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SYNCHRONIZATION STATION LOCATION STUDY SUMMARY

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JULY 1974

SUMMARY FINAL REPORT

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16. Abstract

A technology for evaluating ground station sites on the basis of their effectiveness in disseminating time synchronization has been produced. Analytical studies have complemented the concept of computer modeling of the airspace population, then quantitatively assessing the effectiveness of various configurations and deployment of time disseminating equipments. A unique feature is the ability to evaluate the effectiveness of ground based master stations with and without aircraft to aircraft hierarchal time synchronization relay. These techniques were applied in making trade-off evaluations of the overall synchronization effectiveness of various levels of airborne vs. ground-based hierarchal time relaying equipage. It is shown that the number of ground-based master stations required to supply a desired level of synchronization coverage to all aircraft (hierarchal and non-hierarchal) increases rapidly as the number of hierarchal aircraft is diminished.

Trade-off studies were made of the master station site selection algorithms. It was shown that the priority algorithm is very near optimum. However, choices of alternate sets of sites (such as air traffic hubs) can be used to obtain operational or economic advantages with only slight reduction in overall effectiveness.

Analytic and computational processes were utilized to evaluate the effects of atypical operations: master station outages, reduction in hierarchal support due to airline outage and weather. In all cases, hierarchal interconnection provided a high degree of redundancy and the overall system exhibited little loss in effectiveness under these abnormal conditions.

NTSB records of aircraft collisions indicated that a high level of coverage could be supplied by five master stations complemented by larger aircarrier equipped with hierarchal equipment.

Computational capabilities developed during this study include the ability to model total CONUS aircraft population activity as a three-dimensional time variant and to model scheduled aircarrier operations. For a given day and time-of-day, the altitude, latitude and longitude of each aircarrier aircraft is computed. Then, air-to-air and air-to-ground communication linkages are determined. The effectiveness of all potential sites for ground based equipments can be determined and quantitatively compared. These computational and analytic capabilities are directly applicable to assessing the effectiveness and deployment of other ground-based ATC equipments such as surveillance radars, VOR's and TACAN and ATC service demand problems.

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1. INTRODUCTION AND SUMMARY

This is the summary final report on studies developed under contract DOT-FA73-WA-3172. The purpose of this contract has been to determine siteing criteria and installation priority for deployment of master stations that provide precision time service to NAS. The study was performed in three overlapping stages; methodology development, calibration, and trade-off analyses. There has been a major report after each of these phases. Methodology development is documented in reference [1], the calibration phase in reference [2], and trade-off and sensitivity analyses are reported in reference [3].

It is the intent of this summary final report to concisely present methods, analyses and conclusions of the study from a logical point of view which incorporates the benefits of growth changes and iterative work developed during the full course of the study. References [1], [2], and [3] present a historical development of the subject and contain detailed descriptions of total program activities. As such, they should all be considered technical appendices to this document.

1.1 Problem Formulation

A distinguishing feature of this study has been the inclusion of air-to-air relay of time synchronization, called hierarchal chaining, to enhance the effectiveness of master stations. This procedure was first proposed in connection with the design of a time/frequency airborne Collision Avoidance System, reference [4]. Also, synchronized operation has been considered by others in connection with air traffic control systems including navigation, communication, and surveillance. Synchronized aircraft could donate time practically throughout CONUS for a variety of applications, in addition to ATC-related uses. Several time dissemination methods are being considered for these applications, including LORAN, TV broadcast, satellites, and VLF. Hierarchal chaining as a time dispensing mechanism had not previously received the thorough investigation it deserves.

In the time dispensing network described in reference [illegible], time is maintained to an accuracy of approximately ± .5 microseconds, [illegible], by master stations. These stations in turn synchronize all equipped aircraft within approximately a 1[illegible] nautical mile range. See Figure 1-1.

There are two basic classes of aircraft equipments:

(a) Those that receive, retain, and relay time (Hierarchal)
(b) Those that only receive time from ground stations or other aircraft. (Non-hierarchal.)

Equipments that participate in time retention and relay are automatically ordered in a hierarchal system. By means of data encoded in the RF transmissions of the equipments, the hierarchal rank of each available time source is known. Upon being synchronized, the hierarchal equipment assumes a rank one level lower than that of the time donor. In the absence of further synchronization, the equipment demotes in time hierarchy at a rate that is inversely proportional to the timekeeping accuracy of the particular equipment.

While in the system, the hierarchal equipments pass time to:

(a) Other hierarchal equipments of lesser rank (higher hierarchy number).
(b) Non-hierarchal equipments that only receive time.

Thus the hierarchy number of any craft represents the sum of relays and equivalent demote intervals accumulated in the entire process of receiving synchronization from a source master station.

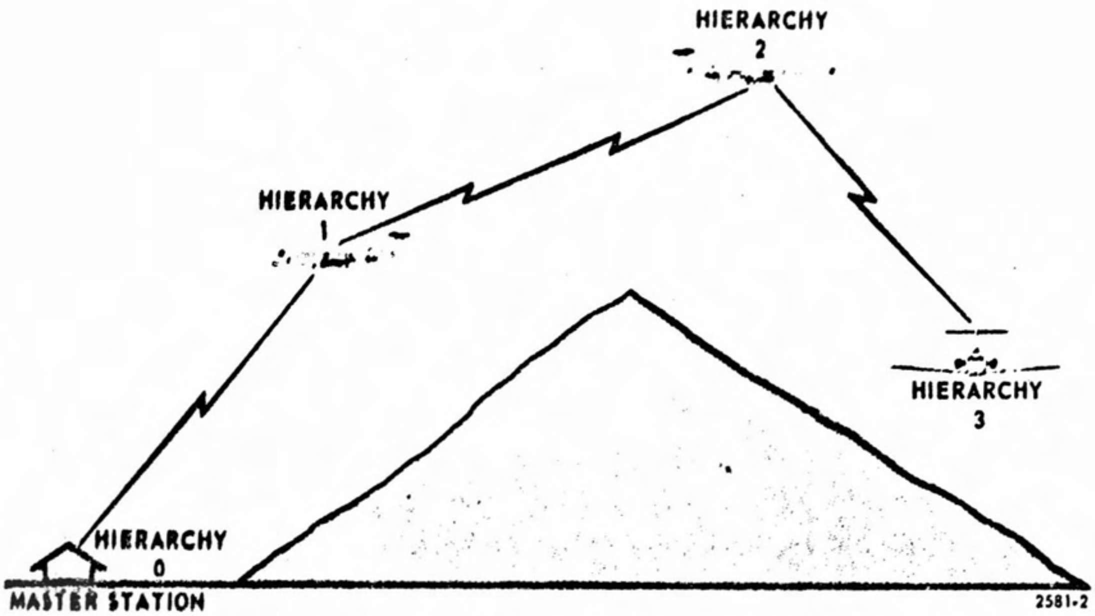


FIGURE 1-1. AIR TO AIR SYNCHRONIZATION LINKED TO A MASTER STATION

With this structuring of the system, a study designed to generate a deployment plan for master stations must:

(a) Model in four dimensions and throughout CONUS, the location of hierarchal aircraft which may establish communication chaining to relay time out from a master station.
(b) Create a similar model of the density and location of non-hierarchal aircraft that would benefit from the synchronization provided either by a hierarchal aircraft or by a master station.
(c) Model the synchronization process among the hierarchal equipments.

Furthermore, implicit in the concern for placement and number of master stations is the need to measure the degree to which a given arrangement achieves time dissemination goals. The desire is not only to deploy enough master stations to achieve these goals, but also to have them efficiently placed so that the goals are satisfied by a near minimum number of stations.

These four requirements lead to the three major technical problems which required resolution.

Due to the stochastic nature of air travel and the need for predictions of traffic flow in planning future deployments, the level of detail which can be accurately represented in a traffic flow model is limited. The most convenient and adequate source of information concerning the location of aircraft is airline schedules. This is fortunate since current thinking has it that airline aircraft are the most likely candidates for hierarchal equipment. Nevertheless, the strength of this program's conclusions have been greatly enhanced by showing that ground station effectiveness is determined by the gross patterns of aircraft travel and does not require great precision in the representation of specific aircraft locations.

While the primary goal prescribed for this study has been to provide aircraft synchronization adequate for collision avoidance purposes, secondary and alternate goals must also be considered. Precise time techniques have been considered, e.g. reference [6], as a basis for an air traffic control system designed to provide communication, navigation and surveillance through one integrated set of airborne and ground based equipment. Furthermore, air traffic control is not the only possible application of precise time techniques. Increasing numbers of non-aviation demands for precise time are appearing, e.g. reference [7]. To increase the usefulness of this study, it was necessary to show that master station deployments can be derived which are simultaneously near optimal with respect to several different possible uses of the system.

Finally, there has been little published analysis of communication systems of the hierarchal type and experimental evidence is restricted to two or at most three interacting airborne systems. Prior to the analysis set forth in this report, it was felt that detailed simulation might be required to determine which aircraft were synchronized and what hierarchy they possessed. We have been able to show that over a broad and typical range of chaining parameters and with very high probability, aircraft are always synchronized whenever a sequence of hierarchal aircraft is available leading to a master station wherein successive members of the sequence are within radio line-of-sight and synchronization range.

1.2 Solution Alternatives

In dealing with each of the following essential ingredients necessa., to successfull master station siteing:

(a) A model of hierarchal and non-hierarchal aircraft locations and resultant hierarchal chaining,

(b) A function of aircraft and master station locations which measures the effectiveness of time distribution and,

(c) An algorithm for optimizing this function with respect to station locations;

there are alternative methods which have been evaluated with respect to accuracy, timeliness of application, and conclusiveness.

In the area of modeling, the alternatives relate to the division between mathematical and numerical representation. For effectiveness measures, the choices require selecting from the various possible measures the one that most adequately represents system goals. Finally, the selection of a siteing algorithm requires a trade-off between the degree of optimality achieved and the cost of application.

Two solution methods were pursued during the course of the program. The main stream effort is a synthesis of mathematical and computer analysis which is accurate and rich in detail. A fully mathematical solution was also achieved which required little additional effort and produced conclusions which agree in every respect with those produced by the more lengthy method. However, the mathematical solution contains questionable assumptions and some apparently gross simplifications. Therefore it lacks the persuasiveness and the wealth of detail present in the more rigorous main stream effort. However, it provides valuable cross checking and verification which adds confidence to study results.

1.3 Summary of Results

The following is a summary of the findings and conclusions derived in the course of the study. In the interest of brevity, no attempt is made in the overview given below to explain qualifying conditions in uniform detail. Information of this type is to be found in the remaining sections of this report and in the technical appendices, references [1], [2], and [3]. The summary is as follows:

A. Time Propagation depends almost solely on hierarchal aircraft pairs being within radio line-of-sight and sync range. Within reason, the following factors have negligible effect;

   Clock quality,
   Communication probability,
   Resync accuracy, and
   Number of hierarchy levels available.

B. On this basis we have shown that chaining effectiveness is mainly a function of hierarchal aircraft density. Nearly all of the benefit of hierarchy is obtained when the heavier aircarrier or an equivalent set of aircraft are hierarchy equipped. Then, almost regardless of specific aircraft locations;

(1) Time propagation potential is CONUS wide.
(2) Less than 15 links are needed if at least three master stations are in operation.
(3) Redundant chaining opportunities are available at almost every craft.
(4) Synchronization range of master stations has little effect.

C. System effectiveness is best defined as a time integral (average) of instantaneous effectiveness;

(1) A measure must be an additive set function on subsets of three dimensional space.
(2) Various measures are appropriate to various system goals;
    Time propagation to aircraft,
    Time propagation to general users, and CAS coverage.
(3) Above a hierarchy density threshold, initial effectiveness per master stations is very high; below this density hierarchy support is small and thus effectiveness per master station is very low.

D. Master station siteing should maximize system effectiveness for the given number of stations;

(1) Practical optimizing algorithms do not exist.
(2) Above the density threshold overall effectiveness is very insensitive to site selection.

(3) Sub-optimal algorithms are very effective simultaneously for location, location priority and relocation.

(4) Sites are readily derived which are simultaneously nearly optimal for several effectiveness measures.

(5) Siteing constraints do not limit achievable effectiveness.

(6) Detailed distributionns for non-hierarchal aircraft are not necessary for selecting efficient sites.

E. Initial Master Station site desirability is;

(1) Determined by the number and patterns of hierarchal aircraft that are serviceable from the site.

(2) Not sensitive to effectiveness measure.

(3) Not sensitive to data interval or data extent beyond one day.

F. Effectiveness of Master Stations with hierarchal support has the properties;

(1) The first station is enormously effective.

(2) The next two or three diminishingly so, but still high.

(3) After the first four or five, over 90% of aircraft hours have synchronization available and hierarchal potential is essentially fully utilized.

(4) Without adequate hierarchal support effectiveness per master station is very low and sensitive to detailed local population distribution.

G. Any reasonable combination of five master stations could have furnished synchronization to 96% of the participants in the collisions recorded by NTSB during the period 1964-1971.

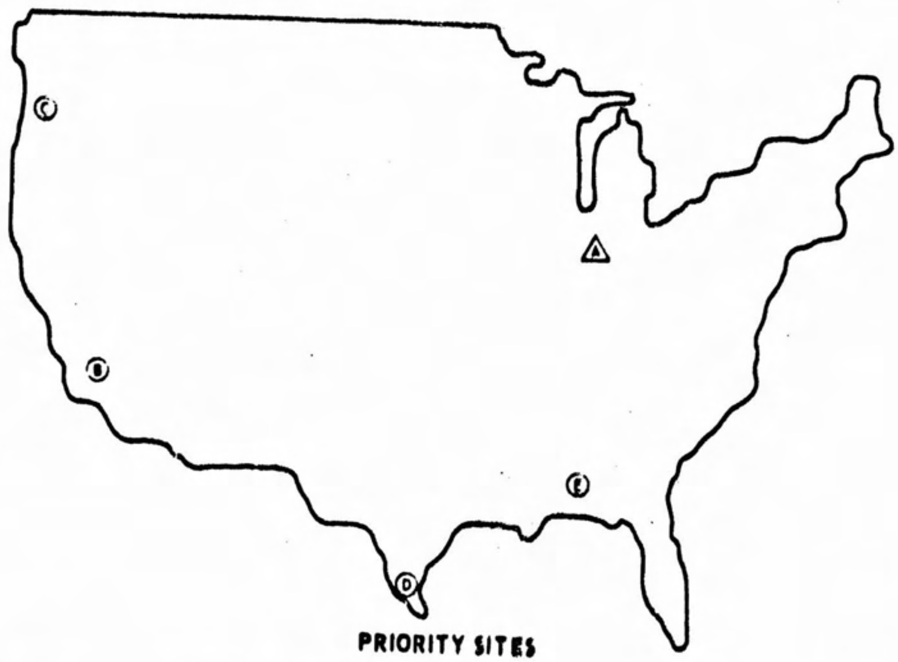
H. Because CONUS is covered by a single communicating cloud of aircraft throughout the high traffic hours and because this cloud can be synchronized at any point of contact with a master station, each master station provides a high level of backup to all other master stations.

I. Weather fronts may cause a subnormal aircraft density in some parts of CONUS and an accompanying density rise in other parts. Communication with a master station may be interrupted in this case but the net effect will be less troublesome than a station outage.

J. The following course of action with respect to any possible future master station siteing is recommended:

> Plans be made for five master stations. When these are deployed, something near 99% of aircraft hours will have precise time available providing only that the heavier airline aircraft are hierarchy equipped. Should additional synchronization service be judged desirable, master station siteing judgements should be based on on-site real time evaluation of synchronization need.

K. While many sets of five stations can produce comparable performance, the two sets shown in Figure 1-2 have been identified as providing the most utility. The priority of installation is indicated alphabetically. While the upper grouping of Figure 1-2 exhibits somewhat higher performance in terms of gross aircraft hours synchronized, the lower grouping (the best selections from major hubs) has the advantage of better service to aircarrier aircraft, 24 hour service to high density traffic, available government sites and service, and less sensitivity to future changes in travel patterns.



PRIORITY SITES

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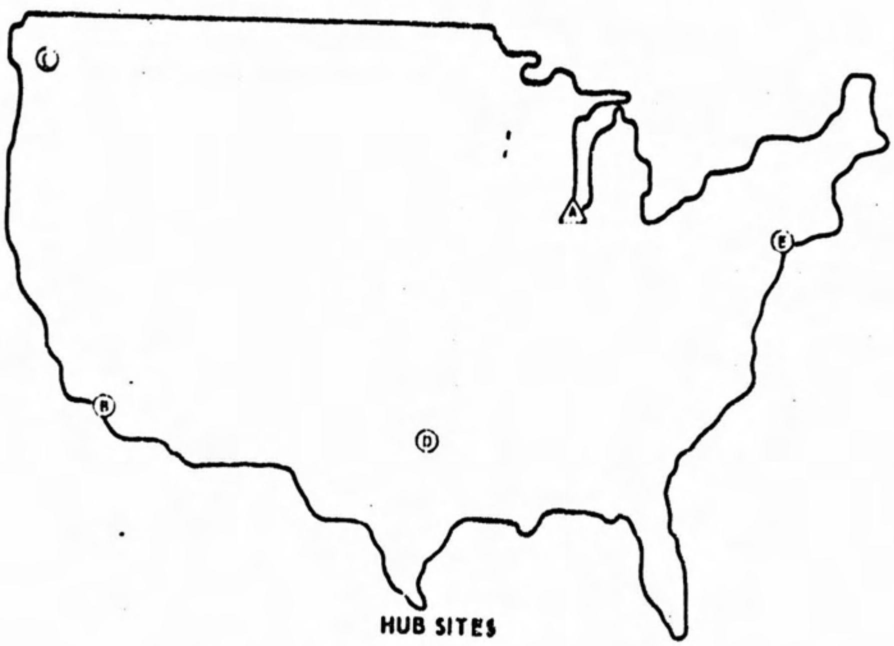

HUB SITES

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FIGURE 1-2. BEST SITEING ALTERNATIVES

1.4 Organization of this Report

Solution methods and supporting mathematical analyses are presented in Section 2 and Section 3 contains conclusions and siteing recommendations.

Appendix A provides a summary of the numerical and graphic results. For the reader desiring additional information; references [1], [2] and [3] provide detailed reporting of activity throughout this study. Appendix B of this report is a section by section cross reference to this material.

2. SOLUTION METHODS

As a result of Phase I studies, a detailed solution method was selected which is a synthesis of mathematical and computer analysis. Section 2.1 below describes the mathematical aspects of the method while Section 2.2 outlines the computer analysis. During the course of the study, and as an outgrowth of investigations into the sensitivity of master station effectiveness to the location and density of hierarchal aircraft, a second, less complete but fully mathematical, method was achieved. A description of this method is included in Section 2.1.4.

2.1 Mathematical Analysis

Four especially important problems in the location of master synchronization stations are analyzed in this section.

(a) How can hierarchal chaining be modelled?
(b) What are adequate measures of master station effectiveness?
(c) What master station location algorithms result in near optimum placement?
(d) What precision in hierarchal aircraft positions is required?

We first identify that hierarchal chaining has a graph theoretical basis in which the nodes of the graph represent the hierarchal equipped aircraft and an edge of the graph is defined between two nodes whenever two aircraft are within radio line-of-sight and synchronization range. At any given instant in CONUS the connected components of this graph (which we call clouds of hierarchal aircraft) describe completely the potential paths available for air-to-air relay of synchronization. If any one of the aircraft (nodes) in a cloud (connected component) has access to synchronization from a master station, then every aircraft in the cloud has the potential for being synchronized.

We show that, over a broad and typical range of problem parameters and with high probability, aircraft in a cloud serviced by a master station are synchronized. This conclusion is shown to be valid whenever:

(a) Airborne clocks have an accuracy greater than 1 part in 10^{-8} (3σ).

(b) The probability of achieving air-to-air relay of synchronization is at least 0.4 per try.

(c) The accuracy with which time is relayed is at least 0.25 microsecond (3σ).

(d) There are at least 20 hierarchy levels available.

It is felt that in any system likely to be installed, all these parameters would lie within the stated range and some combination of them would be greater than the minimums listed.

Based on this result, we determine the clouds of aircraft and thus, which aircraft are synchronized through a computer model of the time positions of all airline aircraft described in Section 2.2.

We then turn to the problem of placing master stations so that synchronization can be spread maximally through CONUS. This requires that measures of the effectiveness of time distribution be derived to reflect the different possible uses of the time distribution system, and accompanying master station siteing algorithms be selected.

Finally, to be assured that the methods will be adequate, we need to know what aircraft position accuracy is required to faithfully represent the hierarchy communications cloud structure in CONUS at any instant. To answer this question, we examine the overall structure of the hierarchal graph from the point of view of stochastic graph theory. In this effort a mathematical model based on the law of large numbers is developed in order to translate the results of stochastic graph theory into properties approximating those of real distributions of aircraft. This investigation shows that during the portions of the day when large numbers of hierarchal aircraft are active, all hierarchal aircraft in CONUS will be in a single cloud almost regardless of their individual positions. The less active portions of the day have been examined by exercising subsequent computer simulation models with the overall result that, for the purpose of modelling time distribution, there is no need to locate aircraft more accurately than to within 30 x 30 $(\text{n.m.})^2$ cells. Throughout the more active portions of the day much less accuracy is required.

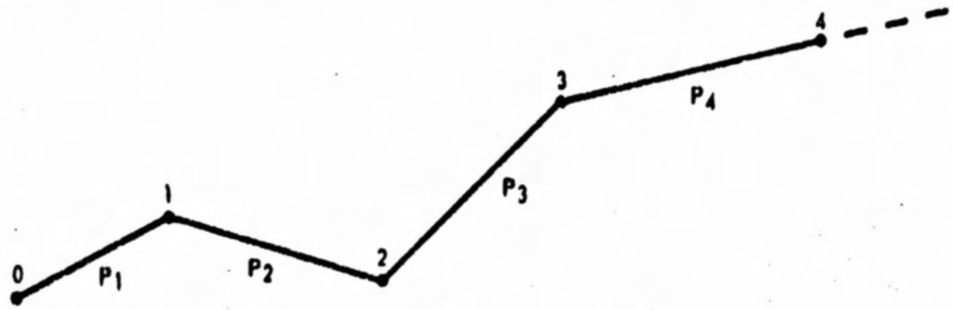
The mathematical model derived to examine the dependence of cloud structure on aircraft locations permits preliminary estimates to be made of the number of master stations required and their effectiveness. Section 2.1 ends with these results.

2.1.1 Hierarchal Chaining

The hierarchy number at each aircraft is the sum of the number of relays of time synchronization from the master station and equivalent demote intervals accumulated by all members between synchronizations. Since each relay contributes a timing error and a demote interval is defined to be that length of time during which the aircraft's clock develops an error equal to that suffered in a relay, each craft's hierarchy number indicates the accuracy of time available aboard the craft.

Depending upon the error introduced in a time relay, and the total error tolerable in the system application being considered, some upper limit, H, to useable hierarchy exists. Aircraft possessing hierarchy less than H are deemed to be satisfactorily synchronized. The maximum useable hierarchy for the airborne CAS defined in reference [4] is forty.

Consider the hierarchal chain depicted in Figure 2-1 where 0 is the master station and the nodes 1, 2, etc., represent aircraft such that each successive pair are within radio line-of-sight and synchronization range. We will take as a model a system implementation which allows synchronization to be attempted at regular intervals as does, for example, the system defined in reference [4]. For a given successive aircraft pair in Figure 2-1 let ρ be the probability of achieving synchronization in one attempt and let ν be number of attempts permitted in a demotion interval. Then, $q = (1-\rho)^{\nu}$ is the probability of not achieving synchronization in a demote interval, while $p = (1-q)$ is the probability of achieving synchronization in a demote interval.



20311 **FIGURE 2-1. A HIERARCHAL CHAIN**

The probability that it takes this pair precisely k demote intervals to transfer synchronization is the probability of (k-1) failures followed by a success, namely $q^{k-1}p$. But if synchronization takes k demote intervals, then the change in hierarchy, Δh, across this link of the chain is precisely k. Thus, the probability that $\Delta h = k$ is $q^{k-1}p$. This probability function is illustrated in Figure 2-2.

Since the probability of some change in hierarchy is unity, we should and do have

$$\sum_{k=1}^{\infty} q^{k-1}p = p/(1-q) = 1. \tag{2-1}$$

Moreover, the average hierarchy change between successive aircraft is:

$$\overline{\Delta h} = \sum_{k=1}^{\infty} kq^{k-1}p = 1/p = 1 + (q/p), \tag{2-2}$$

and the variance of the change is:

$$\sigma^2_{\Delta h} = \sum_{k=1}^{\infty}\left[k - (1/p)\right]^2 q^{k-1}p = q/p^2. \qquad (2\text{-}3)$$

Each link of the chain in Figure 2-1 has its own characteristic probability, p_i, of obtaining synchronization in a single demote interval and the hierarchy at the m^{th} aircraft, h_m, is the sum of all m Δh's in the chain. By the central limit theorem we expect that since h_m is the sum of random variables, it will be approximately normally distributed with mean μ_m, equal to the sum of the means,

$$\mu_m = \sum_{k=1}^{m} (1/p_k) = m + \sum_{k=1}^{m} (q_k/p_k) \qquad (2\text{-}4)$$

and variance, ${\sigma_m}^2$, equal to the sum of the variances,

$${\sigma_m}^2 = \sum_{k=1}^{m} (q_k/{p_k}^2) \qquad (2\text{-}5)$$

The actual distribution is derived in reference [1], page 2-12, and the normal approximation is shown to be valid. The situation is very close to that illustrated in Figure 2-3 for a 23 link chain where the p_i's have been selected at random from the interval [0.994, 0.48].

