W \) = the average waiting time in the system
\( \sigma_A^2 \) = variance of interarrival times
\( \sigma_s^2 \) = variance of service times
\( \sigma_D^2 \) = variance of interdeparture times
\( \tau_A \) = average interarrival time
\( \rho \) = the utilization factor

By substituting Equations (3.8.1) - (3.8.5) into Equation (3.8), we obtain Equation (3.8.6):

\[
\sigma_s = c_s \tau_s
\]

(3.8.1)
If we know the standard deviations of service times, interarrival times and interdeparture times, we can estimate delay in the single queuing system for any general distribution of service times. For example, if we assume the following information for a TLOF:

- mean service time ($t_s$) = 20 seconds
- utilization factor ($\rho$) = 0.8
- $\sigma_s = 10$ seconds, $\sigma_A = 15$ seconds, $\sigma_D = 13$ seconds

Then, the waiting time in the system can be estimated as follows:

First, find $c_s$, $c_A$, $c_D$ from Equations (3.8.1)-(3.8.5):

$$c_s = 0.5, \quad c_A = 0.6, \quad c_D = 0.52$$

Second, calculate the waiting time with Equation (3.8.6)

$$W = \frac{(0.6^2 - 0.52^2 + 2*0.5^2*0.8^2)/(2*0.8*0.2) + 1)}{2*0.8*0.2} * 20 = 56.85 \text{ seconds}$$
3.2.2 Multiple-Servers Queuing Model

Multiple servers queuing models are more difficult to treat mathematically than models with only one server. One of the simpler multiple servers models is the $M/M/k$ model (see Kendall notation in Section 1.5). It assumes that users arrive with exponential interarrival times at an average rate of $\lambda$ users per unit time and receive exponential service times in $k$ parallel servers at an average rate of $\mu$ users per server per unit time. These modeled servers can be TLOFs or gates. The following assumptions are made for the $M/M/k$ queuing model:

a) Both service time and interarrival time are exponentially distributed.

b) If all TLOFs (or gates) are busy, the aircraft joins the single queue from which all servers are fed.

c) The aircraft goes immediately to the free TLOF (or gate) when one TLOF (or gate) is free and all others are busy.

d) The aircraft randomly selects any of free TLOF (or gate) when there are two or more free TLOFs (or gates).

Based on the assumptions, we can estimate the average waiting time in the system with the following Equation (14):

$$W = \frac{p (kp)^k}{k! (1-p)^2} \frac{1}{\lambda} P_0 + \frac{1}{\mu} \quad (3.9)$$

where

$$P_0 = \sum_{r=0}^{k-1} \frac{1}{r!} \frac{(kp)^r}{k!} \frac{1}{1-p} \quad (3.10)$$

and

$W =$ the average waiting time in the system

$p_0 =$ the probability of having no user in the system
\[ k = \text{number of servers} \]
\[ \rho = \text{utilization factor} = \frac{\lambda}{\mu} \]

Table 2 summarizes the equations for the M/M/k queuing model when \( k \) varies from 1 to 4 based on Equations (3.9) and (3.10):

\[ W = \frac{1}{\mu} + \sum_{i=1}^{\infty} e^{-\lambda t} \left[ \sum_{j=1}^{k} \frac{(i\lambda)^j}{j!} - \frac{k}{\lambda} \sum_{j=k+1}^{\infty} \frac{(i\lambda)^j}{j!} \right] \]

where
\( W = \text{the average waiting time in the system (hours/CTR)} \)
\( k = \text{number of servers (TLOFs or gates)} \)
\( \lambda = \text{arrival rate in unit time (arrivals/hour)} \)

If the service consists of the same routine task for all users, it tends to have little variation. When we assume that a queuing system has a Poisson input process with an average arrival rate \( \lambda \) and that all service times have some deterministic value, the M/D/k model can provide reasonable estimates for the average waiting time in the system (18):

\[ W = \frac{1}{\mu} + \sum_{i=1}^{\infty} e^{-\lambda t} \left[ \sum_{j=1}^{k} \frac{(i\lambda)^j}{j!} - \frac{k}{\lambda} \sum_{j=k+1}^{\infty} \frac{(i\lambda)^j}{j!} \right] \]
These queuing models can be used to estimate the delay at each system (terminal airspace, TLOF and gate). After estimating the delay as a function of traffic volume, we can plot the delay-volume curves. From the delay-volume curves, we can find a "practical" TLOF or gate capacity, defined as the volume at which average delay exceeds 4 minutes, which was previously recommended in FAA Advisory Circular AC 150/5060-1A (2).

3.3 Simulation Method

Simulation is a very powerful method, widely used in airport planning and design for the analysis and study of complex systems. The previous sections presented deterministic and probabilistic queuing methods for analyzing capacities and delays. Those approaches are based on simplifying assumptions about the interarrival and service time distributions. However, real world problems cannot always be represented adequately with such assumptions and model forms. Simulation techniques provide ways to analyze more complex systems than previous approaches. We have developed the simulation models to deal with transient demand conditions (e.g. effects of peak period duration and various volume/capacity level).

