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A REVIEW AND ANALYSIS OF THE HONEYWELL COLLISION AVOIDANCE SYSTEM PHASE II

(IDA Study S-460)

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16. Abstract

This is the second analysis of the Honeywell CAS conducted by the Institute for Defense Analyses. Like the first report (A Review and Analysis of the Honeywell Collision Avoidance System, Vols. I and II, October 1973), this report explores critical problems in the Honeywell CAS in a high density environment such as that projected by the FAA in 1971 for the Los Angeles Basin in 1982 (800 aircraft). In the analysis, the Air Transport Association (ATA) collision avoidance logic was used to evaluate the performance of the Honeywell CAS and the suitability of the ATA logic was not questioned. Where possible, this report attempts to build on IDA's previous analyses; however, because of extensive and significant design changes, particularly in the threat evaluation logic, the performance of the Honeywell CAS with regard to detection probability and false alarm probability in a dense environment had to be revised considerably.

Honeywell has proposed two levels of equipment for use in collision avoidance; the AVOIDS-I and AVOIDS-II. The AVOIDS-I is intended for use by high-performance aircraft and can handle encounters between aircraft with closing speeds up to 2000 ft/sec. The threat evaluation logic of the AVOIDS-I is patterned after the ATA logic, but some minor differences exist.

The AVOIDS-II equipment is intended for use by low-performance aircraft and is not patterned after the ATA logic. The AVOIDS-II has a much smaller threat volume than is required by the ATA logic and may not provide adequate protection to general aviation aircraft. For purposes of this report, it was assumed that the AVOIDS-II employed the same logic as AVOIDS-I operating at altitudes below 10,000 ft where maximum speed is restricted. (To provide a larger protected volume, the AVOIDS-II threat logic and power budget would have to be increased.) The results of the analysis indicate the false alarm rate for both the Honeywell AVOIDS-I and assumed AVOIDS-II will not be excessive when operated in the peak density forecast by the FAA for the 1982 Los Angeles Basin. The FAA forecasts about eight false alarms per hour for all aircraft in the 1982 Los Angeles Basin. The average false alarm rate per aircraft is expected to be less than one per 100 hours.

The detection probability of the CAS was assessed assuming the communication range of the equipment was adequate to insure sufficient data for threat evaluation. A missed detection will cause a delay in warning and the probability of a delay in warning for one or two threat evaluation periods (i.e., 3.7 sec to 7.4 sec) is about 2 percent. The probability of a delay in warning of more than two threat evaluations is very small and can be ignored.

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METRIC CONVERSION FACTORS

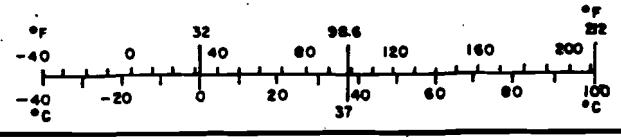
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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I. INTRODUCTION

A. BACKGROUND

Since the late 1950's, industry has proposed several devices for airborne Collision Avoidance Systems (CASs). The earliest proposals were based on radar, but these proved to be technically and economically infeasible for general use. Consequently, most subsequent proposals embodied cooperative techniques. In 1966, under the aegis of the Air Navigation Traffic Control Division of the Air Transportation Association of America (ATA), a CAS Technical Working Group published a report ANTC-117, which described "a standard" cooperative airborne CAS. While ANTC-117 is not a government report, the FAA has encouraged its use by industry developing CASs and has used it in several experimental evaluations of CAS designs as well as in simulations of CAS operations. (See Appendix B for ANTC-117 logic.)

ANTC-117 describes a specific CAS implementation employing airborne clocks, all synchronized to a common time reference. Thus, each CAS can measure one-way range to all others by comparing its local clock time with the arrival time of transmissions from the others. Each CAS can measure the range rate of all others by using the doppler frequency shift of the transmissions. Each CAS encodes its barometric altitude on its own transmissions, which others can decode and compare with their altitude to obtain relative altitude separation. ANTC-117 provides a collision avoidance logic which uses the measured range, range rate, altitude separation, and the altitude rate of the evaluating CAS to define a volume in space within which intruders are possible collision threats.

The size of the ANTC-117 threat volume changes, depending on the conditions of any encounter, to provide sufficient warning time, before a possible collision, for an avoidance maneuver. This maneuver is a climb or dive depending on the relative altitude of the aircraft. Approximately 15 sec is required for the maneuver to achieve safe vertical separation in all cases. However, the time allowance is increased primarily to allow for pilot/aircraft reaction and measurement tolerances but also for horizontal accelerations and vertical rates that may be present at the alarm. ANTC-117 provides for two such alarm volumes: a warning alarm occurs first in the larger volume to effectively reduce aircraft horizontal accelerations and vertical rates, and if the intruder penetrates the second, smaller volume the alarm changes to command the vertical maneuver.

Because the measured parameters used in the ANTC-117 CAS logic are limited to those given above, the volume is unnecessarily large and some alarms are given during perfectly safe encounters. Nevertheless, such alarms are "valid" in terms of the logic used. Other alarms that may be caused by equipment deficiencies are called "false" alarms.

Several manufacturers have built CASs using the implementation and logic of ANTC-117. Other manufacturers, including Honeywell, have used the ANTC-117 logic, but have employed an implementation based on interrogator/transponder techniques. In the latter techniques, the time of arrival of a transponder reply stimulated by an interrogation is used to measure range, and range rate is derived from the change in successive range measurements. The Honeywell CAS determines the altitude of an intruder by altitude coding the interrogation. Transponders reply only to interrogations that have coded altitudes approximately that of the transponder. In addition to permitting the interrogator to determine altitude of the transponder, the latter technique also serves to suppress unwanted replies from transponders at other altitudes. Such replies are a principle source of interference in the interrogator/transponder techniques.

B. HONEYWELL COLLISION AVOIDANCE SYSTEMS

The Honeywell CAS is an asynchronous system, using interrogator/transponder techniques and operates on a single frequency (1600 MHz). Short 65-nanosec pulses with a low-duty cycle are used to reduce mutual interference. Range between aircraft is determined by measuring the two-way propagation delay, and range rate is derived by processing the range measurements. The interrogation waveform is encoded with a reference altitude, biased from the barometric altitude of the interrogator. Only aircraft reply whose altitudes are within ±650 ft of the encoded reference altitude of a received interrogation. By processing the replies from a series of interrogations at different reference altitudes, an interrogator can determine the altitude band of intruders. During a measurement and evaluation period of 3.7 sec, range measurements are processed and the altitude of replying transponders determined for use in the threat logic. An alarm is displayed if evaluations in two successive periods show a threat.

Each Honeywell CAS operates either as an interrogator or a transponder on a time-shared basis. It would be desirable for a single interrogator at a time to probe all other transponders without interference. However, since more than one CAS may be interrogating at the same time, replies elicited by one may interfere with those generated by another. This interference, called fruit, has several effects. For example, it interferes with desired responses and can generate false interrogations. It is a function of the CAS tracker to eliminate the effects of such interference. Once a transponder receives either a valid or a false interrogation, it is blocked to other valid interrogations for a period of time. A CAS is also blocked when it is transmitting its own interrogations and awaiting replies. To overcome blockage and other sources of unreliable communications, redundant interrogations are made to assure sufficient data is collected for an evaluation.

Honeywell has developed two CAS versions: AVOIDS-I for 600 knot aircraft at all altitudes, and AVOIDS-II for aircraft with speeds below 250 knots and operating below 10,000 ft. AVOIDS-I uses the complete ANTC-117 logic. Both CASs measure the same parameters described above. In addition, AVOIDS-I uses the interrogator's altitude rate as required by ANTC-117.

In the AVOIDS-I logic, depending on altitude separation, an alarm is given if the relative range between aircraft in an encounter goes below a threshold value. The range threshold is a variable equal to the relative closing range rate times a constant (τ with dimensions of time) plus a constant (R_o, which has the dimensions of range). Two paired values of τ and R_o are used: for altitude separations up to ±3250 ft, a τ_2 alarm (with τ = 40 sec and R_o = 1.8 nmi) requires the pilot to limit aircraft maneuvers and the rate of climb/dive; for altitude separations up to ±600 ft, a τ_1 alarm (τ = 25 sec and R_o = 0.25 nmi), which will occur at a shorter relative range, requires the pilot to climb or dive, depending on whether the aircraft is higher or lower than the intruder.

The AVOIDS-II differs from the AVOIDS-I in that it has less power, poorer sensitivity, and provides a smaller protection volume. The protection volume of the AVOIDS-I is patterned after ANTC-117 specifications and covers ±3250 ft relative altitude and 102,000 ft in range. The AVOIDS-II protection volume covers only ±1250 ft relative altitude and 25,600 ft in range.

C. IDA'S TASK

IDA previously conducted an analysis of an earlier version of the Honeywell air-derived CAS. The latter work, referred to as Phase I, was presented in Ref. 2. Since the publication of that report, Honeywell has redesigned the CAS.

IDA was requested under Task B of the DOT/FAA contract DOT FA74WA-3498 (see Appendix A) to analyze the revised Honeywell CAS. The new work is referred to as Phase II. Like the

first report (Ref. 1), this report is limited to exploring critical problems and performance of the Honeywell CAS in the most dense aircraft traffic environment projected by the FAA's Los Angeles (LAX) 1982 model--that below 8000 ft and within about 10 miles of the Los Angeles airport.

Where possible, we attempted to build on the Phase I analysis of the Honeywell CAS, but because of extensive and significant design changes (particularly in the target trackers and threat evaluation logic), the portion treating performance of the CAS with regard to detection probability, false track initiation, and false alarm probability in a dense environment, had to be revised.

D. ASSUMPTIONS

The FAA 1982 LAX traffic model used in our analyses provides the density of aircraft as a function of altitude and range from Los Angeles airport. The traffic is most dense at the center and falls off with increasing range from LAX and with increasing altitude. Since the model gives only the static distribution, it is necessary to make assumptions about dynamic parameters, such as speed. Generally, the aircraft were assumed to have random heading and a velocity distribution similar to thosr described in Ref. 1, even though traffic is more structured in the real world. With such an assumption, the range rates can be shown to have a zero mean Gaussian distribution and standard deviation of about 300 ft/sec.

The proposed logic and equipment parameters (power, sensitivity, etc.) for AVOIDS-II are not compatible with ANTC-117 requirements for protection. Therefore, IDA assumed that AVOIDS-II would be modified, i.e., that the power and sensitivity would be improved to permit detection ranges compatible with ANTC-117 threat volumes below 10,000 ft and that its logic would be modified to that of ANTC-117. Determining the cost of such changes to AVOIDS-II is outside the scope of IDA's task assignment and was not addressed.

All aircraft were assumed to be instrumented with the Honeywell CAS, and the equipment was assumed to be reliable and to operate in accordance with Honeywell's present design goals. In addition, a clean electromagnetic environment was assumed in that the effect of electromagnetic interference, other than that induced by other CAS-equipped aircraft, was not considered. Deviation from these assumptions could adversely affect the performance capability predicted for the equipment.

In this report, the evaluation of the CAS has been based on the criteria and specifications, insofar as they are applicable, of ANTC-117. No direct conclusion about the efficacy of the CAS, as such, should be drawn from the fact that the system evaluated herein conforms to the requirements set forth in that document. In another, concurrent, study by IDA (Ref. 2), an evaluation of ANTC-117 itself was made. The results of that study indicate that ANTC-117 requirements do not guarantee the safe avoidance of collision between encountering aircraft in all circumstances, i.e., the desired time allowance for maneuver of about 15 sec may not be realized in all encounters. Moreover, they predict excessive alarm rates (of the order of a few per minute) for an aircraft in high density traffic such as forecast by the FAA for the Los Angeles Basin in 1982.

E. REPORT ORGANIZATION

The summary and conclusions of our analysis are presented next. Chapter III describes the operation of the CAS and includes a list of the design changes that have been incorporated since IDA conducted the Phase I analysis (Section III-E). Chapter IV presents the results of an analysis of the false alarm rate. An analysis of the AVOIDS detection probability is provided in Chapter V.

II. SUMMARY AND CONCLUSIONS

A. SUMMARY

This report presents the results of an analysis of the performance of the Honeywell Collision Avoidance System (CAS) under the high-density conditions forecast for peak air traffic in the Los Angeles (LAX) Basin in 1982.* Our analysis indicates that the Honeywell AVOIDS-I deviates in minor respects from the requirements set forth in ANTC-117. However, the AVOIDS-II equipment, designed for use on slower aircraft with speeds less than 250 knots and at altitudes below 10,000 ft, has a much smaller protection volume than that specified by ANTC-117 for operation below 10,000 ft. For the purposes of estimating detection and false alarm probabilities, the protection volume of AVOIDS-II was increased to make it compatible with ANTC-117. (The protection volume of AVOIDS-I is already designed to be approximately that specified by ANTC-117 for aircraft operating below 10,000 ft and for aircraft speeds up to 600 knots above 10,000 ft.)

The results of our analysis of detection and false warning probabilities indicate that the AVOIDS-I and the assumed modified AVOIDS-II will function satisfactorily in the peak density projected by the FAA air traffic density forecast for the Los Angeles Basin in 1982.

Many factors, such as terrain shielding, ignition noise, multipath, and fine structure in antenna patterns, were not considered. Further, the ANTC-117 specifications require engineering and technical tolerances applicable to a one-way CAS but not

*The forecast was prepared by the Aviation Forecast Division of the FAA, 27 April 1971.

to a two-way system. If the ANTC-117 specifications had cited performance tolerances, such as permissible probabilities of late alarms and false alarms, we might have been able to determine the level of traffic density at which the CAS is adequate. Lacking these specifications, the adequacy must be a judgmental factor.

B. CONCLUSIONS

<u>Conclusion 1</u>: *The Avoids-II, as presently designed, has a much smaller threat volume than that required by the ANTC-117 specifications and may not provide adequate protection to general aviation aircraft.* The threat volume of the AVOIDS-II is ±1250 ft in relative altitude and 25,000 ft in range, whereas ANTC-117 logic has a threat volume of ±3350 ft in relative altitude and about 47,000 ft in range for an aircraft below 10,000 ft. To meet ANTC-117 specifications, the AVOIDS-II threat logic would have to be changed and the power budget would have to be increased by at least 6 dB to support reliable communications.

<u>Conclusion 2</u>: *The AVOIDS-I altitude threat logic has minor discrepancies as compared to ANTC-117.* The ANTC-117 logic covers ±3350 ft in relative altitude versus ±3250 for AVOIDS-I. The boundary between the commands to limit altitude rate ($\dot{h}$) to 2000 and 1000 ft/min is specified as 1850 ft in ANTC-117 but as 2050 ft relative altitude for the AVOIDS-I. The boundary of the limit command between 1000 ft/min and 500 ft/min is specified as 1350 ft in ANTC-117 but 1250 ft for the AVOIDS-I.

In addition, the description provided IDA regarding the manner in which the predicted co-altitude bands work

is not in accordance with ANTC-117.* Under some conditions of high vertical rates and target altitude separation, the AVOIDS logic could generate a "Limit Vertical Rate" command whereas the ANTC-117 logic would generate a "Level-Off, Do Not Turn" command.

Conclusion 3: *For noncollision threats, the AVOIDS tracking filter may prematurely drop a command as the aircraft nears the point of closest approach.* This results from the high indicated acceleration due to changes in geometry. The CAS will not meet ANTC-117 specifications in this regard; however, this effect will not occur in collision encounters, and in noncollision encounters the alarm is dropped no more than 5.5 sec early. This time is too little for the aircraft to maneuver into a collision trajectory.

Conclusion 4: *The results of our analysis indicate that the false alarm rate of the AVOIDS CAS, if operated in the peak density forecast by the FAA for the 1982 Los Angeles Basin, will not be excessive.* About eight false alarms per hour are expected for all aircraft in the 1982 Los Angeles Traffic Model, and the average false alarm rate per aircraft is therefore less than one per 100 hr. Most of these false alarms are due to aircraft with near-threatening values of range rate and are allowed according to ANTC-117 specifications of τ alarm accuracy.

Conclusion 5: *AVOIDS transponder blockage can result in a delay in displaying an alarm, but its effect is relatively small.* The probability of *detecting* a target in a single 3.7 sec evaluation epoch is about 0.99. Since the

*Honeywell states that the description of the predicted co-altitude bands provided IDA was in error and that the current equipment design will work in accordance with ANTC-117 threat logic.

AVOIDS CAS requires that a threat be detected in two successive epochs to display an alarm the probability of *displaying an alarm* at the τ_2 boundary, or of changing an alarm from that for τ_2 to τ_1 at the τ_2 boundary, is o.98. Delays of one-epoch or two-epochs are equally likely, each with probability 0.01, and a three-epoch delay has probability 0.0002. The *expected* delay is a small fraction (0.03) of an epoch, or 0.11 sec. Once an alarm is displayed, the probability of falsely dropping the alarm is 0.001 and can be ignored.

When a one G positive acceleration, i.e., one that is reducing the rate of closure, is present the target may not be detected properly with probability 0.02, but there do not appear to be any encounter circumstances in which this deviation from ANTC-117 has importance.

III. SYSTEM DESCRIPTION

A. INTRODUCTION

The Honeywell CAS is described here in some detail for the reader who may not be familiar with it. Changes since the previous IDA analysis (Phase I) are described in Section E. Analysis of the system performance is presented in Chapters IV and V.

The system employs a transponder ranging technique. It requires no time synchronization between participating aircraft, since the system has both an interrogation and a response mode of operation. During the interrogation mode, a four-pulse altitude-coded waveform is transmitted. This waveform requests other aircraft within communication range and within a specified altitude band (typically ±650 ft of an encoded reference altitude) to respond. An aircraft responds by transmitting a single 65-nsec pulse, which is delayed a fixed amount with respect to the received interrogation waveform.

The interrogation and reply waveforms are transmitted on the same frequency (1.6 GHz). They are distinguished by using two pairs of pulses transmitted during the interrogation mode, and a single pulse transmitted during the reply mode. The pulses of the first interrogation pair, each of 65-nsec duration, are separated in time by 500 nsec. A second pair of 65-nsec pulses in the interrogation waveform have a relative separation of 600 nsec. Altitude is encoded on the interrogate waveform by varying the time separation between the first

and second pulse pairs. A minimum time separation of 32.5 μsec occurs when the lowest reference altitude (-1200 ft) is encoded. For higher altitudes, the time separation is increased by 200 nsec for each 100 ft of altitude. The maximum altitude that can be encoded is 60,000 ft. The minimum separation of 32.5 μsec is intended to prevent ground reflections from interfering with the direct path transmission between aircraft.

Small aircraft will probably be equipped with a single antenna, but larger aircraft will be equipped with two antennas. To avoid producing radiation pattern nulls when two antennas are used, a relative time delay greater than 65 nsec is introduced between the excitation of the two antennas. Also, the pulse separation of the first pair is made different from that of the second pair so that the first pair can be recognized by a decoder as the beginning of an interrogation and the second pair as the end. Multipath, even with a 32.5 μsec guard delay, can lead to ambiguous altitude decoding.

B. AVOIDS INTERROGATION/RESPONSE SEQUENCE

The basic interrogation/response sequence for making a range measurement is shown in Fig. 1. The sequence starts when an interrogation waveform is transmitted by an aircraft (Aircraft 1 in Fig. 1). For purposes of this discussion, we assume that the reference altitude encoded in the transmission is such that a second aircraft (Aircraft 2) responds. This does not ensure that the two aircraft are within ±650 ft of each other, because several altitude bands must be searched during threat evaluation. The altitude search is accomplished by transmitting a series of interrogation sequences in which the reference altitude is changed (this is described in more detail in Section C).