We wish to be assured that, with high probability, h_m is no greater than the maximum useable hierarchy H. Using the normal approximation, we know that when

$$\mu_m + Q\sigma_m \leq H \qquad (2\text{-}6)$$

then $h_m < H$ with at least probability P provided

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Q} e^{-t^2/2}\, dt = P.$$

For example, when Q is 2.05, P is 0.98.

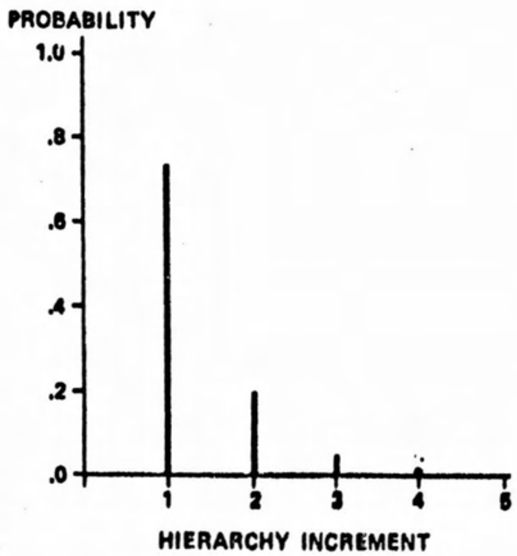


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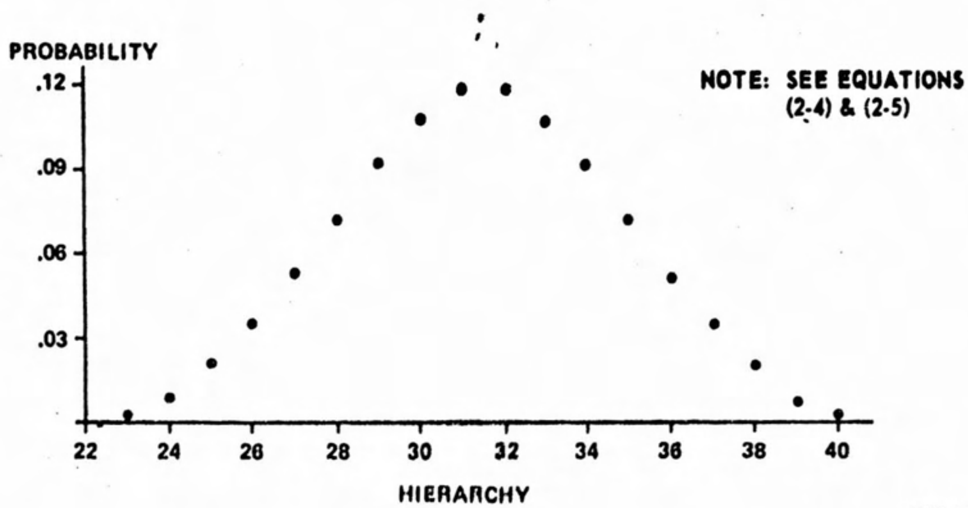
FIGURE 2-2. DISTRIBUTION OF THE HIERARCHAL INCREMENT INTRODUCED BY ONE LINK



2581-15

FIGURE 2-3. PROBABILITY DISTRIBUTION FOR THE HIERARCHY OF THE 23RD MEMBER

When equation (2-6) is satisfied by system chaining parameters, synchronization of all members of the chain up to the m^{th} member is assured with probability no less than P. Having selected a suitably high probability, equation (2-6) can therefore display those sets of possible chaining parameters which provide satisfactory performance. The parameters involved in equation (2-6) are

(a) The error introduced by time relay
(b) The clock accuracy
(c) The interval between synchronization attempts

Together these define the number of synchronization attempts per demote interval, ν.

(d) The minimum useable system accuracy which defines the maximum useable hierarchy, H
(e) The quality of the communications channel which determines the probability ρ that a synchronization attempt is successful.

In reference [1], page 2-15, it is shown that

$$p = \left[\frac{1}{m} \sum_{i=1}^{m} \frac{1}{p_i} \right]^{-1} \tag{2-7}$$

is the effective probability per link of achieving synchronization in one demote interval for a chain that exhibits the individual probabilities p_i. This parameter is used in conjunction with equation (2-5) to produce the summarizing graph in Figure 2-4.

The curve gives the chain length for which we are 98% certain that all chain members have hierarchy less than 40 as a function of the effective probability per link.

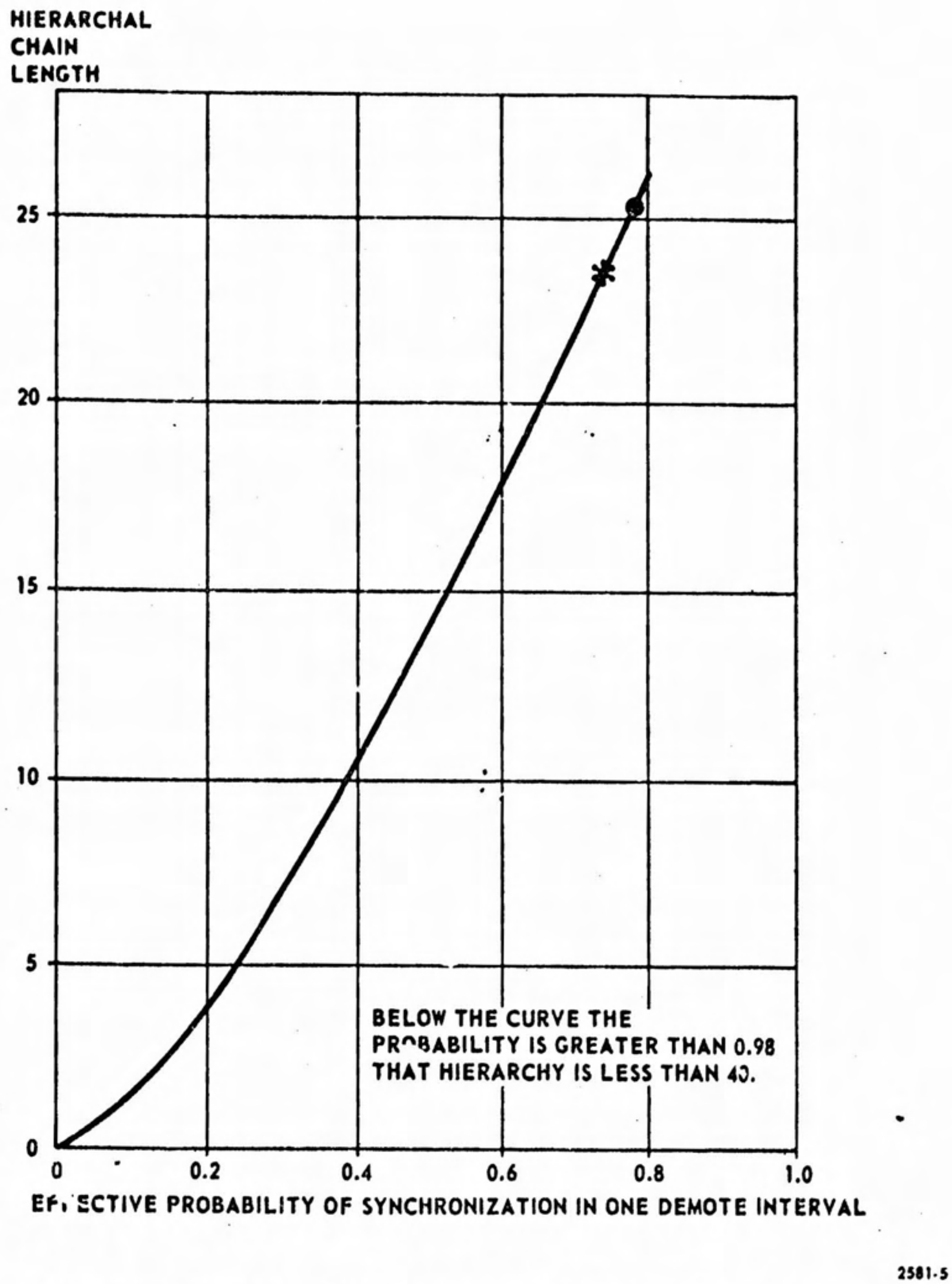


FIGURE 2-4. HIERARCHAL CHAIN LENGTH THAT HAS 98% CERTAINTY OF ALL MEMBERS BEING SYNCHRONIZED

In "Collision Avoidance System Flight Test and Evaluation", Martin Marietta Corporation, April 1970, tests on air-to-air synchronization are reported for various distances and altitudes. When the experimental results are summarized over all altitudes and ranges to find the probability of obtaining synchronization in a 6-second demote interval, one obtains 0.73. A 23-member chain with this probability for each link is 98 percent certain to have all members in hierarchy less than 40. This point is illustrated with the * on the figure. A 6-second demote interval corresponds to 10^{-8}, crystal oscillator, clock.

The various experiments, analyzed individually, exhibited a range of synchronization per 6-second demote interval probabilities from 0.994 to 0.48. These variations showed no particular dependence on range. It appears that signal-to-noise ratios were more than adequate at all ranges tested so that multipath interference and antenna pattern nulls dominate synchronization probabilities. Calculations based on equation (2-7) show that a 25-member chain, in which the individual links exhibit this entire range of probabilities in equal proportion, is 98 percent certain to have all its members in hierarchy less than 40; this point is labeled • on Figure 2-4.

Thus we have shown, on the basis of the best experimental evidence available, that with high probability any chain of less than 20 members will provide synchronization to an aircraft. Based on this result, the solution method utilized in the study assumes that an hierarchal aircraft is synchronized if there exists a sequence of hierarchal craft leading from a master station to the aircraft in which each successive pair are within radio line-of-sight and synchronization range.

We hold this assumption to be conservative on a number of grounds.

(a) With a synchronization range of approximately 100 n.m., as used in this evaluation, a chain of 20 links is on the average 1500 n.m. long. Subsequent analysis shows that three ground stations within

CONUS limit the number of links required to fully utilize hierarchal time distribution to less than fifteen.

(b) The analysis above assumes only a single chain leading to any given aircraft. In point of fact, multiple chaining opportunities exist at nearly every craft. Each alternative chain increases the probability of being synchronized.

(c) The numerical results quoted above assumes that all aircraft are equipped with a 10^{-8} crystal clock. The state of the art in producing precision time pieces is such that we would expect many, if not most, aircraft to be better equipped.

2.1.2 Effectiveness Measures

In this study, we have taken the point of view that the distribution of time throughout CONUS could be a service directed toward a number of possible objectives. As with any service, one needs some means of measuring the degree to which the service meets its objectives.

In the systems we are considering, the primary source of time is the master station. However, the hierarchal aircraft serve to extend the service into regions not directly covered by such stations. Somewhat analogous to more familiar utilities, (e.g., electrical power, natural gas, water, etc.), each hierarchal aircraft serves as a substation with its own particular region of influence. In this case, the region of influence is the subset of CONUS airspace within radio line-of-sight and synchronization range of each synchronized aircraft.

The utility of bringing time into this region should be directly relatable to system objectives. If the objective is to provide time to aircraft, the utility should be measured in terms of the number of aircraft, perhaps of a specific type, within the region. If the objective is to obtain the broadest possible availability of time, then the utility might properly be measured in terms of either the volume of the region or the land surface area it covers. If time service is provided for collision avoidance purposes, then the utility should be measured in terms of the rate of occurrence of near misses and/or collisions in the region.

However, regardless of the specific objective, we wish to define a function which operates on a region (subset of CONUS airspace) and produces a positive real number reflecting the degree of utility provided. That is, we wish to define a function, μ, which operates on subsets, V, of three dimensional space (the service region of a given aircraft) and yields a positive real number α, i.e., $\mu(V) = \alpha$.

Since the region V serviced by given hierarchal aircraft changes with time as does the characteristics of the region which determine $\mu(V)$, we must interpret $\mu(V)$ as the instantaneous utility of providing service to the region V. While instantaneous measures of utility are of interest, we will ultimately desire to integrate this function in time to obtain a measure of the mean utility provided.

A further, and very definitive, property required of the function μ can be discerned. If we have two hierarchal aircraft which provide service to two completely disjoint regions, V_x and V_y, then the utility of providing time to these two regions combined must be the sum of the separate individual utilities. Expressed symbolically, this means:

$$\mu(V_x \cup V_y) = \mu(V_x) + \mu(V_y)$$

whenever the intersection

$V_x \cap V_y$ is the empty set.

A function which associates real numbers with sets and has the above property is called an (finitely) additive set function. It can be shown that when such a function is applied to a set derived by combining (in the sense of set union) a number of over-lapping regions, the regions of overlap are counted only once in the determination of the numerical functional value.

With this background, we can now perceive the steps necessary to specify the utility of time distribution. One begins with a specific number and placement of master stations. First, one must determine for each time instant which hierarchal aircraft are synchronized as the result of this master station placement. The service regions for all synchronized stations, including hierarchal and master stations, must then be derived and combined. Following this, the functional rule μ is applied to the combined region to obtain the numerical representation of instantaneous utility. Finally, the instantaneous utility must be integrated over time to find the time averaged utility. This procedure is summarized in the following symbology:

$$\frac{1}{T} \int_{0}^{T} \mu \left[\bigcup_{x \in S(t)} V_x \right] dt \qquad (2\text{-}8)$$

where $S(t)$ is the set of all stations synchronized at time t.

The computation described in (2-8) suffices for comparison of the performance of alternative master station placements, but does not yield information as to what fraction of the total need is being fulfilled. To reflect this aspect, we must compute the utility of providing synchronization to all CONUS airspace and normalize the expression in (2-5) with respect to this quantity. Thus, with a given set of ground stations M, we associate an effectiveness number E(M) given by

$$E(M) = \frac{\int_0^T \mu \left[\bigcup_{x \in B(t)} V_x \right] dt}{\int_0^T \mu(A) dt} \tag{2-9}$$

where the set A is the total CONUS airspace.

Five different definitions of the set function μ have been investigated. Each of these emphasize a different possible use for time distribution.

I. <u>μ(V) is the number of hierarchal aircraft in V.</u>

With this definition of μ, the effectiveness is the fraction of hierarchal aircraft hours that are synchronized. This is a good measure of utility for master station location purposes regardless of the specific objective of time dissemination. Since the basic time disseminating mechanism is air-to-air relay between hierarchal equipped craft, and since nearly all useful measures of effectiveness are enhanced by the broadest possible dissemination of time, a sequence of master stations well placed to utilize hierarchal chaining potential serves all likely system goals very well.

II. <u>μ(V) is the CONUS surface area contained in V.</u>

With this μ, effectiveness is the fraction of CONUS land area which has synchronization service available. This measure is intended to be indicative of utility to non-aviation users of time.

III. <u>μ(V) is the total number of aircraft in V.</u>

In this case, effectiveness is the fraction of total CONUS flying hours that have synchronization available. Since this definition is felt to be most representative of service to aviation, it has received primary emphasis.

IV. <u>$\mu(V)$ is the aircraft encounter rate in V.</u>

Our use of this definition has been restricted to the assumption that the encounter rate in a given volume of airspace is proportional to the square of the aircraft density and further that the constant of proportionality does not vary throughout CONUS. It provides an approximate indicator of time distribution utility for collision safety purposes since effectiveness by this definition is the fraction of aircraft encounters during which time synchronization is available.

V. <u>$\mu(V)$ is the aircraft collision rate in V.</u>

This definition, which gives effectiveness as the fraction of collisions provided time synchronization coverage, is intended to be supplementary to the measure of effectiveness described in IV above. In application, the collision rate has been determined from NTSB collision records covering the period 1964-1971.

Effectiveness as defined by equation (2-9) depends on time through the synchronized station set $S(t)$. $S(t)$ in turn, is determined by the time profiles of all hierarchal aircraft. Taken as a whole, these time profiles and hence $S(t)$ exhibit strong periodicities. The strongest periodic component is diurnal as flights build up during the day, die out at night, and show characteristic peaking at rush hours. The next strongest component is weekly, with minimum travel on weekends and build up through the week to accommodate the public's travel habits. There is also an annual periodicity to match seasonal air travel patterns.

We can view the function $S(t)$ as being made up of daily, weekly, and yearly components, all modulated by long term trends plus a noise component. Noise enters in two ways. First, on a fine scale, deviations from schedule occur due to weather, flight path choices, delays, etc., and on a larger scale, schedules are altered in response to economic forces. Second, since we cannot represent the function $S(t)$ exactly in our calculations, we will inevitably introduce some numerical noise.

Any high frequency components of noise will be strongly attenuated by the integration process which has a half-power bandpass of only 0.443/T Hz. Only disturbances which persist longer than 2.26 T will pass without significant attenuation. If T is on the order of days, few natural disturbances persist this long. If care is exercised with respect to numerical noise, the noise components will be effectively averaged.

In light of these considerations, the logical choice for a representative period is T equals one week. All daily and weekly variations will be accounted for and noise adequately attenuated.

One characteristic of master station effectiveness is not represented in equation (2-9) as it now stands. Minimizing the effects of master station outages should be a consideration in station deployment. Patterns of redundancy may be possible which enhance the overall effectiveness of the master station set. The best way to assure that such possibilities are properly exploited is to design the effectiveness measures to include the contribution of redundancy to utility.

While the effectiveness measure of equation (2-9) was constructed on the implicit premise that all master stations are available at all times, actually the set M of master stations can be in various operational states. Each operational state is described by the subset X of the stations in M which are "on the air". These subsets (states) X of M range from none of the stations available through all combinations of partial availability to all master stations being simultaneously available. Each state provides a certain level of service E(X), described by equation (2-9), and each possesses a characteristic probability of occurrence, P(X). The mean effectiveness, $\overline{E}(M)$, of the set M is therefore given by

$$\overline{E}(M) = \sum_{X \epsilon \pi(M)} P(X)\, E(X) \qquad (2\text{-}10)$$

where $\pi(M)$ is the power set of M (i.e., the set of all subsets of M).

Under the expected condition that all master stations are of like kind, and that station outages will be independent events, a common probability value can be used for each combination that has the same number of stations available. Expressing availability as α, and non-availability as $\beta = 1-\alpha$, equation (2-10), in this case, reduces to

$$\overline{E}(M) = \sum_{X \epsilon \pi(M)} \alpha^{\hat{X}} \beta^{\hat{M}-\hat{X}} E(X) \qquad (2-11)$$

where the notation $\hat{X}$ means the number of elements in the set X (i.e., the cardinal number of the set X).

Equation (2-10), or as appropriate, (2-11), used in conjunction with equation (2-9) is the most general description of master station effectiveness utilized in computational portions of this study. However, depending on the specific application of the master stations, various emphasis will be applied to operational and economic considerations such as, uninterrupted service to high traffic areas, service to aircarrier aircraft, utilization of available government sites and service, and applicability during early phases of introduction. Although such operational and economic motivations have not been factored into the analytical evaluation of effectiveness, they have been considered in making final judgements on site alternatives.

2.1.3 Algorithms for Master Station Location

$\overline{E}(M)$ in equation (2-10) is a function of the geographic coordinates of the master station locations. The intent is that the stations be located so that their effectiveness is as great as possible. If there are m stations in M, each point in the 2m dimensional space of station coordinates yields a particular value for $\overline{E}(M)$. This suggests the possibility of employing standard multidimensional optimization techniques to locate master stations. These techniques invariably assume the function to be continuous, and most require at least first order derivatives. Unfortunately, $\overline{E}(M)$ is not even continuous in the station coordinate variables. Step discontinuities occur when the last link between a master station and an aircraft cloud is severed. Even if this were not the case, investigations have shown that the function $\overline{E}(M)$ has many relative maxima, so that the result of conventional optimizing techniques is highly dependent on initial solution estimates.

The only method for verifying that a truly optimal station location solution has been reached, is exhaustion of all possibilities. To do this, a mesh of possible station locations must be laid over CONUS fine enough that significant variations in effectiveness do not occur within a cell, and then $\overline{E}(M)$ must be computed for every way that m stations can be placed in these cells. If there are n cells, this requires $\binom{n}{m}$ computations of $\overline{E}(M)$. For example, if an average cell in CONUS covers 700nm^2 (approximately 3000 cells) and there are to be 20 master stations, then

$$\binom{n}{m} = \binom{3000}{20} = 10^{51}$$

computations of $\overline{E}(M)$ would be required.

To reduce the number of computations we have employed a priority algorithm. First we solve the problem by exhaustion for only one master station; that is, we select the most effective single master station. Then we remove all hierarchal aircraft that are synchronized by that station from consideration, and repeat the process; that is, find the most effective ground station that can be adjoined to the one already selected, and continue this process until all m stations are located. The number of computations of $\overline{E}(M)$ involved, is approximately n - (m/2). This would be 2,990 in the above example, and while this is still not a comfortable number, it is within the realm of possibility.

The priority algorithm may not always produce absolutely optimum station locations. Situations can be constructed, in which some hierarchal aircraft are redundantly covered by more than one master station, where master station sites can be found which exhibit somewhat higher performance than the priority selected sites. While such cases can be constructed, reference [1], page 2-47, we have never encountered such a case in an actual application.

Since computation time for the priority algorithm is a strong function of the number n of possible station locations, the best opportunity for holding computation within reasonable bounds is to eliminate all possible locations which are not strong contenders before computation begins. The desireability of a station location depends upon the sizes and the intersection patterns of aircraft clouds. As is well known, human capacity for pattern recognition greatly exceeds computer capabilities at the present time. This suggests that an interactive algorithm with the operator limiting the choices to promising regions, while the computer furnishes the detailed computations of the relative effectiveness of alternatives. At times we have employed this procedure.

2.1.4 Sensitivity to Aircraft Location and Mathematical Solution Method

At any given moment, we can think of the hierarchal aircraft distributed across CONUS as nodes. A graph can be defined on these nodes by placing an edge between every pair of aircraft which are within radio line-of-sight and synchronization range. We call the connected components of this graph clouds of aircraft. By virtue of the way the graph is defined, any two aircraft within the same cloud are assured that there exists a sequence of aircraft leading from the one to the other in which successive pairs are within radio line-of-sight and synchronization range. Thus, according to the conclusion of Section 2.1.1, if any aircraft in the cloud has contact with a master station, then all aircraft in the cloud are synchronized.

By virtue of the particular geometrical patterns developed in air travel over CONUS, different graphs will be developed at different times, so that any of the $2^{\binom{n}{2}}$ graphs possible on n nodes have some probability of occurrence.

Let p be the probability that two aircraft selected at random from the list of hierarchal aircraft active at a given moment are within radio line-of-sight and synchronization range. If aircraft were uniformly distributed over CONUS then to a close approximation

$$p = \pi R^2/A = 0.0125$$

where R is the synchronization range, and A is the area of CONUS. However, some parts of CONUS, (e.g. Maine, southern Texas, the Dakotas and Montana) have much lower densities than average. Since the effective area is somewhat lower than all of CONUS, and there are well known aircraft concentration points, we expect p to be higher than 0.0125.

In reference [3] an argument based on the law of large numbers is presented which shows that, when n is large, the number of edges (q) in the hierarchal graph (i.e. the number of air-to-air communication links) is given by:

$$q = n^2 p/2, \qquad n \text{ large} \tag{2-12}$$

This result is based on a model which allows an arbitrary distribution of aircraft over CONUS. The only requirement is the function which describes the instantaneous density distribution must be integrable.