This simulation model is an event-scanning model where the system status (e.g. TLOF or gate) is updated by events. The simulation model is composed of six subroutines, i.e. Initialization, Scheduler (timing subroutine), Arrival, Departure, Update statistics, and Random number generator (Figure 4). The scheduler, which provides the control for the simulation period, is the heart of the simulation model, and it invokes all other operational routines necessary to process the simulation. This model can handle any distributions for CTR interarrival times and service times. Readers who wish to learn more about simulation methods may consult a basic textbook such as Law and Kelton (17).

Our basic approach is to first develop a deterministic analytic model for delay that is expressed in a simple relation (Equations (3.12) or (3.13)), then simulate the probabilistic system behavior, compute a "stochastic adjustment factor" which is the ratio of simulated delay to deterministic delay, and multiply this stochastic adjustment factor by the previously obtained deterministic delay. We have also developed the simulation models for multiple servers and for various interarrival and service time distributions.
Figure 4. Flow Chart of Simulation Model
3.3.1 Deterministic Delay due to Peak Period Overflow

When we assume that interarrival time and service time are deterministically distributed (e.g., according to a uniform continuous distribution), the total delays can be obtained by analytical methods. Figure 5 shows the diagram of total delay due to peak period overflows. It simplifies the queueing process by assuming it is continuous as well as deterministic. Total delays are affected by peak period duration, peak-hour volume and off-peak hour volume. Given the capacity \( c \), peak period volume \( v_p \), off-peak volume \( v_o \) and peak period duration \( d \), the total delay due to peak period overflows can be described by the triangular area in Figure 5. It is calculated by multiplying the queue length \( L \) by peak period duration plus queue dissipation time. The deterministic delay due to peak overflows can be formulated as follows:

\[
D_c = \frac{1}{2} (L)(d+s) = \frac{1}{2} (d(v_p - c))(d+\frac{d(v_p - c)}{c-v_o})
\]

\[
D_c = \frac{cd^2}{2} (\rho_p - 1) (1+\frac{(\rho_p - 1)}{(1-\rho_o)})
\]

where

- \( D_c \) = total deterministic delay due to peak period overflows (flight-hours)
- \( d \) = peak period duration (hours)
- \( v_p \) = peak period volume or peak period arrival rate (flights/hour)
- \( v_o \) = off-peak period volume or arrival rate (flights/hour)
- \( c \) = capacity or service rate (flights/hour)
- \( \rho_o \) = utilization factor during off-peak period \((= v_o / c)\)
- \( \rho_p \) = utilization factor during peak period \((= v_p / c)\)
Arrival Rate (flights/hour)

\( v_p \)

\( c \)

\( v_o \)

Time (hours)

Flights in Queue

\( L = d (v_p - c) \)

\( (v_p - c) \)

\( (v_o - c) \)

\( l \)

\( d \)

\( s \)

Time (hours)

\( c \) = capacity (flights/hour)

\( d \) = peak period (hours)

\( L \) = queue length (flights)

\( s \) = queue dissipation time (hours)

\( v_p \) = peak period volume (flights/hour)

\( v_o \) = off-peak period volume (flights/hour)

Figure 5. Total Deterministic Delay Due to Peak Period Overflows
3.3.2 **Stochastic Delay Due to Peak Period Overflows**

Simulation models have been developed to estimate delays due to peak period traffic that exceeds capacity. Various statistical distributions are used for inter-arrival times and service times. In order to estimate the delay due to a peak period, simulation runs with and without peak traffic volumes are required. The stochastic delay is the difference between simulated delays with and without peak traffic.

The stochastic adjustment factor $F_s$ is the ratio between the deterministic delay $D_c$ which is computed with Equations (3.12) or (3.13) and the stochastic delay $D_s$ obtained from simulation results:

$$F_s = \frac{D_s}{D_c} \quad (3.14)$$

After determining the stochastic adjustment factor $F_s$ for a range of typical cases, we dispense with further simulation runs, and estimate the stochastic delay quite precisely by multiplying $F_s$ with the deterministic delay $D_c$ given by Equation (3.12). The stochastic adjustment factor $F_s$ may also be formulated as a functional form and estimated using regression analysis.

To reduce the variance of simulation results, we have obtained sufficiently long simulation runs and sufficient numbers of replications. Also, to insure that the simulation reaches stability before results are collected and compared, each simulation run discards the results obtained during its initial stabilization period.

3.3.3 **Average Delay of Multiple Parallel Server System**

It is possible to use queuing theory to determine the delay for multiple parallel servers when both interarrival and service time distributions are exponential (i.e., an $M/M/k$ queuing system). For multiple server systems with general interarrival or service time distributions, queuing theory provides only approximate results. This study has developed a simulation model for systems with multiple parallel servers (e.g., for multiple TLOFs or multiple gates). This
model can estimate the delays with any distribution of service time or interarrival time.