Aircraft within communication range of Aircraft 1 receive the first pulse pair of the interrogation waveform and check

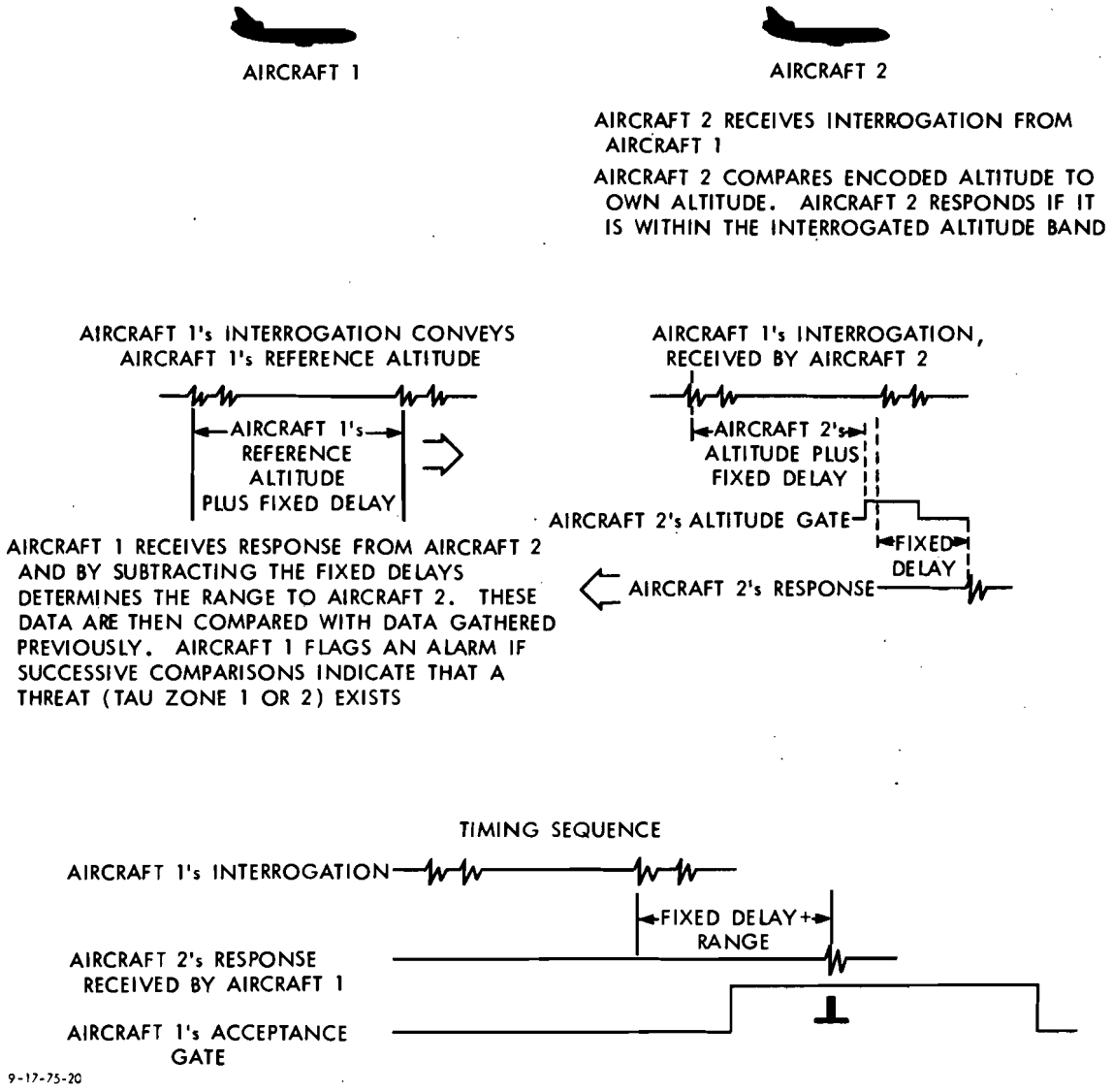

FIGURE 1. Interrogation/Response Sequence of Events (Source: Ref. 3)

for the correct time separation between the pulses for the interrogation first pulse pair; each CAS receiver is equipped with a digital correlation network, matched to the interrogation first pulse pair. An output of the correlation network occurs only if the time between the two received pulses is approximately correct (i.e., 500 nsec for the first pulse pair). When Aircraft 2 receives the first pulse pair of the interrogation waveform, it generates an altitude acceptance window for checking the transmitted reference altitude. If the second pulse pair of the interrogation waveform passes a second correlation network, which checks for a time separation of 600 nsec, and is received within the acceptance window, Aircraft 2 responds. The acceptance window is positioned so that a reference altitude, equal to the receiving aircraft's altitude, appears in the middle of the gate. By making the window 2.6 μsec wide, a pulse pair that is encoded within ±650 ft of the receiving aircraft's altitude can be received. An output from the altitude window causes the transponder to reply with a single 65-nsec pulse. However, the actual time of transmission is delayed by 32 μsec.

After Aircraft 1 transmits its interrogation waveform, its receiver is programmed to accept replies that are returned within a maximum range of interest. According to the ANTC-117 specification, a τ_2 alarm can appear at a maximum range of

$$R_{max} = 10{,}940 + \dot{R}\,(\tau_2)$$

$$= 10{,}940 + (916)(40)$$

$$= 47{,}580 \text{ ft}$$

for an aircraft below 10,000 ft. Above 10,000 ft, the legal speed is greater and R_{max} must be increased.

Honeywell has designed two versions of its CAS: AVOIDS-I for high-performance aircraft and AVOIDS-II for low-performance aircraft. The Honeywell AVOIDS-I design provides a maximum instrumented range of 102,400 ft above altitudes of 10,000 ft so that an encounter between two aircraft, each flying at a speed of 2000 ft/sec, can be detected at the maximum τ_2 warning range of ANTC-117. The AVOIDS-I is patterned after ANTC-117 threat logic specifications. The AVOIDS-II, however, is not. The AVOIDS-II has been designed to provide a smaller threat volume and has a maximum instrumented range of 25,600 ft. There is no accepted specification similar to ANTC-117 for a CAS with the characteristics of the AVOIDS-II. Honeywell believes the AVOIDS-II will be useful for smaller aircraft which cannot afford the full performance of the AVOIDS-I. However, because the FAA task under which IDA conducted this study (see Appendix A) specifies that the CAS should be analyzed within the context of ANTC-117, IDA has chosen to assume that AVOIDS-II would be modified to incorporate the ANTC-117 logic rather than dismiss the AVOIDS-II as inappropiate for analysis. It should be noted that the AVOIDS-II operating in the transponder mode is compatible with AVOIDS-I.

Replies received within the maximum range of interest are detected and assigned to range bins. The data are then stored in binary shift registers for comparison with other successive data sets obtained at a different time. Each range bin is assigned to a specific bit of the shift register and the width of a range bin is 50 ft in range.

Timing for the range measurement sequence is summarized in Fig. 2. The interrogate waveform from Aircraft 1 is transmitted at zero time ($t = 0$) and is received by Aircraft 2 at $t = 5$ μsec, assuming a 5000-ft separation between the two aircraft (all altitudes are barometric, referenced to sea level). The output of the two-pulse correlation network of Aircraft 2 occurs at $t = 5.5$ μsec. Then, the ramp generator is started

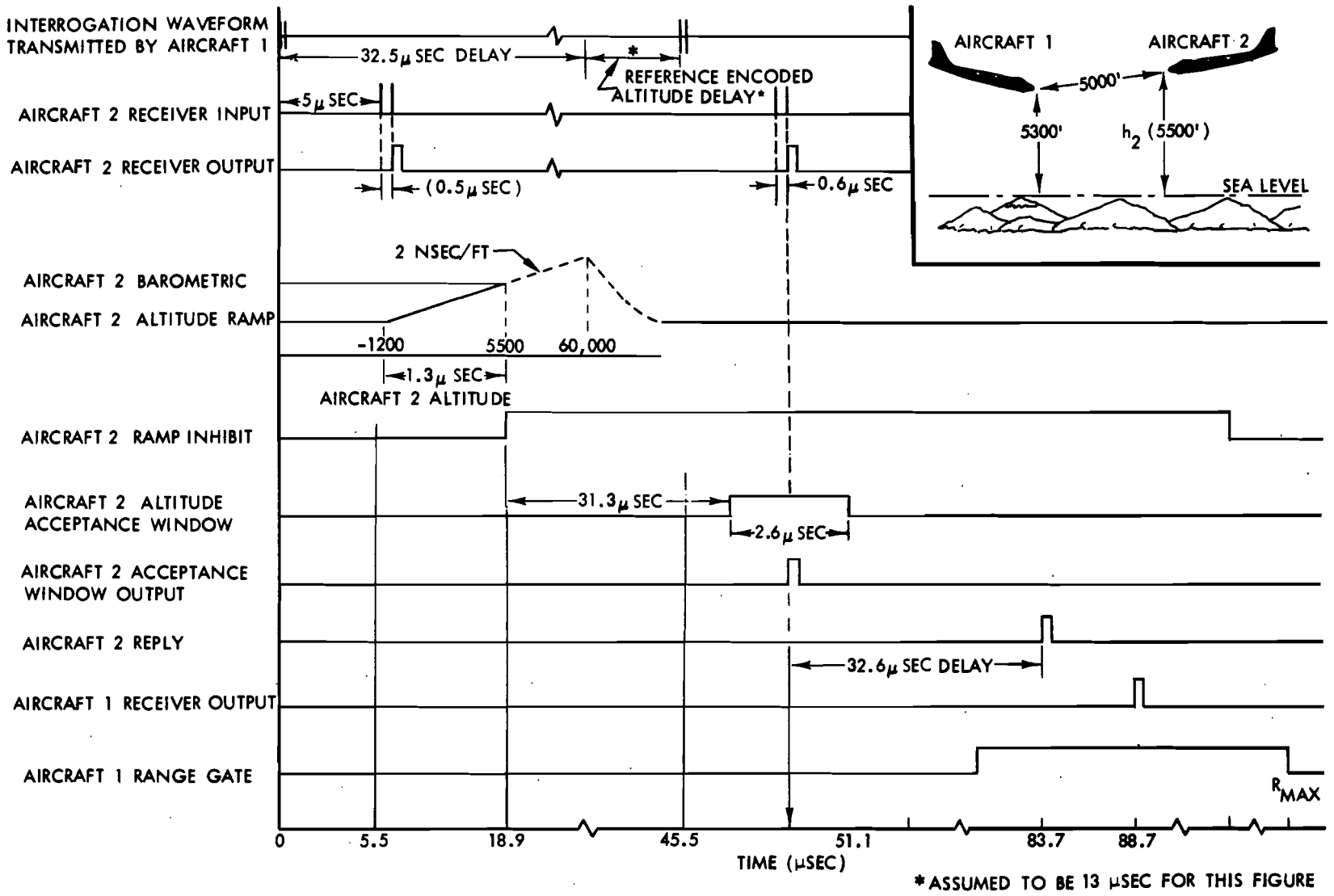

FIGURE 2. Timing Diagram for Interrogate/Response Sequence

to position the altitude acceptance window for receiving the second pulse pair of the interrogate waveform. For the case shown in Fig. 2, the ramp produces a 13.4 μsec delay which is added to the encoding delay of 31.3 μsec; therefore, the acceptance window starts at 50.2 μsec (i.e., 5.5 + 13.4 + 31.3) and continues until t = 52.8 μsec so that reference altitudes within ±650 ft can be received.

The second pulse pair of the interrogation waveform from Aircraft 1 is transmitted at t = 45.5 μsec. This assumes that a reference altitude of approximately 5300 ft has been encoded. Aircraft 2 receives the second pulse pair at t = 50.5 μsec and produces an output of the two-pulse correlator about 0.6 μsec later. The signal is then compared with the acceptance window.

After the signal passes the acceptance window, it is delayed 32.6 μsec, and a single 65-nsec reply pulse is transmitted. This occurs at t = 83.7 μsec. The response is received by Aircraft 1 at t = 88.7 μsec. Since the reply occurs within the transponder's range gate, it is recorded.

Note that in the Honeywell CAS the altitude filtering is performed in the transponder and is set by the 2.6 μsec altitude gate. The interrogator biases its transmissions to probe a particular altitude band; in the case shown in Fig. 2, the bias is zero. Only transponders whose altitude gate passes the second pulse pair respond; however, all other transponders replying to other interrogators and whose transmissions are received by Aircraft 1 while its range gate is open are put into range bin memories. These latter signals are not correct in range or altitude, because they are not synchronized in any way with Aircraft 1's original transmission. Such signals, referred to as transponder or reply fruit, are the most significant cause of interference in the Honeywell CAS, just as they are in any unsynchronized interrogator/transponder system.

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While Aircraft 1 is transmitting its interrogation and waiting for replies, it does not respond to any other aircraft's interrogations. This effect is referred to as interrogator blockage. In fact, during the interrogation mode, any other received interrogations are treated as replies; these are a second, but less important, source of fruit. Transponders can also be blocked if they are already completely occupied in the processes of replying to interrogations when an additional interrogation is received.

C. TAU THREAT EVALUATION LOGIC

The threat evaluation logic equipment detects the presence of threatening aircraft and determines if they meet the Tau criteria. The AVOIDS-I is patterned after the ANTC-117 logic so the equipment determines if an intruder meets either Tau 1 or Tau 2 criteria. The AVOIDS-II design, which is not patterned after the ANTC-117 threat logic, uses three Tau zones to evaluate potential threats. (The latter logic has not been analyzed, as previously explained; however, this logic is explained in this section for the benefit of readers who may be interested.) The threat evaluation logic equipment should also prevent fruit and/or a combination of nonthreatening tracks from being reported as a false threat. For those unfamiliar with the ANTC-117, a short description of the logic is included in Appendix B.

The defining equations for the different Tau zone threats which are incorporated into the AVOIDS designs are presented in Table 1. The boundaries of the Tau zones are a function of range and range rate so that an intruder with a given value of range rate may represent a Tau 1 threat at short ranges, a Tau 2 at intermediate ranges, or no threat at longer ranges. In the remainder of this subsection, the description is mostly devoted to the AVOIDS-I design. The manner in which the data are processed in the AVOIDS-II is similar; however, the parameters used for threat evaluations are different.

TABLE 1. TAU ZONES FOR AVOIDS-I AND -II

Tau Zone	AVOIDS-I	AVOIDS-II
1	$R \leq -25\ \dot{R} + 1500$ ft or $R \leq 3000$ ft	$R \leq 25\ \dot{R}$ or $R \leq 2500$ ft
2	$R \leq -40\ \dot{R} + 10{,}800$ ft	$R \leq 40\ \dot{R}$ or $R \leq 6000$ ft
3	(NOT USED AND NOT PART OF ANTC-117)	$2500 \text{ ft} \leq R \leq 6000$ ft and $-80 \leq \dot{R} \leq 50$

Notes: R = Range to Intruder (ft); $\dot{R}$ = Relative Range Rate (ft/sec).

The range and range rate of an aircraft are the parameters used to determine whether a threat exists. The short pulse-width waveforms used by the Honeywell CAS are ideally suited for measuring range; however, range-rate information cannot be obtained from a single transmission. This requires the use of a long pulse. Instead of using the doppler principle, the AVOIDS-I and -II determine an aircraft's range rate by observing the change in range occurring during a fixed time interval. The observation time for measuring range rate is called an epoch. Since a nominal 3.7-sec epoch is planned for the CAS, the range rate of an aircraft could be calculated by dividing 3.7 into the observed change in range. It is not necessary to actually perform this calculation, as will be seen later. By using range measurements separated in time to ascertain range rate, accurate results may be obtained, but this technique is susceptible to "ghosting" whenever multiple aircraft are within communication range. (Ghosting occurs

when the equipment cannot associate the returns received during an epoch with each aircraft. When this happens, it is impossible to uniquely determine the range rate of the intruding aircraft.)

To reduce the ghosting problem and help prevent the generation of false tracks, the Honeywell CAS obtains a maximum of eight sets of range data within an epoch. The data sets are distributed throughout an epoch and have a time separation of approximately 1/2 sec. The time interval between data sets is called a sub-epoch. For ease of reference, the sub-epochs will be designated A through H, where A is the first sub-epoch within an epoch and H is the last.

The use of multiple data sets within an epoch helps to reduce ghosting. In addition, the threat evaluation logic requires that data be acquired during all eight sub-epochs if a threatening track is to be declared. This helps to prevent the generation of a false track due to fruit and/or a combination of nonthreatening tracks, since a potential false track is eliminated when any one set of the sub-epoch data is missing. The use of this feature requires the system to have a high probability of acquiring an entire data set on a threatening track. Because of the interrogator blockage (described in Chapter IV), it is not feasible to attain an adequate detection probability on a single interrogate/response sequence. As a result, a number of interrogations are made during each sub-epoch. The data acquired from the multiple interrogations are combined in a logical "or," so that a separate memory is not needed to store the data received from each interrogation.

Either two or three interrogations are made during each sub-epoch. During sub-epochs D through H, three interrogations are made. In sub-epochs A through C, three interrogations are made unless no threat has occurred during the previous two sub-epochs.

The threat evaluation logic examines the range data as they are acquired and discards data not contributing to the generation of a threatening track. The techniques for sorting the data are described later; however, the sorting process does not affect the equipment's data storage requirements. In fact, it is possible to store all data obtained during the sub-epochs in raw form and to perform the entire threat evaluation at the end of the epoch. This procedure does not change the end result, and it is much easier to describe. The remainder of this section describes the threat evaluation logic system as if this procedure were followed; note, however, that after the first sub-epoch a new data set in the second sub-epoch is acquired only if a target has been detected in the first sub-epoch. In subsequent sub-epochs (third, fourth, etc.) a new data set is acquired only if a potentially threatening track exists. For example, if the data from the first and second sub-epochs cannot be combined in such a way as to indicate at least one potential threat, the entire threat evaluation process is stopped. Then, interrogations are not made within the altitude band in question until the start of a new evaluation sequence (i.e., the next epoch). This procedure reduces the number of transmissions and, therefore, the amount of generated fruit. However, in a dense traffic environment, the procedure loses most of its usefulness because the probability of having at least one potential threat during most of the sub-epochs is high. This is discussed in more detail in Chapter V.

In general, then, the AVOIDS CAS accumulates range-ordered replies from all other CASs that reply in the eight sub-epochs of an epoch. The data are used to form tracks using various criteria corresponding to the ANTC-117 logic and others to suppress false tracks formed by interactions of fruit and valid track data. Data corresponding to nonthreatening tracks are discarded, and potential alarms corresponding to threatening targets are generated.

The range tracking filter or Tau filter is best described in terms of Fig. 3. We assume the depicted track to be threatening, and it should be detected by the filter. The threatening aircraft is first observed in range bin ℓ; however, during the epoch, the aircraft range decreases and the H data show the aircraft in range bin ℓ-21. The range of the aircraft is not important in this discussion.

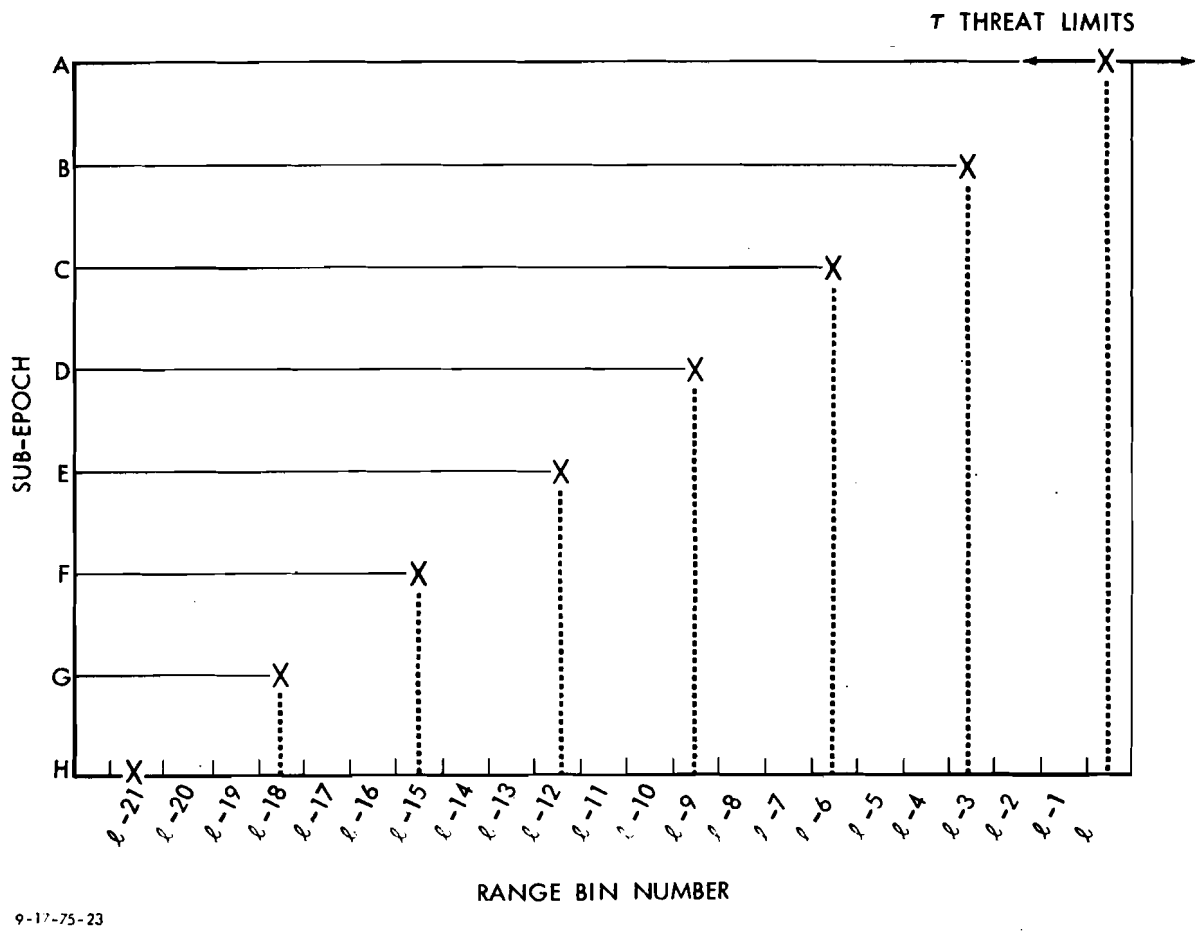

FIGURE 3. Range Track for a Threatening Aircraft with a Range Bin Skip of 21

The tracking filter works as follows. Assume all of the sub-epoch range data have been accumulated and stored in the equipment's memories. The data from sub-epoch H are searched until a return is located. Next, the data from sub-epoch A are checked to see if any returns can be combined with the return which was found in sub-epoch H data to produce a threatening track. Only a limited number of range bins in sub-epoch A can be occupied if the aircraft is threatening. The most distant (i.e., longest range) range bin in sub-epoch A is fixed by the maximum range rate that can be encountered. The closest range in sub-epoch A is determined by the minimum value of range rate that produces a Tau threat. The maximum range rate that can be encountered is 914 ft/sec, if the interrogating aircraft is at an altitude below 10,000 ft and a valid threatening track can move, at most, 64 range bins during an epoch. Above 10,000 ft, the filter for the AVOIDS-I design can detect a maximum range rate of 2029 ft/sec and a valid track is allowed to move inbound 142 range bins.

For the AVOIDS-I, the minimum number of range bins that can be skipped by a threatening aircraft is set by the criterion for the τ_2 zone. The equation defining the boundary of the τ_2 zone is

$$\tau_2 = -\frac{\text{range} - 10{,}940}{\text{range rate}} \quad . \tag{1}$$

In this equation, range is expressed in feet, range rate in ft/sec, and τ in seconds. An aircraft with a τ_2 of 40 must be detected with high probability; however, there may be considerable delay before detection. The equipment is designed so that a threat must be detected in two consecutive epochs before a command or warning is displayed to the pilot. As a result, τ_2 in Eq. 1 is assumed to be set to 46 sec to establish the relationship between a threatening aircraft's range and its highest acceptable range rate.