Figure 2-5 presents an analysis of the value of p exhibited by air-carrier aircraft distributions at 68 points through the day and week. Superimposed is a close approximation to airline activity, see Figure A-1, relative to its peak value for these same points. The dashed line in Figure 2-5

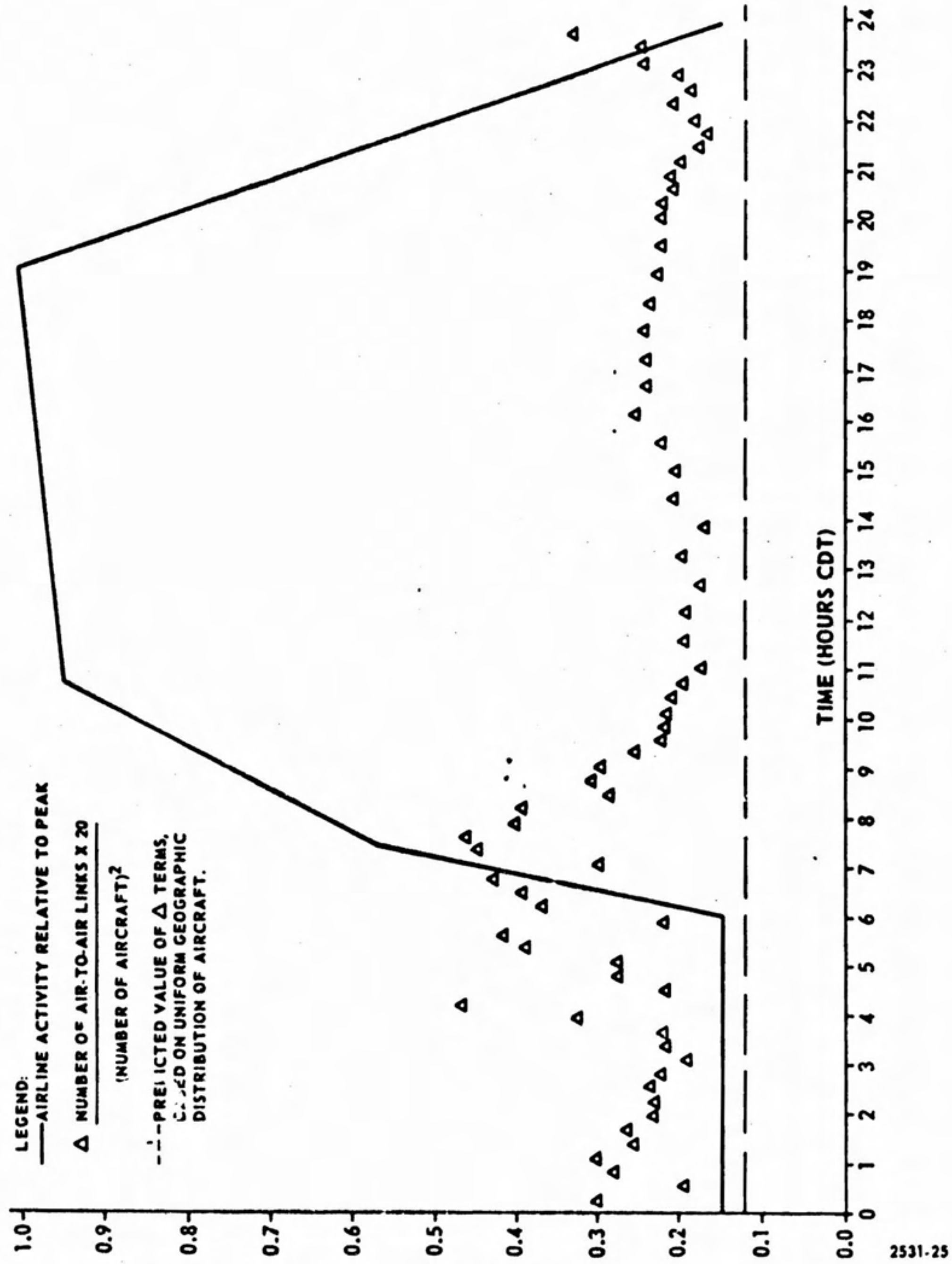


FIGURE 2-5. PROBABILITY THAT TWO RANDOM AIRCRAFT ARE LINKED AND AIRCRAFT ACTIVITY PROFILE

is at $p = 0.0125$, the uniform distribution value. As was expected, the value of p throughout the well-traveled portions of the day is somewhat higher about 0.021. Early morning traffic consists mostly of freighters flying between major city pairs. The concentration of aircraft along these major airways is reflected in a higher p (about 0.025) during this time. Beginning at about 7 o'clock Eastern Time, traffic in the eastern half of the United States quickly attains its full daylight value while the western half of the United States is, by comparison, bare of traffic. During this period, since the aircraft distribution is concentrated in only half the United States, p is about double that of the mid-day value.

In reference [5], E. M. Wright has shown that if n, the number of aircraft nodes, is large, then the probability, P, that all aircraft are in a single cloud is

$$P = 1 - ne^{-2q/n} \qquad (2\text{-}13)$$

where q is the number of edges in the graph. Thus, substituting from equation (2-12), we have the following expression for the probability that at a given instant, all active hierarchal aircraft are in a single cloud.

$$P = 1 - ne^{-np} \qquad (2\text{-}14)$$

Before discussing the application of this equation, it must be noted that Wright's formula, (2-13), is based on the assumption that all graphs with q edges on n unlabeled nodes are equi-probable. We must accept this hypothesis, because, without it, no comparable result is available. For one thing, this assumption requires that there be no insular centers of aircraft activity, in the sense that there is no air travel between them. Since the airlines provide point to point service, virtually arbitrarily throughout CONUS, we know that this is not the case. There are, however, instances which approximate this condition. We will, at least in part, compensate for this possibility by selecting a value of p which is a compromise between the less active portions of CONUS (<0.0125) and the aircraft population as a whole (≈0.025).

Figure 2-6 is a plot of the probability, as given by equation (2-14) with p = 0.015, that all hierarchal aircraft are in a single cloud as a function of the number of active hierarchal aircraft. The figure clearly shows a sharp threshold with respect to the efficiency of hierarchal chaining. According to this graph, when more than 750 hierarchal aircraft are active, complete communication linking of these craft is very likely. As the number decreases from 750, the likelihood of this condition fades rapidly. Figure 2-7 dramatically illustrates this phenomena. The situations illustrated are two of the 68 sample points referred to in connection with Figure 2-5.

If we chose for a reference the case when all aircarrier aircraft greater than 40,000 lbs./gross weight are hierarchal equipped, then the peak value in Figure 2-5, represents 1330 aircraft. Thus, more than 750 aircraft are active from 0722 to 2135 Central Time, and during this period we expect virtually all of these aircraft to be in a single communicating cloud. (See, for example, Figure 2-8, which is another in the series of sample points mentioned.)

Since the ordinate in Figure 2-5 is number of hierarchal aircraft and the abscissa is measured in hours, integration under the illustrated curve gives hierarchal aircraft hours. Therefore, we can obtain an estimate of the total fraction of hierarchal aircraft hours which would be synchronized by a single ground station by determining the area under the curve between 0722 and 2135, and dividing it by the total area under the curve. The number so obtained is 86%. Much more refined calculation based on computer modeling of air traffic activity, described in Section 2.2 and Appendix A (Figure A-23), yields 85% for this same percentage. Presumably the single station may miss a few aircraft hours during the high density periods, but it also picks up a few in the off hours, and these effects are nearly compensating.

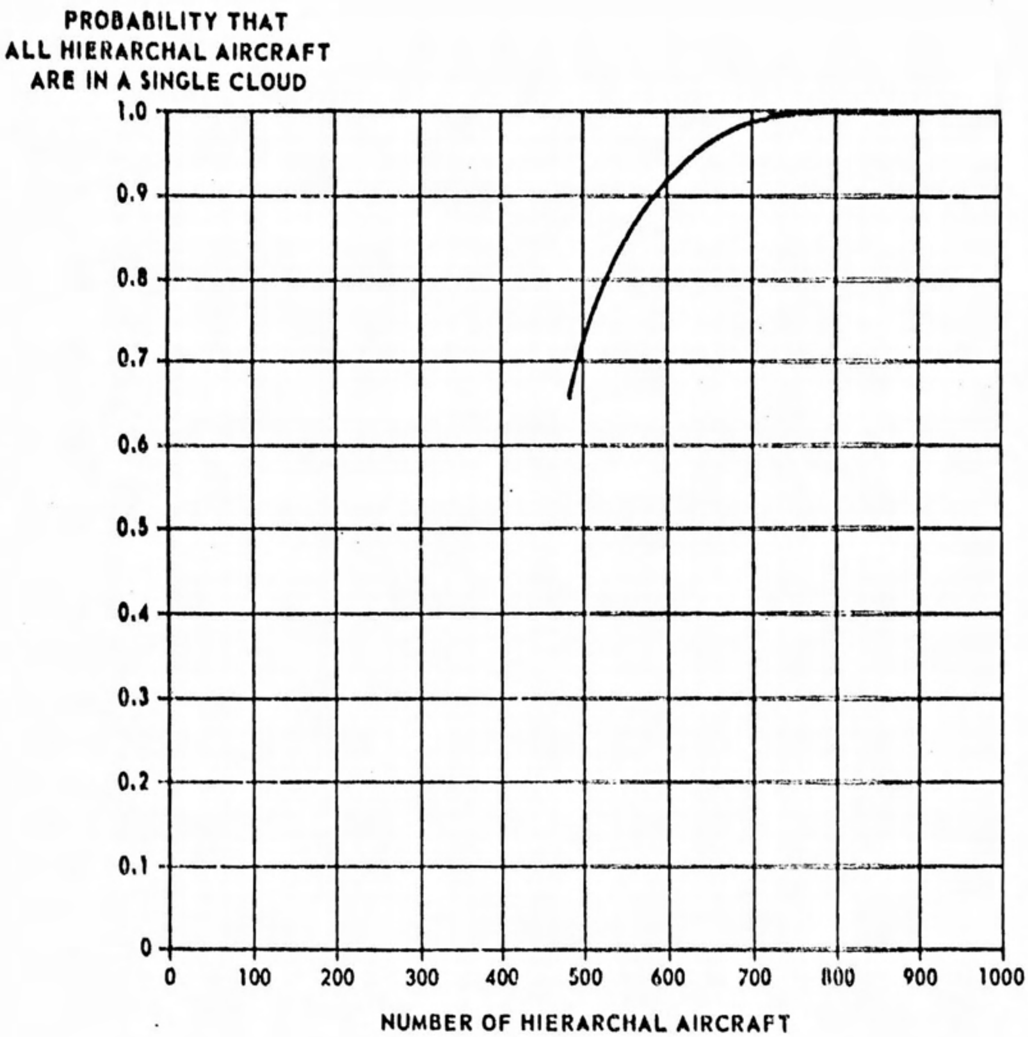


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FIGURE 2-6. DENSITY THRESHOLD FOR HIERARCHAL AIRCRAFT

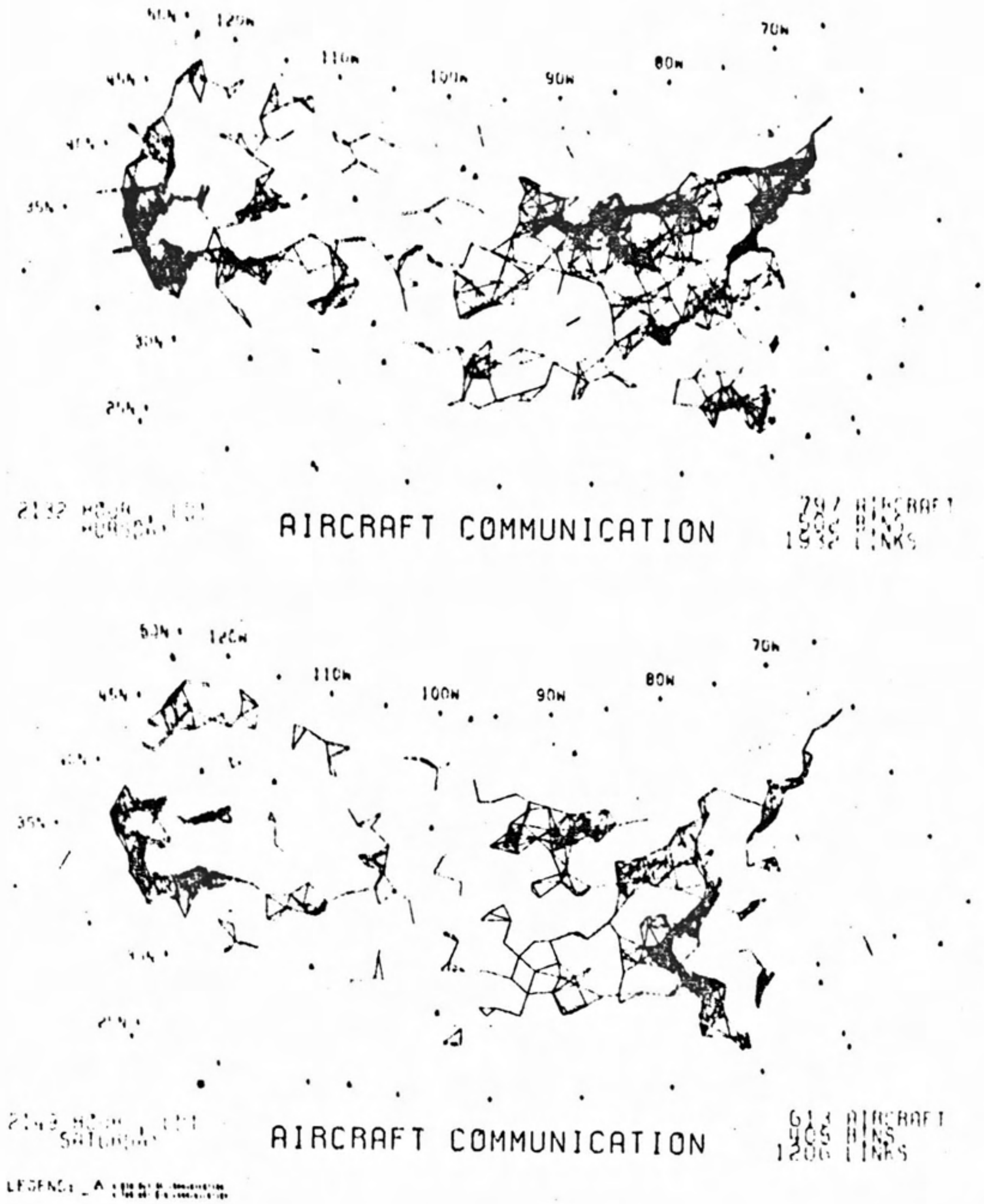

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FIGURE 2-7.

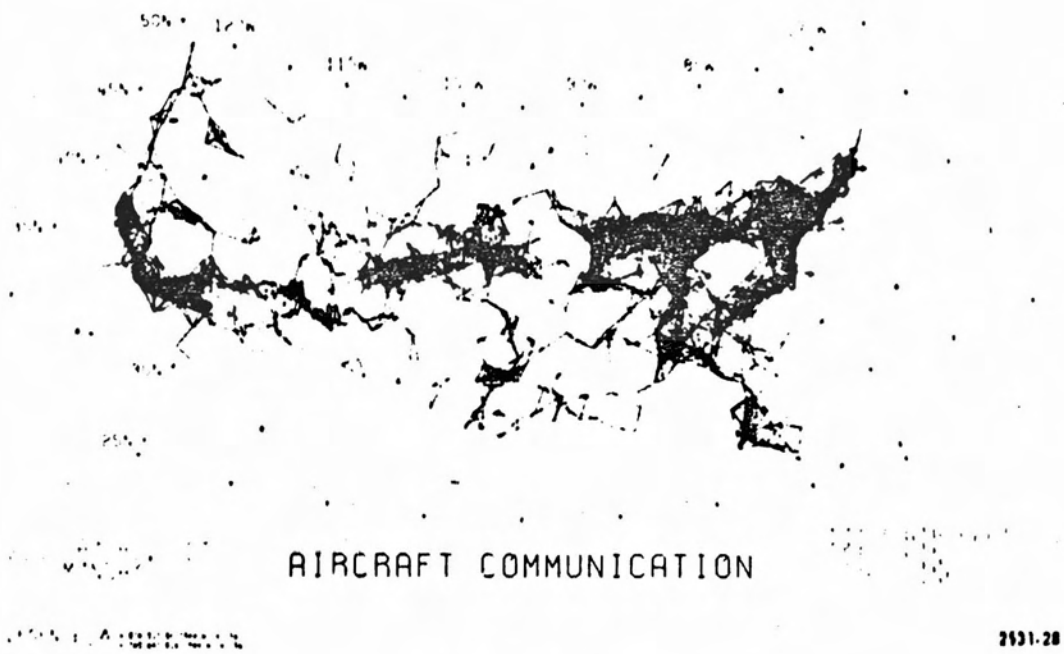

FIGURE 2-8.

During the transitions to and from nighttime traffic, i.e., 0600 to 0722 and 2135 to 2400 hours, the large cloud gradually fragments into several minor clouds centered on major hubs (e.g., Chicago, Los Angeles, Seattle, New York). On this basis, we expect that about 4 master stations could synchronize virtually all hierarchal aircraft during these periods. Repeating the integration for this new time period indicates that these stations would provide synchronization to 94% of the hierarchal aircraft hours. As before, more refined calculations based on computer modelling of air traffic activity produce a very similar number; namely, 93% (see Appendix A, Figure A-23).

Since we do not expect total aircraft distributions to be greatly different from hierarchal aircraft distributions, these same two numbers, 86% for one station, and 94% for four stations should also approximately equal the fraction of total aircraft hours synchronized; more refined calculations produce 84% and 90% respectively (see Appendix A, Figure A-25).

If we assume that the large single cloud of hierarchal aircraft which persists from 0722 to 2135 hours provides synchronization service to 90% of the CONUS land area, we reach the conclusion that on the average 53% of CONUS land area could receive synchronization from a single master station and 68% from 4 master stations. The more refined air traffic modelling numbers for these quantities are 54% and 67%, respectively (see Appendix A, Figure A-24).

By repeating the above calculations for different hierarchal aircraft equippages than the reference population assumed (1330 peak aircraft), we can derive a relationship between percentage of hierarchal aircraft and master station effectiveness. Figure 2-9 shows the result. The ordinate is the percentage of hierarchal aircraft hours that can be synchronized by a single master station. The abscissa is the percent of hierarchal equipped aircraft relative to the reference population. Three comparison points have been computed by the more refined method, at 100%, 85% and 67% relative equippage. In each case the derived values do not differ from those given in Figure 2-9 by more than 1%. The reference population is well positioned. Higher percentages of hierarchal aircraft involve diminishing returns. While small percentage reductions can be tolerated, e.g. due to an airline out of service, lower percentages involve rapid fall-off in effectiveness.

All the results quoted in this section have been derived with no reference to specific aircraft locations. Throughout, comparisons have been made both graphically and numerically, with results obtained by a methodology that locates all aircraft to within ± 15 n.m. No significant differences can be discerned. Furthermore, it has not been necessary to refer to specific master station locations except in broad regional terms. Whereas, the more refined method selects the best station sites from 3300 alternatives. These facts are compelling evidence that neither specific hierarchal aircraft positions nor master station sites have great influence on time dissemination effectiveness. The ruling parameter is hierarchal aircraft density.

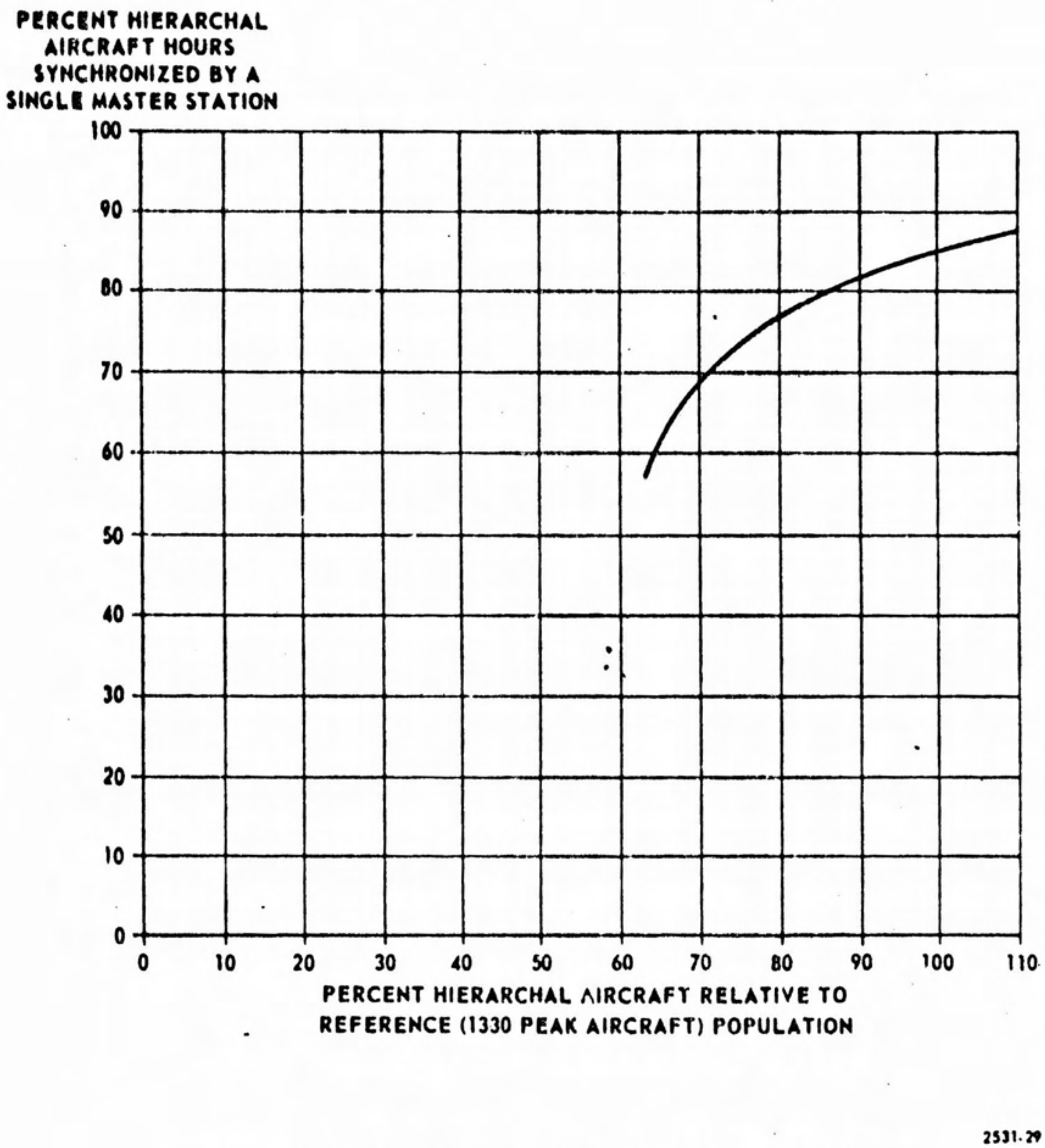

FIGURE 2-9. DEPENDENCE OF MASTER STATION EFFECTIVENESS ON HIERARCHAL AIRCRAFT DENSITY

2.2 Computer Processes

Extensive use of digital computer data processing was employed to implement the analytical processes described in Section 2.1. In addition to the ability to process the large data base needed to represent the entire CONUS air traffic, the computer programs provide reference and summary data that give the user an overview of the properties and patterns that lead to the results. Thus, the user is provided information necessary for effective interaction in the site selection and trade-off process. The programs are applicable to master station only synchronization, and functions such as VORTAC or radar surveillance, and master stations operating with any level of air-to-air communication support.

The major computer tasks were to:

(a) Model, in four dimensions throughout CONUS, the patterns of hierarchal aircraft which may establish communication chaining to relay time from a master station.

(b) Model the communication/synchronization process among the aircraft hierarchal equipment and ground-based master stations.

(c) Determine what portion of CONUS airspace receives synchronization as a result of a given hierarchal and master station pattern.

(d) Apply worth rules to quantitatively evaluate the utility of supplying synchronization to each specific cell of airspace, at specific times of the day and week. To compute some worth rules a four-dimensional model of the total aircraft population, including non-hierarchal aircraft must be provided.

(e) Assess the effectiveness of competing potential sites and apply automatic or user interactive selection algorithms to select the best sites.

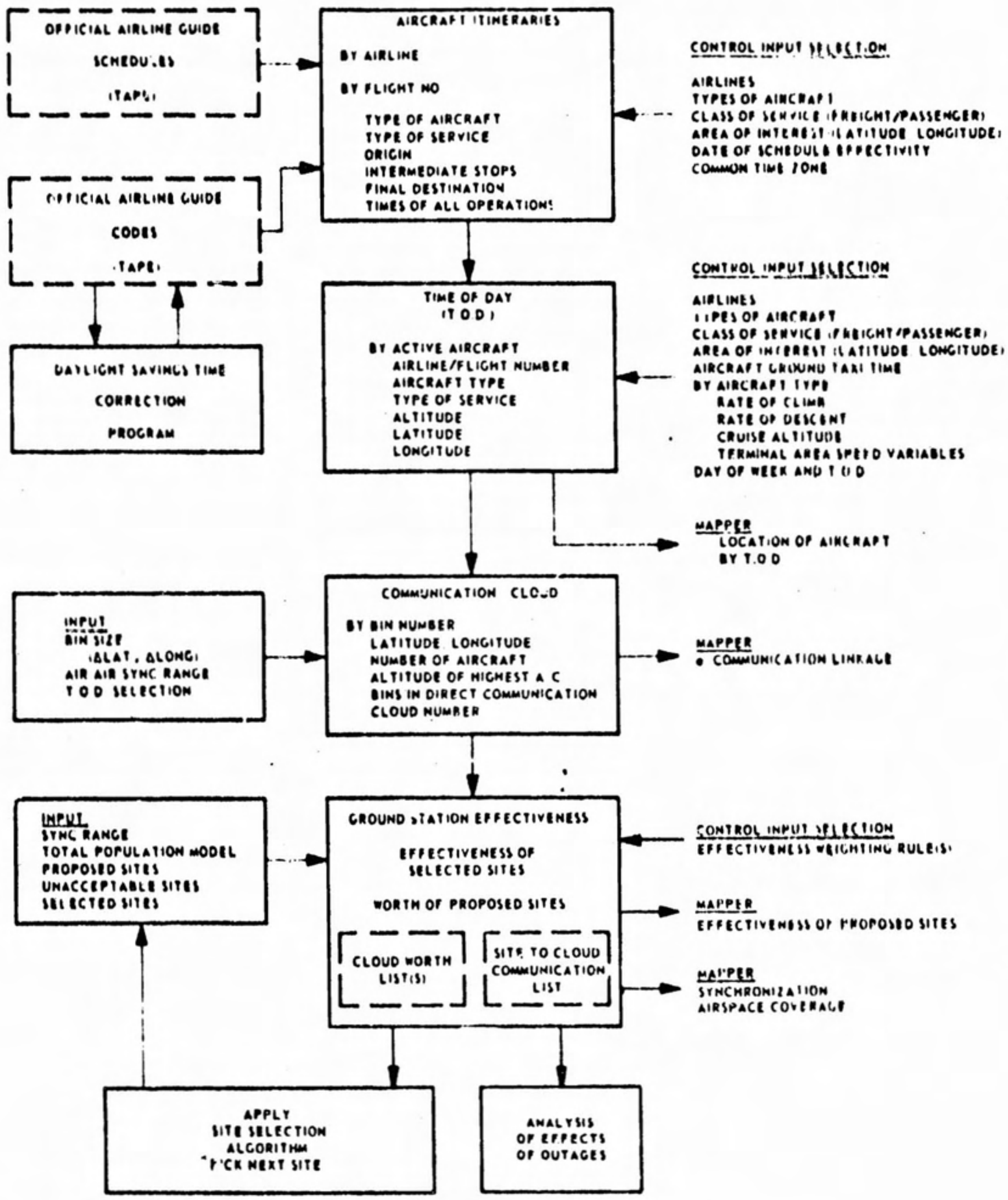

FIGURE 2-10. ORGANIZATION OF COMPUTER PROCESS

These tasks were accomplished by means of a series of programs listed in Figure 2-10. Each provides necessary input data to the next program in addition to direct user reference outputs. Overall functions, key features and samples of outputs are described in the following sections.