To check the logic of simulation model, its results were compared to the theoretical results (Table 2 and Figure 9) from queuing theory. The comparison showed a close match between theory and simulation, as shown in Table 3.

**Table 3.** Comparison of Average Waiting Time in the Queue Between Theory and Simulation Results (M/M/k)

<table>
<thead>
<tr>
<th>V/C</th>
<th>Theoretical Results</th>
<th>Simulation Results</th>
<th>Simulation Results†</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k = 1</td>
<td>k = 2</td>
<td>k = 3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.08</td>
<td>0.02</td>
</tr>
<tr>
<td>0.4</td>
<td>1.33</td>
<td>0.38</td>
<td>0.16</td>
</tr>
<tr>
<td>0.6</td>
<td>3</td>
<td>1.12</td>
<td>0.59</td>
</tr>
<tr>
<td>0.8</td>
<td>8</td>
<td>3.56</td>
<td>2.16</td>
</tr>
<tr>
<td>0.9</td>
<td>18</td>
<td>8.52</td>
<td>5.45</td>
</tr>
</tbody>
</table>

† These are average waiting times in the queue. The results were obtained from Table 2 (as plotted in Figure 9) by subtracting the mean service time (assumed to be 2 minutes) from the mean time in the system.

‡ These are resulted from simulation model (same as Table 10)
4. CAPACITY AND DELAY DETERMINATION

4.1 System Capacity Estimation

We present the capacity estimation results obtained with the models developed in Section 3 for each subsystem in Figure 3.

4.1.1 Terminal Airspace Capacity

In order to determine the terminal airspace capacity, we assume here that CTRs come in only one size. Based on survey results from the pilots and air traffic controllers, the assumed minimum in-trail separation distance varies from 3 nautical miles to 6 nautical miles. These distances depend on pilots’ perceptions and responses. Based on this information and Equation (3.1), we can estimate the terminal airspace capacity.

If we know the weighted minimum separation distance (optimistically 3 nautical miles and pessimistically 6 nautical miles) and the approach speed (120 knots from pilot interviewed), we can translate the minimum separation distance into time headways using an approach speed of 120 knots. The headways are:

\[
\begin{align*}
3 \text{ nautical miles} \times \left(\frac{3600 \text{ sec.}}{120 \text{ knots}}\right) &= 90 \text{ seconds (optimistic)} \\
6 \text{ nautical miles} \times \left(\frac{3600 \text{ sec.}}{120 \text{ knots}}\right) &= 180 \text{ seconds (pessimistic)}
\end{align*}
\]

Based on Equation (3.1), capacities are determined as follows:

\[
\begin{align*}
\text{Optimistic terminal airspace capacity} &= \frac{3600}{90} = 40 \text{ flights/hour} \\
\text{Pessimistic terminal airspace capacity} &= \frac{3600}{180} = 20 \text{ flights/hour}
\end{align*}
\]

These capacities are based on the minimum separation distance and approach speed obtained from our surveys.

Next, we can also estimate the terminal airspace capacity based on the approach path profile developed by the Boeing Defense Space Group (see Figure 1). From the first and fourth
columns in Table 1, the headways for minimum separation (i.e. 3 and 6 nautical miles) can be obtained as follows:

Headway for 3 nm = 2.1 min + (0.4 min)(3 nm - 2.32 nm)/(3.05 nm - 2.32 nm) = 2.47 minutes
Headway for 6 nm = 3.5 min + (0.7 min)(6 nm - 5.55 nm)/(8.01 nm - 5.55 nm) = 3.63 minutes

Based on these headways, the terminal airspace capacity is determined by Equation (3.1):

Optimistic terminal airspace capacity = (60/2.47) ≈ 24.3 ≈ 24 flights/hour
Pessimistic terminal airspace capacity = (60/3.63) ≈ 16.5 ≈ 16 flights/hour

Two different headways were used to estimate the airspace capacity. One was obtained from the interviewed test pilot and the other was obtained from Boeing Defense Space Group. The later one is recommended for determining airspace capacity since it is based on a complete approach path profile. With more accurate information about the minimum separation and speed profile, the capacity can be estimated more precisely.