The maximum value of range rate depends strongly on the aircraft's range. In fact, an aircraft with a range of less than 10,940 ft is allowed to have an opening range rate, and the search range of sub-epoch A is less than the range bin location of the aircraft in sub-epoch H. For the AVOIDS-I design, a threatening intruder at short range may move outbound by 28 range bins so that the sub-epoch H data can be 28 range bins more distant than the A data.

If a return in the sub-epoch A data is found within a range bin separated from the H datum corresponding to a potential track which is threatening, a series of range gates (containing a variable number of range bins determined by the threat logic) is established to check whether the data accumulated during the inner sub-epochs can form a threatening track. Actually, two range bin patterns are used to search the inner sub-epoch data. However, they work independently and can be discussed separately. The range gate pattern which would be used to detect the threatening aircraft range track of Fig. 3 is shown in Fig. 4. The range gate pattern has three range bins in each of the inner sub-epoch range gates. This particular pattern is referred to as the low-acceleration pattern. A valid track requires a return within each range gate, so that the track depicted in Fig. 4 would be classified as threatening for the epoch shown. Note that the location of the range track in the inner sub-epochs could move one range bin to the right or left and still be classified as a threatening track. However, if the range datum for any of the sub-epochs is missing i.e., no return was received in the range gate for any of the two or three interrogations of a sub-epoch, or if the range track for one of the inner sub-epochs is displaced by more than one range bin from the locations shown in Fig. 4, the intruder will be classified as nonthreatening and rejected. Figure 4 shows the position of the range gates that correspond to a target with a nominal range rate of 314 ft/sec and that corresponds

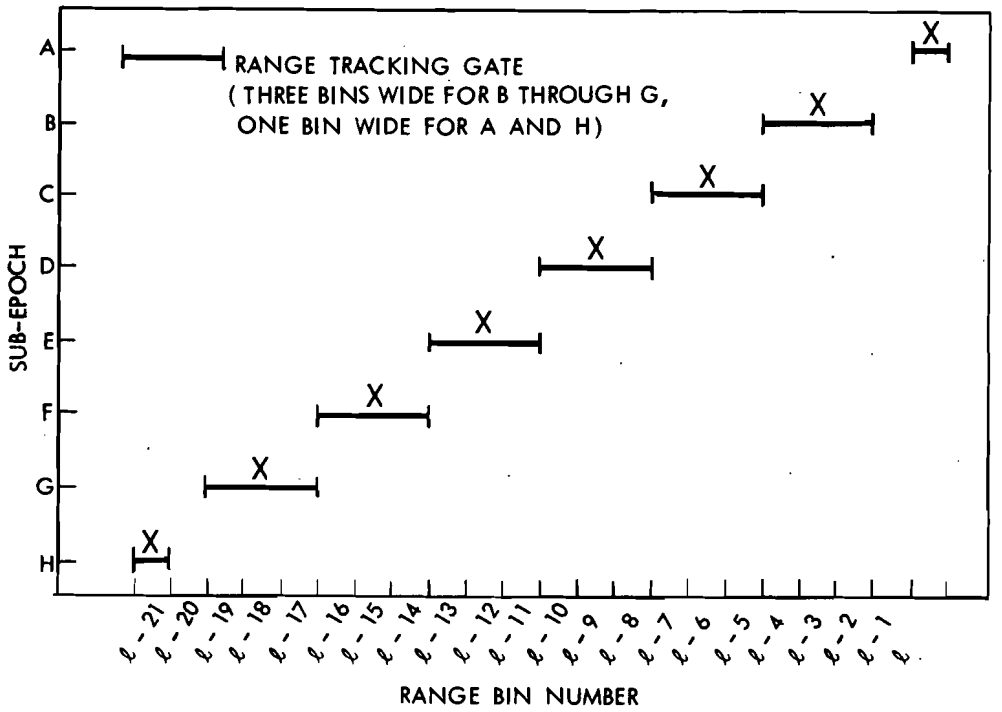

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FIGURE 4. Range Gate Placement for a Target Skip of 21

to the range bin skip of 21 from sub-epochs A to H. If the number of bins skipped between A and H changes, the position of the range gates for the intermediate sub-epochs would change. To understand the positioning of the intermediate gates, which corresponds to forming the tracking filter, first consider a threatened aircraft which produces a stationary range track. The range track would appear in the same range bin number in the sub-epoch A and H data and the intermediate sub-epoch range gates for detecting such a threatening aircraft would be centered about the same numbered range bin positions. This position of the range gates will be referred to as the zero reference position.

To track a moving aircraft requires a repositioning of the range gates so that they follow the average value of observed range rate. For example, in Fig. 4, the center range bin of the range gates of sub-epochs B through H have been incremented by a 3i bin relative to the position of the

sub-epoch A range bin, with i = 1, 2 ... 7 corresponding to sub-epochs B, C ... H. The observed average range rate is found by measuring the number of range bins skipped between the A and H data. When making this measurement, the equipment uses a reference clock that counts in a number system to the base seven. For each clock cycle, the range gates move one range bin between successive sub-epochs from the zero reference position. For example, an aircraft skipping 21 range bins between A and H is measured as a clock cycle of three. (This is the case depicted in Figs. 3 and 4.) The range gate for sub-epoch B is then placed three range bins (3i with i = 1) from its zero reference position, the range gate for sub-epoch C is moved six range bins (3i with i = 2), and so on.

However, the above process does not test for all possible range rates, since the range rates tested jump in steps of one range bin per sub-epoch or 50 ft per half second (100 ft/sec). Of course, the finite width of the range bin allows one to detect a spread of range rates at each clock cycle. The spread corresponds to ± one range bin per epoch, i.e., ±14 ft/sec. One must then provide for testing for the other range rates which would correspond to the intermediate steps of the range gate in sub-epoch H, i.e., gate steps of only one range bin in sub-epoch H rather than steps of seven. In testing for the other range rates, one is faced with the problem of moving the intermediate gates, which, of course, likewise can move only in increments quantized to one bin. Therefore, when the sub-epoch H gate is moved by one bin, the intermediate (B through G) bins may or may not move one bin. These intermediate gate movements must be programmed to move so as to approximate the average range rate being tested.

Another point to be made is that the track of a real target need not correspond to constant range rate because the

target may have acceleration or apparent acceleration if it is not on a collision course. The effects of acceleration plus the quantizing require that the intermediate sub-epoch gates be three bins rather than one bin wide.

To accommodate all range rates that must be tested, the measured phase or portion of a clock cycle also produces a movement of the range gates. Clock phase (θ) is the number of range bins an aircraft moves in excess of an integer multiple of seven:

$$\theta = \left| \ell_A - \ell_H \right| - 7 \times \mathrm{INT}\left(\frac{\left| \ell_A - \ell_H \right| - 1}{7}\right) \qquad \ell_A - \ell_H \neq 0 \qquad (2)$$

$$= 7 \qquad \ell_A - \ell_H = 0$$

In this equation, ℓ_A and ℓ_H are the aircraft's position in sub-epochs A and H, respectively, and INT ( ) indicates the integer value of the enclosed quantity. If for example, $\ell_A - \ell_H = 17$, then $\theta = 3$. The range gate movement for each sub-epoch that results from each of the seven possible clock phases is given in Fig. 5. Actually, however, the data are moved relative to the range gates, but the effect is identical. In the matrix shown, a "1" indicates that during that clock phase the data stored in the particular sub-epoch range bin are moved one range bin position with respect to the stationary H data. A zero indicates the data are not moved during a particular clock phase. The shift is always from left to right in the direction in which range increases. The incremental movement repeats after seven values of range rate are checked, so that only seven shifts, or clock phases, of the data are shown. After seven clock phases, the data are displaced one range bin between sub-epochs. On the eighth shift of the data (that is, one clock phase cycle plus θ_1) the data are shifted one range bin between sub-epochs plus the shift indicated in Fig. 5 for θ_1.

Sub-Epoch Data Registers							
A_i	1	1	1	1	1	1	1
B_i	1	0	1	1	1	1	1
C_i	0	1	1	0	1	1	1
D_i	0	1	0	1	1	0	1
E_i	0	1	0	0	1	0	1
F_i	0	0	0	1	0	0	1
G_i	0	0	0	0	0	1	0
H_i	0	0	0	0	0	0	0
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7

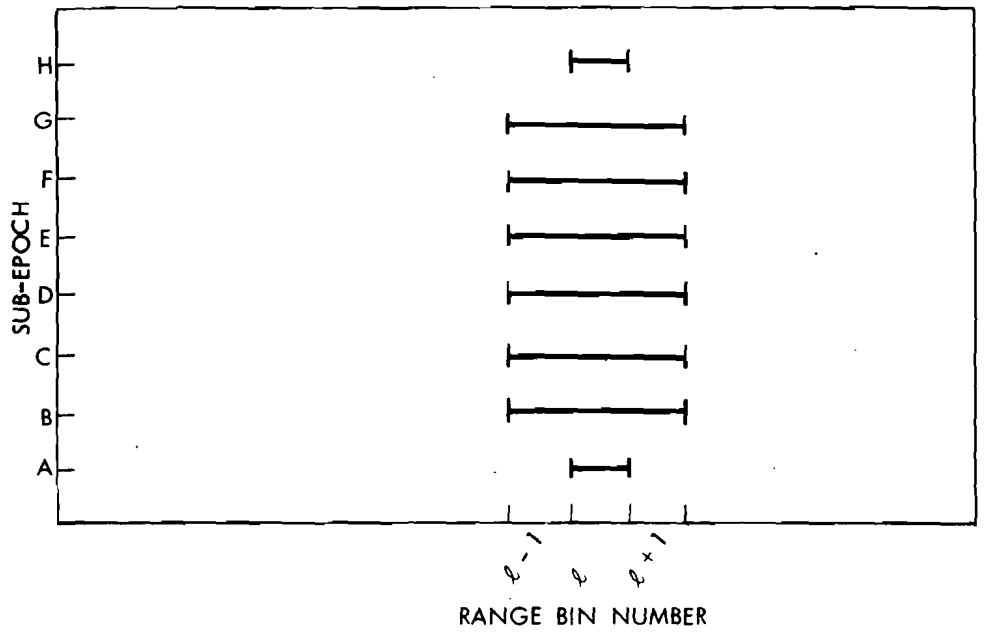
Clock Phase, θ_i

FIGURE 5. Data Shift as a Function of Clock Phase

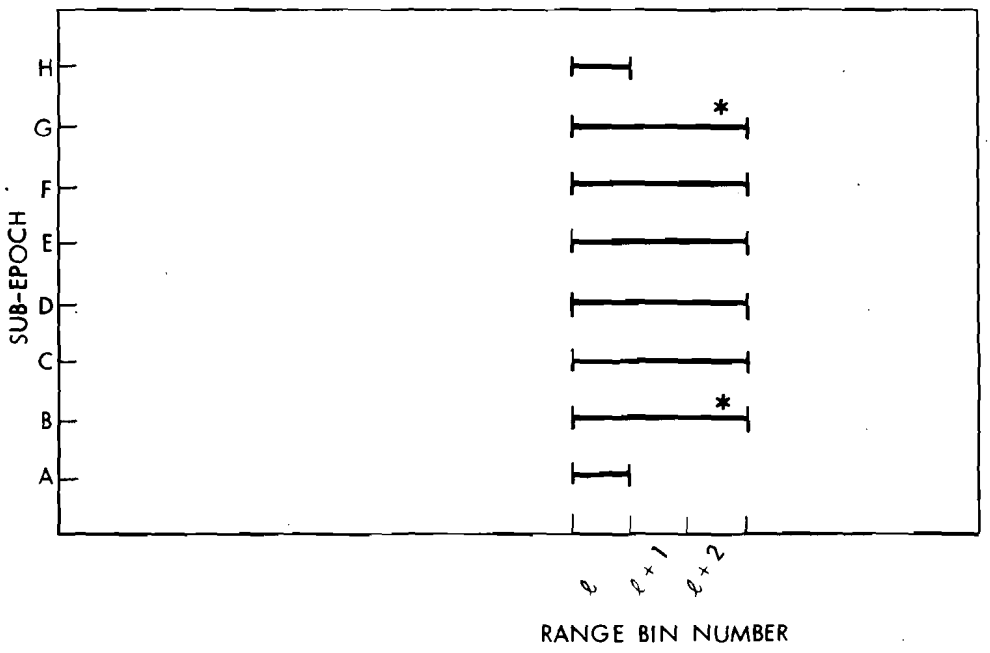
Effectively, a new value of range rate is checked during each clock phase. During clock phase θ_1, the A and B sub-epoch data are moved one range bin and the C through H data are held fixed. During clock phase θ_2, the A data are moved an additional range bin and the data in the shift register for sub-epoch C, D, and E are moved one bin and those for B and F through H are not moved. The data in H remain fixed and those in sub-epoch A are moved one bin so that each clock phase checks for a new value of range rate which is 50/3.5 ft/sec different than the previous value.

The range tracker requires a separate memory location for each range bin (i.e., 50 ft of range) in each sub-epoch for each altitude band in which a Tau determination is made. The memory for the AVOIDS-I CAS consists of a number of 2048-bit shift registers (each bit represents a range bin). Forty-eight shift registers are needed just for Tau evaluation. The shorter range AVOIDS-II require twenty-four 512-bit shift registers for Tau evaluation. To increase the instrumented range of the AVOIDS-II to allow the detection of a threat at maximum Tau 2 warning range would require doubling the bit length of the shift register.

The range or Tau tracker uses a set of fixed range gates to determine if a threatening range track is present. The stored data collected during an epoch are moved within the shift register memory to check for an occurrence of a threatening pattern. A single pattern, used in an older design, consisted of one range bin in the first and last sub-epoch and either two or three range bins in the inner sub-epoch. The new design has two patterns which are depicted in Fig. 6. Range gates are shown in their zero reference positions. The symmetrical pattern shown in Fig. 6(a) is the same one as was displayed in Fig. 4 for a clock cycle of three. The pattern consists of a single range bin in the first and last sub-epochs and three range bins in the inner six sub-epochs. This pattern cannot always detect aircraft which are accelerating or decelerating at 1 G. Target acceleration causes the range track of an aircraft to deviate from linearity, and, as a result, the inner sub-epoch range data to bulge with respect to a line drawn to connect the sub-epoch A and H data. When added to measurement noise, the bulge in the range data can cause the inner sub-epoch data to appear outside the tracker range gate pattern, in which case, a threatening aircraft is not detected. To partially overcome this problem, the asymmetrical



(a) PATTERN FOR TRACKS WITH LOW ACCELERATION



(b) PATTERNS FOR 1 G ACCELERATION

*These range bins are not included for clock phases θ_1, θ_6, or θ_7.

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FIGURE 6. Sub-Epoch Range Bin Patterns for an Average Range Rate of Zero

pattern shown in Fig. 6 (b) is used to insure that a threatening intruder with a 1-G acceleration can always be detected, assuming all sub-epoch range measurements are obtained. This pattern is referred to as the high-acceleration pattern.

The range data which are stored in the shift registers are passed by the fixed range bin acceptance patterns and the occurrence of a return (i.e., an interrogation reply) in all eight sub-epoch range gates for either pattern constitutes a potential threatening track for that epoch. One basic way of implementing this procedure is described below. The manner in which the AVOIDS equipment performs this task is described in the next paragraph.

Initially, the data within the eight shift registers are arranged so that, for a given shift register position, the A data are 28 bins or 1400 ft closer in range than the H data. In other words, an intruder outbound at 400 ft/sec would occupy the same shift register location in the A and H memories. This value of range rate is the most outbound rate of interest and represents a threat only at short ranges. If the eight sub-epoch memories are shifted in parallel past the fixed acceptance gates, the entire data set would be checked for intruders at all ranges of interest with +400 ft/sec range rate. To check for other values of range rate, the relative position of the data can be changed and the eight shift registers can again be recirculated in parallel. Such a process can be continued until all values of range rate of interest are checked. For the AVOIDS-I, the data are checked for a relative range bin shift from +28 bins to -84 bins between sub-epochs A and H if the interrogator is below an altitude of 9600 ft and between +28 and -142 bins at altitudes above 9600 ft. The AVOIDS-II design checks a relative shift between +7 and -64 range bins.

To reduce the number of operations and the time required to check the possible combinations, the actual equipment operates in a manner slightly different than that described above. All of the shift registers are moved in parallel until a return is located in the H data. All range-rate values of interest for possible tracks terminating in the H datum are then checked by changing the position of the A through G data while holding the H datum fixed. At the conclusion of this process, the A through G data are returned to their original positions and all eight sub-epoch data sets are again shifted in parallel until the next H datum is found. The process of changing the relative position of the sub-epoch data is then repeated.

It is sometimes easier to visualize the operation of the tracking filter by considering the stored data to be fixed and the acceptance range bin patterns to move. This is displayed in Fig. 7 for the seven possible clock phases. The top diagrams are for the low-acceleration pattern and the bottom shows the high-acceleration pattern. Phase 7 is shown first, since it corresponds to the case in which the acceptance gates are in line and a check is made for an intruder with a zero value of range rate. Note that during the θ_7, θ_1, and θ_6 clock phases, only two range bins are included in sub-epochs B and G for the high-acceleration pattern.

D. ALTITUDE COMMAND LOGIC

In addition to detecting Tau zone threats, the CAS determines the relative altitude of the intruder aircraft so that proper commands are displayed to each pilot. The altitude threat logic recommended in the ANTC-117 specification for a two-aircraft encounter is shown in Fig. 8. The altitude of interest is within ±3400 ft of the interrogating aircraft. Within these limits, which require different instructions to be

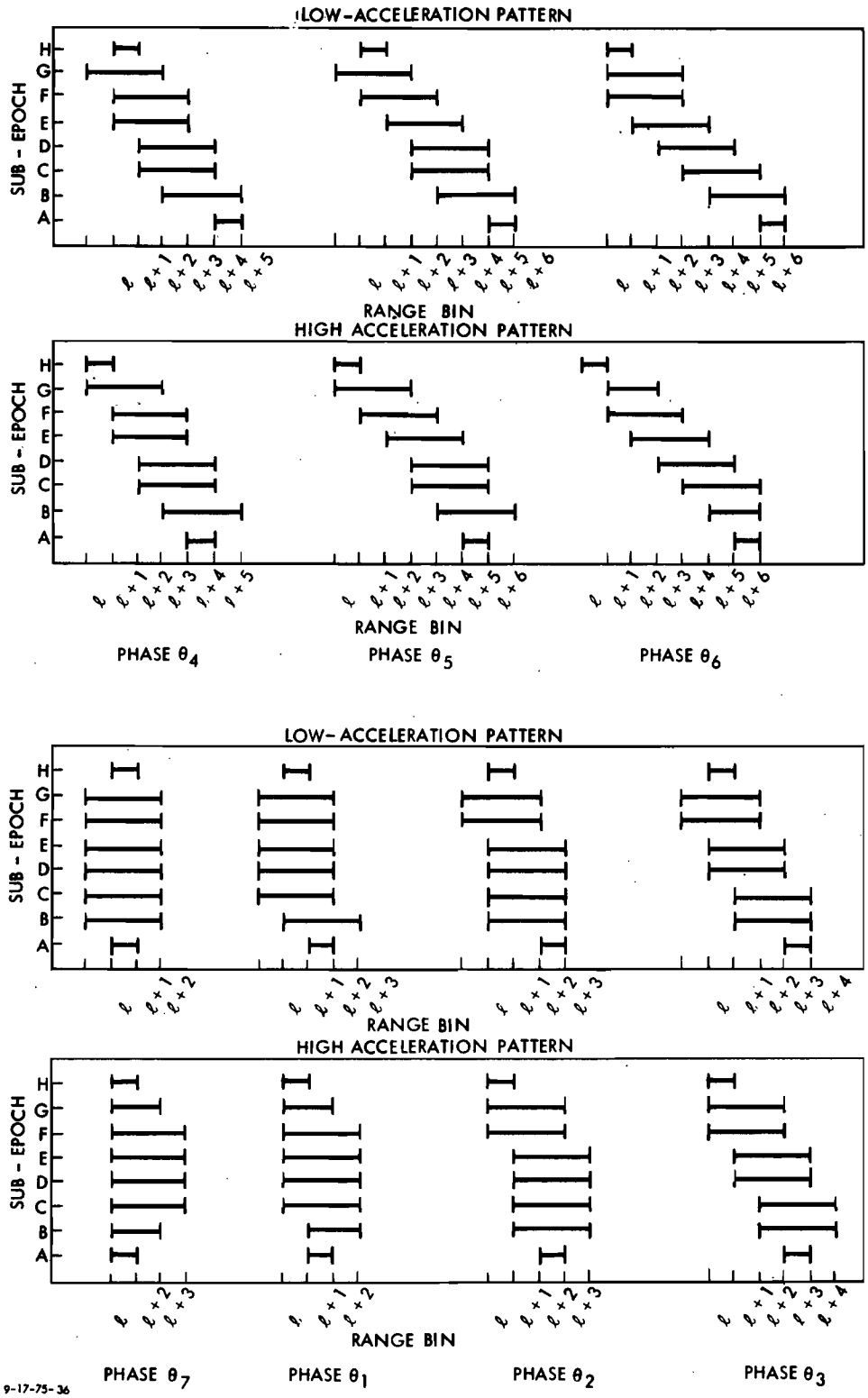
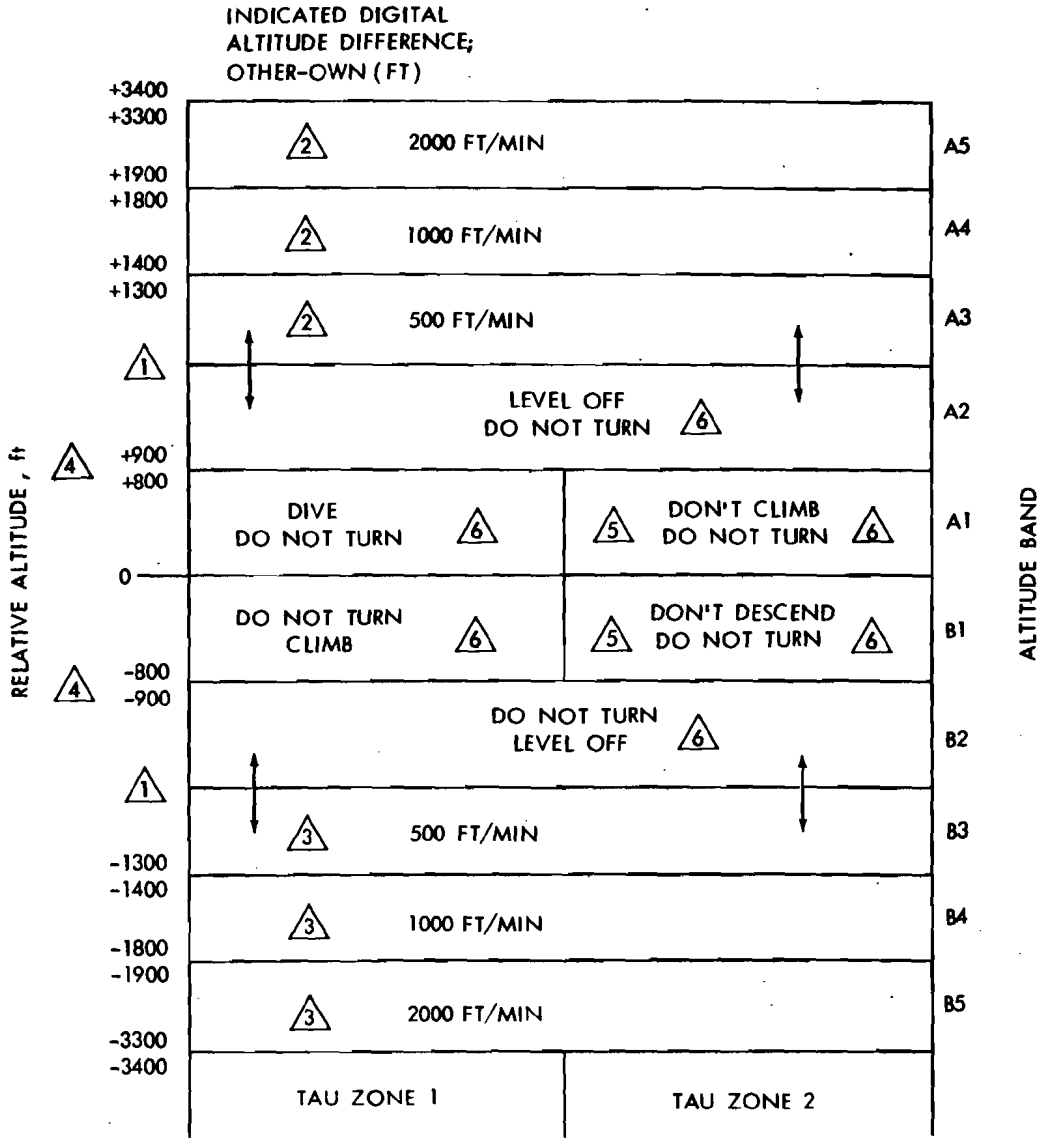