2.2.1 Aircarrier Aircraft Itineraries

This program performs initial selection and sorting of the Official Airline Guide data set which covers world-wide operations. Latitude, longitude of the departure and arrival airports are written into the record of each leg of a flight, and all legs of each specific flight are collected and linked to form a common itinerary record.

2.2.2 Hierarchal Aircraft Location Versus Time-of-Day (T.O.D.)

To determine the spatial patterns of communication as a time variant, it is necessary to produce a one-for-one model of the flight and ground operations of each aircraft involved in the process of using and/or disseminating time synchronization. Basic considerations were choice of lateral flightpath; correct cruise speed, speed reduction in the terminal area; time of the day at origin, intermediate and final airports; and altitude profiles as they affect line-of-sight cutoff of RF communication.

Altitude - With the variations in aircraft performance and effects of ATC, it was decided to input average climb rate, descent rate, and cruise altitude for each type of aircraft, Figure 2-11. Although most cruise altitudes are well above typical line-of-sight cutoffs (e.g., air-ground range of 98 nautical miles requires an aircraft altitude of only 8,500 feet), it was decided to include a full modeling of climbout, cruise, and descent for continuity in modeling and possible future applications. On short flights where there is insufficient time to achieve cruise altitude, the profile goes directly from climbout to descent. All the values selected, particularly

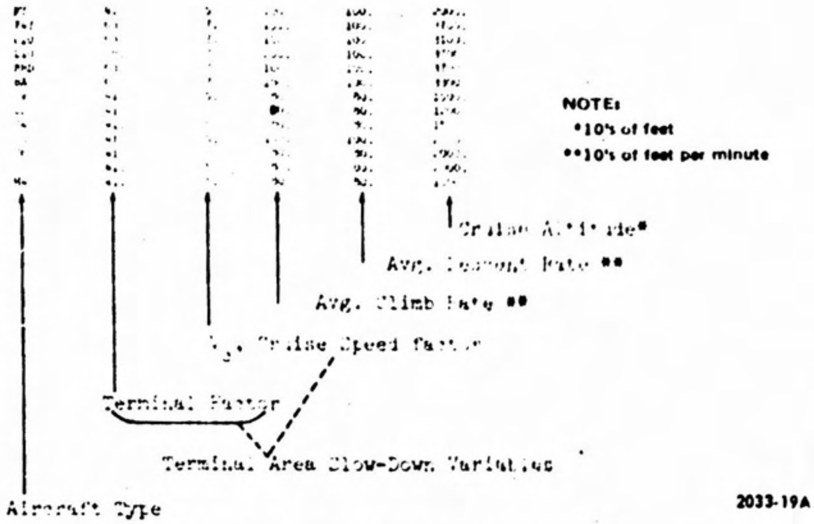

FIGURE 2-11. AIRCRAFT/ATC PERFORMANCE VARIABLES

descent rate, were based on knowledge of normal ATC operations as well as aircraft performance.

Another factor in the altitude and speed-along-route models is time on the ground during taxi time or time at an intermediate stop of a multi-legged flight. In our model, the aircraft synchronization equipment becomes active at gate time at the first airport of an itinerary: the time listed in the OAG. Take-off occurs and climbout starts at gate time plus taxi time; touchdown occurs at the next airport gate time minus taxi time. Taxi time is an input variable, which was set at 5 minutes. This choice was made on the basis of average operating experience of the major aircarriers. Although the taxi time selected is applied uniformly to all operations at all airports, it is considered representative of the general case. Where long taxi times are experienced (e.g., JFK, ORD, or LAX), aircraft densities will be correspondingly high and the shorter taxi time in the model will have little effect.

Lateral Flightpath - Using the latitude, longitudes of the departure/arrival airports and takeoff/touchdown times, a linear interpolation of latitude, longitude is used to approximate a great circle route for north/south flights and shorter east/west flights. On the longer flights there are two additional considerations; the flight path bowing effects of winds aloft vs. flight cost optimization and the fidelity of modeling a great circle route.

On the basis of an airline practice of not applying least cost optimization on flights less than 900 miles, our computer programs make a similar determination on the basis of longitude. When separation between origin/destination airports is less than 15° delta longitude, a linear interpolation is used for both directions. For flights of 15° delta longitude or greater the linear interpolation is used on the east-bound flights, this results in a bowing to the south. For west-bound flights, a simple quadratic approximation of a great circle route, reference [1] - Section 4.3.2, was adjusted to produce bowing to the north, Figure 2-12. On even longer intercontinental flights, where the delta longitude is greater than 53°, the great circle algorithm is applied to all flights. Specifically, this provides a good approximation of sub-polar flights between CONUS and Europe or Asia.

Outputs from this model were reviewed in relation to airway route structures. In general, the model produces aircraft paths that are representative of preferred routes or plausible airway-route selections for nominal weather conditions. Deviations due to dog-legs in VOR/Tacan airways are small compared to the variety of choices of airways and the scope of modeling the entire CONUS in a single overview.

Speed Along Route - Consideration was given to the effects of slower speeds in the terminal areas. An unmodified linear interpolation of position, relative to takeoff/touchdown times, would represent the aircraft as spending more than actual time enroute, providing more time dissemination capability to these intermediate zones than a more accurate model. Thus, a terminal area slow-up has been included. Applying a simple quadratic, adjusted to aircraft-ATC performance characteristics, by aircraft type per Figure 2-11, the average radial speed of a jet aircraft to and from the airport is modulated as shown

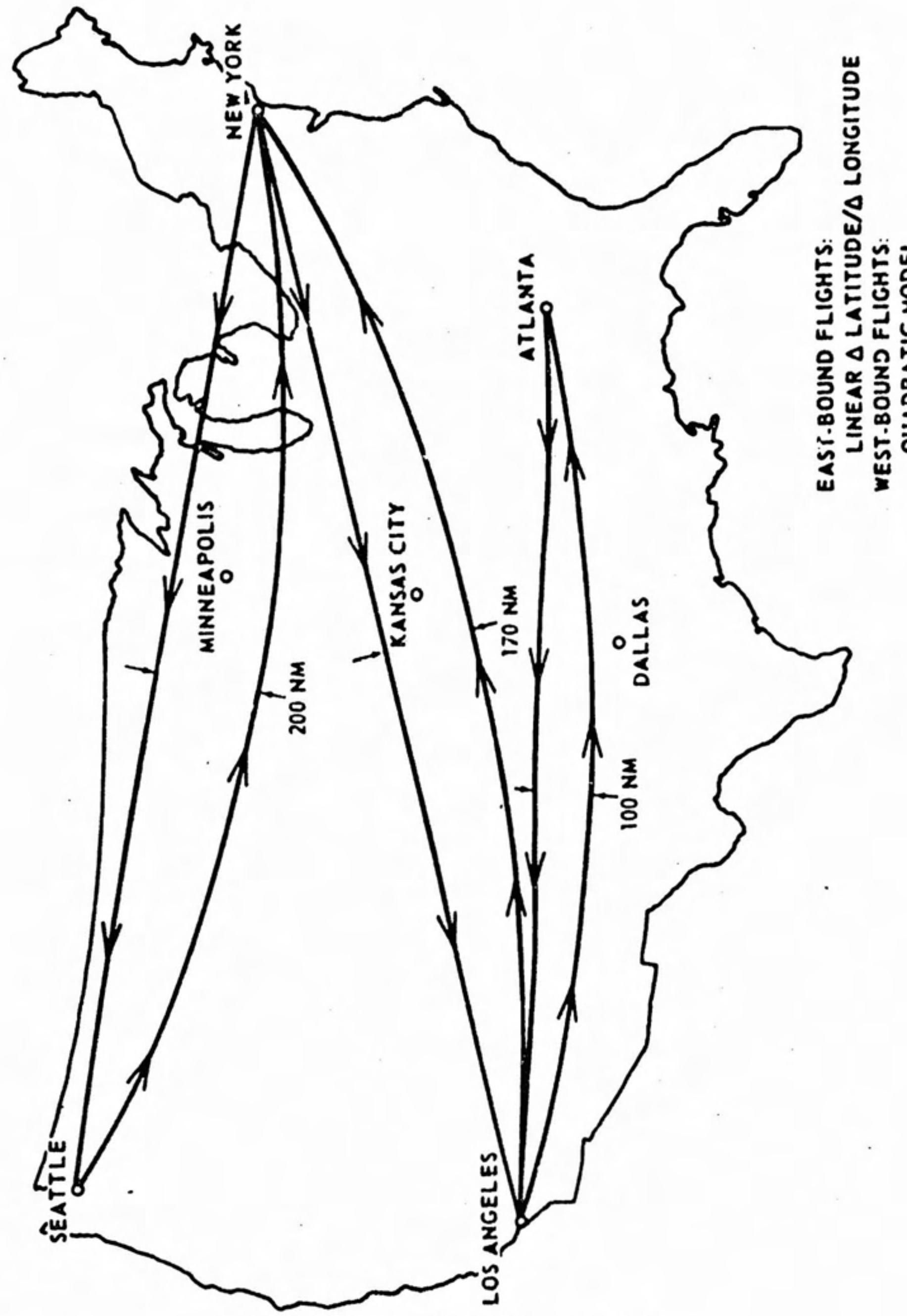

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FIGURE 2-12. FLIGHT PATH MODELING

in the example of Figure 2-13. Additional details are provided in reference [1], Section 4.3.3.

Thus we have the capability to determine the location of each aircarrier aircraft at any selected time of any selected day. Figure 2-14 is a sample printout of the record written for each aircraft. In addition to being supplied to succeeding computer programs, this is mapped as shown in Figure 2-15.

2.2.3 Hierarchal Communication Linkage and Clouds

In order to conserve computer processing time, the population model of hierarchal aircraft is simplified and condensed by redefining the overall discrete 1:1 population as unit populations of geographic cells. This means that a small latitude, longitude bin has been communication-modeled as a single communicator, at the center of the bin at an altitude equal to that of the highest hierarchal aircraft in the bin. In Phase I, reference [1], it was demonstrated that $0.5^\circ \times 0.5^\circ$ latitude, longitude bins provided adequate resolution, yet afforded worthwhile savings in computer costs. Using a latitude, longitude bin address notation, a file is created for each occupied bin. For each bin, the number of hierarchal aircraft and the altitude of the highest aircraft is recorded. Then in a roll call, latitude bin number by longitude bin number, communication checks are made relative to all other occupied bins.

When communication exists it is noted in the records of the respective bins. Then, after all bins have been tested, the cloud definitions and relationships are determined. The computer program sequentially assigns a cloud number and tests for bins that have either direct or indirect communication linkage. The respective cloud number is written in the record of each bin and a table is established to record the overall characteristics of each cloud. For visualization of these results a map output, Figure 2-16, is provided.

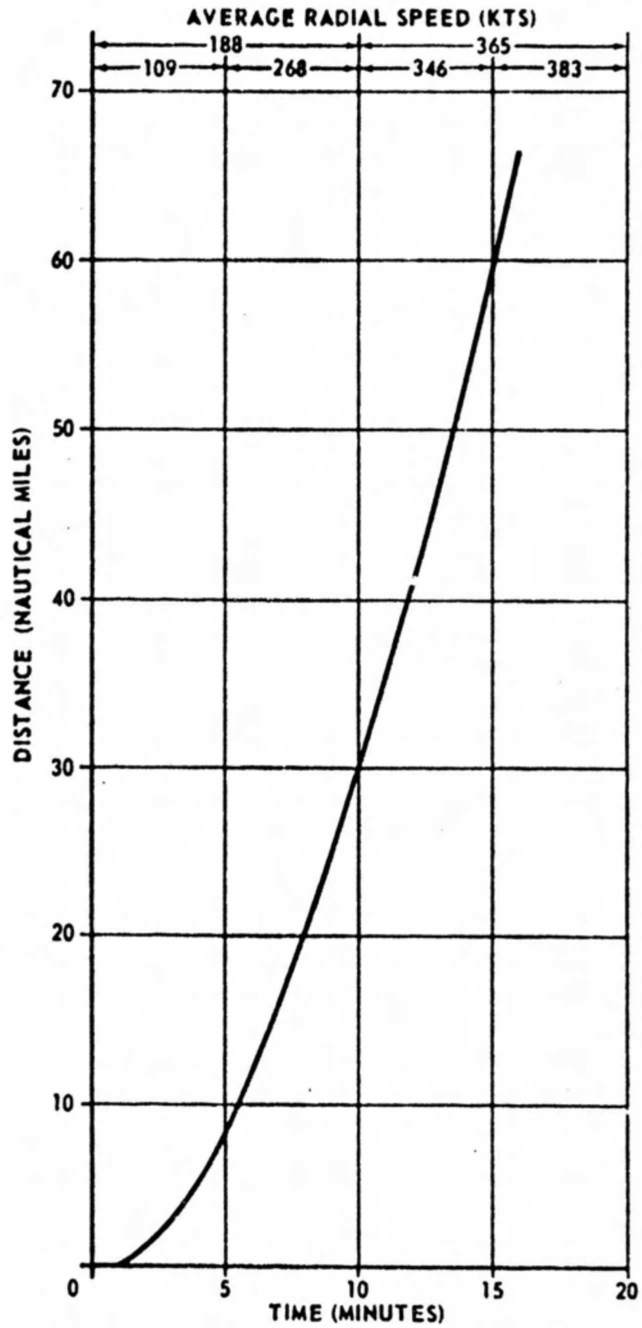

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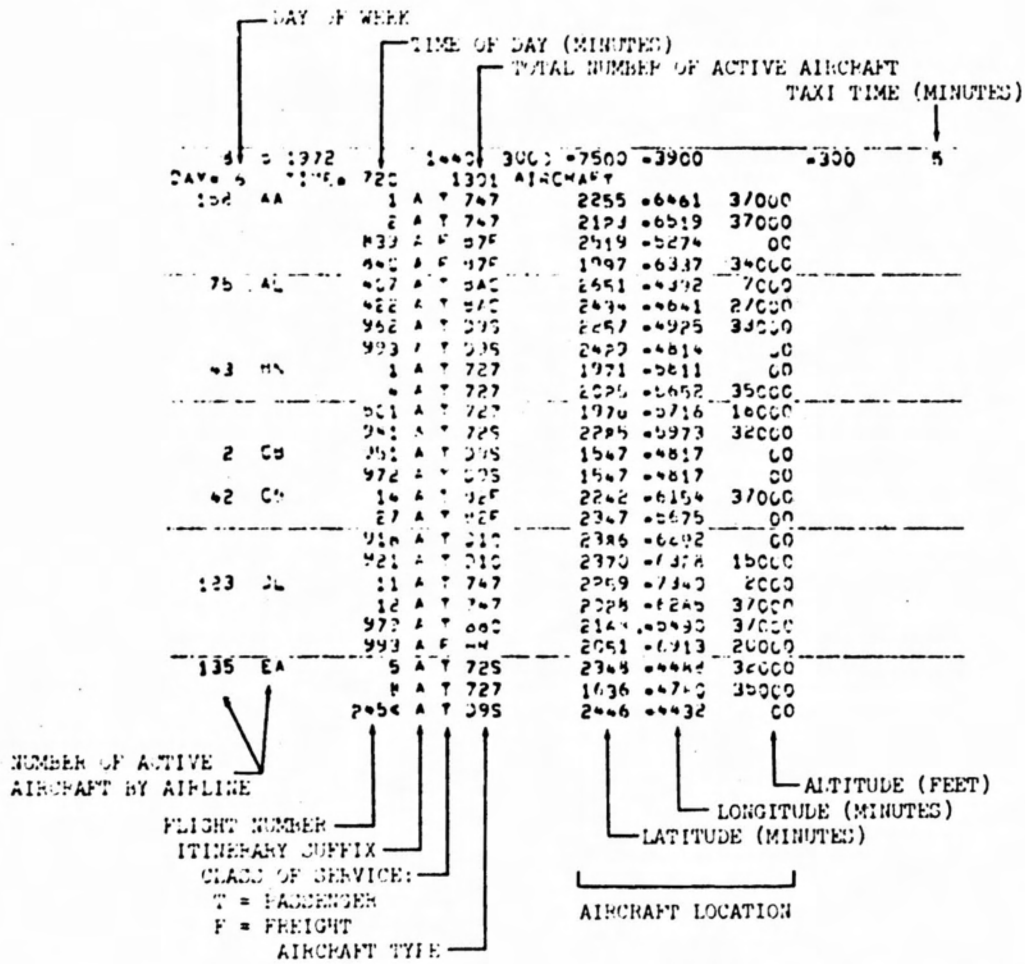
FIGURE 2-13. DISTANCE FROM TAKEOFF/TOUCHDOWN VS. TIME



NOTE: ONLY FIRST TWO AND LAST TWO RECORDS PER AIRLINE ARE PRINTED OUT AS SPOT CHECKS; SAMPLE DATA.

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FIGURE 2-14. AIRCRAFT LOCATION VS. TIME OF DAY

FIGURE 2-15.

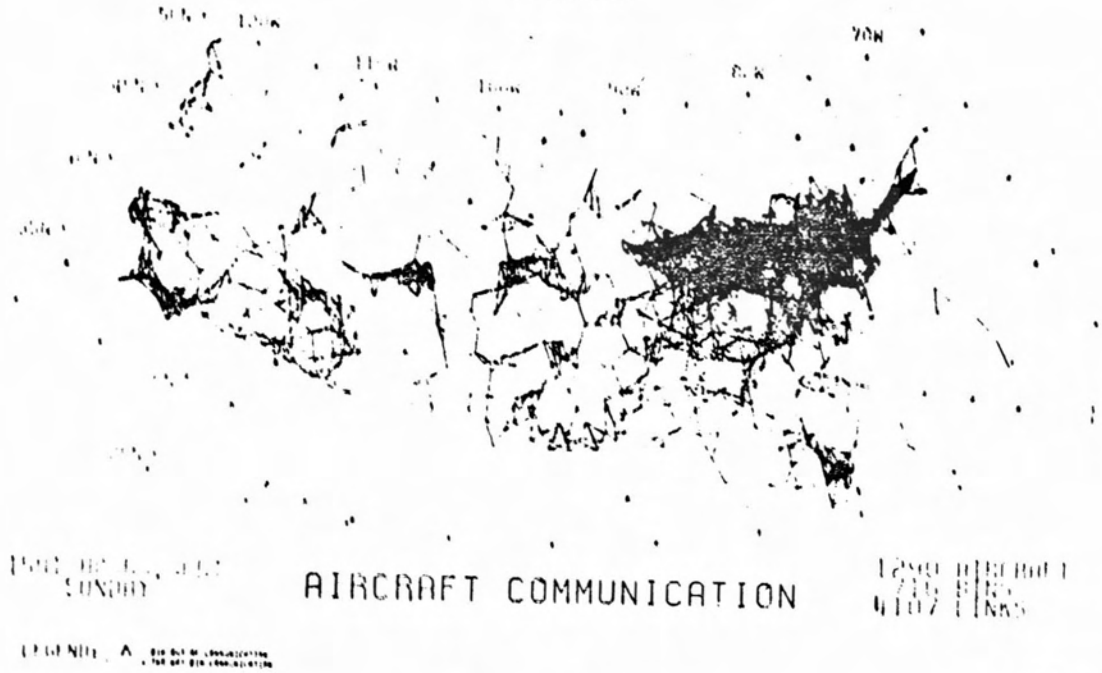

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FIGURE 2-16.

2.2.4 Airspace and Total Population Modeling

Airspace above the CONUS has been subdivided into three-dimensional cells for the purposes of determining communication patterns and producing a quantified total population model that is readily adaptable to computer modeling.

Altitude strata vertically divides the space in the same manner that CONUS area has been subdivided into latitude longitude bins. The cells that a ground-based unit can communicate with are determined by the following simple range test.

Bin to Bin Range	Strata Within L.O.S.	Altitude
Zero	#0 and up	Zero and up
Less than 45 n.m.	#1 and up	1K ft. & up
Less than 74 n.m.	#2 and up	3K ft. & up
Less than 98 n.m.	#3 and up	8K ft. & up

As shown in Figures 2-17 and 2-18, this provides an approximation of earth curvature line-of-sight cutoff. The communication of hierarchal aircraft is modeled in a similar manner.

Cell Population - This modeling was developed during Phase II, and is described in detail in reference [2], Appendix A. Simply, a two-dimensional peak instantaneous latitude, longitude distribution of the total aircraft population is supplied as an input to our program. For a specific time-of-day, the local respective bin peak value is modulated on the basis of a local activity, which is a function of local time. The resulting bin/area population is then distributed vertically in accordance with FAA aircraft vs. altitude profiles, reference [2] Appendix A. Thus for each time snapshot there is a modeled total aircraft population for each cell of CONUS airspace.

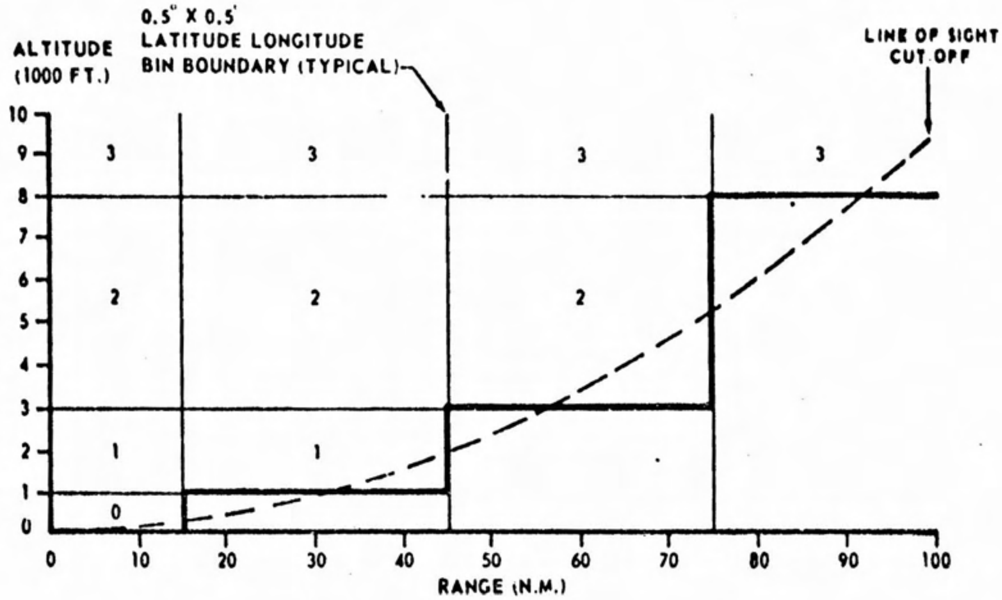
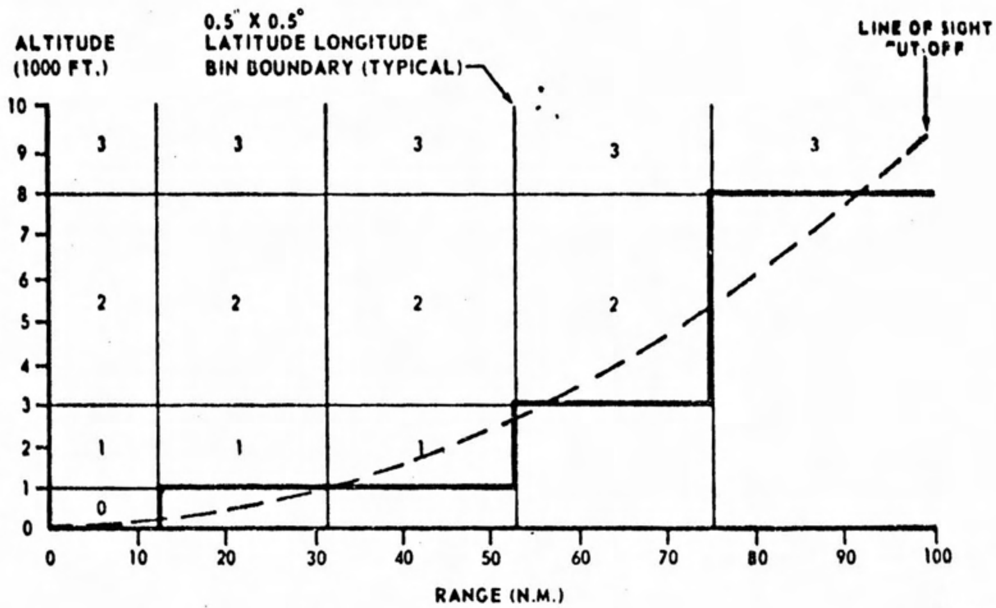


FIGURE 2-17. AIRSPACE CELLS IN NORTH-SOUTH DIRECTION



2314-2A

FIGURE 2-18. AIRSPACE CELLS IN EAST-WEST DIRECTION (45° LATITUDE)

2.2.5 Worth Computation

Each of the cells of airspace, described in the preceding section, have a unique worth value in terms of the utility of supplying synchronization to that cell. As described in detail in Section 2.1.2, four worth rules for evaluating this utility have been programmed. These are:

- I The number of hierarchal aircraft
- II The airspace synchronized
- III The total aircraft (hierarchal and non-hierarchal)
- IV The encounter exposure; proportional to the square of the total aircraft population density.