4.1.2 TLOF Capacity

The TLOF capacity is determined as a minimum value of two capacities, i.e. TLOF ground capacity and TLOF approach path capacity, (Equation (3.2), $C_{TLOF} = \text{Min} \{ C_g, C_{app} \}$) since the TLOF capacity is affected by terminal airspace capacity. The TLOF ground capacity is determined by the landing speed, TLOF clearance time for landing, average distance between landing point and exit point, and exit turn-off speed. Unfortunately, such information was not found in the published sources. Based on our survey of pilots and air traffic controllers, the minimum TLOF occupancy time varies from 10 seconds to 20 seconds. However, we conservatively assume that minimum TLOF occupancy time varies from 15 to 30 seconds. With the minimum TLOF occupancy time, the TLOF ground capacity can be computed as follows:

Optimistic TLOF ground capacity = (3600/15) = 240 operations/hour
Pessimistic TLOF ground capacity = (3600/30) = 120 operations/hour
Since the TLOF capacity is strongly dependent on the terminal airspace capacity, the approach path capacity can be obtained using the headways that used in section 4.1.1.

Optimistic TLOF approach path capacity = \((60/2.47) = 24.3 \approx 24 \text{ flights/hour}\)

Pessimistic TLOF approach path capacity = \((60/3.63) = 16.5 \approx 16 \text{ flights/hour}\)

The TLOF capacity is the minimum of the capacities based on ground occupancy times and approach headways, as formulated in Equation (3.2). Therefore, the TLOF capacity is same as the terminal airspace capacity.

Optimistic TLOF capacity = 24 flights/hour
Pessimistic TLOF capacity = 16 flights/hour

Compared to the TLOF approach path capacity, the TLOF ground capacity far exceeds the capacity of TLOF approach path, as discussed in Section 3.1.2. Based on the surveyed data, the interarrival time of landings or departures (determined by approach path) ranges from 90 seconds to 180 seconds and the occupancy time of TLOF varies from 15 to 30 seconds. Since each approach path serves one TLOF, the interarrival times at a TLOF are limited by approach path in-trail separation to no less than 90 seconds, which implies a capacity of 40 landings per TLOF per hour.

However, we can increase the TLOF operation by inserting a take-off between any two landings without affecting landing intervals. Hence, the number of operations per TLOF might be doubled during periods with similar numbers of take-offs and landings by alternating such operations on one TLOF.

4.1.3 Gate Capacity
The gate capacity is mainly determined by gate occupancy time. The gate occupancy time can be obtained from Equation (2.3). We can estimate the gate capacity from Equation (3.3) based on the following assumptions:

a) CTR's come in only one size \((k=1)\).
b) Gates of only one size can accommodate all CTRs.
c) CTR arrivals generate two operations (landing and take-off).
d) CTR operations at adjacent gates can be conducted independently.

Table 3. Gate Capacity (CTRs/hour)

<table>
<thead>
<tr>
<th>Utilization factor</th>
<th>Number of gates</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>†U=0.5</td>
<td>†T = 30 min</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>25 min</td>
<td>1.2</td>
<td>2.4</td>
<td>3.6</td>
<td>4.8</td>
<td>6</td>
<td>7.2</td>
</tr>
<tr>
<td></td>
<td>20 min</td>
<td>1.5</td>
<td>3</td>
<td>4.5</td>
<td>6</td>
<td>7.5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>15 min</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>10 min</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>†U=0.8</td>
<td>30 min</td>
<td>1.6</td>
<td>3.2</td>
<td>4.8</td>
<td>6.4</td>
<td>8</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>25 min</td>
<td>1.92</td>
<td>3.84</td>
<td>5.78</td>
<td>7.68</td>
<td>9.6</td>
<td>11.52</td>
</tr>
<tr>
<td></td>
<td>20 min</td>
<td>2.4</td>
<td>4.8</td>
<td>7.2</td>
<td>9.6</td>
<td>12</td>
<td>14.4</td>
</tr>
<tr>
<td></td>
<td>15 min</td>
<td>3.2</td>
<td>6.4</td>
<td>9.6</td>
<td>12.8</td>
<td>16</td>
<td>19.2</td>
</tr>
<tr>
<td></td>
<td>10 min</td>
<td>4.8</td>
<td>9.6</td>
<td>14.4</td>
<td>19.2</td>
<td>24</td>
<td>28.8</td>
</tr>
</tbody>
</table>

†U = gate utilization rate
‡T = gate occupancy time

Table 3 shows the gate capacity with different parameter values. The gate capacity increases as the gate utilization increases and the gate occupancy time decreases. Figure 6 shows the gate capacity for a given number of gates and gate occupancy time when the utilization factor is 0.8.
Figure 6. Effect of Gate Numbers and Occupancy Time on Gate Capacity (0.8 Utilization)
4.1.4 Relations Among Airspace Capacity, TLOF Capacity and Gate Capacity

The vertiport capacity is determined by the minimum capacity among its subsystems (terminal airspace, TLOFs, taxiways and gates). Since one independent approach path serves one TLOF, we can consider them as a pair and find which one has the limiting capacity. Based on surveyed in-trail separation in airspace and TLOF occupancy time, the TLOF ground capacity is up to six times larger than the airspace capacity. If the vertiport has five gates with 80% gate utilization and occupancy times of 30 minutes (see Table 4), then the gate capacity is 8 CTR operations per hour. Eight gate operations generate 16 flights (8 landings and 8 departures). Comparing the two capacities, the gate capacity is critical one, i.e. 16-24 flights/hour for terminal airspace capacity, 16-24 flights/hour for TLOF capacity and 16 flights/hour for gate capacity (6 gates with 0.5 utilization and 20 minutes gate occupancy time). Thus, the vertiport capacity is determined to be 16 flights/hour. If we want to increase the vertiport capacity, the gate capacity should be expanded by adding new gates or reducing the gate occupancy time. The occupancy time of one gate can be reduced by improving the critical time in Equation (2.3).