FIGURE 7. Relative Position Sub-Epoch Range Gate for Different Clock Phases



NOTES:

1. PREDICTED CO-ALTITUDE = 800 + 30 SEC × OWN $\dot{h}$ (BELOW 10,000 FT, PCA = 600 + 30 SEC * OWN $\dot{h}$)
2. DO NOT EXCEED CLIMBING $\dot{h}$ VALUE SHOWN
3. DO NOT EXCEED DESCENDING $\dot{h}$ VALUE SHOWN
4. THESE VALUES ARE DECREASED 200 FEET WHEN AT 10,000 - FT ALTITUDE OR BELOW
5. COCKPIT DISPLAY OF A VERTICAL SPEED LIMIT WITHIN THE RANGE OF 0 TO 400 FEET PER MINUTE IS ACCEPTABLE
6. RESTRICT BANK ANGLE TO ± 10 DEGREES OR LESS

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FIGURE 8. Altitude Threat Logic Output--Two-Aircraft Encounter (Source: ANTC-117)

displayed to the pilot, there are 10 altitude bands. For ease of explanation, the five bands above the interrogating aircraft are labeled A1 through A5 and the five below the aircraft are labeled B1 through B5.

According to the altitude threat logic, a command "Dive Do Not Turn" is displayed to the pilot when a CAS transponder detects an aircraft in A1 (indicated altitude difference of 0 to 800 ft above the interrogating aircraft), which represents a τ_1 zone threat; however, when the intruder is a τ_2 threat the pilot is told "Don't Climb Do Not Turn."

An important feature of the logic is that the boundaries between A1 and A2 and between B1 and B2 change by 200 ft if the interrogating aircraft descends below 10,000 ft. Also, the boundary between A2 and A3 and B2 and B3 depends on the rate of climb or descent of the interrogating aircraft. For example, the A2/A3 boundary is set at 1050 ft when the interrogating aircraft is climbing at a rate of 500 ft/min, but it is set at 1300 ft when the rate of climb increases to 1000 ft/min.

While the ANTC-117 altitude logic was developed with the assumption that each aircraft would always transmit its own quantized barometric altitude, this is not the case for the AVOIDS equipments. Rather, an AVOIDS interrogator transmits a coded altitude interrogation. The coded altitude consists of the interrogator's altitude plus a bias. Other aircraft. reply only if the coded altitude is within ±650 ft of their own altitude. Thus, to cover the necessary total altitude band of ±3400 ft specified in ANTC-117, an interrogator must transmit several interrogations, each with a different bias. A minimum of five such biases are needed to cover ±3250 ft, with each interrogation covering 1300 ft.

In addition to covering the total altitude spread, it is necessary in accordance with ANTC-117 to determine in which of the altitude bands shown in Fig. 8 a target lies. Therefore, given the limitation that each altitude layer probed by an AVOIDS equipment covers 1300 ft, more (overlapping) layers must be probed to determine in which of the ANTC-117 bands the target is. Thus, the general approach taken by Honeywell is to probe a number of partially overlapping 1300-ft layers and by a logical process determine approximately in which ANTC-117 altitude band the target lies. Actually, the AVOIDS-I equipment does not duplicate exactly the boundaries of the ANTC-117 altitude bands in order that the number of biased altitude layers probed can be kept to a minimum. This discrepancy is discussed later.

The AVOIDS altitude command logic works as follows. The commands displayed to the pilot of the interrogating aircraft are based on the determination by the equipment of the threatening aircraft's Tau zone and its relative altitude. The relative altitude between the interrogator and the intruder is quantized into a number of altitude threat zones (nine for the AVOIDS-I and five for the AVOIDS-II). The position, in relative altitude, of the altitude threat zones is presented in Table 2 for each of the AVOIDS designs. The equipment must also determine when a threatening aircraft is within ±450 ft of its altitude so that an altitude bias in the direction of an intended maneuver can be added to a threatened aircraft's interrogation waveform to prevent both aircraft from performing the same maneuver. In addition, the AVOIDS-I has two predicted co-altitude threat zones, designated PCA and PCB, which are used if the interrogating aircraft's rate of climb or dive is 500 ft/min or greater. The PCA band is used by a climbing

aircraft and PCB is used by a diving aircraft. The locations of the PCA and the PCB altitude bands are centered in relative altitude at one-half the climb rate in ft/min and extend 650 ft above and below this point.

TABLE 2. ALTITUDE THREAT ZONES

Altitude Threat Zone	Altitude Range	
	AVOIDS-I	AVOIDS-II
P(20)	+2150 to +3250	Not Used
P(10)	+1250 to +2150	Not Used
P(5)	+650 to +1250	+650 to +1250
TA*	+50 to +650	+50 to +650
TE	-50 to +50	-50 to +50
TB*	-650 to -50	-650 to -50
M(5)	-1250 to -650	-1250 to -650
M(10)	-2150 to -1250	Not Used
M(20)	-3250 to -2150	Not Used

*At an altitude of 9600 ft or higher, the TA and TB zones are changed from ±650 ft to ±850 ft.

The altitude threat zones of the AVOIDS-I are patterned after the altitude threat logic recommended in ANTC-117, although some differences exist. A comparison of the ANTC-117 and the AVOIDS-I altitude threat logic is presented in Fig. 9. Only the upper half of the threat region is shown. For the ANTC-117 logic, the altitude of interest is within ±3400 ft of the interrogating aircraft and is slightly larger than the ±3250 ft of AVOIDS-I design. Within the ±3400-ft limit, the ANTC-117 logic defines ten altitude zones. The AVOIDS logic has nine altitude threat zones and two predicted co-altitude bands for a total of eleven. However, a separate command is

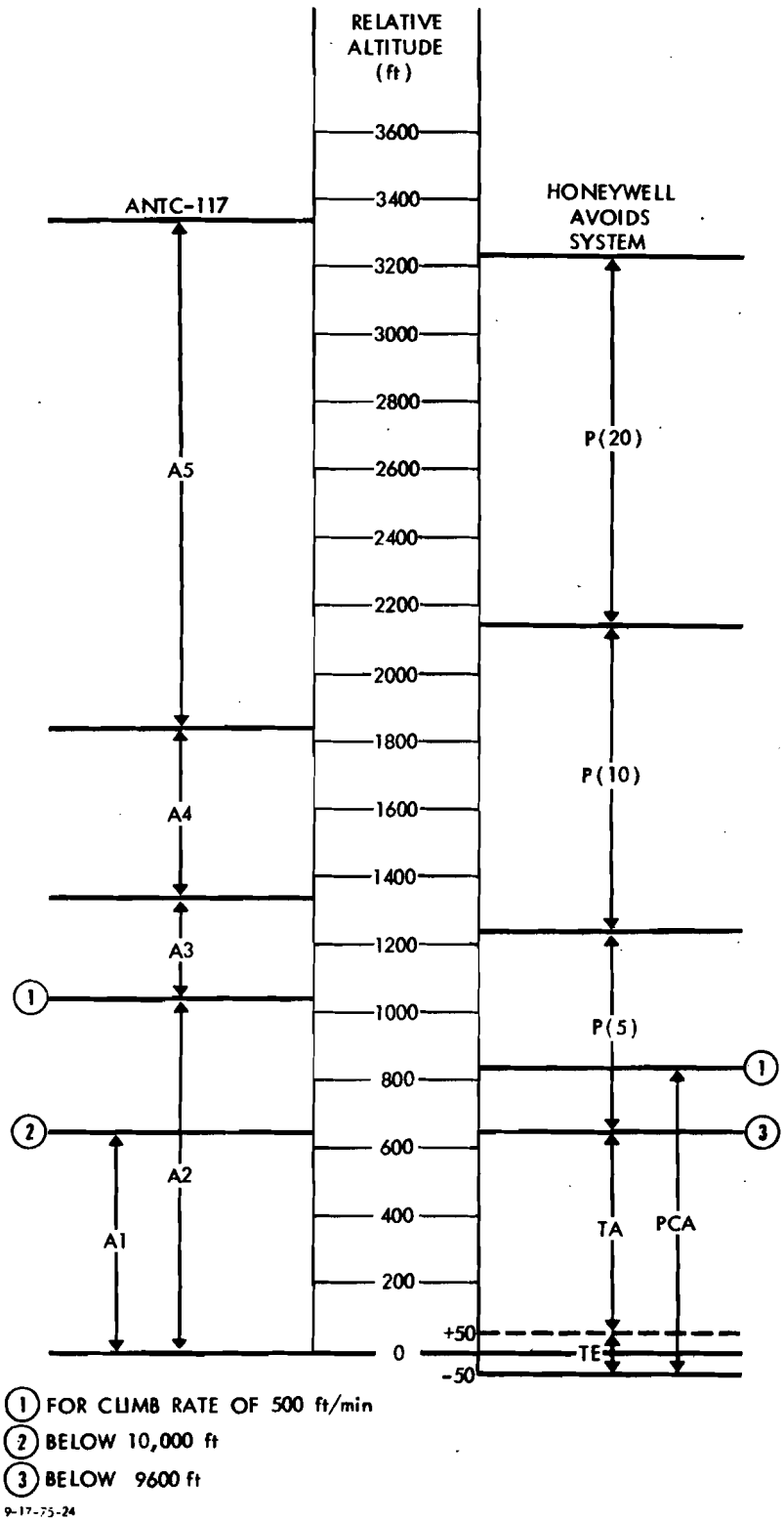


FIGURE 9. Comparison of ANTC-117 and AVOIDS Altitude Zones

not provided for the TE zone of Table 2, so that both the ANTC-117 and the AVOIDS-I logic have the same number of threat zones, although their boundaries differ somewhat in relative altitude.

In Fig. 9, the movable altitude threat zone, A2 for the ANTC-117 logic and PCA for the AVOIDS logic, is shown for an interrogating aircraft which is climbing at 500 ft/min. The difference in the upper boundary for the two logics is 200 ft for this value of $\dot{h}$ (altitude rate). The lower boundary of the AVOIDS logic is correct for this $\dot{h}$. However, for higher values of $\dot{h}$, the lower boundary of the PCA band can rise above the upper boundary of the TA threat zone. It is then possible for a threat to appear between the TA and PCA bands and to produce a limit $\dot{h}$ command rather than the level-off command suggested by ANTC-117.*

The AVOIDS equipment determines the altitude threat zone of a threatening aircraft by transmitting in several altitude bands during an epoch. An altitude band covers ±650 ft and can be positioned at any desired altitude by properly encoding the time separation between the pulse pairs of the interrogation waveform. An AVOIDS-I uses 11 discrete altitude bands, each placed at a different relative altitude with respect to the interrogating aircraft's altitude, for determining a threatening aircraft's altitude threat zone. However, not all the altitude bands are interrogated during any epoch. The bands which are interrogated depend on the altitude rate and the threat conditions which exist within the interrogating aircraft's protected volume. This will be discussed in more detail later in this subsection. The total altitude searched

*Honeywell has informed IDA that the system description they provided was in error and that this condition can no longer occur. Also, the upper boundary of the predicted co-altitude bands is now positioned in accordance with ANTC-117.

by an AVOIDS-I is ±3250 ft. An AVOIDS-II provides a smaller protected volume and uses only five altitude bands to search ±1250 ft in altitude. More than the minimum number of altitude bands are provided for searching the altitude of interest, so that the desired spacing and width of the altitude threat zones can be established. The placement of the interrogated altitude bands is shown in Fig. 10 for the AVOIDS-I, and those for the AVOIDS-II are presented in Fig. 11. On the right side of these figures are displayed the altitude threat zones discussed earlier. The altitude bands for the AVOIDS-I are labeled I(+6), I(+15), I(+26), I(-6), I(-15), I(-26), I(0), I(+4), I(-4), PCA, and PCB. Those for the AVOIDS-II are labeled I(+6), I(-6), I(0), I(+4), and I(-4). Note that at least one boundary of the altitude bands which are interrogated coincides with a boundary of an altitude threat zone. By checking for the presence or absence of a detected threat in the interrogated altitude band, it is possible to uniquely determine an intruder's altitude threat zone.

The Tau zone of threatening aircraft within the relative altitude covered by the protected volume must be checked and may require a full set of eight sub-epoch range measurements. There is considerable overlap in the altitude bands so a Tau measurement in each band is not necessary. For the AVOIDS-I, a Tau measurement is made in each of the six altitude bands of the left side of Fig. 10 [i.e., those marked I(+6), I(-6), I(+15), I(-15), I(+26), and I(-26)] and an AVOIDS-II measures Tau in only two of the altitude bands marked I(+6), I(-6) in Fig. 11. In the remaining bands altitude correlation data are acquired and a full set of range measurements is not needed, i.e., in each of the eight sub-epochs.

The altitude bands in which altitude correlation data are obtained are only interrogated if a potential threatening

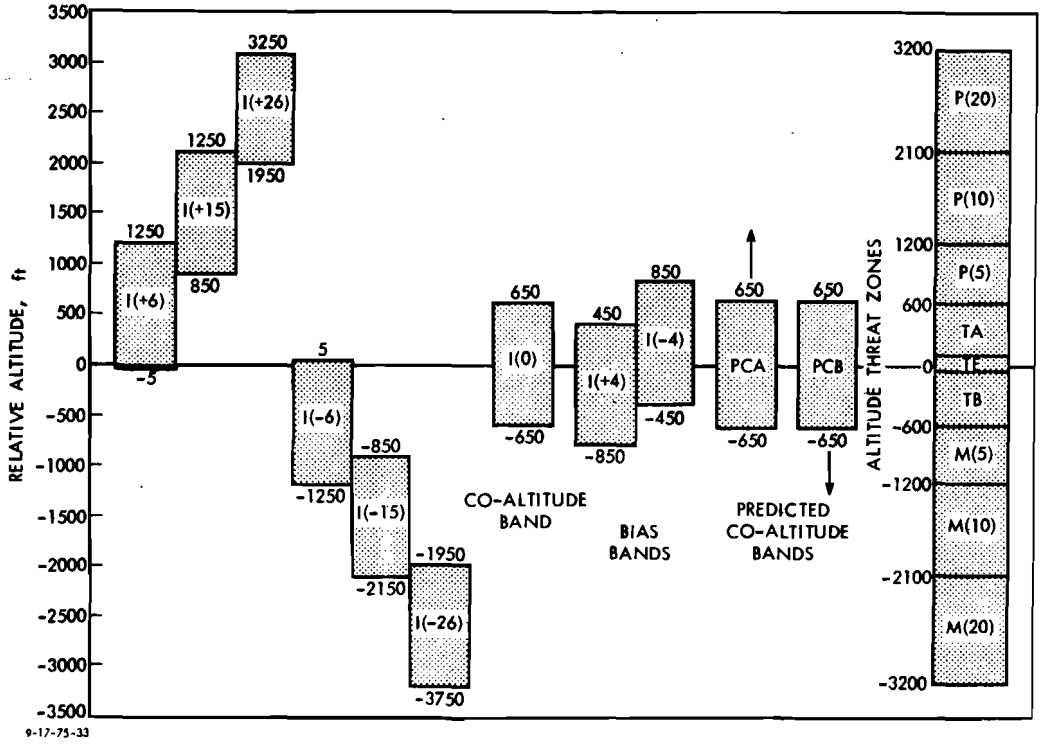


FIGURE 10. Interrogation Bands and Altitude Threat Zones for AVOIDS-I Below Altitudes of 9600 ft
Note: Above altitudes of 9600 ft, the co-altitude band I(0) is not used.

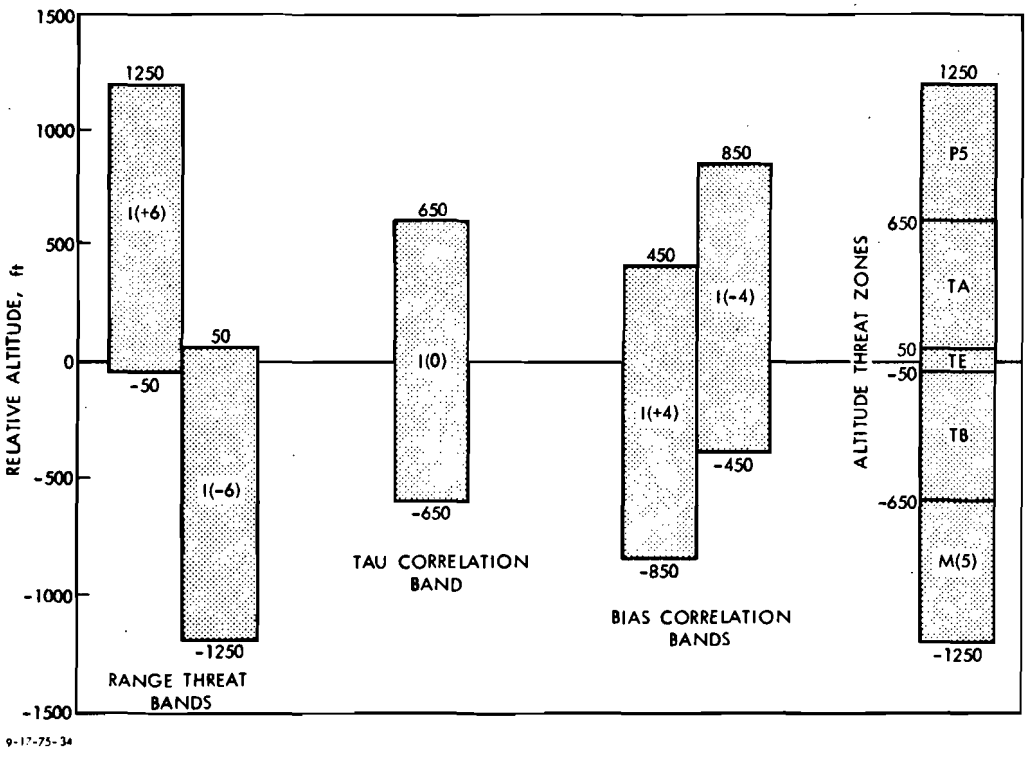


FIGURE 11. AVOIDS-II Interrogation Altitude Bands

track exists in an altitude band in which a Tau measurement is in progress. Even then, a potential threat must be observed for three sub-epochs before an attempt is made to refine its location in altitude by interrogating the appropriate altitude bands for correlation. The manner in which the equipment uses the range data obtained from the different altitude bands can best be explained with an example. Suppose a potential threat exists 500 ft above an aircraft with an AVOIDS-I CAS which is flying below 9600 ft and that this is the only aircraft within communication range. Of the six altitude bands interrogated in sub-epoch A, a reply would only be received in I(+6). During sub-epochs B and C, only the I(+6) altitude band would be interrogated and a potential threatening response would be observed. At this point in the threat evaluation process, the altitude of the threat could be anywhere between -50 ft and +1250 ft and could, therefore, appear in altitude threat zone TE, TA, or P(+5). To resolve this ambiguity, altitude bands I(0), I(-6), and I(+6) would be interrogated in sub-epoch D. The Tau evaluation would continue in I(+6). For example, suppose that in sub-epoch D the threat occurred in range bin ℓ. Then, data from I(0) and I(-6) would be checked for a return in range bins ℓ-1, ℓ, and ℓ+1. This process is referred to as altitude correlation. For the example being considered, the data received in the designated bins ℓ-1 to ℓ+1 from I(0) would correlate with the potential threatening track in I(+6), but those from I(-6) would not, unless, of course, a fruit pulse appeared in I(-6) in the appropriate bins. As a result, the equipment would decide the threat was in altitude threat zone TA, and during sub-epoch E, only altitude bands I(+6) and I(0) would be interrogated. I(+6) would continue to be interrogated as long as the range track fell within the Tau filter range gate pattern and I(0) would be interrogated during the remaining sub-epoch as long as received response correlated with the threatening track in I(+6).