Based on selected rule and population model, a worth term is calculated for each cell, for each time snapshot, as shown in the example of Figure 2-19(a).

2.2.6 Master Station Site Evaluation and Selection

For a given time-snapshot, the potential effectiveness of a ground station site is the sum of the worth values of all cells within communication range, and the worth of any clouds of hierarchal aircraft that are within direct communication range of the site, Figure 2-19b. As described in reference [1], Section 3, a potential ground station site can acquire the total worth of a cloud by being within communication range and L.O.S. of any hierarchal aircraft member of the cloud.

The overall effectiveness of a site is the sum of the effectiveness subtotals for all of the snapshots in the data base. These totals are accumulated for each site and recorded in the computer programs for mapping of the current relative effectiveness and automatic selection of the "best" site,

2-19(a) CELL WORTH LIST

CELL LOCATION		ALTITUDE STRATA			
LAT.	LONG.	3	2	1	0
11	82	71	54	39	8
11	83	143	108	79	17
11	84	143	108	79	17
11	85	359	271	199	43
11	86	359	271	199	43
11	87	575	436	319	69
11	88	575	436	319	69
11	89	143	108	79	17

RESPECTIVE CELL WORTH VALUE

2-19(b) COMMUNICATION LIST SITE TO CLOUD

SITE NO.		IDENT NUMBER OF CLOUDS		
LAT.	LONG.			
8	85	3	0	0
8	86	3	0	0
8	87	1	3	0
8	88	1	3	0
8	89	1	3	0
9	51	0	0	0
9	52	0	0	0
9	53	0	0	0

2-19(c) CLOUD VALUE LIST

1	56637	2	0	3	34783	4	0	5	2076
6	1383	7	36380	8	54196	9	19756	10	10640

CLOUD NO. — VALUE

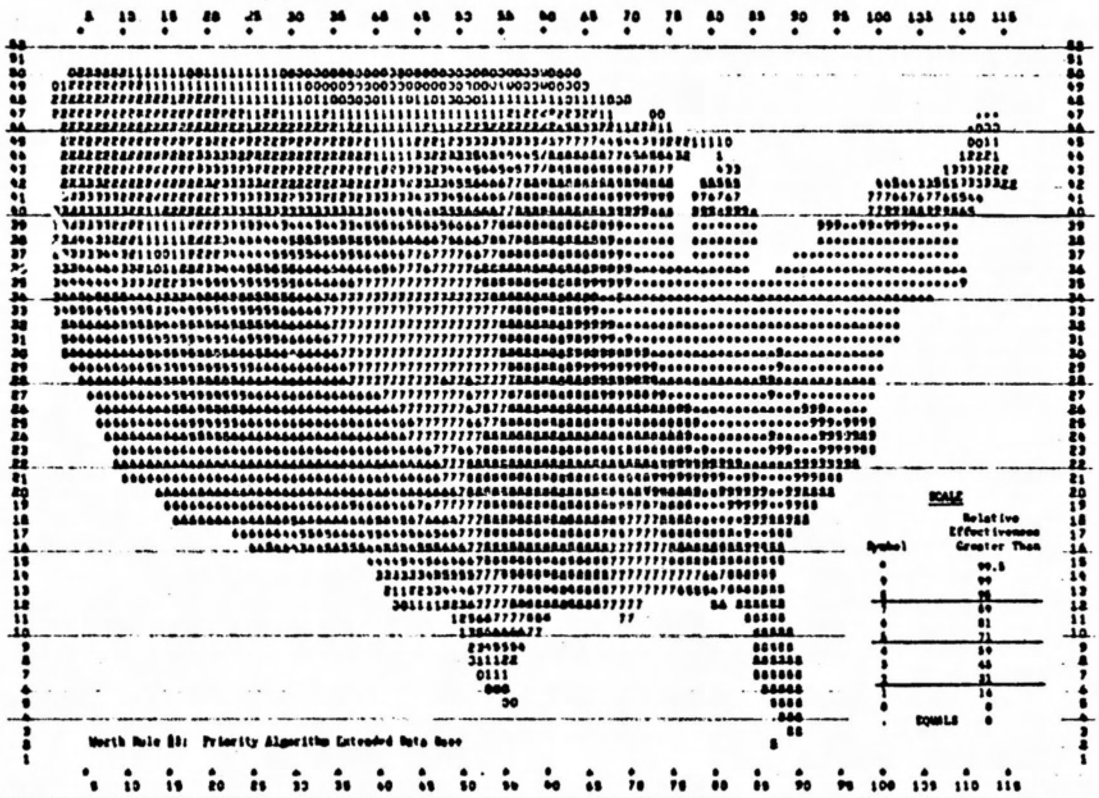
2-19(d) POTENTIAL SITE VALUE LIST

SITE NO.		POTENTIAL WORTH
LAT.	LONG.	
8	85	34354
8	86	35429
8	87	92066
8	88	91905
8	89	91420
9	51	21468
9	52	30508
9	53	47810
9	54	62365
9	55	67384

2314-3A

FIGURE 2-19. INTERMEDIATE COMPUTER OUTPUTS

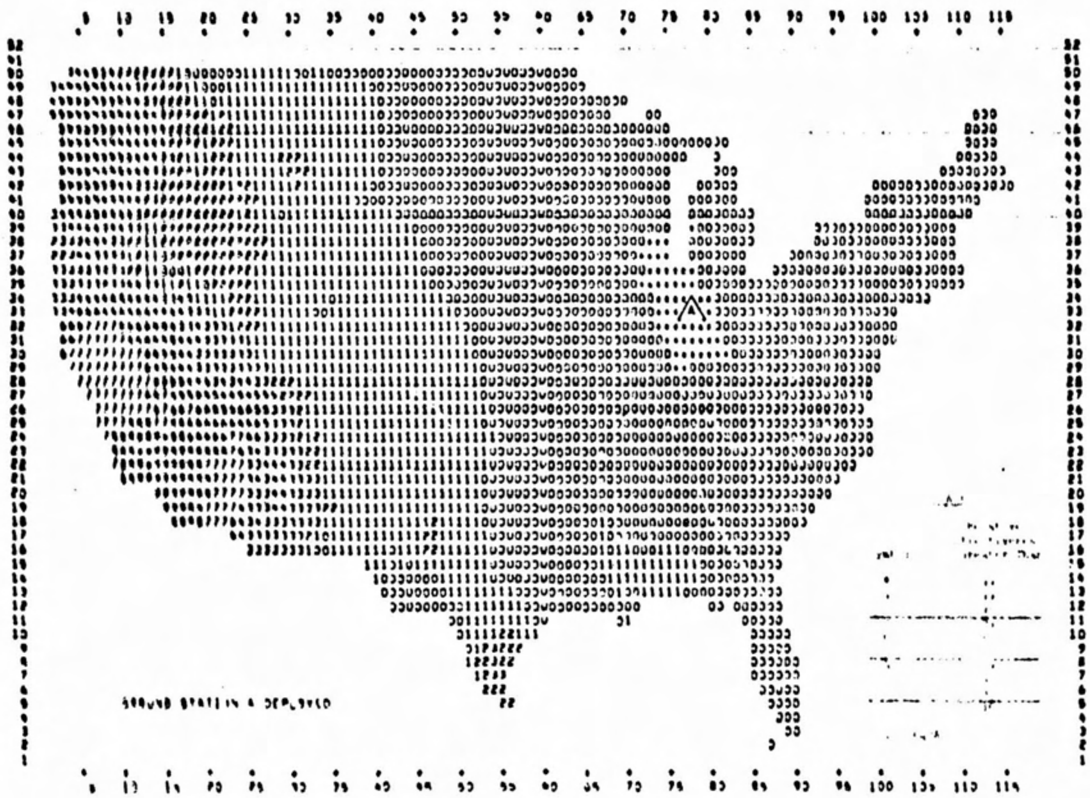
or a preselected choice of the user. The map, as shown in Figure 2-20, is always normalized to the "best" site as 100 and the alternate sites are presented on the non-linear scale shown in the figure. The total value acquired by the selected site, time snapshot worths and clouds captured are also output listed.



2314-77

FIGURE 2-20. RELATIVE EFFECTIVENESS MAP PRIOR TO FIRST SITE SELECTION

After a site is selected the effectiveness values for all the remaining sites are adjusted. Sites, which are within communication range of cells of airspace or clouds of hierarchal aircraft acquired by the last selected site, have their effectiveness value recomputed to reflect the downward change in utility due to coverage provided by the preceding selection(s). Once again the maps are normalized and the automatic priority selection chooses the "best" site, Figure 2-21. Where "best" is determined on the basis of providing the most new coverage.



2314-78

FIGURE 2-21. RELATIVE EFFECTIVENESS MAP AFTER FIRST SITE SELECTION

2.2.7 Program for Analysis of the Effects of Master Station Outage

As shown in the data processing flow diagram, Figure 2-10/page 2-29, output from the site selection program, "Ground Station Effectiveness", is saved for use in the Outage Analysis. This program evaluates the effects of all combinations of single and dual outages of all the master stations in the deployed set. This information is computed for each time snapshot, summed for the total data set and presented in a summary printout, as shown in Figure 2-22.

NOTE: ALL VALUES ARE PERCENTAGE REDUCTION IN EFFECTIVENESS UNDER STATED OUTAGE CONDITIONS

CONCURRENT OUTAGE \ INITIAL OUTAGE	A	B	C	D	E
B	2.54				
C	1.83	4.36			
D	0.69	2.63	1.92		
E	3.04	2.51	1.80	0.66	
NONE	0.30	2.24	1.53	0.39	0.27

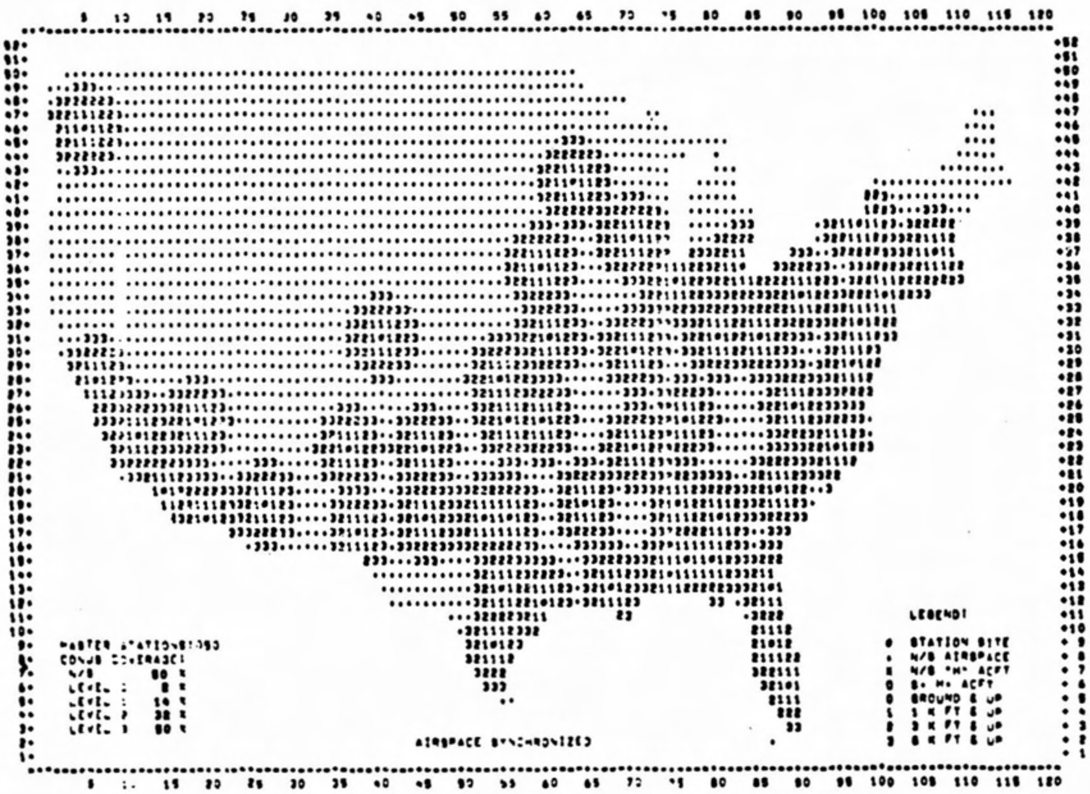
2531-12

FIGURE 2-22. REDUCTION IN EFFECTIVENESS DUE TO STATION OUTAGE

With this data and an assigned value of availability for each master station the mean effectiveness can be determined, in accordance with equation (2-11).

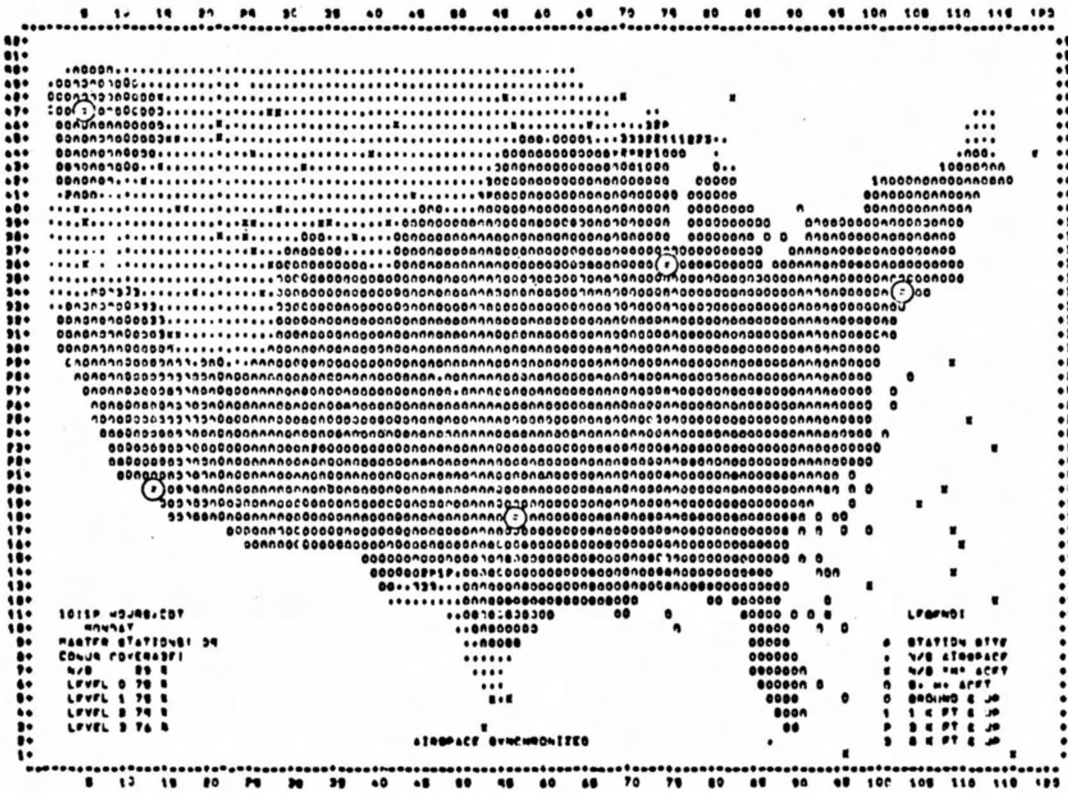
2.2.8 Synchronization Coverage Mapper Program

As a graphic aid to the user, this program determines by altitude strata the areas of CONUS that are synchronized. For each selected time snapshot and deployment of master stations the computer performs a communication range and line-of-sight check to all cells of airspace over CONUS. The line-of-sight test checks eart' curvature cutoff as a function of the altitude of the hierarchal aircraft. Output consists of a map showing the synchronization status of each latitude longitude bin and a summary, by altitude strata, based on percentage of the CONUS surface. Samples are shown in Figures 2-23 and 2-24, with and without hierarchal aircraft.



2581-1

FIGURE 2-23. MAP OF SYNCHRONIZED AIRSPACE
FIFTY MASTER STATIONS WITHOUT HIERARCHY SUPPORT



2314-6

FIGURE 2-24. MAP OF SYNCHRONIZED AIRSPACE
FIVE MASTER STATIONS WITH HIERARCHY SUPPORT

3. CONCLUSIONS AND RECOMMENDATIONS

Section 1.3 summarized the significant findings and conclusions of this study and Section 2 described the methods used to arrive at these results. The specific outputs of these methods which support each finding and conclusion are too extensive for inclusion in this summary document. A survey of these outputs has been prepared and appears as Appendix A of this report. Sufficient detail has been included in Appendix A to illustrate the evidence supporting each conclusion. However, in the interest of brevity, explanatory narrative is held to a minimum. The reader is encouraged to review this material and, if necessary, utilize the cross reference between this Final Summary Report and references [1], [2] and [3] provided as Appendix B to obtain more complete explanations.

In this section, the major conclusions and recommendations of the study and the rational leading to them are presented in some detail.

3.1 Master Station Sites

The most significant conclusion of this study is that, with adequate hierarchal support, only a few master stations are needed to attain high levels of coverage and the precise placement of these stations is not critical. We are in the comfortable position of being able to make refined choices between alternatives, all of which fulfill basic siteing requirements.

Considering all the evidence developed during the study we believe the best course of action is to install master station in order at: Chicago (ORD), Los Angeles (LAX), Seattle (SEA), Dallas (DFW) and New York (JFK). Concurrently, we recommend that U.S. certified aircarrier aircraft greater than forty thousand pounds gross weight be equipped with hierarchal capability.

When these two steps have been taken, approximately 90% of total aircraft hours will have synchronization available. The worst drop in coverage due to a single station outage (LAX) or an airline out of service (UAL) is little more than 2%.

Other performance indicators are:

1. 93% aircarrier hour coverage
2. 64% average CONUS landmass coverage, and
3. 95% coverage of NTSB recorded collisions; 1964 to 1971.

The order selected produces better coverage at each successive stage of implementation than any other possible ordering.

The most attractive alternative set of sites that have been identified is shown in Figure 1-2. This set achieves 1.6% higher total aircraft hour coverage than the set of Hubs and has equivalent station and airline outage characteristics. The authors feel, however, that the superiority of the Hubs with respect to operational and economic considerations such as;

1. Uninterrupted 24-hour service to high traffic areas
2. Superior service to aircarrier aircraft
3. Utilization of available government sites and service
4. Superior coverage during early phases of introduction, and
5. Permanent significance in air travel patterns

more than offsets their small deficiency in total aircraft hour coverage.

Our studies have shown that after four ground stations have been deployed, nearly all the time distribution potential of hierarchal relay has been utilized. For the most part, succeeding stations provide service through direct synchronization and single hierarchal relay. Their contribution to effectiveness per station is nearly negligible compared to that obtained by the first four stations through hierarchal chaining.

Since master station selections at this level mainly serve the local population, their relative effectiveness is very sensitive to the local distribution of aircraft. Although small in absolute terms, the effectiveness gained per site can vary by factors of three or more. Because we are unable to infer precise future distributions of aircraft on the local scale required, we feel further siteing judgments should be based on on-site, real-time evaluation of synchronization needs. The fifth station recommended above, has been selected as the best suited for maintaining high levels of service during periods of master station or airline outages. Since four stations are required to fully utilize hierarchal chaining, its purpose is to provide the needed redundancy.

Fortunately, the five master stations which we feel we can justifiably recommend, provide a very high, and possibly adequate, level of service.

While we have recommended that aircarrier aircraft greater than forty thousand pounds gross weight be hierarchy equipped, equal performance could be obtained from any collection of aircraft which exhibit equivalent or superior numbers of airborne aircraft and CONUS wide distribution. However, it is difficult to see how any alternate collection of aircraft can compete with airliners for this purpose. The recommendation requires that 2300 airline aircraft be hierarchal equipped. Because of the high utilization of airliners, more than half of these are active in CONUS during most of the day. The airline mission insures a consistently wide distribution across CONUS. They can, for the most part, be counted on to repeat this performance daily without significant variations.

Given that airline aircraft are the most logical choice to provide hierarchal service, there still remains the question of the optimum equipage level. The recommended level of equipage provides nearly the full potential of hierarchal chaining; increasing the level does not yield significant returns in performance. A fifteen percent lower level of equipage also can provide nearly all the benefits of hierarchal chaining (a loss of about 2%), but this margin is needed when an airline goes out of service.

We have based our recommendation mainly on a trade-off between the number of ground stations required to obtain satisfactory coverage and the level of hierarchal equipage. By reviewing the various levels that have been analyzed in the course of this study, and the associated number of master stations required to furnish greater than 90% aircraft hour coverage, we can estimate the slope of the equi-performance curve, Figure 3-1, in the neighborhood of the recommended level. These estimates indicate that

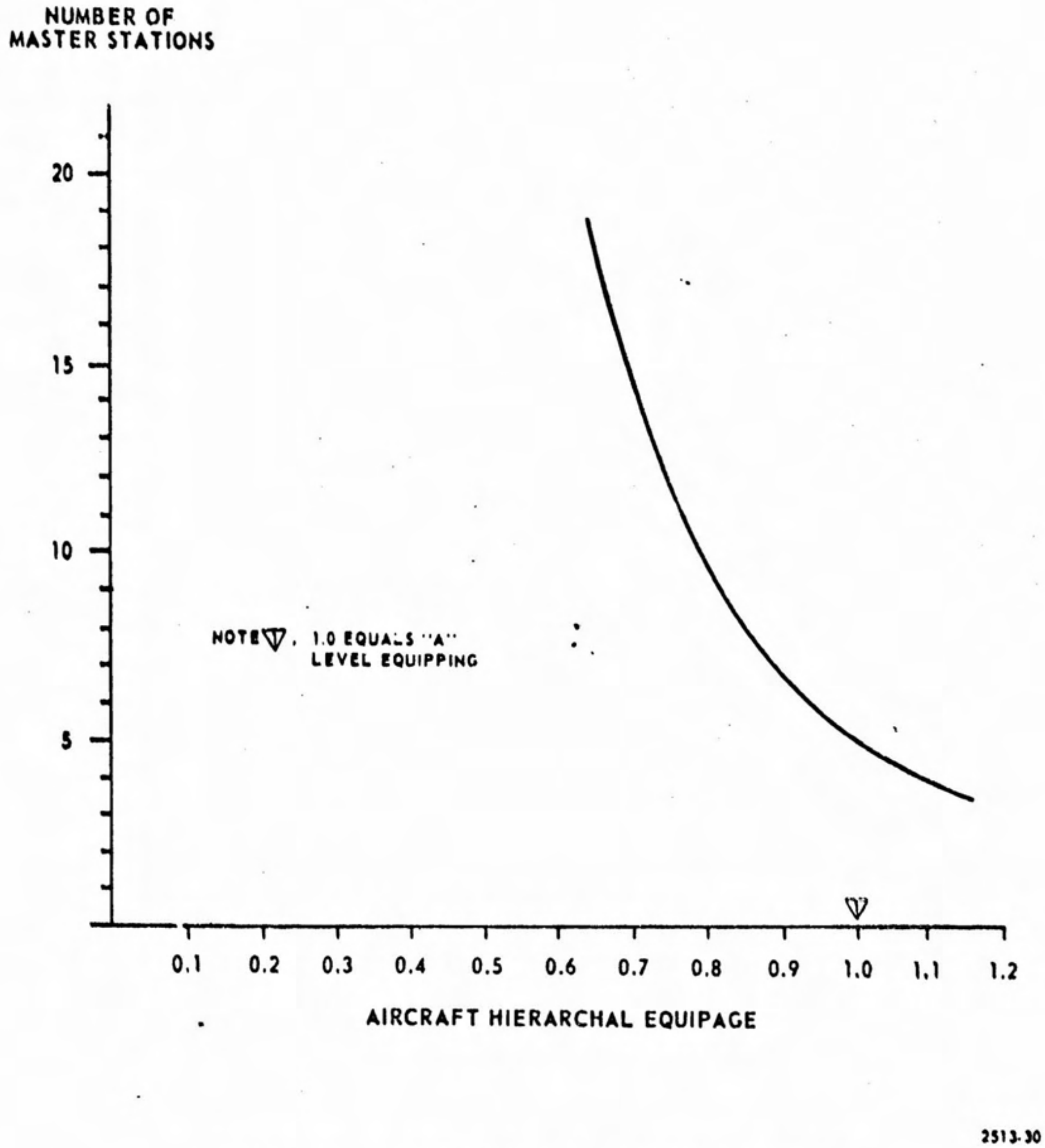


FIGURE 3-1. NUMBER OF MASTER STATIONS REQUIRED TO ATTAIN 92% TOTAL AIRCRAFT HOUR COVERAGE

so long as the cost of a master station is more than the cost of 150 hierarchal installations, no reduction in equipage should be contemplated. The hierarchal cost pertinent to this consideration is the cost differential between a hierarchal and a non-hierarchal installation, all other characteristics being equal. While we have not investigated this cost ratio in any detail, we feel that a ground station probably will cost more than 150 times the cost differential of adding hierarchal capability to aircarrier time-frequency equipment. Thus we are not motivated to reduce the hierarchal percentage.