Based on the given capacity for airspace and TLOF capacity, we can determine the required numbers of gates. These gates should handle at least 20 flights per hour (10 arrivals and 10 departures) for the pessimistic view and 40 flights per hour (20 arrivals and 20 departures) for optimistic view. The required number of gates can be obtained based on following equation:

\[ G = \frac{C \times T}{U} \]  

(4.1)

where 

- \( G \) = the required number of gates
- \( C \) = maximum volume of aircraft arrivals (aircraft/hour)
- \( T \) = gate occupancy time (hours/aircraft)
- \( U \) = gate utilization factor (no units)

We can compute the required number of gates which can accommodate all arrivals and departures for an hour and we add a reserve factor as suggested by De Neufville (10). Table 5 shows the required number of gates for given utilization factors, gate occupancy times and
volumes. The adjusted number of required gates (G') includes a reserve factor to compensate for schedule deviations and is rounded up to the nearest integer.

From the table, the required number of gates, assuming with 50% gate utilization and 20 minutes of gate occupancy time, is 25 gates for 30 arrivals per hour and 10 gates for 10 arrivals per hour.

Table 4. Required Number of Gates (G')

<table>
<thead>
<tr>
<th>Occupancy time</th>
<th>30 minutes</th>
<th>20 minutes</th>
<th>15 minutes</th>
<th>10 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>U=0.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 arrivals / hour</td>
<td>10</td>
<td>14</td>
<td>6.7</td>
<td>10</td>
</tr>
<tr>
<td>30 arrivals / hour</td>
<td>30</td>
<td>36</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>U=0.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 arrivals / hour</td>
<td>6.25</td>
<td>9</td>
<td>4.17</td>
<td>7</td>
</tr>
<tr>
<td>30 arrivals / hour</td>
<td>18.75</td>
<td>24</td>
<td>12.5</td>
<td>17</td>
</tr>
</tbody>
</table>

† required number of gates with reserve factor (G' = \lceil N + \sqrt{N} \rceil), \lceil \rceil means rounded up value.

4.2 Estimation of Delay

Queuing models were proposed for estimating delays at vertiports in Section 3.2. These models were developed for systems with a single server and with multiple parallel servers.
4.2.1 Delay at a Single Server Queuing System

The queuing models are based on several assumptions already discussed. Such queuing systems can represent one TLOF or a gate at a vertiport. The M/M/1 queuing model assumes that service times and interarrival times are exponentially distributed. The M/D/1 model has exponentially distributed interarrival times and deterministic service times. The M/G/1 model has exponentially distributed interarrival times and generally distributed service times.

In order to estimate the delay for single TLOF, we use equations (3.5), (3.6) and (3.7) developed in Section 3.2. We compute the average waiting time in the system (delay time plus mean service time $t$). Then we plot the average waiting time as a function of the utilization factor $\rho (\lambda/\mu)$. Figure 7 shows the waiting time in the system as a function of the utilization factor.

From Figure 7, we can estimate the delay at a TLOF or gate for a given utilization factor. For example, if the TLOF utilization factor is 0.8 and if service times and interarrival times are exponentially distributed, the average waiting time is $5t (=5$ times the mean service time). We can also find the "practical" capacity which is defined as the volume having four minutes of average delay (2). For example, in Figure 7, if we assume that the mean service time ($t_s$) is one minute, we can estimate the practical capacity in a M/M/1, M/D/1, or M/G/1 system. The horizontal dotted line in Figure 7 indicates the average delay of four minutes. Thus, $5t (=t+4t) = \text{mean service time} + \text{delay}$. For a M/M/1 system (e.g. a TLOF or gate), the $\rho$ value projected to the x-axis is 0.8. The practical capacity can be estimated as follows:

\[
0.8 = \rho = \frac{\lambda}{\mu}, \\
\mu = 1/(1 \text{ minute}) = 60 \text{ flights/hour} \quad \therefore \lambda = 48 \text{ flights/hour} = \text{practical capacity}.
\]

Similarly, we can estimate the practical capacity for other queuing systems. It is $52.8 \approx 53 \text{ flights/hour}$ for an M/D/1 system and $44.4 \approx 44 \text{ flights/hour}$ for an M/G/1 system whose standard deviation of service times ($\sigma_s$) = $1.2t_s$. Figures 7 and 9 can also be used if other delay values are preferred for defining practical capacity.
Figure 7. Average Waiting Time for a Single Server Queuing System.
4.2.2 Delay at a Queuing System with Multiple Parallel Servers