Table 3 shows the relationship between a threatening aircraft's altitude threat zone and the altitude bands in which it will respond. Both the AVOIDS-I and AVOIDS-II designs are shown.

TABLE 3. AVOIDS ALTITUDE CORRELATION LOGIC

Altitude Threat Zone	Altitude Correlation Logic	
	AVOIDS-I	AVOIDS-II
P(20)	I(+26) and Not I(+15)	Not Used
P(10)	I(+15) and Not I(+6)	Not Used
P(5)	I(+6) and Not I(0)	I(+6) and Not I(0)
TA	I(+6) and I(0) and Not I(+4)	I(+6) and Not I(-6)
TA(BIAS)	I(+6) and I(0) and I(+4)	I(+6) and I(+4) and Not I(-6)
TE	I(+6) and I(-6) and I(0)	I(+6) and I(-6) and I(0)
TB(BIAS)	I(-6) and I(0) and I(-4)	I(-6) and I(-4) and Not I(+6)
TB	I(-6) and I(0) and Not I(-4)	I(-6) and I(0) and Not I(+6)
M(5)	I(-6) and Not I(0)	I(-6) and Not I(0)
M(10)	I(-15) and Not I(-6)	Not Used
M(20)	I(-26) and Not I(-15)	Not Used

Note: At an altitude of 9600 ft or higher, the co-altitude zone (TA and TB) is changed from ±650 to ±850 ft for AVOIDS-I. This is accomplished by replacing the I(0) altitude band with the I(+4) and I(-4) bands.

The logic for deciding which altitude bands will be interrogated is presented in Table 4 for the AVOIDS-I and Table 5 for the AVOIDS-II. In the tables, T(X) indicates a potential threatening range track exists in altitude band X; H(X)

TABLE 4. AVOIDS-I ALTITUDE BAND DECISION LOGIC

Altitude Band	Sub-Epoch Time Periods							
	A	B	C	D	E	F	G	H
PCA[1]	0	0	0	T(+6) T(+15) T(+26)	H(+6) H(+15) H(+26)	H(+6) H(+15) H(+26)	H(+6) H(+15) H(+26)	H(+6) H(+15) H(+26)
U(+26)	$\dot{h}$ > 1000 ft/min	T(+26)	T(+26)	T(+26)	T(+26)	T(+26)	T(+26)	T(+26)
I(+15)	$\dot{h}$ > 500 ft/min	T(+15)	T(+15)	T(+15) T(+26)	T(+15) H(+26)	T(+15) H(+26)	T(+15) H(+26)	T(+15) H(+26)
I(+6)	1	T(+6)	T(+6)	T(+6) T(-6) T(+15)	T(+6) H(-6) H(+15)	T(+6) H(-6) H(+15)	T(+6) H(-6) H(+15)	T(+6) H(-6) H(+15)
I(+4)	0	0	0	T(+6) T(-6)[2]	H(+6) H(-6)[2]	H(+6) H(-6)[2]	H(+6) H(-6)[2]	H(+6) H(-6)[2]
I(0)[3]	0	0	0	T(+6) T(-6)	T(+6) T(-6)	T(+6) T(-6)	T(+6) T(-6)	T(+6) T(-6)
I(-4)	0	0	0	T(-6) T(+6)[2]	H(-6) H(+6)[2]	H(-6) H(+6)[2]	H(-6) H(+6)[2]	H(-6) H(+6)[2]
I(-6)	1	T(-6)	T(-6)	T(-6) T(+6) T(-15)	T(-6) H(+6) H(-15)	T(-6) H(+6) H(-15)	T(-6) H(+6) H(-15)	T(-6) H(+6) H(-15)
I(-15)	$\dot{h}$ < -500 ft/min	T(-15)	T(-15)	T(-15) T(-26)	T(-15) H(-26)	T(-15) H(-26)	T(-15) H(-26)	T(-15) H(-26)
I(-26)	$\dot{h}$ < -1000 ft/min	T(-26)	T(-26)	T(-26)	T(-26)	T(-26)	T(-26)	T(-26)
PCB[4]		0	0	T(-6) T(-15) T(-26)	H(-6) H(-15) H(-26)	H(-6) H(-15) H(-26)	H(-6) H(-15) H(-26)	H(-6) H(-15) H(-26)

Notes: T(X) indicates a valid track through previous sub-epoch in band X and H(X) indicates a valid track through previous sub-epoch in altitude correlation band X. 0 indicates that no interrogations are made during the sub-epoch, and 1 that they have been made.

[1]Used only when altitude rate > 500 ft/min.

[2]Only above 9600 ft.

[3]Not used above 9600 ft.

[4]Used only when altitude rate is < -500 ft/min

indicates that valid (i.e., within ±1 range bin of the potential threatening Tau track) altitude correlation data have been obtained in altitude band X. Multiple symbols at a row and column juncture indicate the particular altitude band is interrogated if any of the conditions are true. For example, I(+6) is interrogated in sub-epoch E, if, in the previous sub-epoch, a potential threat was being tracked in I(+6) or if the previous range data responses in I(+6) correlated with a potential threatening track in altitude bands I(-6) or I(+15). It should be remembered that the interrogation of an altitude band is stopped for the remainder of the epoch time if none of the conditions specified at a row and column juncture are correct.

TABLE 5. AVOIDS-II INTERROGATION DECISION LOGIC

Altitude Band	Sub-Epoch Time Periods							
	A	B	C	D	E	F	G	H
I(+6)	1	τ(+6)	τ(+6)	τ(+6) or τ(-6)	τ(+6) or H(-6)	τ(+6) or H(-6)	τ(+6) or H(-6)	τ(+6) or H(-6)
I(+4)	0	0	0	τ(+6)	H(+6)	H(+6)	H(+6)	H(+6)
I(0)	0	0	0	τ(+6) or τ(-6)	H(+6) or H(-6)	H(+6) or H(-6)	H(+6) or H(-6)	H(+6) or H(-6)
I(-4)	0	0	0	τ(-6)	H(-6)	H(-6)	H(-6)	H(-6)
I(-6)	1	τ(-6)	τ(-6)	τ(-6) or τ(+6)	τ(-6) or H(+6)	τ(-6) or H(+6)	τ(-6) or H(+6)	τ(-6) or H(+6)

Notes: τ(X) indicates a valid track through previous set in band X and H(X) indicates a valid track through previous set in band X which correlated in altitude.

Source: AVOIDS-I Collision Avoidance System, System Description, Honeywell Inc., Government and Aeronautical Products Division, unpublished document.

Table 4 also shows the dependence on the altitude rate of interrogating aircraft. Altitude bands I(+15) and PCA are only interrogated if $\dot{h}$ is greater than 500 ft/min and I(+26) only if $\dot{h}$ is greater than 1000 ft/min. Altitude bands I(-15), PCB, and I(-26) are used only if an aircraft is diving at the indicated rate. The placement of PCA and PCB was previously discussed.

The previous discussions described the manner in which the Tau threat zone and the altitude threat zone of a potential threat intruder are determined. In order to display a detected P(5), M(5), or co-altitude threat, the threat must occur in two consecutive epochs and be correlated in range between epochs. The range correlation requirement is satisfied if, upon starting a new epoch, the A data of a range track are one bin outbound to nine bins inbound of H data of the threatening track in the previous epoch. For P(10), P(20), M(10), and M(20) threats, the threat must occur in two consecutive epochs but range correlation is not required.

E. SYSTEM DESIGN CHANGES

A number of design changes have been made to the Honeywell CAS since IDA's Phase I review. The more significant changes, which affect the operation of the equipment and prevent the false alarm rate from becoming excessive in a dense environment, are listed below.

1. The inhibit/suppression logic has been eliminated. This very complicated logic was used to reduce the likelihood that multiple nonthreatening range tracks could combine to produce a false threatening track. The logic also reduced detection probability. The elimination of this circuitry will improve detection probability and increase the single-epoch false track probability.

2. The epoch time has been increased from 3.5 sec to about 3.7 sec. In addition, the number of sub-epochs has been increased from seven to eight. These changes will reduce the false alarm rate but will also reduce the system's data rate.

3. All range bins are now 50 ft wide; in the previous design they were 50 ft wide out to 30,000-ft range and then increased as a function of range. Fruit and nonthreatening aircraft reply densities are proportional to range bin width and will be reduced beyond 30,000-ft range in the new design. This should have a favorable impact on false alarm rate.

4. The new design requires a threat to appear in two consecutive epochs before a command is issued. The likelihood of a false alarm in two consecutive epochs is much less than in one epoch. In addition, range correlation, described in Section D, is required for the inner altitude threat zones. These features should greatly reduce false alarms and were needed after the inhibit/suppression logic was eliminated.

5. A second pulse pair decoder has been added to the CAS receiver, thus increasing the time the transponder is available to receive and process interrogations from other aircraft. In the previous design, an interrogation waveform to which the transponder was not to respond would tie up the circuitry for a considerable length of time and could prevent the transponder from receiving an interrogation to which it should reply.

6. The time separation between the pulses in the interrogation waveform's two-pulse pair has been made different to reduce the chances of an erroneous response.

IV. ANALYSIS OF FALSE WARNING PROBABILITY IN THE CAS

A. INTRODUCTION

In IDA's Phase I analysis of the Honeywell CAS, considerable effort was devoted to evaluating the probability of detection and probability of false warning. The results of the analysis indicated that both parameters were marginal. The elimination of the inhibit/suppression logic in the new design should improve the probability of detection so that this parameter is essentially limited by the system's ability to obtain the sub-epoch range measurements needed for the Tau tracking filter. However, elimination of this logic will also increase the probability of false warning. To compensate, other design changes have been made. These include the following:

1. The range bin size beyond 30,000 ft has been reduced to 50 ft.
2. The number of data sets within an epoch has been increased by one.
3. Most importantly, a Tau threat must be present for two consecutive epochs before a command or warning is issued.

In this section, the false warning probability of the new design is estimated. A false range track must persist for two consecutive epochs to produce a false warning. In the outer altitude threat zone [P(20), P(10), M(20), M(10)], the false tracks can appear anywhere in range so the probability of false warning is equal to the probability of a single-epoch false track squared. For the other altitude threat zones, range

correlation between the successive single-epoch range tracks is required and a second single-epoch false track must start within one bin outbound to nine bins inbound of the last range bin position of the single-epoch false track in the first epoch.

A false warning requires two consecutive single-epoch false tracks, so the first step to estimating the false warning probability is to understand the phenomena which produces false tracks. The Tau tracker uses a series of range gates to search the sub-epoch range data for threatening range tracks and each of the eight sub-epoch range gates must be occupied to produce a false track. One way for this to happen is for fruit or a group of randomly distributed responses from nonthreatening aircraft to accidentally fall into the eight range gates of the Tau filter. If the sub-epoch range data are randomly distributed, the probability that any particular set of eight sub-epoch range gates will be occupied can easily be calculated. However, the Tau filter checks a large number of range gate patterns that overlap each other and are therefore correlated. This complicates the analysis so that a simplified model of the AVOIDS tracking filter was devised for estimating false warning due to randomly received responses. This work is presented in Section B.

Another, more serious, kind of false warning results when the sub-epoch range responses are not randomly distributed. These false warnings are caused by nonthreatening aircraft having a range rate close to that considered threatening. A nonthreatening aircraft will contribute to only one sub-epoch of a Tau filter pattern if there is a large mismatch between the aircraft's range rate and the average slope of the pattern. As the range rate of the nonthreatening intruder approaches the value sought after by the filter pattern, the responses from the aircraft contribute to a greater number of the sub-epoch range gates, and if the intruder has an indicated

range rate skip of one range bin less than that considered threatening, it is possible for seven of the eight sub-epoch range gates of the lowest velocity threatening pattern to be occupied. A fruit pulse or a response from another nonthreatening aircraft could land in the remaining range gate and create a single-epoch false track. The analysis of this type of false track is presented in Section C.

As a check on the analysis presented in Sections B and C, a small-scale computer simulation of the AVOIDS tracking filter was prepared; the results are presented in Section D. Aircraft density in the simulations was twice that of the 1982 Los Angeles traffic model. However, the analysis of Sections B and C was extrapolated to the higher density and good agreement between the analytic results and the computer simulation were obtained.

Section E summarizes the work and results and estimates the overall false warning rate for all aircraft in the 1982 Los Angeles traffic model. In deriving this estimate, all aircraft were assumed to be equipped with an AVOIDS transponder and the protected volume of the AVOIDS-II equipment was increased to that of the AVOIDS-I.

B. PROBABILITY OF A FALSE TRACK DUE TO RANDOMLY DISTRIBUTED REPLIES

This section is concerned with the possible random distribution of responses satisfying the Tau filter requirement. The operation of the Tau filter is explained in Chapter III. Basically, a series of range gates is set up to check for the occurrence of a return in each of the eight sub-epochs. Two range gate patterns are used for each value of range rate tested. The low-acceleration pattern has a single range bin in the first and last sub-epochs and three range bins in each of the inner epochs. The high-acceleration pattern has a single range bin in sub-epochs A and H, either two or three range bins in sub-epochs B and G, and three range bins in the C, D, E, and F.

In estimating the system false alarm rate, first consider the probability, P(T), that a group of random replies will produce a single-epoch false track in any one of the low-acceleration range gate patterns. This probability, as a function of reply density, μ, is

$$P(T) = \mu^2 \, (3\mu)^6, \tag{3}$$

since only a single range bin must be occupied in two of the sub-epochs while any one of three range bins may be occupied in the other six sub-epochs. In the equation, the density (μ) is the expected number of replies in a 50-ft range bin, which is equivalent to the probability of one or more replies in a 50-ft range bin if $\mu << 1$.

The density of aircraft below 10,000 ft and within 10 miles of Los Angeles, as projected by the FAA 1982 model, can be fitted with the following function:

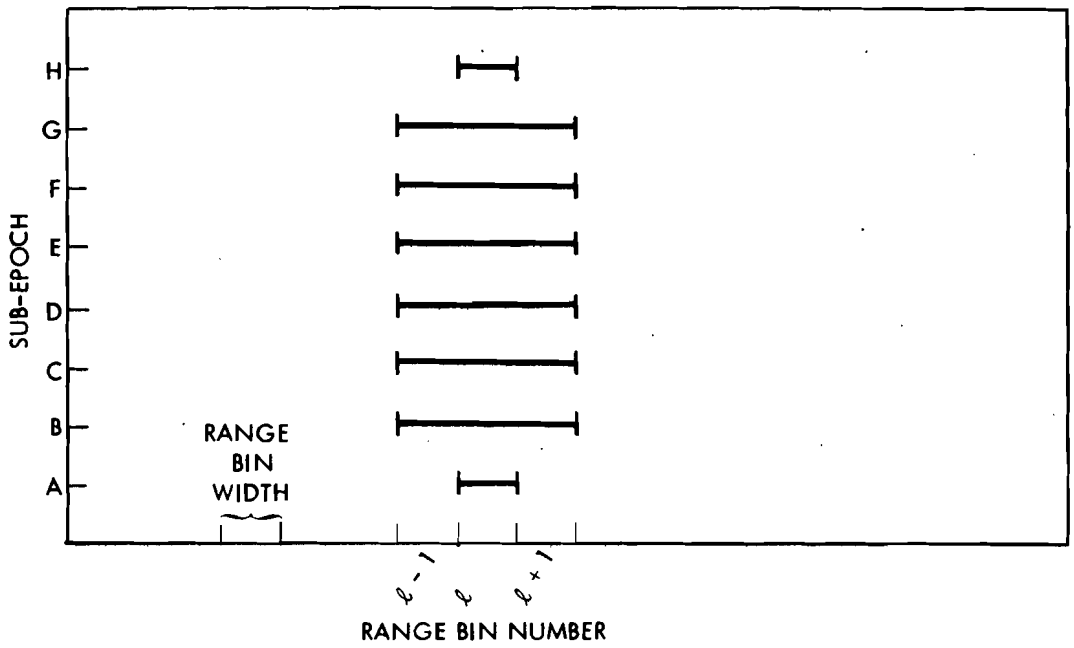
$$\rho = \rho_o e^{-(R/R_1)^2}, \tag{3a}$$

in which $\rho_o = 0.84$, $R_1 = 9$ miles and R is the range in question. The density at the most densely populated altitude band is about one-fifth of this value. Assuming a B-737 antenna pattern, an AVOIDS-I in the densest region of the Los Angeles traffic would observe a peak density of about 0.03 aircraft per 50-ft range bin at a range of approximately 30,000 ft. Based on IDA's Phase I analysis (Ref. 1), the fruit density observed by such an aircraft would be about 0.02 returns per range bin, so that the overall density would be about 0.05 replies per range bin. The density of replies is a function of range but remains fairly constant between 20,000 ft and 35,000 ft. The

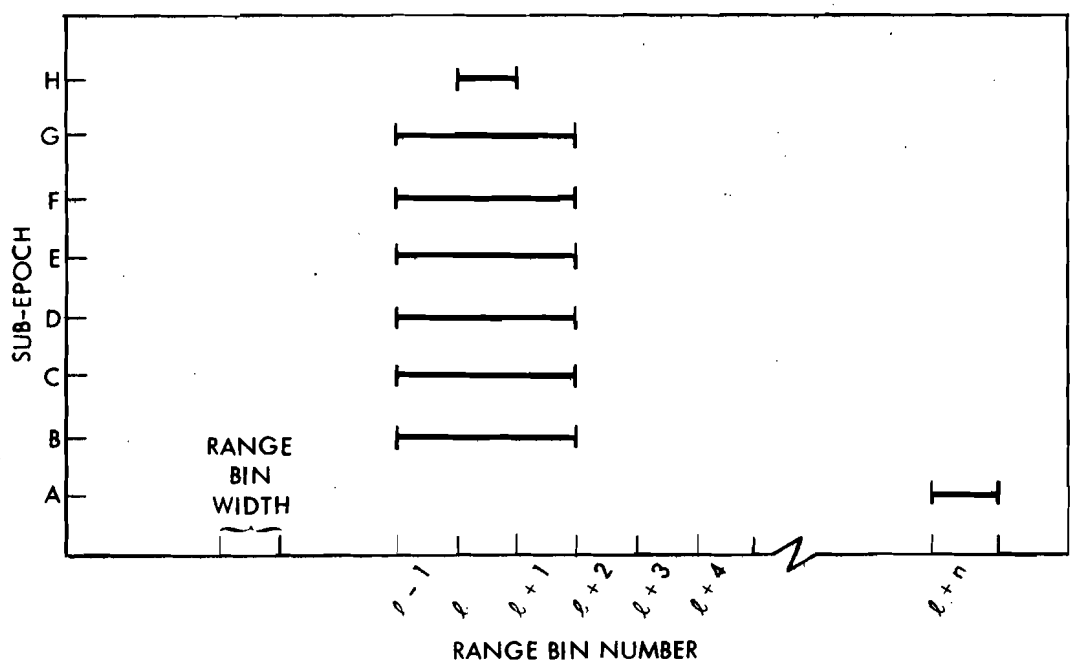
expected density observed by an AVOIDS-II is about 0.025 non-threatening aircraft and 0.01 fruit replies per range bin. The density is fairly flat over the same region as that of AVOIDS-I.

If the individual range gate patterns were independent, the per-epoch false alarm rate could be obtained by summing the probability for each pattern tested by the Tau filter, and for a constant-density environment, Eq. 3 could simply be multiplied by the total number of patterns searched. However, the patterns are not independent, so the per-epoch false track probability will be analyzed by first estimating the equivalent number of independent range gate patterns searched by the AVOIDS tracking filter. A simplified model of the AVOIDS tracking filter, shown in Fig. 12, will be used for this purpose. Remember that the random distribution of replies is being considered so that the specific location of the range gate patterns within the sub-epochs is not of prime importance.