In any event, the reduction could not be large. For a 15% reduction to be justified, a master station would have to cost less than about 70 hierarchal installations. More than 15% reductions are probably not tolerable, due to the large loss in service that would occur during an airline outage.

3.2 Trade-Off Data

Should alterations or departures from the recommendations made in 3.1 be contemplated, data are available within this document and references [1], [2], and [3] which permit further trade-off analyses. Furthermore, the effects of changing many of the fundamental parameters descriptive of the system can also be discerned.

If changes in relative costs motivate further investigation the curve of Figure 3-1 can be used to evaluate the trade off between numbers of master stations and numbers of hierarchal equipments. Changes in the location or priority of master station sites can usually be evaluated by examining the sequential master station site desirability maps such as Figures 2-20 and 2-21. The benefits of increasing the number of ground stations are given in a large variety of circumstances by the effectiveness vs. number of master station curves, e.g., Figure A-23 thru A-26.

Prospective changes in synchronization parameters (as for example; number of available hierarchy levels, minimum useable accuracy, frequency of synchronization attempts, etc.) can be examined through the use of equation (2-6) as described in Section 2 of this document. Trade-offs between effectiveness and master station availability can be conducted on the basis of equation (2-11) and the associated loss matrices of reference [3], page 3-15.

The model described in Section 2.1.4 not only permits trade-offs between hierarchal equipage and effectiveness but also allows changes in the daily pattern of aircarrier activity to be related to effectiveness.

If changes more drastic than can be accommodated by these methods need to be examined, the computer analysis programs described in Section 2.2 very likely could provide the capability to perform any necessary trade-off studies.

3.3 Applicability to Other Problems

This study has produced the capability and data base required to model the total aircraft population of the CONUS, quantatively assess the merit of all available sites for ground based equipments, and determine the airspace coverage supplied by derived deployments of equipments. This can be directly applied to the problem of assessment and management of the deployment of ATC surveillance radars, communication, navigation and control systems. With this capability it is possible to evaluate the effectiveness of present applications, assess the advantages of re-deployment of existing equipments and provide useful information with which to guide the introductory deployment of new equipments, such as the Discrete Address Beacon System (DABS).

Locating master stations for synchronization presents the same siteing questions that occur in navigation systems, communication systems and surveillance systems.

- How many sites should there be?
- Where should they be?
- In what order should they be installed?
- What coverage is supplied?
- What is the resulting effectiveness?
- Can alternate sites be use``

These questions apply to evaluation of present applications, formulation of improved deployments of existing equipments and the introduction of new systems.

The techniques developed in this study provide the capability to address these questions in terms of the percent of aircarrier flying hours provided coverage, the percent of aircraft encounters providing coverage, and total airspace coverage. Further, coverage can automatically be compared to the NTSB collision records to obtain a direct measure of collision prevention utility. In conjunction with each of these measures, a siteing algorithm is available which automatically selects sites for near optimum coverage per site, and sequences deployment according to utility of the sites. The key elements in providing this capability are:

A 7-dimensional (position, velocity, and time) aircarrier activity model from the Official Airline Guide and synthetic flight profiles.

The evaluation of service effectiveness as an additive set function.

A siteing algorithm which locates stations for near optimum effectiveness.

A 4-dimensional total aircraft activity model.

An aircraft encounter frequency model.

A large variety of unique computer graphics which aid analysis and interpretation.

In addition to these potential applications, the aircarrier model could be the source of answers to many questions concerning airline operations and safety. It can be used to study the suitability of present route structures, needs for area navigation routes and service demand patterns on ATC sectors and terminal areas.

Due to the direct applicability of the systems analysis, computer modelling and data processing that has become available during this study, it is recommended that these methods be utilized in studies of ATC equipment deployments. So far as we are aware, these capabilities are unique, and have not yet been applied either for evaluation of current ATC performance or for planning future installations. We believe that the FAA, having made the investment to generate these capabilities, will judge their application to other ATC problems to be useful and efficient both in terms of information content, cost effectiveness and timeliness.

APPENDIX A
NUMERICAL RESULTS

A.1 Introduction

This section provides the reader an overview and summary of the computer outputs which support the findings, conclusions, and analyses reported in this document.

Computer generated graphics have been used extensively throughout the study to present numerical outputs in a form that permits rapid assessment of their meaning and pattern.

Since interpretations of this material are provided in Sections 1.0, 2.0, and 3.0 of this report, they are presented here with a minimum of narrative description. Reader interpretation is encouraged. Detailed data and complementary analysis which were developed and used in the process of producing this information is documented in references [1], [2], and [3]. Appendix B is a detailed cross reference index between this report and the cited references.

A.2 Hierarchal Communication

Early in the study it was recognized that scheduled aircarrier aircraft provide highly predictable geographic/time patterns of operations that would be advantageous for air-to-air hierarchal relay of time synchronization. Conversely, the irregularity of the operation of military, unscheduled aircarrier and larger general aviation aircraft reduces interest in depending on their support for relaying time; however, this does not imply that these aircraft will not be major users of time as supplied by master stations and scheduled aircarrier aircraft.

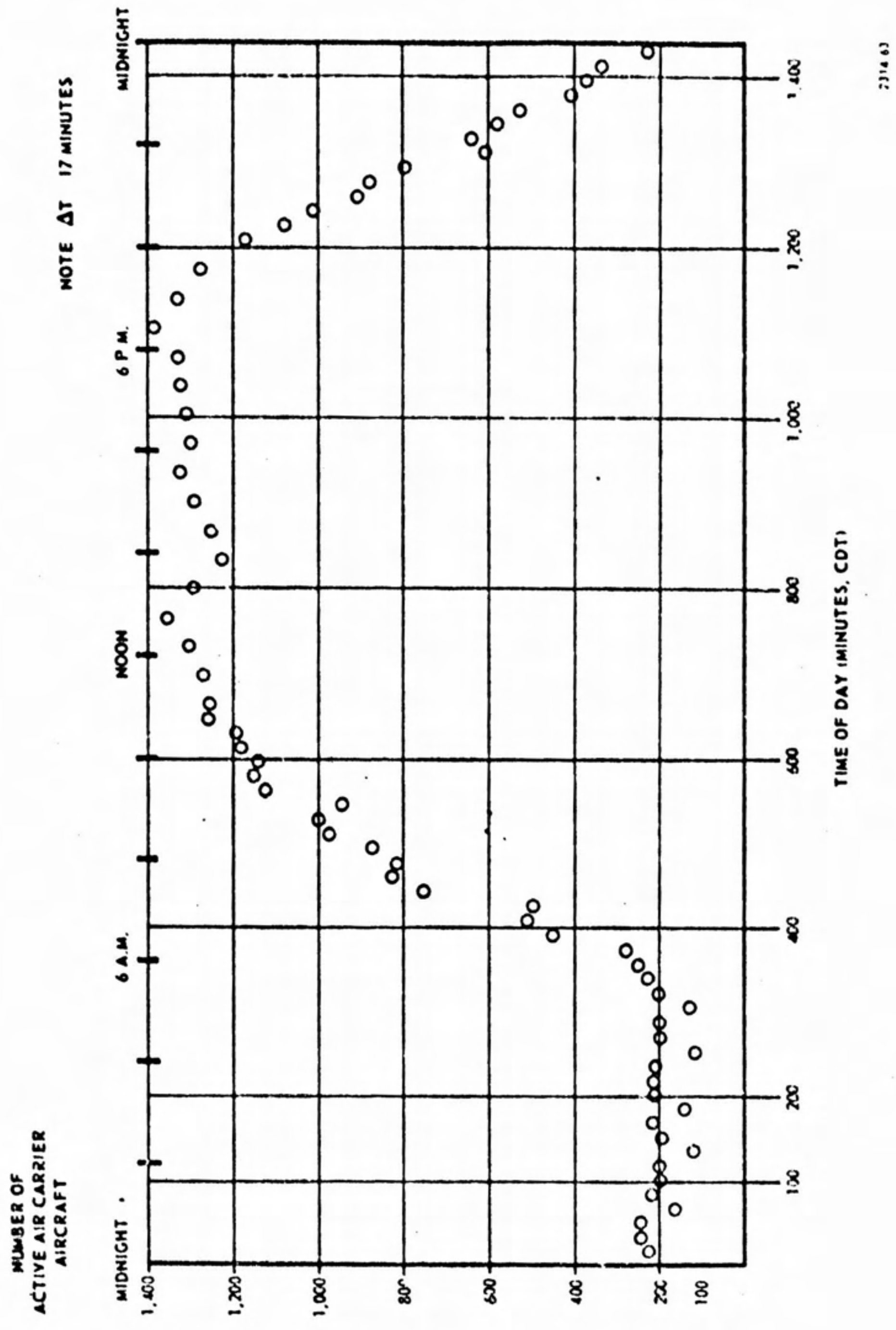


FIGURE A-1. AIRCARRIER AIRCRAFT ACTIVITY VS TIME-OF-DAY

Approximate Gross Wt. (1000 lbs.)	Aircraft Type Code	Number of Aircraft	Percent of Total	Cumulative Percentage
750	747	47	3.2	3.2
555	D10	33	2.3	5.5
430	L10	7	0.5	6.0
335	D8S	40	2.8	8.7
335	D8F	67	4.6	13.4
335	DC8	19	1.3	14.7
329	B3F	38	2.6	17.3
329	B7F	86	5.9	23.2
257	707	12	0.8	24.0
234	B2F	21	1.4	25.5
229	720	13	0.9	26.4
188	880	31	2.1	28.5
175	72S	211	14.5	43.0
165	727	290	20.0	63.0
127	LE	3	0.2	63.2
114	737	106	7.3	70.5
110	D9S	179	12.3	82.9
98	DC9	71	4.9	87.7
85	BAC	20	1.4	89.1
58	C5	77	5.3	94.4
55	YS	17	1.2	95.6
49	C4	1	0.1	95.7
46	C6	14	1.0	96.6
44	FH	20	1.4	98.0
42	F7	15	1.0	99.8
42	M4	13	0.9	99.9
-	Jet	1	0.1	100.0
		1452		

Set C

Set B

Set A (See Text)

2581-16

FIGURE A-2. ACTIVE AIRCARRIER AIRCRAFT BY TYPE
6 P.M. CDT, FRIDAY 11 AUGUST 1972

Typical levels of CONUS activity for the larger aircraft, 40K lbs. and up, of U.S.A. airlines is shown in Figure A-1. These activity numbers are based on time snapshots taken throughout all days of the week, at regular increments of the time-of-day. To evaluate air-to-air hierarchy performance at reduced levels of aircraft equippage; this set, referred to as the "A" set, was sub-divided as shown in Figure A-2 to produce 33 and 66 percent reductions in the number of active aircraft. The respective total number of U.S. air-carrier aircraft (active and inactive) that would need to be hierarchy equipped are: A $\approx$ 2300 aircraft, B $\approx$ 1640 aircraft, C $\approx$ 860 aircraft.

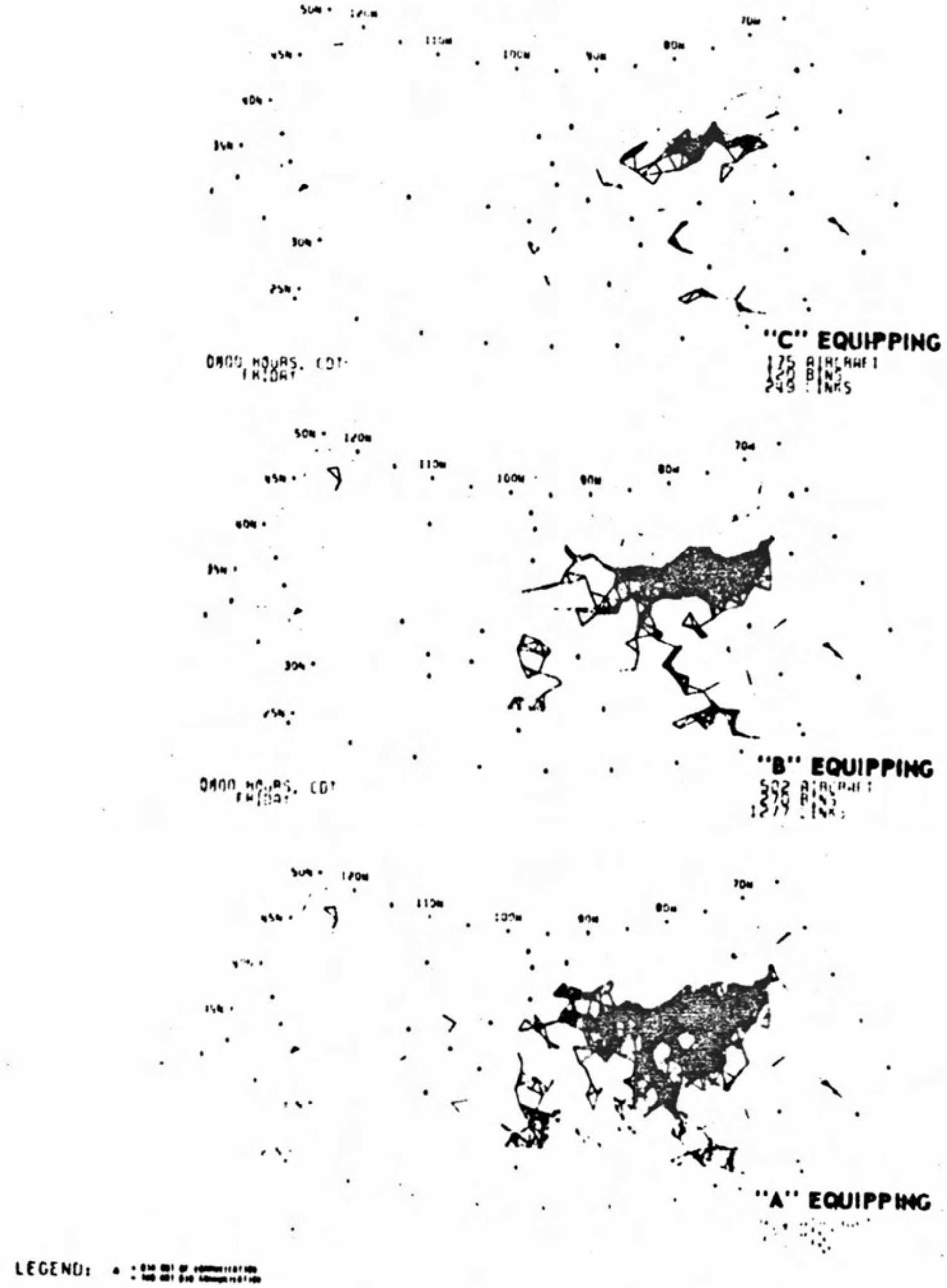
Comparative patterns of the hierarchal aircraft-to-aircraft communication linkage are presented in Figures A-3 through A-6, with the higher levels of equipping demonstrating the capability to provide synchronization to large areas of the CONUS. As a result of the changes in patterns of communication synchronization service provided, the various equipping levels exhibit significant differences in sensitivity to choice of the site for the first master station. Figures A-7 through A-10 report the results of testing the A, B, C, and Zero hierarchy levels, using worth Rule III (total aircraft hours synchronized). The higher equipping A and B levels provide extensive communication coverage and exhibit a general insensitivity to choice of first site. This means that a site could be picked at random, or picked to satisfy any operational preference, in the eastern third of CONUS still provide within 5% the effectiveness offered by the "best" site. As the equipping is reduced and the communication coverage is fragmented, the freedom of selection for the first "C" site is reduced to very nearly that of zero hierarchy: master station only synchronization.

Examination of the freedom of selection of the "best" site after deployment of a few master stations provides additional insight to hierarchy process. Figure A-11 is based on "A" level equipping. After deployment of only three master stations the area of "best" sites has been reduced to approximately that provided initially by Zero or "C" level equipping. This is due to the

LEGEND:

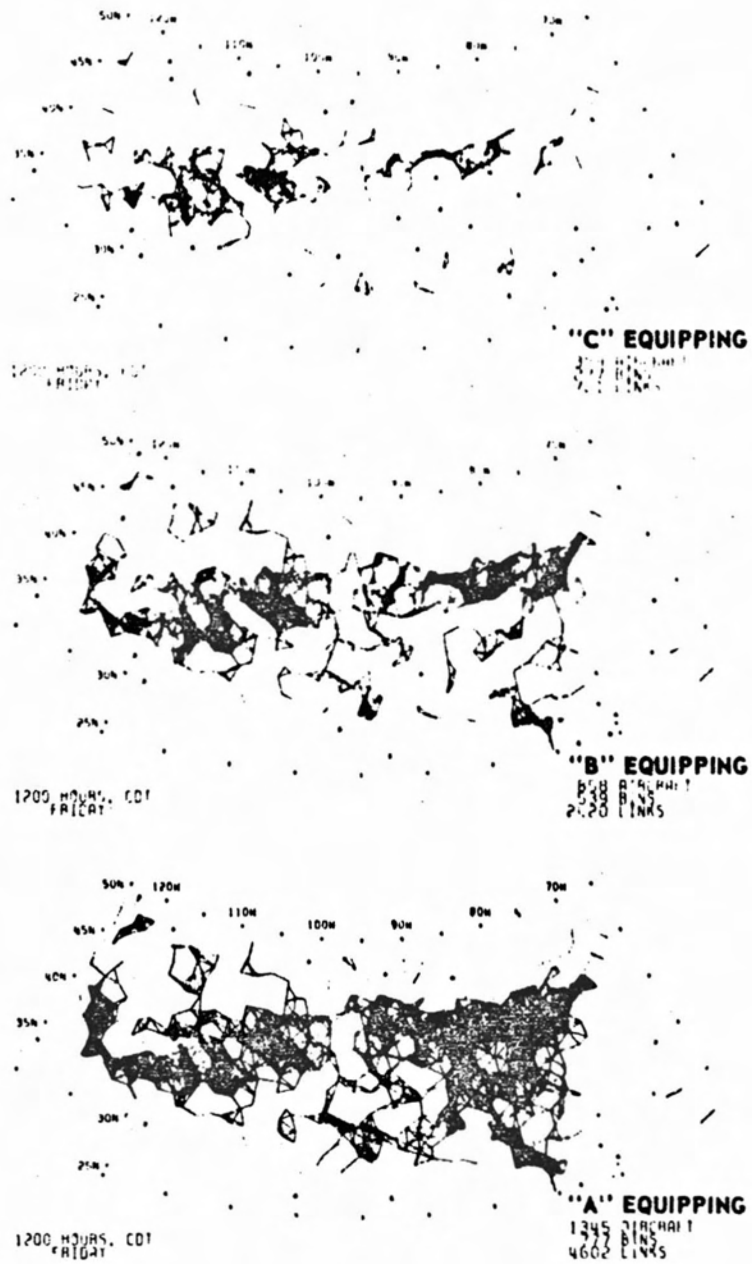
2314-42

FIGURE A-3. COMPARATIVE AIRCRAFT COMMUNICATION MAPS
0001 HOURS C.D.T.



2314-46

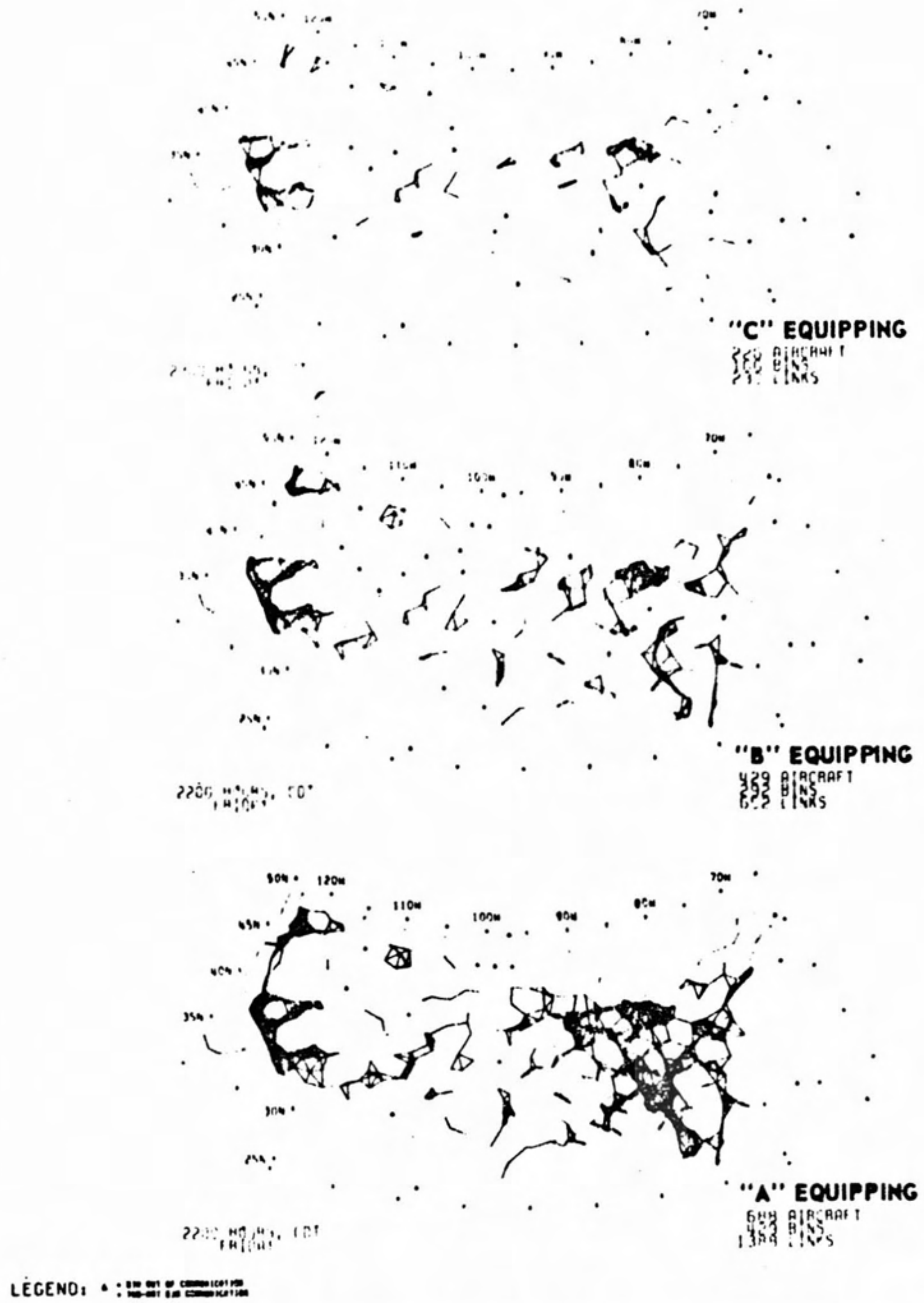
FIGURE A-4. COMPARATIVE AIRCRAFT COMMUNICATION MAPS
0800 HOURS C.D.T.



LEGEND:

2314-48

FIGURE A-5. COMPARATIVE AIRCRAFT COMMUNICATION MAPS
1200 HOURS C.D.T.



2314-53

FIGURE A-6. COMPARATIVE AIRCRAFT COMMUNICATION MAPS
2200 HOURS C.D.T.