We can estimate the average waiting time in such a system using the equations in Table 2. Figure 8 shows the value of $P_0$ (probability that all servers are unused) when $k$ (i.e., number of servers (gates)) varies from 1 to 4. Figure 9 shows the average waiting time in the system. From this figure, the average waiting time can be estimated when the utilization factor is given. For example, assume that gate utilization ($\rho = \lambda/\mu$) is 0.85. Then, we can estimate the delay of system in each case (i.e. for each number of gates). The average waiting time is 6.5 times the mean service time ($= 7.5t-t$) for one gate and 1.5 times the mean service time ($=2.5t-t$) for four gates.
Figure 8. Idle System Probability ($P_0$)
Figure 9. Average Waiting Time for Queuing System with Multiple Parallel Servers
4.3 Simulation Results

The simulation models have been developed to estimate delays due to excess volumes during limited periods, as discussed in Section 3.3.1 and 3.3.2. The simulation models were also used to estimate delays in multiple parallel server systems (i.e., multiple TLOFs or multiple gates) for various interarrival and service time distributions in Section 3.3.3. This section discusses the results of those analyses.

4.3.1 Total Delay for Deterministic and Stochastic Traffic Flow

For continuous deterministic flow, Equation (3.12) or (3.13) in Section 3.3.1 were developed to estimate delays due to peak overflows during peak periods. It provides the total deterministic delay for various peak period durations, peak traffic volumes, off-peak volumes, and capacities. With Equation (3.13), we obtained the deterministic delays for several cases. For a base case, we chose the utilization factors $p_o = 0.5$ during the off-peak hour, and $p_p = 1.5$ during the peak hour. The sensitivity of delays to different parameter values ($p_o = 0.2$, $p_o = 0.8$, $p_p = 1.2$, and $p_p = 2.0$) was analyzed.

Table 6. Deterministic Delay due to Peak Overflows (flight hours)

<table>
<thead>
<tr>
<th>Peak Duration (hours)</th>
<th>(I) 0.5/1.5†</th>
<th>(II) 0.2/1.5</th>
<th>(III) 0.8/1.5</th>
<th>(IV) 0.5/1.2</th>
<th>(V) 0.5/2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.5</td>
<td>1.05</td>
<td>0.17</td>
<td>1.8</td>
</tr>
<tr>
<td>0.4</td>
<td>2.4</td>
<td>1.95</td>
<td>4.2</td>
<td>0.67</td>
<td>7.2</td>
</tr>
<tr>
<td>0.8</td>
<td>9.6</td>
<td>7.8</td>
<td>16.8</td>
<td>2.69</td>
<td>28.8</td>
</tr>
<tr>
<td>1.2</td>
<td>21.6</td>
<td>17.6</td>
<td>37.8</td>
<td>6.05</td>
<td>64.8</td>
</tr>
<tr>
<td>2.0</td>
<td>60</td>
<td>48.8</td>
<td>105</td>
<td>16.8</td>
<td>180</td>
</tr>
</tbody>
</table>

† off-peak utilization ($p_o = \lambda_o/\mu$)
‡ peak utilization ($p_p = \lambda_p/\mu$)
Table 6 and Figure 10 show the deterministic delays due to peak period overflows for five different cases. Figure 10 shows that delays increase approximately with the square of the peak period duration.

Figure 10. Deterministic Delay for Peak Period Overflows
The delays for stochastic traffic are estimated from the simulation models discussed in Section 3.3.2. In order to estimate the delay due to peak period overflows, simulation models were run with and without such overflows. The difference between those two delays is the delay due to peak overflows. In order to reduce the variance of simulation results, simulations were repeated 30 times for each case. To simulate different cases, the capacity (or maximum service rate) was fixed at 30 flights/hour and volumes were varied in each case. Table 7 shows the stochastic delays due to excess volumes during a peak period for the M/M/k and M/N/k cases. Figures 11 and 12 show the deterministic and stochastic delay for each case.