Figure 12a shows the range gate locations of the simplified model that would be used to check for a threatening track with a zero value of range rate. This is the same pattern as that of the AVOIDS tracking filter's low-acceleration pattern. For other values of range rate, the AVOIDS tracking filter changes the relative positions of the sub-epoch range gates so that they will follow the average slope of the range track being sought. However, the simplified model simulates this action by changing only the position of the range gate in sub-epoch A, as depicted in Fig 12b. This difference actually results in a higher correlation between range rate patterns of the simplified model than between the AVOIDS tracking filter patterns. For the simplified model, the conditional probability that a false track with a skip of n range bins occurs is equal to μ if a false track with some other skip has occurred. This is simply the probability of receiving a reply in a single range bin. Because of this, it is relatively easy to compute



(a) RANGE GATE PATTERN FOR ZERO RANGE RATE



(b) RANGE GATE PATTERN FOR A RANGE BIN SKIP OF n

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FIGURE 12. Range Gate Patterns for Simplified Tau Filter

the probability of at least one false track from a number of overlapping, and therefore correlated, range gate patterns. The general form for the probability of the sum of N nonexclusive sets is (from Ref. 5)

$$P(T_1 \text{ or } T_2 \text{ or } \ldots \text{ or } T_N) = \sum_{i=1}^{N} P(T_i) - \sum_{\substack{ij \\ i<j}} P(T_i, T_j)$$

$$+ \sum_{\substack{i,j,k \\ i<j<k}} P(T_i\ T_j, T_k) - \ldots \pm P(T_1, T_2, \ldots T_N).$$

For the simplified model of the tracking filter

$$P(T_i, T_j) = \mu\ P(T),\ P(T_i, T_j, T_k)$$

$$= \mu^2\ P(T) \text{ etc., and } P(T_1, T_2, \ldots T_N)$$

$$= \mu^{N-1}\ P(T).$$

Substitution gives Eq. 4.

$$P(T_1 \text{ or } T_2 \text{ or } \ldots T_N) = NP(T) - \left(\frac{N}{2}\right) \mu P(T) + \left(\frac{N}{3}\right) \mu^2 P(T) - \ldots + \left(\frac{N}{N}\right) (-\mu)^{N-1}\ P(T) \tag{4}$$

In Eq. 4, P(T) is the probability that all range gates in any particular pattern will be occupied, μ is the reply density per range bin and $\left(\frac{N}{x}\right)$ is the number of combinations of N things taken x at a time. In this equation, the first term is the probability for N independent patterns, the second term takes into account the correlation between the patterns taken two at a time, the third term takes into account correlation between the patterns taken three at a time, etc. Equation 4 can be rewritten as

$$P(T_1 \text{ or } T_2 \text{ or } \ldots \text{ or } T_N) = P(T) \sum_{n=1}^{N} \left(\frac{N}{n}\right) (-\mu)^{n-1} \quad . \tag{5}$$

The equivalent number of uncorrelated patterns, K, can be obtained by dividing each side of Eq. 5 by P(T). That is,

$$K = \sum_{n=1}^{N} \left(\frac{N}{n}\right) (-\mu)^{n-1} \quad . \tag{6}$$

This equation has been evaluated and the results presented in Table 6 for three values of μ and for several values of N. K approaches $1/\mu$ since the probability of a false track for any particular pattern can be written as

$$P(T) = \mu P'(T), \tag{7}$$

where $P'(T)$ is the probability of a return in all but sub-epoch A. The probability of a track in at least one of a large number of correlated patterns approaches $1/\mu$ P(T), which equals $P'(T)$, as it should.

TABLE 6. RELATIONSHIP BETWEEN CORRELATED AND UNCORRELATED PATTERNS

Correlated	Uncorrelated		
	Reply Density (Replies per 50-ft Range Bin)		
N	μ = 0.1	μ = 0.05	μ = 0.01
15	7.9	13.6	14.0
30	9.8	15.7	26.0
50	9.9	18.5	39.5
75	10.0	19.6	52.9
100	10.0	19.9	63.0
150	10.0	20.0	77.9
200	10.0	20.0	86.6

For each sub-epoch H range bin position, a large number of threatening values of range rate are checked by the AVOIDS tracking filter; based on the simplified model, these will be considered correlated. The correlation between the range gate pattern used to test for threatening tracks at different ranges and for the high and low target acceleration will also be based on the simplified model.

Figure 13 shows the range gate patterns of the simplified model for checking adjacent range bins. The worst-case overlap is shown in that the range rates of the two patterns differ so that they both require a reply in the same range bin of sub-epoch A. The number of reply combinations in the inner sub-epochs that will satisfy both patterns is small (i.e., 64) compared to the number of combinations for one of the patterns (i.e., 729), and therefore the correlation between them should be small. As a result, the patterns used to check each range cell in the dense region will be considered independent in the analysis.

The AVOIDS equipment uses two acceleration patterns for each range bin tested. However, there is considerable overlap if the high acceleration pattern for range bin ℓ is compared with the low acceleration pattern for range bin $\ell+1$. This condition, shown in Fig. 14, indicates that either the acceleration or the adjacent range patterns should be considered independent, but not both.

To estimate the equipment's behavior based on the simplified model, the different range-rate patterns should be considered correlated and the patterns used to test for a threatening intruder in adjacent range bins should be considered independent if the acceleration patterns are not. To take into account the acceleration patterns the number of correlated patterns will be assumed to be equal to twice the number of range rate patterns.

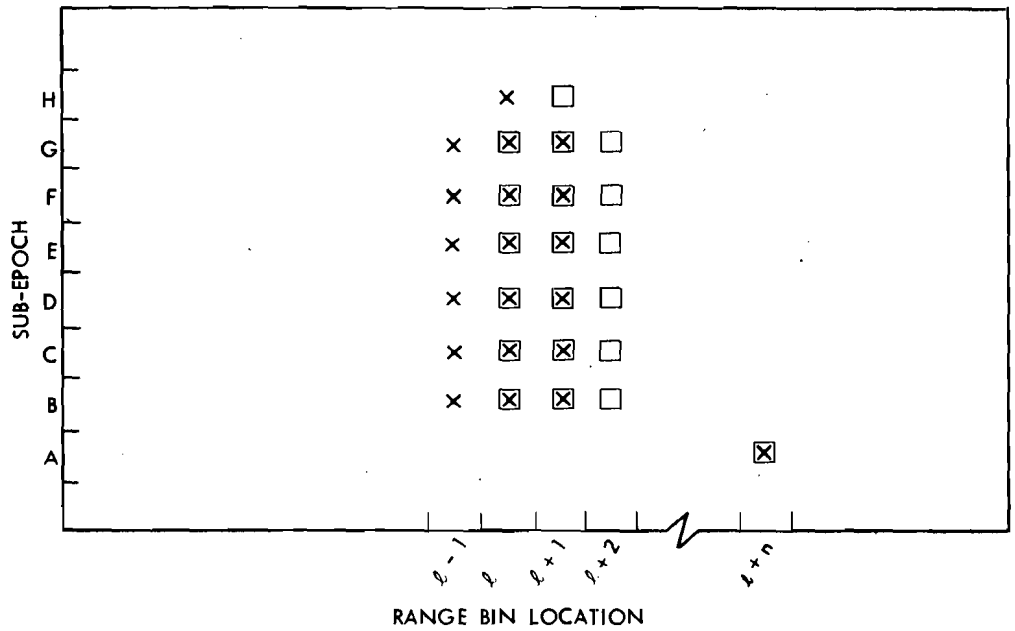
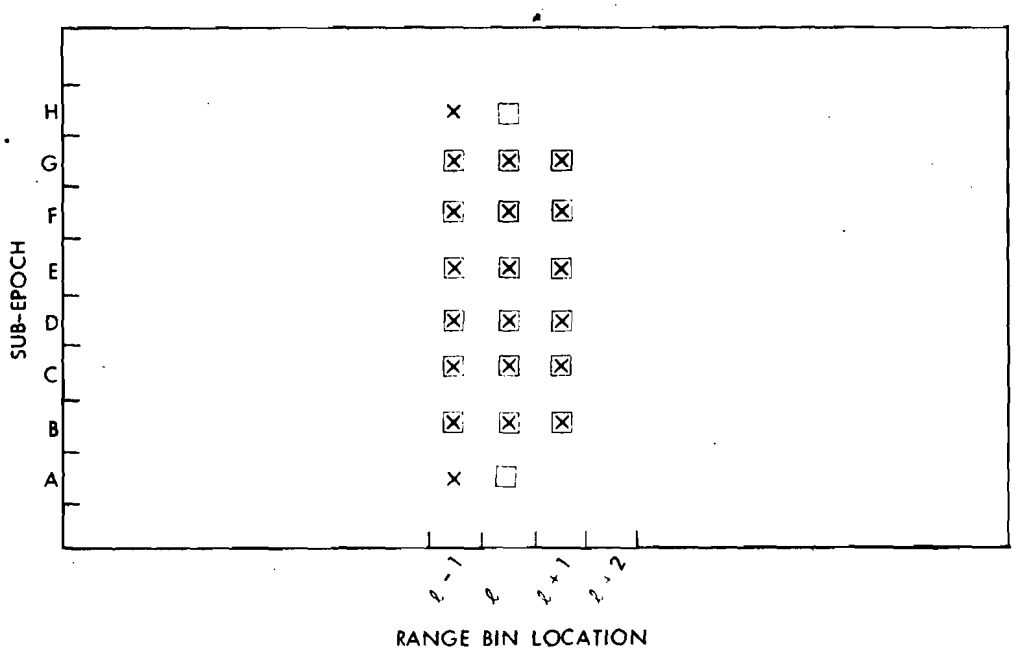


FIGURE 13. Range Gate Patterns of the Simplified Tau Filter for Adjacent Range Bins



X's SHOW THE RANGE BIN LOCATION FOR THE PATTERN AT RANGE BIN $\ell - 1$ AND □'s THE PATTERN AT RANGE BIN ℓ

FIGURE 14. Overlap Between Different Acceleration Patterns for Adjacent Range Bins

Only the range bins within the high density region (20,000-ft to 35,000-ft range) need be considered, since the probability of a false track is a function of μ^7 (see Eq. 3). Within this region, there are 300 range bins which should be considered independent as far as the generation of false tracks is concerned. Below an altitude of 10,000 ft, the equipment must test for a maximum range rate of 914 ft/sec, which is a range bin skip between sub-epoch A and H of 64 bins. The minimum skip of interest for the AVOIDS-I threat logic is 15 range bins at 20,000-ft range, 23 range bins at 25,000-ft range, 30 range bins at 30,000-ft range and 38 range bins at 35,000-ft range. The number of different values of range rate which are tested at the different ranges are therefore 50, 42, 35, and 27, respectively. Taking the acceleration patterns into account, the AVOIDS is estimated to have 100 correlated patterns at 20,000-ft and 54 at 35,000-ft range. The equivalent number of independent patterns is, respectively, 18.5 and 19.9 (see Table 6), giving an average of 19.2 for each 50 ft of range investigated. The total number of equivalent independent patterns tested is therefore about 6×10^3 (i.e., 19.2 x 300) during an epoch.

For random replies, the single-epoch false warning probability in an altitude band interrogated for Tau evaluation can be obtained by substituting the appropriate density into Eq. 3 and multiplying 6×10^3. In the peak density projected for Los Angeles in 1982, the single-epoch probability of a false track per altitude band would be about 1.7×10^{-4} for an AVOIDS-I and about 9.5×10^{-6} for the AVOIDS-II if the AVOIDS-II were designed to search the same protected volume as the AVOIDS-I. The AVOIDS-II false track probability is much lower than that of the AVOIDS-I because of the lower expected reply density, due to its power, sensitivity, and shorter range performance.

The probability of a false warning in an altitude band in which range correlation is not required is the square of the single-epoch probability of a false track. For the AVOIDS-I, the false warning in such an altitude band due to random replies is about 2.9×10^{-8} and for the AVOIDS-II is about 9×10^{-11}. For the altitude bands in which range correlation is performed, the false warning probability should be reduced by a factor of 11/300 or 0.037. This reduction results from the fact that there are 300 high-density range bins in an epoch, yet, with range correlation, the second false track needed to produce a false warning must start within the 11 range bin range correlation windows. With range correlation between epochs, the AVOIDS-I density gives an expected false warning probability of about 1.2×10^{-9} and the AVOIDS-II density yields a probability of about 3.8×10^{-12}.

C. FALSE TRACKS DUE TO NONTHREATENING AIRCRAFT WITH AN $\dot{R}$ CLOSE TO THREATENING

This section deals with false warnings which result when a single nonthreatening aircraft occupies most, but not all, of the sub-epoch range gate patterns of the Tau filter. For this to happen, the range track of the aircraft must have a slope which is about the same as the threatening range gate pattern in which the false track is to occur. As a result, these false tracks only occur in the range rate patterns which have relatively low values of range bin skip and involve nonthreatening aircraft with a range rate close to threatening.

The range rate which is considered threatening varies with range. However, the critical parameter which determines the likelihood that a nonthreatening aircraft will cause a false track is the difference in range bin skip between the intruder's range track and the Tau filter range gate pattern. The analysis, therefore, need take into consideration only one

range which has arbitrarily been set at 20,000 ft. At this range, a range bin skip of 15 range bins is considered threatening, while a skip of 14 is not.

To analyze the false track/warning probability, nonthreatening aircraft with a range rate of -175 ft/sec and -150 ft/sec will be considered. The range rate of -175 ft/sec is very close to threatening, in that an aircraft at 20,000-ft range with a range rate of -230 ft/sec represents a Tau 2 threat according to ANTC-117 specifications. Since two epochs may lapse before a warning is displayed, the equipment should actually detect as threatening an aircraft at 20,000-ft range having a range rate of 211 ft/sec. Stated another way, an aircraft with a range rate of -175 ft/sec does not become a Tau 2 threat until it reaches a range of 17,800 ft. The second of two successive detections should occur at this range, so the first single-epoch detection should occur at a range of 18,450 ft. The first detection is early by 8.9 sec or 2.4 epochs if it occurs at 20,000 ft. At 20,000 ft, an aircraft with an $\dot{R}$ of -175 has a "time to go" (i.e., $R/\dot{R}$) of 114 sec. The "time to go" for a nonthreatening aircraft at 20,000-ft range with an $\dot{R}$ of -150 ft/sec is 133 sec.* Based on ANTC-117 logic, such an aircraft does not represent a Tau 2 threat until it reaches the range of 16,800 ft, but for the AVOIDS design, the first Tau filter alarm should be generated one epoch earlier, at about 17,300 ft. A Tau 2 alarm on this target at 20,000 ft is about 4.8 epochs or 17.6 sec early. It will be shown later that the probability of a false track decreases rapidly as the range rate of the nonthreatening aircraft increases, so that a range rate of less than -150 ft/sec was not considered in the analysis.

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*The much larger values of $R/\dot{R}$ of 114 and 133 sec as compared to τ_2 = 40 sec or τ_2 = 47 sec arise because of the offset range of R_o = 1.8 nmi.

To further understand how a nonthreatening aircraft with a range rate of -175 ft/sec can produce a high false alarm rate, consider the low acceleration range gate pattern with a skip of 15 which is plotted in Fig. 15. The sloping lines represent a range rate of -175 ft/sec and the area between any two lines separated by one range bin will be referred to as a range rate corridor. Since a target with a range rate of -175 ft/sec is confined to one of these corridors, it cannot simultaneously pass through the gate pattern shown in Fig. 15 and cause an alarm by itself. An idealized range track from an intruder with this range rate would be confined to one of the corridors. The AVOIDS tracking filter checks all range bins in the vicinity of 20,000-ft range for a threatening range track with a range bin skip of 15, so the relative geometry between the range gate pattern and the intruder's range track changes during the threat evaluation sequence and an idealized range track would, in relationship to the range gate pattern, alternately appear in range corridors a, then b, and so on.

The conditional probability that an aircraft with a range rate of -175 ft/sec will result in a Tau filter alarm with a skip of 15 can be found from Fig. 15 by noting which range gates are not occupied. For example, an intruder with a range rate of -175 ft/sec in range bin H_ℓ will be in range rate corridor b of Fig. 15. At this range, the filter range gates in sub-epochs C through H will be occupied, and the conditional probability of a Tau filter alarm for this alignment between the target and range gate pattern is equal to the probability of a reply in sub-epochs A and B due to other aircraft or fruit. There are three range bins in sub-epoch B and only one in sub-epoch A, so the conditional probability is

$$P(T)\Big|_{-175,\ 15} = 3\mu^2 \ . \qquad (8)$$

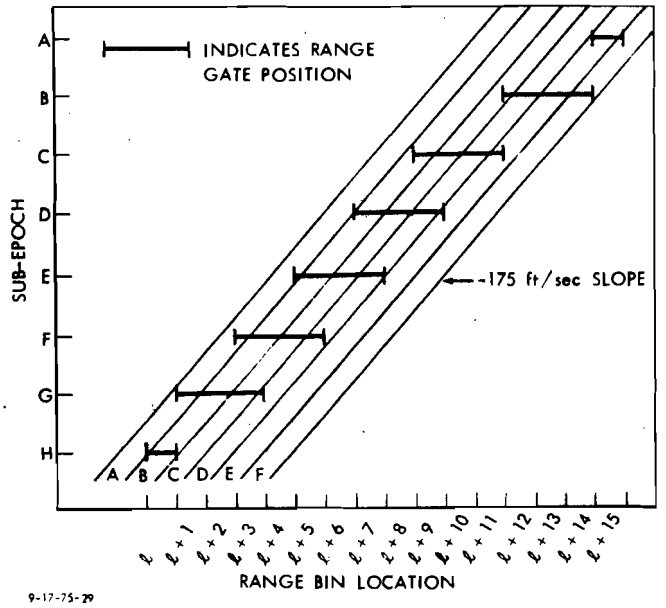

FIGURE 15. Single-Epoch Low-Acceleration Pattern

The overall conditional probability that the target will cause a Tau filter alarm is obtained by summing the conditional probabilities obtained by considering the target in the different range rate corridors. This assumes that an independent trial, in the probability sense, is created when the data are shifted in parallel by one range bin. For the conditions of Fig. 15, the overall conditional probability is approximately

$$P(T)_{LA}\Big|_{-175,\ 15} = 3\mu^2 + 3\mu^3 + 9\mu^4 \ . \qquad (9)$$

The conditional probability for the high acceleration pattern and a range rate of -175 ft/sec and for the high- and low-acceleration patterns and a range rate of -150 ft/sec were computed in a similar way. The diagrams for these cases are presented in Appendix C and the results are summarized in Table 7. The equations from this table have been evaluated for a reply density of 0.035, 0.05, and 0.08 and the results are shown in Table 8. A reply density of 0.035 is expected for an AVOIDS-II in the Los Angeles Traffic Model, a density of 0.05 is expected for an AVOIDS-I, and the higher-than-expected density of 0.08 will be used in the next section to compare the predicted results of Table 8 with a computer simulation of the AVOIDS tracking filter.

TABLE 7. SINGLE-EPOCH CONDITIONAL FALSE TRACK PROBABILITY AS A FUNCTION OF REPLY DENSITY

Acceleration Pattern	Conditional Probability
Near-Threatening Intruders With $\dot{R}$ = -175 ft/sec	
Low	$3\mu^2 + 3\mu^3 + 9\mu^4$
High	$0.14\mu + 1.64\mu^2 + 5.6\mu^3$
Near-Threatening Intruders With $\dot{R}$ = -150 ft/sec	
Low	$3\mu^3 + 27\mu^4$
High	$1.7\mu^3 + 18.7\mu^4 + 35\mu^5$

TABLE 8. SINGLE-EPOCH CONDITIONAL FALSE TRACK PROBABILITY FOR NEAR-THREATENING AIRCRAFT

	Reply Density		
	0.035	0.05	0.08
Range Rate of -175 ft/sec and a Filter Skip of 15 Range Bins			
Low-Acceleration Pattern	3.8×10^{-3}	7.9×10^{-3}	2.1×10^{-2}
High-Acceleration Pattern	7.2×10^{-3}	1.2×10^{-2}	2.5×10^{-2}
Estimated Overall[1]	1.0×10^{-2}	2.0×10^{-2}	4.6×10^{-2}
Range Rate of -150 ft/sec and a Filter Skip of 15 Range Bins			
Low-Acceleration Pattern	1.7×10^{-4}	5.4×10^{-4}	2.6×10^{-3}
High-Acceleration Pattern	1.0×10^{-4}	3.4×10^{-4}	1.8×10^{-3}
Estimated Overall[1]	2.7×10^{-4}	8.8×10^{-4}	4.4×10^{-3}

[1]Sum of the low- and high-acceleration patterns.

There is correlation between the high- and the low-acceleration patterns. The conditional probabilities from adjacent range bins were summed to obtain the equations presented in Table 7. However, these equations do not include the contribution from range gate patterns used to check for threatening aircraft with range bin skip greater than 15. As a result, the overall conditional probability should be greater than that for either of the acceleration patterns but less than the sum of both. To be conservative, the sum of the conditional probabilities for the high- and the low-acceleration patterns will be used in the remainder of our analysis. Later, these estimates are compared with a computer simulation of the tracker with the same density of traffic (Section D).

The data presented in Table 8 give the conditional probability of a false track in a single epoch. The conditional false warning probability in an altitude band in which range correlation is not performed is equal to the square of the single-epoch probability. A lower probability of false warning should occur in the altitude bands in which range correlation is required.

A graphical method similar to that described for the single-epoch case was used to calculate the conditional probability of false warning when range correlation is checked. The graphs used to perform the analysis are presented in Appendix C. The range gate patterns for two epochs had to be considered in the analysis and the time separation between the epochs was set at 0.25 sec.* The range gate patterns were aligned so that the first range gate of the pattern for the second epoch was one bin outbound of the last range gate of the pattern in the first epoch. This alignment was used since it provides the highest conditional probability of false warning. The high- and the

*The separation between epochs is a random variable, varying between 0 sec to 0.4 sec.

low-acceleration patterns were considered so that four combinations of range gate patterns are analyzed for both values of nonthreatening range rate. A summary of the results is presented in Table 9 for each combination of the high- and low-acceleration patterns.

TABLE 9. FALSE WARNING ESTIMATES AS A FUNCTION OF REPLY DENSITY FOR DIFFERENT COMBINATIONS OF ACCELERATION RANGE GATE PATTERNS

Acceleration Pattern Combination	Conditional Probability
	Near-Threatening Intruder With $\dot{R}$ = -175 ft/sec
Low-Low	$P_{FLL}(T) \mid_{-175,\ 15} = 0.75\mu^4 + 3.75\mu^5$
Low-High	$P_{FLH}(T) \mid_{-175,\ 15} = 7.7\mu^6 + 15.4\mu^7$
High-Low	$P_{FHL}(T) \mid_{-175,\ 15} = 0.14\mu^3 + 0.57\mu^4 + 3.0\mu^5$
High-High	$P_{FHH}(T) \mid_{-175,\ 15} = 2.1\mu^4 + 1.7\mu^5 + 20.6\mu^6$
	Near-Threatening Intruder With $\dot{R}$ = -150 ft/sec
Low-Low	$P_{FLL}(T) \mid_{-150,\ 15} = 104\mu^9 + 1145\mu^{10}$
Low-High	$P_{FLH}(T) \mid_{-150,\ 15} = 38.6\mu^7 + 162\mu^8$
High-Low	$P_{FHL}(T) \mid_{-150,\ 15} = 69.7\mu^8 + 347\mu^9$
High-High	$P_{FHH}(T) \mid_{-150,\ 15} = 122\mu^8 + 608\mu^9$

The equations presented in Table 9 have been evaluated for the three reply densities discussed earlier and the results are presented in Table 10. The correlation between the different combinations of acceleration patterns should be less than for the single-epoch case because of the increased length of the sequence. In addition, different combinations of range rate skip have not been considered, and false warnings which occur in the high-acceleration pattern have only been included when

the sub-epoch B and G range gates are three range bins wide. To compensate for some of these omissions, the overall probability of false warning has been estimated as the sum of the different acceleration pattern combinations and is presented in the last row for each $\dot{R}$ of Table 10.