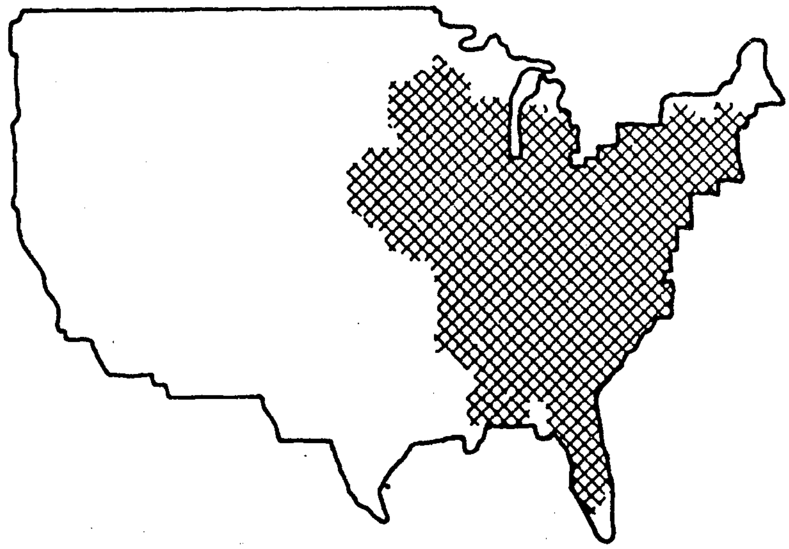
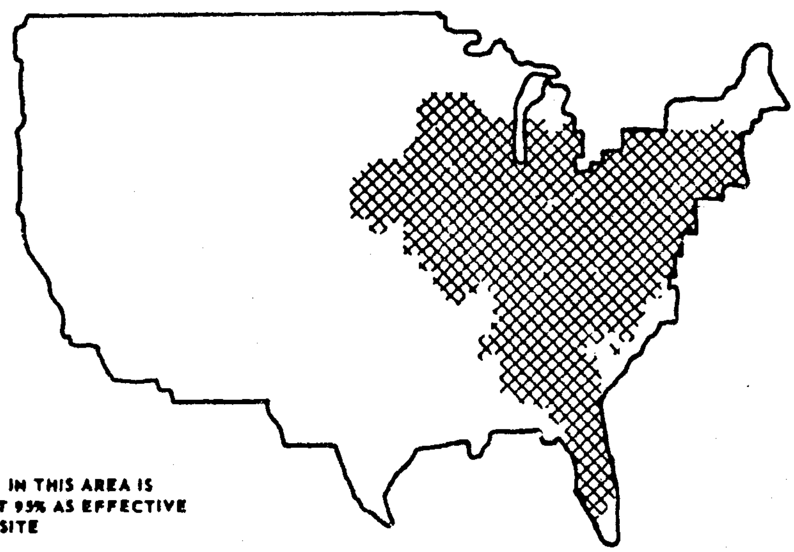


FIGURE A-7. SENSITIVITY TO CHOICE OF 1ST SITE "A" HIERARCHAL EQUIPPING



LEGEND:
ANY SITE IN THIS AREA IS AT LEAST 95% AS EFFECTIVE AS BEST SITE

2581-6

FIGURE A-8. SENSITIVITY TO CHOICE OF 1ST SITE "B" HIERARCHAL EQUIPPING

FIGURE A-9. SENSITIVITY TO CHOICE OF 1ST SITE
"C" HIERARCHAL EQUIPPING

FIGURE A-10. SENSITIVITY TO CHOICE OF 1ST SITE
ZERO HIERARCHAL EQUIPPING

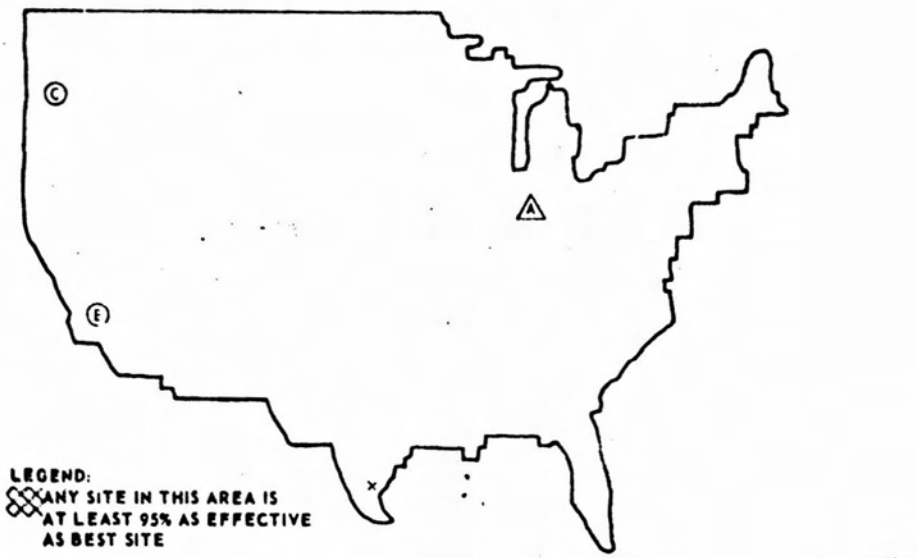


FIGURE A-11. SENSITIVITY TO SELECTION OF "BEST" FOURTH SITE "A" HIERARCHAL EQUIPPING fact that the extensive interlocking hierarchal coverage already provided by the first three stations has reduced the next site selection to being a local process, with only local hierarchy support.

A.3 Sensitivity to Worth Rules

With the diversity of goals of the four worth rules, Section 2.2.5, the question arises as to the magnitude of the effect the rule selection has upon the site selection. Specifically does the use of a particular rule produce a fixation of a specific set of goals, which eliminates flexibility to accomodate other secondary objectives. Freedom of first site selection as a function of the worth rule being used, is presented in Figures A-12 through A-15. All present common areas of choice and a general insensitivity to choice of rule.

An additional evaluation of site sensitivity was made relative to data base size. Figures A-16 and A-17 compare the Rule III results obtained with the original 12 snapshot and the expanded 68 snapshot data bases; there are no significant differences.

A.4 Effects of Site Selections

The insensitivity to site selection, illustrated in Section A.3, means that many sites will be numerically scored as being very close to equal by the computer. In applying the priority selection algorithm, the next site is often selected by the computer on the basis of these very slight differences. This has produced large translations in the choice of site, due to very small differences introduced by such factors as aircraft population modeling or worth rule selection, without a significant change in the effectiveness of the selection. However, as shown in Figures A-18 through A-21, the overall patterns of sets of priority selected sites can vary considerably.

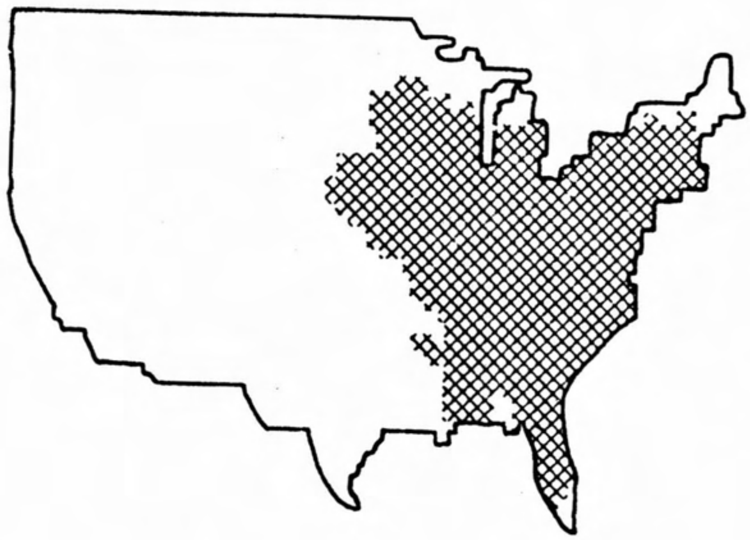
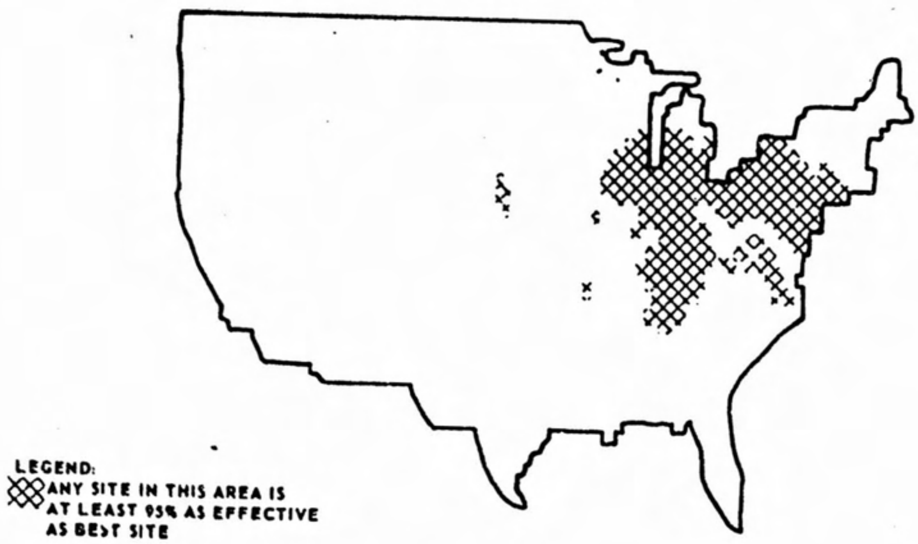


FIGURE A-12. SENSITIVITY TO CHOICE OF 1ST SITE
WORTH RULE #1: HIERARCHAL AIRCRAFT



2581-9

FIGURE A-13. SENSITIVITY TO CHOICE OF 1ST SITE
WORTH RULE #2: AIRSPACE

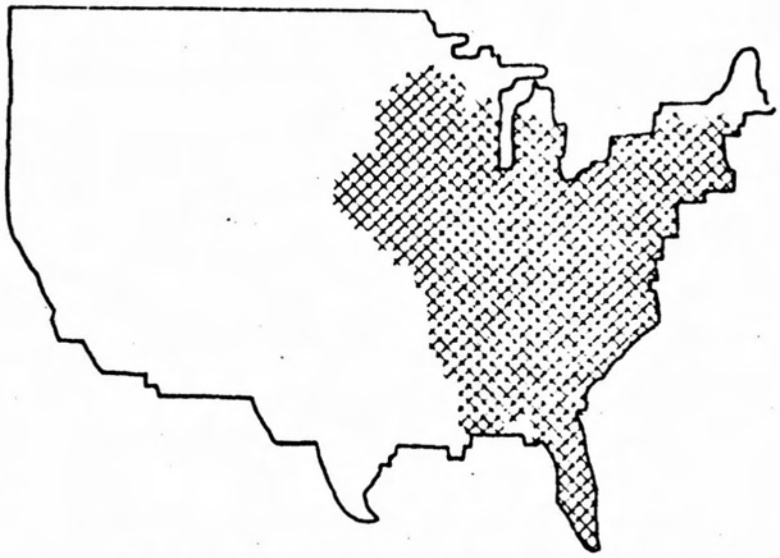


FIGURE A-14. SENSITIVITY TO CHOICE OF 1ST SITE
WORTH RULE #3: TOTAL AIRCRAFT

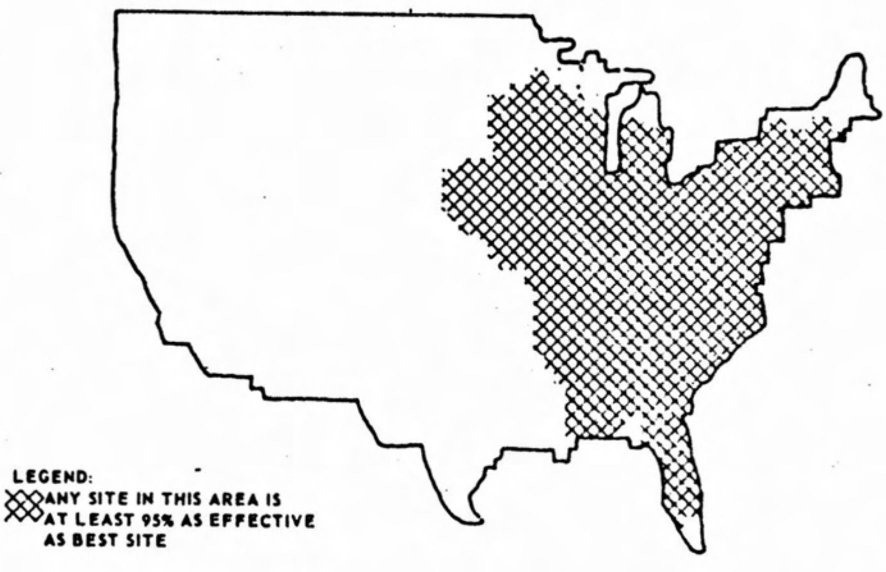


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FIGURE A-15. SENSITIVITY TO CHOICE OF 1ST SITE
WORTH RULE #4: COLLISION EXPOSURE

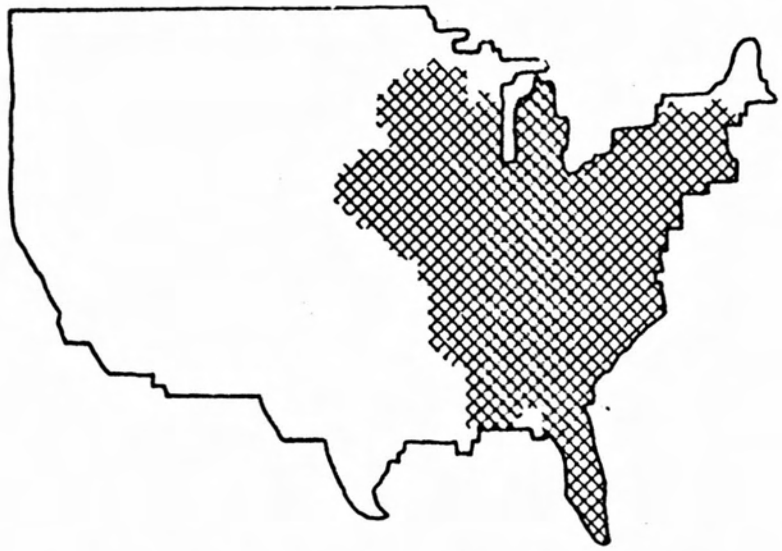


FIGURE A-16. SENSITIVITY TO CHOICE OF 1ST SITE
12 SNAPSHOT DATA BASE

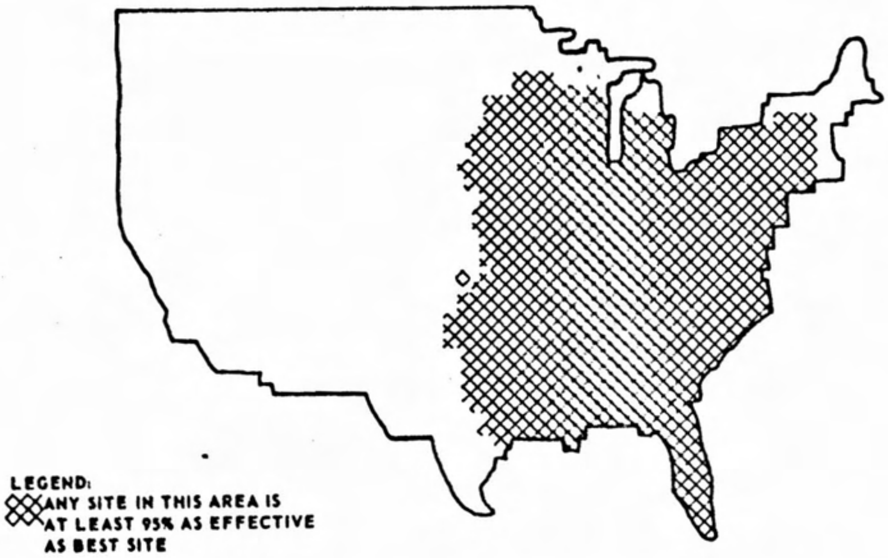


FIGURE A-17. SENSITIVITY TO CHOICE OF 1ST SITE
68 SNAPSHOT DATA BASE

FIGURE A-18. RULE 1 PRIORITY SITES

LEGEND:
△ 1ST STATION

2581-3

FIGURE A-19. RULE 2 PRIORITY SITES

FIGURE A-20. RULE 3, PRIORITY SITES

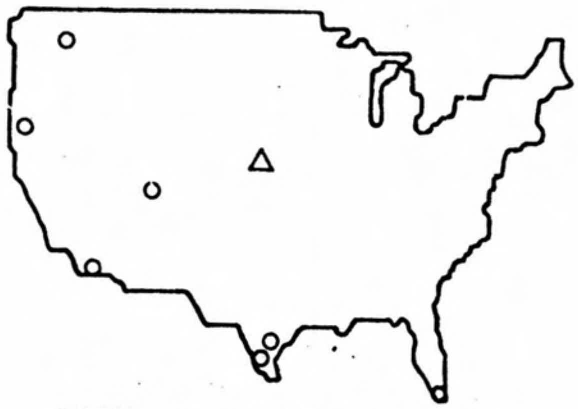


FIGURE A-21. RULE 4, PRIORITY SITES

LEGEND:
△ 1ST STATION

2581-4

FIGURE A-22. AIRTRAFFIC HUBS

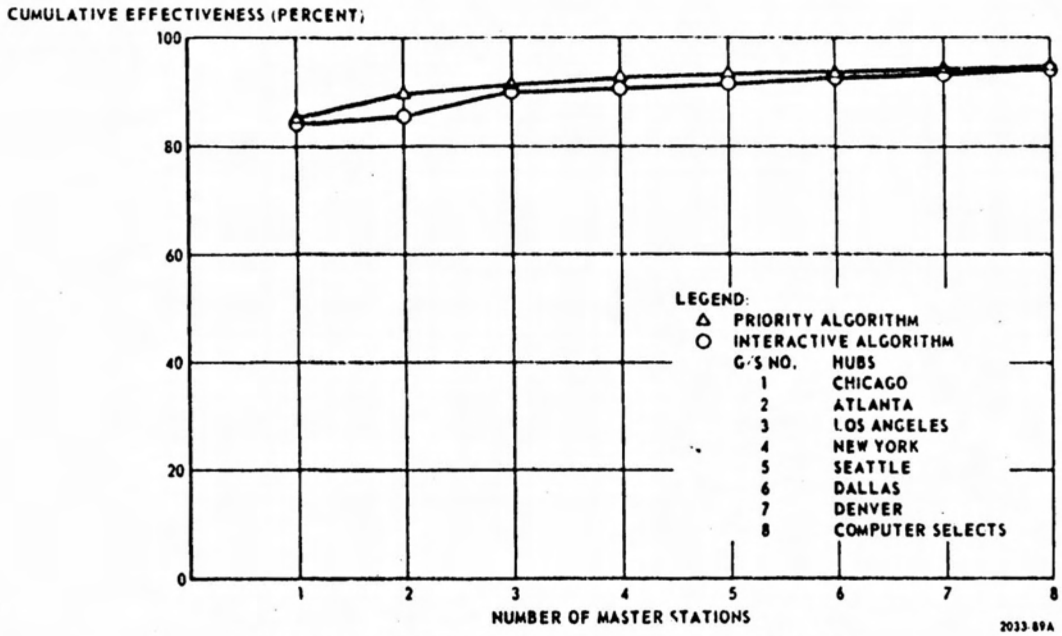


FIGURE A-23. RULE 1 COMPARISON OF PERFORMANCE

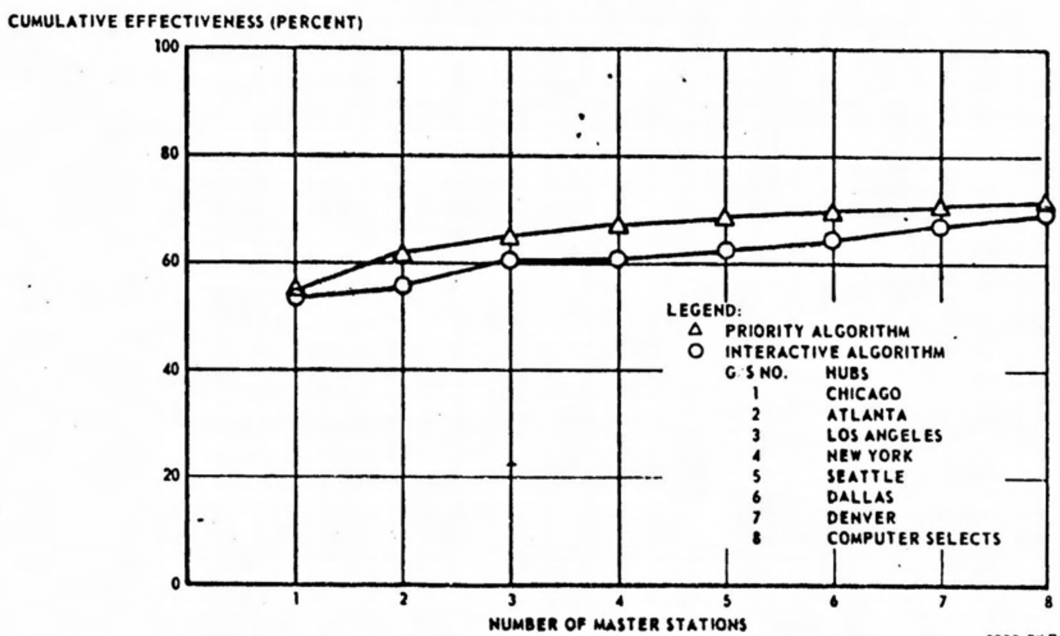


FIGURE A-24. RULE 2 COMPARISON OF PERFORMANCE

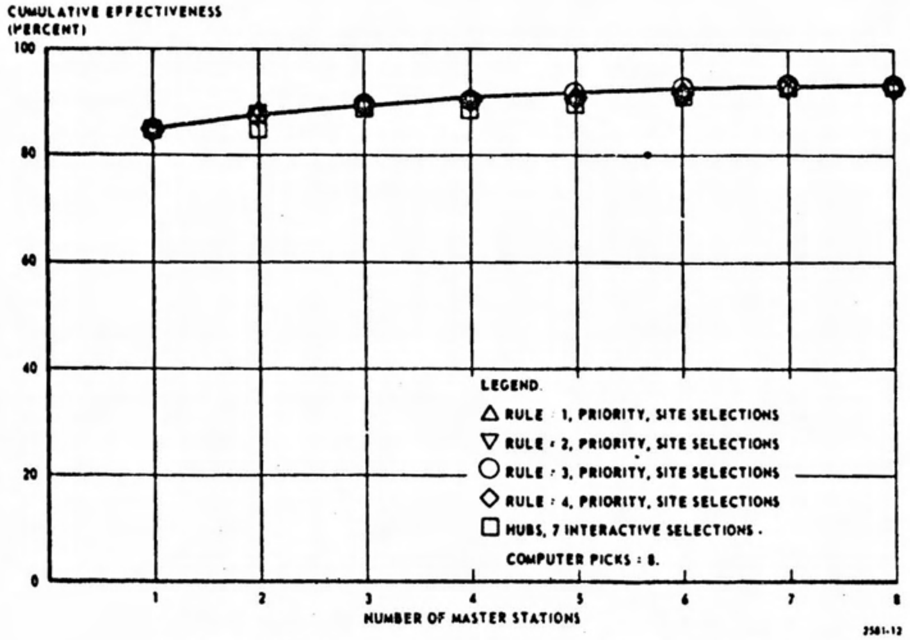


FIGURE A-25. RULE 3 COMPARISON OF PERFORMANCE

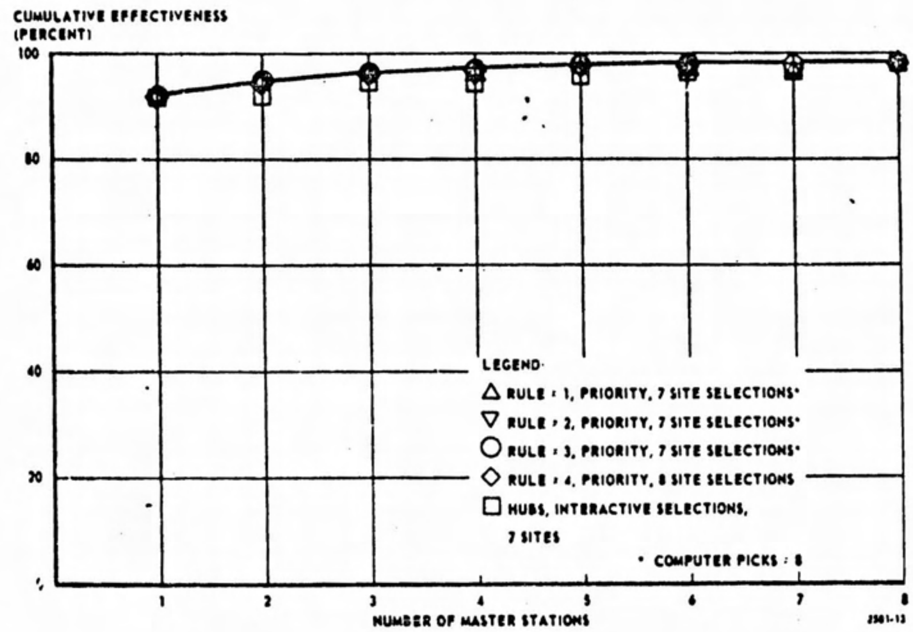


FIGURE A-26. RULE 4 COMPARISON OF PERFORMANCE

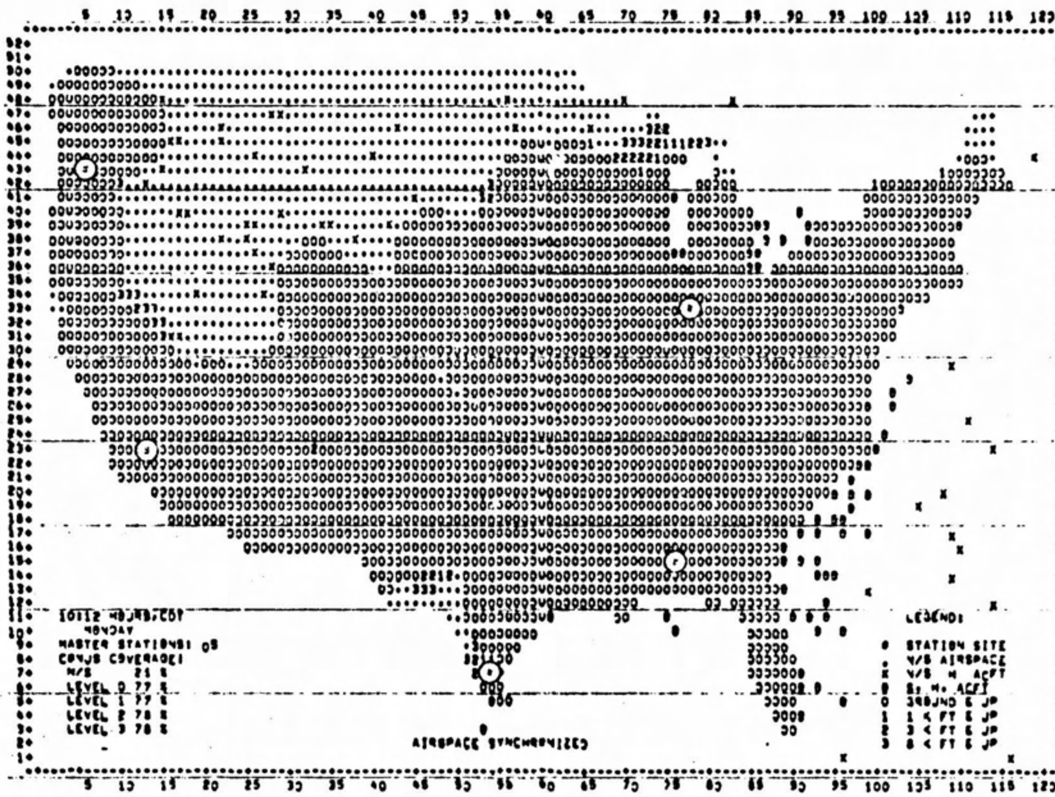
Also of interest is the sensitivity of alternate sets of sites, such as air traffic hubs, Figure A-22. The question is: Is there any significant difference in performance of these ensembles. Using the "A" level of hierarchal equipping, these five sets of sites were cross-evaluated on the basis of the various worth rules; Figures A-23 through A-26. There is no significant difference in performance, which indicates a general insensitivity to the set of sites selected. This provides latitude to consider and accommodate secondary preferences in site selections.