**Table 7. Stochastic Delays for Peak Period Overflows (flight-hours)**

<table>
<thead>
<tr>
<th>Peak Duration (hrs)</th>
<th>M/M/k Case</th>
<th>M/N/k Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>0.5†</td>
<td>0.2</td>
<td>0.8</td>
</tr>
<tr>
<td>1.5‡</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>0.2</td>
<td>1.1</td>
<td>1.5</td>
</tr>
<tr>
<td>0.4</td>
<td>3.6</td>
<td>2.8</td>
</tr>
<tr>
<td>0.8</td>
<td>12</td>
<td>9.1</td>
</tr>
<tr>
<td>1.2</td>
<td>25</td>
<td>19</td>
</tr>
<tr>
<td>2.0</td>
<td>67</td>
<td>52</td>
</tr>
</tbody>
</table>

† off-peak utilization ($\rho_o = \lambda_o/\mu$)
‡ peak utilization ($\rho_p = \lambda_p/\mu$)
Figure 11. Deterministic and Stochastic Delays Due to Peak Overflows (M/M/k)
Figure 12. Deterministic and Stochastic Delays Due to Peak Overflows (M/N/k)
4.3.2 Estimation of Stochastic Adjustment Factors

The stochastic adjustment factor \( F_s \) was defined as the ratio of the simulated stochastic delay \( D_s \) and deterministic delay \( D_d \) in Section 3.3.2. This factor can be used to estimate the stochastic delay without new simulations, according to Equation (3.12). Table 8 shows the values of stochastic adjustment factors for five different cases and different peak period durations. Figures 13 and 14, based on Table 8, show that stochastic adjustment factors are decreasing exponentially as the peak duration increases, asymptotically approaching 1.0. The dotted lines in upper figures (Figures 13 and 14) show the estimated exponential functions for the stochastic adjustment factor using regression analysis.

Table 8. Stochastic Adjustment Factor \( (F_s = D_s/D_d) \)

<table>
<thead>
<tr>
<th>Peak Duration (hrs)</th>
<th>M/M/k Case</th>
<th>M/N/k Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5/1.5</td>
<td>0.2/1.5</td>
</tr>
<tr>
<td></td>
<td>0.8/1.5</td>
<td>0.5/1.5</td>
</tr>
<tr>
<td></td>
<td>0.5/1.2</td>
<td>0.5/2.0</td>
</tr>
<tr>
<td>0.2</td>
<td>1.80</td>
<td>2.17</td>
</tr>
<tr>
<td></td>
<td>6.11</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td>1.61</td>
<td>1.27</td>
</tr>
<tr>
<td>0.4</td>
<td>1.50</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>3.73</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>1.22</td>
<td>1.21</td>
</tr>
<tr>
<td>0.8</td>
<td>1.23</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>2.42</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>1.16</td>
<td>1.13</td>
</tr>
<tr>
<td>1.2</td>
<td>1.14</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td>2.01</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>1.14</td>
<td>1.05</td>
</tr>
<tr>
<td>2.0</td>
<td>1.11</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>1.71</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>1.13</td>
<td>1.03</td>
</tr>
</tbody>
</table>
Figure 13. Stochastic Adjustment Factor for M/M/k Case
Figure 14. Stochastic Adjustment Factor for \( M/N/k \) Case
4.3.3 Estimation of Average Delay for Multiple Server Systems

Simulation models were developed to estimate the average delay for queuing systems with multiple parallel servers. The proposed models can be used to estimate delays for multiple TLOFs or multiple gate operations. These models can simulate various arrival and service time distributions. Tables 9 and 10 describe the average delay for M/M/k, M/N/k D/M/k, and D/N/k cases. Each case was simulated with various combinations of utilization rates (ρ) and numbers of servers (k). 30 simulation replications were used to reduce the variance of results and the first 1000 minutes in each simulation run were discarded to insure stable results.

Figures 15 and 16 illustrate the results in Tables 9 and 10. They show that the M/N/k case has lower average delays than the M/M/k case and that the D/N/k case has the least average delays among the four cases.

Table 9. Average Delay in Multiple Server Systems (M/M/k and M/N/k) in minutes/flight

<table>
<thead>
<tr>
<th>V / C</th>
<th>M/M/k Case</th>
<th>M/N/k Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k = 1</td>
<td>k = 2</td>
</tr>
<tr>
<td>-------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>0.2</td>
<td>0.49</td>
<td>0.09</td>
</tr>
<tr>
<td>0.4</td>
<td>1.4</td>
<td>0.39</td>
</tr>
<tr>
<td>0.6</td>
<td>3.0</td>
<td>1.1</td>
</tr>
<tr>
<td>0.8</td>
<td>8.1</td>
<td>3.5</td>
</tr>
<tr>
<td>0.9</td>
<td>17</td>
<td>8.2</td>
</tr>
<tr>
<td>0.95</td>
<td>30</td>
<td>16</td>
</tr>
</tbody>
</table>

53
Table 10. Average Delay in Multiple Server Systems (D/M/k and D/N/k) in minutes/flight

<table>
<thead>
<tr>
<th>V / C</th>
<th>D/M/k Case</th>
<th>D/N/k Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>k = 1</td>
<td>k = 2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>0.25</td>
<td>0.04</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0</td>
<td>0.29</td>
</tr>
<tr>
<td>0.8</td>
<td>3.5</td>
<td>1.4</td>
</tr>
<tr>
<td>0.9</td>
<td>8.0</td>
<td>3.8</td>
</tr>
<tr>
<td>0.95</td>
<td>16</td>
<td>8.5</td>
</tr>
</tbody>
</table>
Figure 15. Average Delay for Multiple Server Systems (M/N/k and M/M/k Cases)
Figure 16. Average Delay for Multiple Server Systems (D/N/k and D/M/k Cases)
5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