TABLE 10. CONDITIONAL FALSE WARNING PROBABILITY

Acceleration Pattern Combination	-175 ft/sec Intruder			-150 ft/sec Intruder		
	Density			Density		
	0.035	0.05	0.08	0.035	0.05	0.08
Low-Low	1.3×10^{-6}	5.9×10^{-6}	4.3×10^{-5}	1.5×10^{-10}	2.6×10^{-9}	1.2×10^{-7}
Low-High	1.5×10^{-8}	1.3×10^{-7}	2.3×10^{-6}	1.1×10^{-11}	3.1×10^{-10}	4.6×10^{-8}
High-Low	6.8×10^{-6}	2.2×10^{-5}	1.0×10^{-4}	2.6×10^{-9}	3.7×10^{-8}	1.1×10^{-6}
High-High	3.4×10^{-6}	1.4×10^{-5}	9.7×10^{-5}	1.8×10^{-10}	3.4×10^{-9}	1.7×10^{-7}
$\sum$	1.2×10^{-5}	4.2×10^{-5}	2.4×10^{-4}	2.9×10^{-9}	4.3×10^{-8}	1.4×10^{-6}

D. COMPUTER SIMULATIONS OF FALSE ALARM RATE

A computer program was prepared which simulates the operation of the AVOIDS Tau filter. The primary purpose was to check the false alarm rate so that only the equipment's operation in the expected high-density region was modeled. A constant density of nonthreatening aircraft was randomly distributed among 3000 simulated range bins for sub-epoch A data. Each aircraft was assigned a value of range rate which was selected from a truncated Gaussian distribution. The standard deviation of the distribution was 325 ft/sec, but values below -180 ft/sec and above 900 ft/sec were rejected. Noise was not included in range

bin positions of the aircraft returns nor were bin splitting* effects included. Random replies were added to the sub-epoch data to simulate fruit. Range tracks with an indicated range rate skip of between 15 and 64 range bins were reported as false tracks. Basically, these conditions simulate the operation of an AVOIDS-I below an altitude of 9600 ft and at a fixed range of 20,000 ft. Each run simulated 3000 range bins at this range; however, the threat filter did not check the first or last 100 bins in order to avoid edge effects.

1. Single-Epoch Simulations at Twice Expected Reply Density

A set of 170 runs was made with the computer program in which the density was 0.06 nonthreatening aircraft per range bin and the fruit density was 0.02 replies per range bin. This is the fruit density for an AVOIDS-I predicted by the 1982 Los Angeles traffic model but the density of nonthreatening aircraft is about twice that expected. The 170 runs simulated 476,000 range bins, which is equivalent to about 1600 epochs, since there are about 300 high-density range bins per epoch for the AVOIDS-I design. There were 63 false tracks reported at double the expected AVOIDS-I density of nonthreatening aircraft. Table 11 shows how the occurrence of a false report depended on the range rate of the nonthreatening aircraft which

*Bin splitting occurs when the sub-epoch range data from an aircraft occupy two adjacent range bins. The combination of range data from multiple interrogations produces this effect. Bin splitting will contribute to the generation of false tracks. Previous calculations (Appendix E of Ref. 1) show that the bin splitting of real target data (not fruit) occurs with probability 0.08. This gives an average probability of 0.15 that two bins are filled if blockage is 0.14. Without transponder blockage, the probability is about 0.18.

occupied the greatest number of sub-epoch range gates. The maximum number of sub-epoch range gates which were occupied by a single target is also given in the table. There were 14 false tracks involving nonthreatening aircraft with values of range rate substantially different (i.e., a range rate greater than -100 ft/sec) than that of a threatening aircraft. This indicates an occurrence rate of 2.9×10^{-5} per range bin. (A range rate of -100 ft/sec at 20,000 ft range is a Tau of 92 sec as compared with the ANTC-117 Tau 2 zone of 40 sec.)

TABLE 11. FALSE REPORTS AS A FUNCTION OF NONTHREATENING AIRCRAFT VELOCITY

Nonthreatening Aircraft Velocity (ft/sec)	No. of False Reports	Maximum No. of Range Gates Occupied by a Single Aircraft					
		1	2	3	4	5	6
+900 to -100	14	13	1	-	-	-	-
-100 to -120	6	-	1	4	1	-	-
-120 to -140	5	-	-	4	0	1	-
-140 to -160	6	-	1	1	2	1	1
-160 to -170	12	-	-	1	3	5	3
-170 to -180	20	-	-	2	5	4	9

For a density of 0.08 nonthreatening aircraft and fruit replies per range bin, Section B predicts 1.2×10^{-6} false alarms per range bin per independent range gate pattern in an epoch. The simplified tau filter model discussed in that section suggests that there are 19.2 independent range gate patterns per range bin so the overall false alarm occurrence rate should be about 2.3×10^{-5} per range bin. This is very close to the false alarm rate (2.9×10^{-5}) observed in the computer simulation which enhances the creditability of the formulation presented in Section B. It would be difficult to observe false tracks with the computer simulations if the lower (i.e., expected 1982 Los Angeles Basin) density were used, since the analysis

presented in Section B indicates the number of false tracks will drop by a factor of 40 in going from a reply density of 0.08 to 0.05.

Most of the false reports observed in the computer simulations were due to nonthreatening aircraft with a range rate close to threatening. At the higher-than-expected density, 78 percent of the false tracks were due to nonthreatening aircraft having a range rate of less than -100 ft/sec. This percentage should increase (although the number of false tracks will decrease) as the density approaches that of the 1982 Los Angeles traffic model, since false tracks for aircraft not close to a threatening value of range rate should be proportional to μ^8, whereas, based on the analysis presented in Section C, the conditional probability for an aircraft having a range rate near threatening is proportional to something like μ^2. There were 20 false tracks due to aircraft with a range rate between 170 to 180 ft/sec. About 412 such aircraft were generated within the operating range of the Tau filter, so that the conditional probability of a false track due to aircraft with this value of range rate is 4.8×10^{-2}. This compares favorably with the probability of 4.5×10^{-2} which is predicted by the analysis presented in Section C. The conditional probability for aircraft with a range rate of -150 ft/sec is predicted to be about 4.4×10^{-3} in Section C, as compared to 8×10^{-3} observed in the computer simulations. These probabilities are for a higher-than-expected density of nonthreatening aircraft. A lower, but still relative, high conditional probability will occur at the lower peak reply densities suggested by the 1982 Los Angeles traffic model. Section C predicts for an AVOIDS-I a conditional probability of 2.0×10^{-2} for an aircraft with a range rate of -175 ft/sec and 8.8×10^{-4} for an aircraft with a range rate of -150 ft/sec. The corresponding values for an AVOIDS-II are 1.0×10^{-2} and 2.7×10^{-4}, respectively.

To verify the AVOIDS-I estimates, the computer model of the AVOIDS tracking filter was rerun with the density of nonthreatening aircraft at 0.03 and the density of fruit at 0.02. The range rate of the nonthreatening aircraft was again obtained from a Gaussian distribution with a standard deviation of 325 ft/sec, but the lowest value allowed was -100 ft/sec (as compared with the previous discussed runs which used a lower limit of -150 ft/sec.) In addition, every tenth aircraft was assigned a range rate of either -150 ft/sec or -175 ft/sec, depending on the objective of the particular run. This seeding of nonthreatening aircraft did not affect the overall density, but increased the population of near-threatening aircraft so that an estimate of conditional probability could be made without requiring an excessive amount of computer time. Computer run time was further decreased by restricting the simulations so that only range gate patterns for a range rate skip of between 15 and 25 range bins were tested. A near-threatening aircraft has little effect on the likelihood of a false track in a range gate pattern with a higher range bin skip.

About 2680 seeded aircraft were used to estimate the conditional probability of a false track for aircraft with a range rate of -175 ft/sec. Thirty-five false tracks were reported, so the estimated conditional probability is 1.4×10^{-2} as compared with 2.0×10^{-2} which was estimated in Section C. Only two false reports occurred when every tenth aircraft was assigned a range rate of -150 ft/sec, even though the number of seeded aircraft was increased to 3860. These statistics indicate a conditional probability of 5.2×10^{-4} as compared with the estimate in Section C of 8.8×10^{-4}.

2. Two-Epoch Simulations of Twice Expected Reply Density

The computer simulations were also run for two epochs at a density of 0.06 nonthreatening aircraft per range bin and 0.02 fruit replies per range bin to check the false warning

estimates in Section C in those altitude bands in which range correlation is performed. To do this, the program was run as described earlier to simulate the operation of the equipment during the first of the two epochs. The position and velocity of the nonthreatening aircraft were stored in memory. The simulated range bin registers were cleared, and the program was rerun with the location of the nonthreatening aircraft advanced 3.7 sec in time. The range bin locations for the nonthreatening aircraft in each sub-epoch were determined and the simulated range bin registers were reconstructed. Random fruit pulses were again added and the range tracker again tested the data for false tracks. The results from the two epochs were compared to see if a false warning occurred which satisfied the range correlation requirement. The predicted occurrence of a false warning is small so the simulations were only run with seeded aircraft with a range rate of -175 ft/sec (every tenth aircraft) and a density of 0.06 nonthreatening aircraft and 0.02 fruit pulses per range bin. Only two false warnings were observed, although approximately 37,000 nonthreatening aircraft were simulated (about 3680 had a range rate of -175 ft/sec). This indicates a conditional probability of false warning of 5.4×10^{-4}. The conditional false warning probability predicted in Table 10 for a density of 0.08 replies per range bin is 2.4×10^{-4}.

E. SUMMARY OF AVOIDS FALSE ALARM ANALYSIS

Excluding the effect of near-threatening aircraft, the analysis presented in Section B predicts a false track probability of 1.7×10^{-4} per epoch per altitude band for an AVOIDS-I in the peak density of the 1982 Los Angeles traffic model. An AVOIDS-II in this environment and with ANTC-117 Tau threat logic would have a false track probability of 9.5×10^{-6}. It was not practical to check these probabilities with the computer simulation. However, at a density of 0.08

replies per range bin, which is higher than that of the 1982 Los Angeles Traffic Model, the computer simulation indicated an AVOIDS-I false track probability of 2.9×10^{-5} per range bin. The false track probability predicted for this high reply density by the analysis in Section B is 2.3×10^{-5} per range bin.

For the expected AVOIDS-I density in the 1982 Los Angeles Traffic Model, the conditional probability of a false track given a nonthreatening aircraft with a range rate of -175 ft/sec is predicted to be 2.0×10^{-2}, whereas the simulations indicated 1.4×10^{-2}. For a nonthreatening aircraft with a -150 ft/sec range rate, the difference between the predicted and observed conditional probability was 8.8×10^{-4} and 5.2×10^{-4}, respectively. In addition, it was observed that the occurrence of false tracks in the simulations remained flat for a range rate between -100 ft/sec and -160 ft/sec, but increased rapidly as the range rate approached -180 ft/sec.

The number of near-threatening aircraft is difficult to estimate, but must be known to determine the false track rate. If aircraft have random headings and their speeds are between 150 ft/sec and 400 ft/sec (assuming they are below 10,000-ft altitude), the range rate can be estimated with a normal distribution with zero mean and a standard deviation of 325 ft/sec, but the distribution is truncated at 800 ft/sec (see Appendix K of Ref 1). However, the use of this distribution of range rate is not entirely satisfactory, since it leads to threatening intruders within the vicinity of an interrogating aircraft. Ground control of the aircraft is instrumental in preventing collisions and may distort the range rate distribution so as to reduce or eliminate τ_1 (and possibly the longer range τ_2) threats. Even so, the occurrence of near-threatening aircraft may not be reduced or eliminated. To overcome this quandary, the number of near-threatening aircraft will be parameterized in the analysis presented below.

If the range rate of reply aircraft follows the truncated normal distribution, about 1.1 percent of the aircraft would fall within a 10 ft/sec band centered around a range rate of -125 ft/sec and about 6.6 percent of the aircraft would have a range rate between -100 and -160 ft/sec. Between 20,000-ft and 35,000-ft range, the peak 1982 Los Angeles density would include, on the average, 0.59 such aircraft for an AVOIDS-I and 0.4 such aircraft for an AVOIDS-II if operated over the same range. About one third as many aircraft would have a range rate between -160 ft/sec and -180 ft/sec.

The false warning probability in an altitude band in which range correlation between epochs is not used is estimated in Table 12 for the 1982 Los Angeles traffic model densities. The results are strongly affected by the range rate of the nonthreatening aircraft. To show this, several conditions are presented. Item A of Table 12 estimates the false warnings if near-threatening aircraft are excluded. These estimates were obtained by squaring the single-epoch false track probabilities derived in Section B for random replies. Item B of Table 12 estimates the false warning if the nonthreatening aircraft have a truncated Gaussian range rate distribution as discussed above. Two cases are presented in which the aircraft with a range rate of less than either -160 ft/sec or -180 ft/sec are excluded. These are values of range rate at 20,000-ft range where a skip of 15 range bins or more is considered threatening (i.e., a Tau 2 threat). Excluding the effects of noise, the minimum range rate which should produce this skip is 200 ft/sec and the average value is 214 ft/sec. An aircraft with a range rate of -180 ft/sec would travel 630 ft during an epoch, and its range track should change by no more than 13 range bins. An aircraft with a range rate of -160 ft/sec should have a skip of no more than 12 range bins during

TABLE 12. SINGLE-AIRCRAFT AVOIDS FALSE WARNING PROBABILITY IN AN ALTITUDE BAND WITHOUT RANGE CORRELATION

	Conditional Probability		Number of Near-Threatening Aircraft	Expected False Warnings Probability	
	AVOIDS-I $P^2(T)$	AVOIDS-II* $P^2(T)$		AVOIDS-I	AVOIDS-II*
A. No near-threatening aircraft	--	--	--	2.9×10^{-8}	9×10^{-11}
B. Gaussian distribution of $\dot{R}$					
1. No $\dot{R}$ below -160 ft/sec**	7.7×10^{-7}	7.3×10^{-8}	0.59	4.8×10^{-7}	4.3×10^{-8}
2. No $\dot{R}$ below -180 ft/sec**	2.3×10^{-4}	5.6×10^{-5}	0.2	4.6×10^{-5}	1.1×10^{-5}
C. Half of near-threatening aircraft in "B" above					
1. No $\dot{R}$ below -160 ft/sec**	7.7×10^{-7}	7.3×10^{-8}	0.3	2.6×10^{-7}	2.2×10^{-8}
2. No $\dot{R}$ below -180 ft/sec**	2.3×10^{-4}	5.6×10^{-5}	0.1	2.3×10^{-5}	5.6×10^{-6}

*With ANTC-117 threat logic.

**At 20,000-ft range.

an epoch. A skip of 15 is considered threatening at a range of 20,000 ft, so the mismatch between the threatening filter pattern and the near-threatening aircraft is at least two range bins for $\dot{R}$ = -180 ft/sec and three range bins for $\dot{R}$ = -160 ft/sec.

The false warnings listed under Item B in Table 12 are cumulative, in that Item B(1) includes the estimated false warning from Item A and Item B(2) includes those for Item A and B(1). Item C follows the same format as Item B, but the number of near-threatening aircraft is halved.

The false warning probabilities for Items B(1) and C(1) were obtained by multiplying the expected number of aircraft with a range rate between -100 ft/sec and -160 ft/sec by the square of the estimated conditional probability derived in Section C for an aircraft with a range rate of -150 ft/sec.

To calculate Items B(2) and C(2), an aircraft with a range rate between -160 ft/sec and -180 ft/sec was assumed to have a single-epoch conditional probability of a false track equal to 0.75 of that for an aircraft with a range rate of -175 ft/sec. Computer simulations of densities of 0.05 and 0.08 substantiate this approximation.

Table 13 shows the estimated false warning in an altitude band in which range correlation is performed. The same format and assumptions were used as in Table 12.

In the 1982 Los Angeles Traffic Model, there are about 650 aircraft below 10,000-ft altitude. Table 14 estimates the false warning rate if 100 of these aircraft are equipped with AVOIDS-I and 550 with an AVOIDS-II. Ten percent of the aircraft are assumed to be climbing or diving at 1000 ft/sec so that they would interrogate four altitude bands for Tau filter measurement and another 10 percent of the aircraft are assumed

TABLE 13. SINGLE-AIRCRAFT AVOIDS FALSE WARNING RATE IN AN ALTITUDE BAND WITH RANGE CORRELATION

	Number of Near-Threatening Aircraft	Expected False Warnings Probability	
		AVOIDS-I	AVOIDS-II*
A. No near-threatening aircraft	--	1.2×10^{-9}	3.8×10^{-12}
B. Gaussian distribution of $\dot{R}$			
1. No $\dot{R}$ below -160 ft/sec**	0.59	2.6×10^{-8}	1.7×10^{-9}
2. No $\dot{R}$ below -180 ft/sec**	0.2	7.6×10^{-6}	2.4×10^{-6}
C. Half of near-threatening aircraft in "B" above			
1. No $\dot{R}$ below -160 ft/sec**	0.3	$\Sigma = 1.4 \times 10^{-8}$	$\Sigma = 8.6 \times 10^{-10}$
2. No $\dot{R}$ below -180 ft/sec**	0.1	3.8×10^{-6}	1.2×10^{-6}

*With ANTC-117 threat logic.

**At 20,000-ft range

TABLE 14. FALSE WARNINGS IN 1982 LOS ANGELES BASIN MODEL

	NUMBER OF ALTITUDE BANDS INTERROGATED											
	2			3			4					
	Number of Aircraft	False Warnings per Aircraft per Epoch	False Warnings per Hour	Number of Aircraft	False Warnings per Aircraft per Epoch	False Warnings per Hour	Number of Aircraft	False Warnings per Aircraft per Epoch	False Warnings per Hour	Total AVOIDS-I False Warnings per Hour	Total AVOIDS-II False Warnings per Hour	Total AVOIDS-I and II False Warnings per Hour
AVOIDS-I												
A. No near-threatening aircraft.	80	2.4×10^{-9}	1.9×10^{-4}	10	3.1×10^{-6}	3.1×10^{-4}	10	6.0×10^{-8}	6×10^{-4}	1.1×10^{-3}	-	-
B. Gaussian Distribution of $\dot{R}$												
1. No $\dot{R}$ below -160 ft/sec*	80	5.2×10^{-8}	4.2×10^{-3}	10	5.3×10^{-7}	5.3×10^{-3}	10	1.0×10^{-6}	1.0×10^{-2}	2.0×10^{-2}	-	-
2. No $\dot{R}$ below -180 ft/sec*	80	1.5×10^{-5}	1.2	10	6.1×10^{-5}	0.61	10	1.1×10^{-4}	1.1	2.9	-	-
C. Half of near-threatening in "B" above												
1. No $\dot{R}$ below -160 ft/sec*	80	2.8×10^{-8}	2.2×10^{-3}	10	2.9×10^{-7}	2.9×10^{-3}	10	5.5×10^{-7}	5.5×10^{-3}	1.1×10^{-2}	-	-
2. No $\dot{R}$ below -180 ft/sec*	80	7.6×10^{-6}	0.6	10	3.1×10^{-5}	0.31	10	5.4×10^{-5}	0.54	1.5	-	-
AVOIDS-II**												
A. No near-threatening aircraft	440	7.6×10^{-12}	3.3×10^{-6}	55	9.7×10^{-11}	5.3×10^{-6}	55	1.9×10^{-10}	1.1×10^{-5}	-	0.2×10^{-5}	1.1×10^{-3}
B. Gaussian Distribution of $\dot{R}$												
1. No $\dot{R}$ below -160 ft/sec*	440	3.4×10^{-9}	1.5×10^{-3}	55	4.6×10^{-8}	2.5×10^{-3}	55	8.9×10^{-8}	4.9×10^{-3}	-	8.9×10^{-3}	2.9×10^{-2}
2. No $\dot{R}$ below -180 ft/sec*	440	4.8×10^{-6}	2.1	55	1.6×10^{-5}	0.88	55	2.7×10^{-5}	1.5	-	4.5	7.4
C. Half of near-threatening in "B" above												
1. No $\dot{R}$ below -160 ft/sec*	440	1.7×10^{-9}	7.5×10^{-4}	55	2.4×10^{-8}	1.3×10^{-3}	55	4.6×10^{-8}	2.5×10^{-3}	-	4.6×10^{-3}	1.6×10^{-2}
2. No $\dot{R}$ below -180 ft/sec*	440	2.4×10^{-6}	1.1	55	8.0×10^{-6}	0.44	55	1.3×10^{-5}	0.72	-	2.3	3.8

*At 20,000-ft range.

**Assuming ANTC-117 threat logic.

to interrogate three altitude bands for this purpose (i.e., they are climbing or diving at a rate between 500 ft/min and 1000 ft/min). The remaining 80 percent of the aircraft interrogate only two altitude bands [I(+6) and I(-6)].