As an extension of this investigation, all Civil Aviation mid-air collisions, January 1964 through December 1971 as reported by the National Transportation Safety Board were included in our analysis. The performance of Rule III priority selected sites was compared to that provided by stations located at Air Traffic Hubs. Excepting crop dusters, formation flights, etc., each collision was used as an evaluation criterion for various numbers of master stations: i.e., would the aircraft have received synchronization from a master station or synchronized hierarchal aircraft at that time of day, at that altitude, at that location. The results as obtained from maps such as Figure A-27 are:

Mid-Air Collision Coverage

Number of Master Stations	Priority Sites	10 Hubs, then Priority Sites
5	96%	96%
10	97%	97%
15	98%	98%
20	99%	98%

Once again system performance is insensitive to the site selection.



2531-14

FIGURE A-27. AIRSPACE SYNCHRONIZED 5 MASTER STATIONS AT PRIORITY SITES

As a continuation of the above review of mid-airs, the coverage supplied by master stations only, without airborne hierarchal support, was investigated.

Two deployments of 100 master stations were evaluated. In the first the sites were located using the priority algorithm. For the second deployment, eighty master stations were located at the sites of ATC control towers with the highest annual activity; then the priority algorithm was applied to select the sites for twenty additional master stations. Without hierarchy support the coverage of the mid-airs was:

100 @ Priority Sites	80 @ Hubs plus 20 @ Priority Sites	
29%	50%	Covered
56%	36%	Within range but below earth curvature line of sight cutoff
3% ⚠1	5% ⚠1	Out of range
12%	9%	Could not be evaluated due to collision altitude being unknown.

Note: ⚠1 Increasing range of master station would not increase coverage due to L.O.S. cutoff.

Although this is a limited deployment, it does provide a point of comparison of the ground base only coverage and air-to-air hierarchy supplemented coverage. While not pursued in this study, the capability illustrated by these two examples could be applied to other ATC functions. For example, by resetting the communication range, in the computer model, and adding more ground stations the coverage provided by ATC surveillance or navigation facilities could be evaluated.

In addition to the basic evaluation provided by the various worth rules, redundancy can be used as a secondary figure of merit. Thus, the performance of the ensembles of stations under conditions of station outages and weather disturbances were evaluated. Weather disturbances are included because a master station being neutralized by virtue of weather dispersing the hierarchal aircraft is, at worst, equivalent to an outage of the master station. However, this is worst case because in an outage the aircraft population is unperturbed; with a weather disturbance the total airborne population will be dispersed away from the neutralized station, where they are more likely to receive synchronization.

Using data, such as shown in Figures A-28 and A-29 the short term effects of weather and equipment outage were assessed. The long term effects due to equipment outages are computed on the basis of hardware availability[a] and the short term data.

Under the extreme condition that each master station has a low availability, $\alpha = 0.99$[b]; the mean effectiveness losses due to outages are:

Total Number of Stations	Weighted Outage Losses Priority Sites	Hubs
5	0.084%	0.048%
6	0.087	0.047
7	0.092	0.050
8	0.083	0.051
9	0.081	0.044
10	0.083	0.038

[a] Availability is defined to be mean time between failure divided by the sum of mean time between failure and mean time to repair. Thus this term is the long term probability that any one master station will be functional.

[b] FAA contract DOT-FA73WA-3239, required that the Collision Avoidance Synchronization Station equipment delivered under the contract provide an availability of 0.9995.

NOTE: ALL VALUES ARE PERCENTAGE REDUCTION IN EFFECTIVENESS UNDER STATED CONDITIONS

CONCURRENT OUTAGE	A	B	C	D	E	F	G	H	I	J
B	3.40									
C	2.18	3.58								
D	2.52	3.66	2.44							
E	3.57	2.95	1.74	2.07						
F	1.61	2.75	1.53	1.87	1.16					
G	1.53	2.66	1.45	1.78	1.08	0.87				
H	2.67	2.62	1.40	1.74	1.03	0.89	0.74			
I	1.45	2.58	1.86	1.71	1.00	0.80	0.72	0.67		
J	1.45	2.58	1.37	1.70	1.00	0.82	0.71	0.66	0.63	
NONE	1.13	2.27	1.05	1.39	0.68	0.48	0.39	0.35	0.32	0.31

INITIAL OUTAGE (STATION)

2531-13

FIGURE A-28. EFFECTS OF OUTAGES OF PRIORITY STATIONS

NOTE: ALL VALUES ARE PERCENTAGE REDUCTION IN EFFECTIVENESS UNDER STATED CONDITIONS

CONCURRENT OUTAGE	A	B	C	D	E	F	G	H	I	J
B	0.54									
C	1.81	1.79								
D	0.67	0.65	1.52							
E	0.79	0.53	1.80	0.66						
F	0.49	0.45	1.72	0.58	0.46					
G	0.50	0.47	1.74	0.60	0.49	0.41				
H	0.52	0.49	1.76	0.62	0.51	0.43	0.48			
I	0.48	0.55	1.72	0.59	0.47	0.40	0.41	0.43		
J	0.47	1.32	1.72	0.58	0.46	0.38	0.40	0.42	0.39	
NONE	0.28	0.26	1.53	0.29	0.27	0.19	0.21	0.23	0.20	0.19

INITIAL OUTAGE (STATION)

2531-14

FIGURE A-29. EFFECTS OF OUTAGES OF HUB STATIONS

Although there are relative differences, the loss terms are negligible when compared to the overall effectiveness of over 90% for both sets. This indicates a high degree of freedom from outage effects.

A.5 Sensitivity to Aircraft Population Model

Having shown, in Section A.4, that a wide variety of distributions of master stations can effectively serve the NAS population distribution model, reference [2], Appendix A, it is natural to inquire whether this property can be extended so that they also furnish synchronization efficiently to any foreseeable distribution of aircraft. Since a master station configuration which efficiently utilizes hierarchal chaining potential is nearly maximally effective for any time distribution purpose, one would expect that, so long as hierarchal aircraft are distributed similarly (or more liberally), this same set of master stations would be nearly maximally effective for furnishing precise time to any likely future distribution of aircraft.

For a test, four widely different distributions of the total aircraft populations were synthesized. They are:

(1) distributed in direct proportion to hierarchal aircraft
(2) uniformly distributed in time and area over CONUS
(3) distributed in direct proportion to the NAS model
(4) distributed in proportion to the square of the NAS model

The service available to these populations was then evaluated for various sets of master stations; i.e., those that were originally selected to best serve hierarchal aircraft, worth Rule I, and the new sets selected to best serve the new population. The results are shown in Figure A-30. Although the geographic patterns of the sets of sites varies considerably, there is no significant difference in effectiveness. Through this whole range of aircraft distributions, one set of eight master stations elected to synchronize hierarchal aircraft

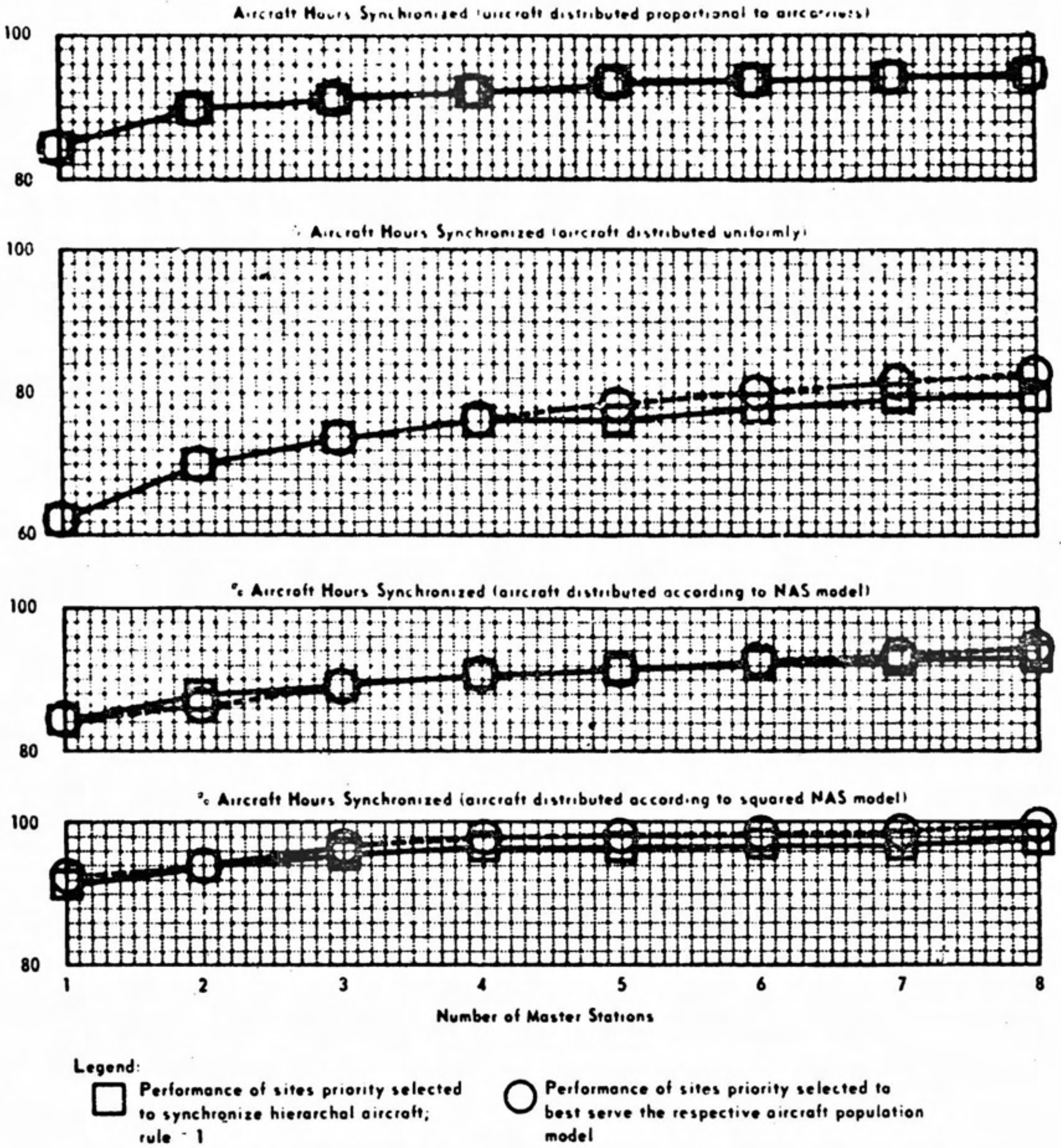


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FIGURE A-30. RELATIVE PERFORMANCE OF RULE 1 PRIORITY SITES FOR FOUR AIRCRAFT POPULATION MODELS are nearly maximally effective. This demonstrates that master stations can be selected which will efficiently serve the aircraft population whatever (within reason) its distribution might turn out to be. The only requirement is that hierarchal aircraft be distributed as liberally as in the JAG model we have utilized.

A.6 Sensitivity to Hierarchal Equipping

An aircraft hierarchal unit at altitude possesses a significant advantage, in area coverage, compared to a ground-based master station. However, to make optimum use of this advantage requires support from some minimum number of master stations. The objective of this portion of the study was to evaluate the interaction of the performance of airborne units vs. master stations.

An effectiveness trade-off at various levels of hierarchal equipping, Section A.3, vs. number of master stations was performed on the basis of total aircraft hours synchronized; worth Rule III. The results, Figure A-31 and A-32 exhibit the threshold effects of hierarchy density. Above this threshold, at the A and B levels, initial effectiveness per master station is very high. Below this threshold, levels C and Zero, hierarchy support is critically reduced and effectiveness per master station is very low. A continuation of this threshold effect, in succeeding master stations as exhibited by the difference in A or B vs. C or Zero performance is shown in Figure A-32. These observations are complemented by the independent analytical studies reported in Section 2.1.4.

If aircarrier aircraft are utilized for air-to-air time relay the level of hierarchal support could be significantly changed due to outage, such as a strike. To test this possibility, "A" level system performance, with five master stations was evaluated under the conditions of a 15% reduction in hierarchal aircraft. This was achieved by modeling an outage of United Air Lines. Reduction in effectiveness was less than 3% in both the five priority sites and the five air traffic Hub sites.

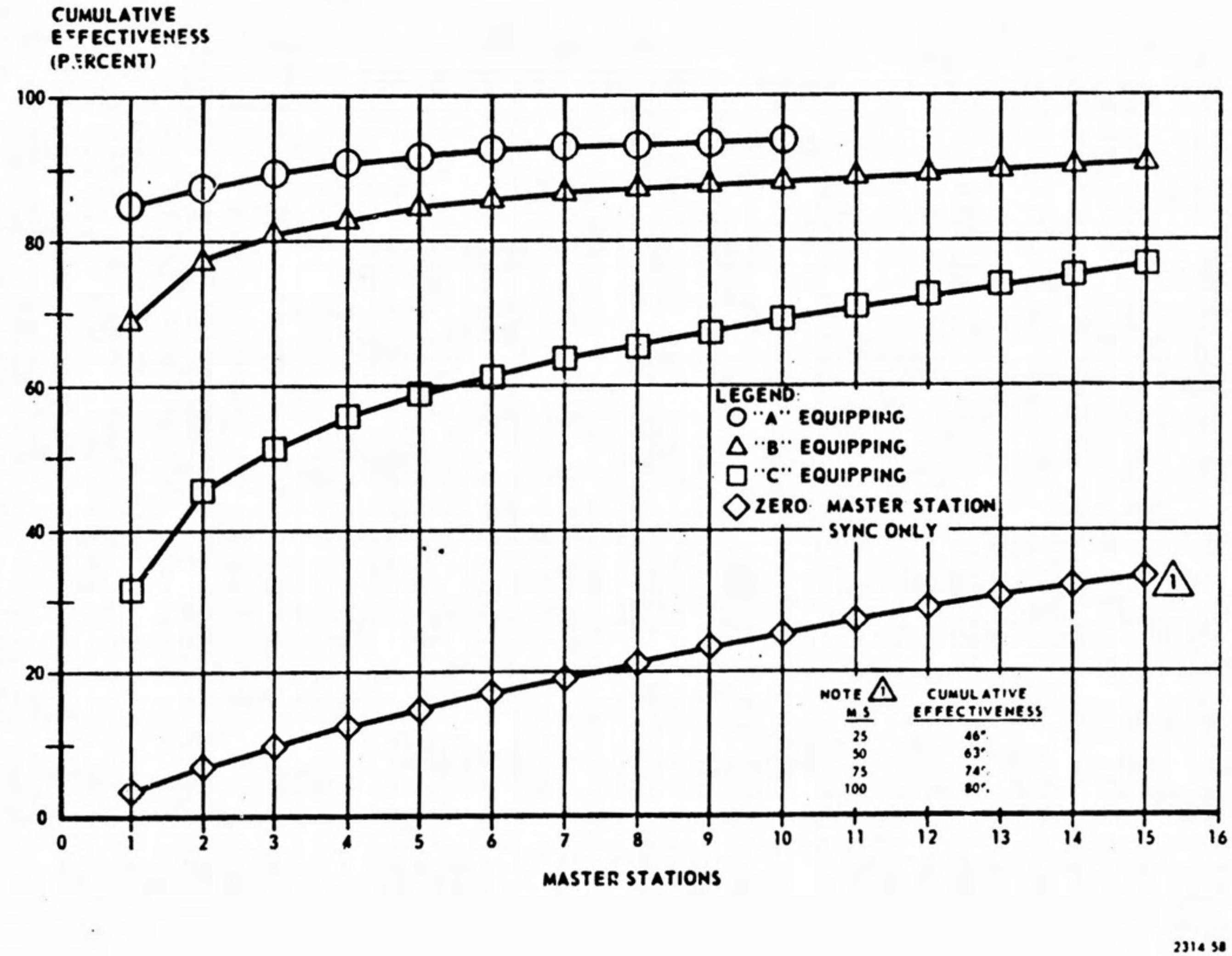

FIGURE A-31. EFFECTS OF HIERARCHAL EQUIPPING ON RESPECTIVE RULE 3 PRIORITY SELECTED SITES

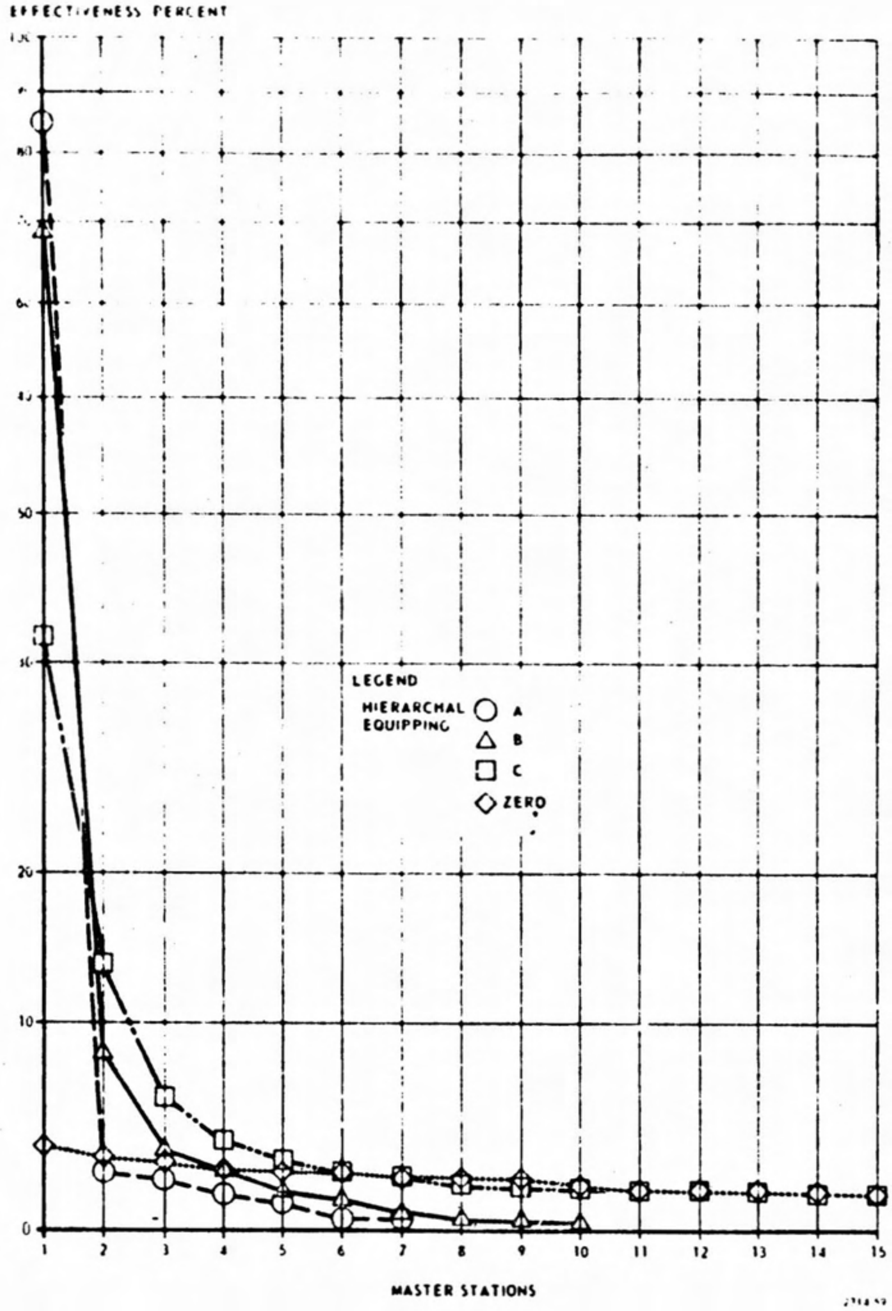

FIGURE A-32. EXCLUSIVE EFFECTIVENESS ACQUIRED BY EACH MASTER STATION

APPENDIX B

DOCUMENTATION CROSS REFERENCE

For the reader desiring additional information the detailed results of Phases I, II and III, as reported in references [1], [2] and [3], have been indexed relative to this, the summary final, report.

Summary Final (section)	Phase I (section)	Phase II (section)	Phase III (section)
2. Solution Methods	2 & 3	2	2
2.1 Mathematical Analysis	2	2	2
2.1.1 Hierarchal Chaining	2.1	---	6
2.1.2 Effectiveness Measures	2.3	2.1	2
2.1.3 Algorithms for Master Station Location	2.4	---	3.1
2.1.4 Sensitivity to Aircraft Location and Mathematical Solution Method	1.4 & 2.2	6	6
2.2 Computer Processes	3 & 4	2 & App. A	App. A
2.2.1 Aircarrier Aircraft Itineraries	4.2	---	---
2.2.2 Hierarchal Aircraft Location Versus Time-of-Day (T.O.D.)	4.3	---	---
2.2.3 Hierarchal Communication Linkage and Clouds	4.4	2.2.3	---
2.2.4 Airspace and Total Population Modeling	---	2.2.1, App. A & App. B	---
2.2.5 Worth Computation	---	2.2.1	---
2.2.6 Master Station Site Evaluation and Selection	4.5 & 4.6	2.2.1	---

Summary Final (section)	Phase I (section)	Phase II (section)	Phase III (section)
2.2.7 Programs for Analysis of the Effects of Master Station Outage	---	---	2.2 & App. A
2.2.8 Synchronization Coverage Mapper Program	---	2.2.2	---
3. Conclusions and Recommendations	6	8	7
3.1 Master Station Sites	5	3, 5 & 7	3 & 4
3.2 Trade-Off Data	6.2 & 6.3	6 & 8.1.3	3, 5 & 6
3.3 Applicability to Other Problems	6.5	2.2 & 8.1.1	4
Appendix A - Numerical Results	5	3, 4, 5 & 6	3, 4, 5 & 6
A.1 Introduction	---	---	---
A.2 Hierarchal Communication	5	6	5
A.3 Sensitivity to Worth Rules	5	3 & 5	3
A.4 Effects of Site Selections	5	3, 4, 5, 6 & 7	2, 3, & 4
A.5 Sensitivity to Aircraft Population Model	---	5	---
A.6 Sensitivity to Hierarchal Equipping	---	6	5 & 6

REFERENCES

1. "Synchronization Station Location Study - Phase I - Methodology Development," Department of Transportation, Federal Aviation Administration; Report FAA-RD-73-100, July 73. AD [illegible] NTIS $ [illegible]

2. "Synchronization Station Location Study - Phase II - Calibration Phase," Department of Transportation, Federal Aviation Administration; Report FAA-RD-73-173, October 1973. AD [illegible] NTIS $[illegible]

3. "Synchronization Station Location Study - Phase III - Tradeoff Phase," Department of Transportation, Federal Aviation Administration; Report FAA-RD-74-117, May 1974 AD NTIS $.

4. "Airborne Collision Avoidance System," Air Transport Association of America, ATA Report 117, Rev 13, May 1971.

5. E. M. Wright, "The Probability of Connectedness of A Large Unlabelled Graph," Bulletin of the American Mathematical Society, Vol. 79, No. 4, July 1973, pp 167-169.

6. Report of Department of Transportation Air Traffic Control Advisory Committee, Vol. 2, Appendix F, December 1969. GPO Vol I $[illegible] Vol II $[illegible]

7. Proceedings of the Fourth Annual NASA and D.O.D. Precise Time and Time Interval (PTTI) Planning Meeting, 14-16 November 1972, NASA Report X-814-73-72.

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