A methodology for estimating vertiport capacity and delays has been developed. Using information obtained from the literature and surveys, we have proposed deterministic models to estimate the capacity of each of three subsystems: terminal airspace, TLOF, and gates. The smallest capacity found among these subsystems is assumed to be the total system capacity. Probabilistic queuing models are also developed to estimate the delay at queuing systems with a single server and multiple parallel servers. The findings are summarized below:

1. With the minimum in-trail separation distance (from 3 nautical miles to 6 nautical miles), headways are 2.47 minutes and 3.63 minutes respectively. The terminal airspace capacity for one approach path ranges from 16 to 24 flights/hour.

2. TLOF capacity is limited by the TLOF ground capacity and TLOF approach path capacity, of which the latter is more constraining. It varies from 16 operations/hour (pessimistic capacity) to 24 operations/hour (optimistic capacity). However, the above TLOF capacity can be increased by up to 100% by inserting a take-off between any two landings without affecting the landing intervals.

3. Gate capacity increases as the gate utilization factor increases and gate occupancy time decreases, as shown in Table 4.

4. From the relations among terminal airspace, TLOFs and gates (Equation 4.1), we can estimate the required number of gates or TLOFs.

5. The vertiport capacity is determined by the minimum capacity among the capacities of subsystems in series, as shown in Figure 3. To increase the vertiport capacity, the subsystem having the minimum capacity should be expanded. Preliminary results show that the airspace capacity or gate capacity are more likely to be critical than TLOF ground capacity.

6. From the delay curves in Figures 7 and 9, we can find the practical capacity, which is defined as the traffic volume resulting in four minutes of average delay. Figures 7
and 9 can also be used if other delay values are preferred for defining practical capacity.

7. Gate capacity will be significantly reduced if CTRs cannot operate "independently" (simultaneously) at adjacent gates. At vertiports where both capacity and land costs are important issues, the use of "jetways", or some similar structures for passenger loading/unloading, would probably be the most cost-effective solution.

8. A method for determining delays due to peak period overflows is developed in Section 3.3 by combining a deterministic queuing model with simulation results.

9. The delays due to various peak period overflow conditions are presented in Tables 6 and 7.

As in previous capacity analysis methods for conventional airports, this study focuses on aircraft rather than passengers. To translate aircraft capacities and delays into corresponding passenger numbers, we can multiply aircraft by their seat capacities (approximately 40 for early CTR's) and load factors (approximately 65%, based on conventional airline operations). Further studies may consider the mix of aircraft sizes and variations in load factors for various periods and situations.

The proposed methodology can provide useful guidelines to vertiport planners, designers and policy makers, even though CTR aircraft are still in the developmental stage and information about them is based on very little experience. This methodology should be also useful in evaluating the commercial feasibility of particular vertiport and the effects of vertiports on a larger air transportation system.

5.2 Recommendations

This report provides several recommendations based on the results of our analysis.

1. The results in this report are based on some parameter values estimated by experts in the absence of real experience with commercial CTR operations. The results of this analysis should be reexamined soon after commercial CTR operations begin.
2. To improve the reliability, precision and usefulness of models for CTR capacity and delay, the following additional research tasks are recommended:

a. Develop methods for relating aircraft operations capacity to passenger capacity for vertiports.
b. Investigate efficient TLOF and gate configurations for vertiports.
c. Develop analysis methods to evaluate the incremental benefits and costs of additional gates and recommend improved gate utilization and reserve factors.
d. Develop methods to analyze capacity and delay for mixed operations of CTR and conventional aircraft.
e. Develop methods to estimate the effects of interdependent operations at gates on vertiport capacity and delays.
f. Investigate interdependencies in series of vertiport components.
g. Statistically estimate functions for the stochastic adjustment factors tabulated in Table 8.
h. Determine empirically the appropriate distributions of service times at vertiport TLOF's and gates.

3. To deal with deviations from schedules, a reserve factor should be considered in determining the number of gates required.

4. Demand fluctuations should be considered in analyzing vertiport capacity.
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APPENDIX A. ACRONYMS

ADS: Automatic Dependent Surveillance
ATC: Air Traffic Control
CTR: Civil Tiltrotor
FAA: Federal Aviation Administration
FIFO: First-in First-out
GPS: Global Positioning System
IFR: Instrument Operating Rules
IMC: Instrument Meteorological Conditions
MLS: Microwave Landing System
PANYNJ: Port Authority of New York & New Jersey
PRM: Precision Runway Monitor
TLOF: Touchdown and Lift-off surface
TERPS: Terminal Instrument Approach Procedures
VFR: Visual Operating Rules
VMC: Visual Meteorological Conditions