The data presented in Table 14 again show that the occurrence of false alarms will be mainly due to near-threatening aircraft if the range rate distribution of nonthreatening aircraft follows the truncated Gaussian distribution discussed above. Reducing the number of near-threatening aircraft by half results in approximately a 50 percent decrease in expected false warnings. While this is significant, it appears that the shape of the range rate distribution is much more important than its amplitude. For example, if nonthreatening aircraft with a range rate between -160 ft/sec and -180 ft/sec (at 20,000-ft range) are excluded, the false alarm rate drops by more than two orders of magnitude.

The phenomenon which causes false Tau 2 warnings will also lead to Tau 1 false warnings by aircraft which represent a true Tau 2 threat. The range rate difference between the Tau zones is about 450 ft/sec at 20,000-ft range. Aircraft within 50 ft/sec to 100 ft/sec of the Tau 1 boundary should have about the same conditional probability of creating a Tau 1 warning as a nonthreatening aircraft with a range rate between -100 ft/sec and -160 ft/sec has of creating a false Tau 2 warning. The Tau 1 false warning rate due to aircraft with a true Tau 2 warning, of course, depends on the number of such aircraft which are present and may result in a substantially lower number of false warnings than the data presented in Table 14 imply.

The results of our analysis indicate that essentially all of the AVOIDS false warnings are due to near-threatening aircraft. The estimate of the false warnings due to these aircraft was based on a reference range of 20,000 ft, where a range rate of -230 ft/sec or -136 knots represents a Tau 2 threat. The

AVOIDS CAS requires an intruder to be detected as threatening in two consecutive epochs before being declared a threat. Ideally, then, an aircraft with a range rate of -211 ft/sec or -125 knots should be detected at 20,000-ft range, since during the following epoch its range will decrease sufficiently (i.e., by 780 ft) so that the aircraft becomes a Tau 2 threat. Based on the data presented in Table 8, an AVOIDS-I has a 2 percent chance of detecting as a Tau threat an aircraft at 20,000-ft range with a range rate of -175 ft/sec, or -104 knots. The range rate difference between this aircraft and what should be detected is 36 ft/sec, or 21 knots. This difference in range rate represents an error in the measurement of Tau boundary and will be compared with the ANTC-117 for Tau alarm accuracy.

The ANTC-117 states in the section entitled "Tau Alarm Accuracy" that readings (used to estimate Tau) shall not exceed the greater of (a) 28 knots or (b) 7 percent of the difference between the nominal threat line $\dot{R}$ value and $\dot{R}_o$. The standard deviation of the set of alarm points shall not exceed the greater of 11 knots or 1.5 percent of the nominal range rate. For a Tau 2 intruder at 20,000 ft, the appropriate values for the parameters specified are 28 knots for the mean and 11 knots for the standard deviation.

The detection of a -175 ft/sec intruder at 20,000 ft represents an error of 21 knots. However, it is not clear if this error should be compared with the standard deviation or some combination of the standard deviation and mean specified in ANTC-117. Also, the specification uses the phrase "shall not exceed" rather than specifying the characteristics of the distribution. We will compare the measurement error for the near-threatening false warning against the standard deviation of the specification and assume a Gaussian distribution.

The detection of the -175 ft/sec (-104 knots) intruder at 20,000-ft range is an error of 21 knots and compared with ANTC-117 is a 1.9σ error. Based on the cumulative normal distribution function, the probability of an error in excess of 1.9σ is about 3 percent. The equipment satisfies the specification since the calculated probability of detecting the -175 ft/sec range rate at 20,000 ft is only 2 percent.

The detection of a range rate of -150 ft/sec or 89 knots represents an error of 49 knots or 4σ. The probability of detecting this value of range rate is 8.8×10^{-4} (see Table 8) as compared to a 4σ error for a Gaussian distribution which gives a probability of 3×10^{-5}. The tail of the error distribution for the AVOIDS design may be somewhat higher than that of Gaussian distribution. However, it should be remembered that the probability of detecting a near-threatening aircraft has been compared to the standard deviation of the specification and the mean error allowed by the specification has not been taken into account. Furthermore, the data presented in Table 14 indicate that the false alarm rate for all aircraft in the Los Angeles Basin is only 2.9×10^{-2} per hr (an average per aircraft of less than 4×10^{-5} per hr) if the false warnings due to range rates which are more negative than -160 ft/sec are excluded. These false warnings should not be included since they are allowed by the ANTC-117 specification.

V. DETECTION PROBABILITY OF TRUE THREATENING AIRCRAFT

A. INTRODUCTION

The detection of an aircraft that is a true threat will be less than unity because (1) the threatening aircraft's transponder is busy or blocked and cannot reply to provide a full set of range data for the Tau measurements, (2) the acceleration of the threatening aircraft places its range bin data outside the Tau filter range gate patterns, and (3) the communication range between the aircraft is inadequate and a full set of range measurements cannot be obtained. These three subjects will be discussed in Sections B through D. A summary is provided in Section E.

B. TRANSPONDER BLOCKAGE

In IDA's earlier report (Ref. 1), transponder blockage was shown to be the main cause of missed detections. The equipment design has been changed by adding a second decoder channel in the receiver which greatly reduces the blockage time. The reception of an interrogation waveform which does not require a response blocks the second response channel for a maximum of 16 μsec in the absence of multipath. If the received interrogation waveform does not require a response but multipath blocks the second channel, the transponder is blocked for a time, depending on the altitude, e.g., 46.3 μsec if the aircraft is at an altitude of 5000 ft. If a response to a received interrogation waveform is required, the second channel blockage without multipath is increased to 19 μsec and with multipath the transponder may be tied up until 3 μsec after the response is

transmitted, e.g., 70.6 μsec at an altitude of 5000 ft. A transmitted interrogation blocks the response circuits from transmission of the first pulse until 112 microseconds after the last pulse is transmitted, so that an aircraft at an altitude of 5000 ft is blocked for 157 μsec.

The fraction of time a transponder is blocked from accepting an interrogation will be denoted by β. Considering only response-mode blockage, the transponder is unblocked as long as it has not entered the response mode in the last τ_R seconds, where τ_R is the time spent in the response mode. If events that cause the transponder to enter the response mode occur at an average rate of f_R, and if the times of occurrence are assumed to be governed by the Poisson distribution, then the probability that the transponder is unblocked at the time an event occurs is the probability that no such event has occurred in the previous τ_R seconds. The probability that the receiver is unblocked at the time an interrogation is received is therefore

$$P_{\mu B} = \exp\,(-f_R\ \tau_R)\ . \tag{10}$$

The fraction of time that the transponder is blocked is

$$\beta = 1 - \exp\,(-f_R\ \tau_R)\ . \tag{11}$$

In these equations, the occurrence rate f_R depends on the number of aircraft within communication range and the average interrogation rate. The average interrogation rate will be discussed first. The range rate difference between a nonthreatening aircraft and the range rate slope of the Tau filter pattern affects the number of sub-epochs which must be interrogated before the aircraft is dismissed as not being a threat. The computer simulation discussed in Chapter IV was run with a truncated Gaussian range rate distribution for nonthreatening aircraft which was cut off at -150 ft/sec and the number of

sub-epochs that had to be processed to dismiss an intruder was noted. For a nonthreatening aircraft density of 0.015 and a fruit density of 0.01 replies per range bin, it was found that almost 70 percent of the detections in sub-epoch A remained as potential threatening tracks after evaluation of the data from the second sub-epoch and 22 percent remained after the third sub-epoch. The percent remaining after the fourth through the seventh sub-epochs were, respectively, 5.4 percent, 2.1 percent, 0.3 percent, and 0.1 percent. If a threatening aircraft is present in one or more altitude bands, transmissions are needed throughout the epoch.

The most dense portion of the Los Angeles 1982 Traffic Model occurs below 5000 ft. Between 0 and 5 miles of the center of the model there are 47 aircraft below 5000 ft and between 5 miles and 10 miles from the center there are 61 aircraft below 5000 ft. The range of interest for an aircraft in the center of the model would be about 8 miles, so such an aircraft would observe about 86 aircraft at altitudes below 5000 ft. If the aircraft are distributed uniformly in altitude, there will be about 20 aircraft within the interrogation altitude band. Between altitudes of 5000 ft and 10,000 ft, there are about 33 aircraft between 0 and 10 miles range, so an interrogation altitude band would contain about 6 aircraft within a range of 8 miles.

The probability that an altitude band will be interrogated during a sub-epoch with no valid threatening aircraft is shown in Table 15. The first row of data gives the probability that a particular nonthreatening aircraft will require that a group of interrogations be made in the indicated sub-epoch. Interrogations are always transmitted in the first sub-epoch which is not shown. These data, obtained from the computer simulations discussed earlier, are based on a range of 20,000 ft and a Gaussian distribution of range rate which was truncated at -150 ft/sec. The next three rows of data are for an interrogation

altitude band below 5000 ft; the last three rows are for an altitude band between 5000 ft and 10,000 ft. If P is the probability that an aircraft will appear as a threat during a sub-epoch, then the term $(1-P)^{20}$ is the probability that 20 aircraft do not require a group of interrogations to be made in a given sub-epoch, $P_1(20)$ is equal to $1-(1-P)^{20}$, which is the probability that at least one of the aircraft will require an interrogation to be made in the sub-epoch, and the term $P_2(20)$ is the probability that one out of two altitude bands requires an interrogation to be made. The need for the latter term will be explained later. The data for an altitude band between 5000 and 10,000 ft follow the same format and differ only in the number of aircraft expected to reply.

TABLE 15. EXPECTED NUMBER OF SUB-EPOCHS INTERROGATED BELOW AN ALTITUDE OF 10,000 FT

Interrogation Probability (All Aircraft)	Number of Sub-Epochs Interrogated						
	2	3	4	5	6	7	8
	Where P = 0.696	Where P = 0.225	Where P = 0.054	Where P = 0.021	Where P = 0.003	Where P = 0.001	Where P = 0.000
			0 to 5000 ft Altitude				
$(1-P)^{20}$	4×10^{-11}	0.006	0.329	0.654	0.942	0.980	1
$P_1(20)$	1	0.994	0.671	0.346	0.058	0.020	0
$P_2(20)$	1	0.988	0.450	0.120	0.003	0	0
			5000 to 10,000 ft Altitude				
$(1-P)^{6}$	7×10^{-4}	0.217	0.717	0.880	0.982	0.994	1
$P_1(6)$	0.999	0.783	0.283	0.120	0.018	0.006	0
$P_2(6)$	0.998	0.613	0.080	0.014	0	0	0

Table 4 in Chapter III shows the conditions under which interrogations within an altitude band are made. Below 9600 ft, an AVOIDS-I aircraft in level flight will interrogate two altitude bands for Tau measurements and possibly three altitude bands for altitude correlation data. The AVOIDS-II is not designed to meet the ANTC-117 threat logic and does not probe as many altitude bands as the AVOIDS-I. However, we will assume

that the ANTC-117 logic will be followed and that, as a result, the AVOIDS-II will be analyzed as if it used the same altitude band probing as the AVOIDS-I.

Table 16 gives the expected number of interrogations within a sub-epoch for the altitude bands which would be probed by an aircraft in level flight below 10,000 ft. Only nonthreatening aircraft replies are assumed. P_1 and P_2 are the appropriate probabilities from Table 15 where P_1 is either $P_1(20)$ or $P_1(6)$ and P_2 is either $P_2(20)$ or $P_2(6)$. Sub-epoch A would always be interrogated in altitude bands I(+6) and I(-6). Only two interrogations are indicated as would be the case if a threat were not indicated in the previous two sub-epochs. In sub-epoch B, two replies are transmitted in I(+6) and I(-6) if a potential threatening aircraft is indicated by the data collected in sub-epoch A. The average number of replies in sub-epoch B is therefore $2P_1$. The same is true for sub-epoch C. In sub-epoch D, three interrogations are transmitted in I(+6) and I(-6) if the data from either of the altitude bands indicate a potential threatening track at the end of sub-epoch C. The data collected in these two altitude bands should be independent, since for all practical purposes, they do not overlap in altitude. As a result, the probability of transmitting a group of interrogations in I(+6) or I(-6) in sub-epoch D is

$$P\,[I(6)]|_D = 2\,P_1 - P_2 \qquad (12)$$

where P_2 is the probability of a potential threat in either I(+6) or I(-6). The average number of interrogations in I(+6) and I(-6) for sub-epoch D is three times the value given in Eq. 12, as indicated in the table. The average number of interrogations for other sub-epochs and altitude bands can be obtained with the aid of Table 4 in Chapter III.

The terms presented in Table 16 have been evaluated with the data presented in Table 15 to give the expected number of interrogations in a sub-epoch and the results are presented in Table 17. An AVOIDS-equipped aircraft in the densest portion

TABLE 16. AVOIDS-I INTERROGATIONS EXPECTED FOR LEVEL FLIGHT BELOW 9600 FT

Altitude Band	Sub-Epoch							
	A	B	C	D	E	F	G	H
I(+6)	2	$2P_1$	$2P_1$	$6P_1-3P_2$	$3P_1$	$3P_1$	$3P_1$	$3P_1$
I(+4)	0	0	0	$3P_1$	P_1	P_1	P_1	P_1
I(0)	0	0	0	$6P_1-3P_2$	$6P_1-3P_2$	$6P_1-3P_2$	$6P_1-3P_2$	$6P_1-3P_2$
I(-4)	0	0	0	$3P_1$	P_1	P_1	P_1	P_1
I(-6)	2	$2P_1$	$2P_1$	$6P_1-3P_2$	$3P_1$	$3P_1$	$3P_1$	$3P_1$

Note: Overlap between I(+6) and I(-6) is ignored and no threatening aircraft are assumed.

TABLE 17. AVERAGE INTERROGATION RATE FOR AVOIDS CAS IN LEVEL FLIGHT (NO THREAT)

	Sub-Epoch								
	A	B	C	D	E	F	G	H	Σ
Altitudes Between 0 and 5000 ft									
I(+6)	2	2	1.99	2.68	1.04	0.17	0.06	0	9.94
I(+4)	0	0	0	2.01	0.35	0.06	0.02	0	2.44
I(0)	0	0	0	2.68	1.72	0.34	0.12	0	4.86
I(-4)	0	0	0	2.01	0.35	0.06	0.02	0	2.44
I(-6)	2	2	1.99	2.68	1.04	0.17	0.06	0	9.94
Σ	4	4	3.98	12.06	4.50	0.80	0.28	0	29.62*
Altitudes Between 5000 ft and 10,000 ft									
I(+6)	2	2	1.56	1.46	0.36	0.05	0.02	0	7.45
I(+4)	0	0	0	0.85	0.12	0.02	0.01	0	1.00
I(0)	0	0	0	1.46	0.68	0.11	0.04	0	2.29
I(-4)	0	0	0	0.85	0.12	0.02	0.01	0	1.00
I(-6)	2	2	1.56	1.46	0.36	0.05	0.02	0	7.45
Σ	4	4	3.12	6.08	1.64	0.25	0.10	0	19.19**

*8.01 per sec.

**5.18 per sec.

of the 1982 Los Angeles Traffic Model would be expected to have an average interrogation rate of 8.01 interrogations per second if it were flying below 5000 ft. This result assumes level flight (an altitude rate of less than 500 ft/min) and only nonthreatening aircraft within the range of interest. For these same conditions, an aircraft between 5000 and 10,000 ft is expected to have an interrogation rate of 5.19 per sec.

The average interrogation rate will increase if the aircraft is climbing or diving at a rate of 500 ft/min or more. For $|\dot{h}|$ (i.e., altitude rate) between 500 and 1000 ft/min, a Tau measurement is made in altitude band I(+15) or I(-15) and interrogations are made in either the PCA or PCB altitude bands for altitude correlation data. A potential threatening track in I(+15) or I(-15) will also cause the transponder to probe either I(+6) or I(-6) if interrogations in these bands are not scheduled for other reasons. The number of interrogations in either I(+15) or I(-15) will be two in sub-epoch A, $2P_1$ in B and C, and $3P_1$ in other sub-epochs. Except for sub-epoch D, the average number of interrogations will be the same as either I(+6) or I(-6) in Table 17. During sub-epoch D, the average number of interrogations in I(+15) or I(-15) will be less than in I(+6) or I(-6) in Table 17 so that below 5000 ft, I(+15) or I(-15) will average about 9.25 interrogations per epoch and between altitudes of 5000 and 10,000 ft will average 6.85 interrogations per epoch. Probing I(+15) or I(-15) will increase its average rate by 2.5 interrogations per second and 1.85 interrogations per second, respectively.

The PCA or PCB altitude bands (predicted co-altitude above or below) will also be probed if $|\dot{h}|$ is greater than 500. The location of the predicted co-altitude will mostly overlap the combination of I(+6) ~~and~~ I(+15) for PCA and I(-6) or I(-15) for PCB. To interrogate the co-altitude band during sub-epoch D requires a potential threat in I(+6) or I(+15) for PCA and I(-6) or I(-15) for PCB. There is about a 300-ft, or 25 percent,

overlap between I(+6) and I(+15) [and also I(-6) and I(-15)], so the average number of interrogations in the predicted co-altitude band will be about

$$\bar{f}_{PC} = 3(1.75\ P_1 - 0.75\ P_2) \qquad (13)$$

since the probability of a potential track in I(+15) which is not in I(+6) is about 0.75 P_1.

In sub-epochs F through H, the average number of interrogations in the predicted co-altitude band will be about $3P_1$*. Table 18 gives the average number of interrogations per sub-epoch for a predicted co-altitude band. The increase in interrogation rate is about one interrogation per second at altitudes below 5000 ft and about 0.5 interrogations per second at altitudes between 5000 ft and 10,000 ft.

TABLE 18. PREDICTED CO-ALTITUDE AVERAGE INTERROGATIONS

Altitude of Interrogator	Sub-Epoch						
	D	E	F	G	H	Σ	$\Sigma \div 3.7$
0 to 5000 ft	2.51	1.03	0.17	0.06	0	3.77	1.01
5000 to 10,000 ft	1.3	0.36	0.06	0.03	0	1.75	0.47

For a climbing aircraft, the number of interrogations in I(+6) will increase in the presence of a potential threat in I(+15). There is about 25 percent overlap in these altitude bands, so in sub-epoch D the number of interrogations in I(+6) would be about

*This assumes overlap between the PCA band and either I(+6) or I(+15) for a climbing aircraft and between PCB and I(-6) or I(-15) for a diving aircraft. If not, the average number of interrogations will be less by a factor equal to the percent overlap.

$$\bar{f}_{+6}\,|_{+15} \approx 8.25\ P_1 - 7.5\ P_1^2 + 2.25\ P_1^3 \quad . \tag{14}$$

The number of interrogations expected in sub-epoch D for I(+6) due to interrogation in I(+15) is 2.84 for an aircraft below 5000 ft and 1.79 for an aircraft at altitudes between 5000 ft and 10,000 ft. This is an increase of 0.17 and 0.34 interrogations, respectively, over the level flight case. In sub-epochs E through H, the number of interrogations in I(+6) should not increase, since altitude correlation with a track in I(+15) only occurs if the track is already a potential Tau threat in I(+6). A summary of the increase in interrogations due to $|\dot{h}|$ between 500 ft/min and 1000 ft/sec is presented in Table 19.

TABLE 19. INCREASE IN INTERROGATIONS DUE TO AIRCRAFT $|\dot{h}|$ BETWEEN 500 FT/MIN AND 1000 FT/MIN

	Aircraft Altitude (ft)	
Altitude Band	0 to 5000	5000 to 10,000
I(+15) or I(-15)	9.25	6.85
PCA or PCB	3.77	1.75
I(+6)	0.17	0.34
$\sum$	13.19	8.94
$\sum \div 3.7$	3.56	2.41

An aircraft with $|\dot{h}|$ > 1000 ft/min will probe I(+26) or I(-26) in addition to the altitude bands that are interrogated when $|\dot{h}|$ is between 500 ft/min and 1000 ft/min. The expected number of interrogations in I(+26) or I(-26) is the same as in I(+15), so if the interrogated band is at an altitude below 5000 ft, the average number of interrogations within the band would be 9.25 and if the altitude were between 5000 ft and

10,000 ft, the average number of interrogations would be 6.85. This represents an increase in average interrogation rate of 2.5 and 1.85 interrogations per sec, respectively. In addition, the average number of interrogations in I(+15) or I(-15) would increase from $3P_1$ to $6P_1 - 3P_2$. If the altitude bands in question were below 5000 ft, the increase in the number of interrogations per epoch in I(+15) or I(-15) would be 0.66, and the average interrogation rate would increase by 0.18. If the altitude bands were between 5000 ft and 10,000 ft, the corresponding increase would be 0.61 interrogations per epoch or 0.17 interrogations per sec. An aircraft flying below 5000 ft and with $|\dot{h}|$ greater than 1000 ft/min will transmit 9.91 interrogations per epoch more than if $|\dot{h}|$ were between 500 ft/min and 1000 ft/min. For altitudes between 5000 ft and 10,000 ft, the increase is 7.46 interrogations per epoch.

A summary of the average interrogations is presented in Table 20. These data assume that a threat is not present. If a threat is present, the interrogation rate will increase. For example, a threat in I(+6) below altitudes of 5000 ft will increase the interrogations within this altitude band to 24; those in I(+4) to 7.27 (i.e., 7 plus 2/3 of the expected interrogations in sub-epochs E through H when no threat is present); those in I(0) to 10.08 (i.e., 9 plus half the interrogations in E through H when no threat is present); and those in I(-6) to 10.24. Between altitudes of 5000 and 10,000 ft, the interrogations in the respective bands are increased to 24, 7.1, 9.41, and 9.01, respectively. A summary of the expected interrogations with a threat present is given in Table 21.

The average interrogation rate expected for the Los Angeles Basin in 1982 is calculated in Table 22. Eighty percent of the aircraft are assumed to have $|\dot{h}|$ of less than 500 ft/min, 10 percent to have $|\dot{h}|$ between 500 ft/min and 1000 ft/min, and 10 percent to have $|\dot{h}|$ greater than 1000 ft/min. The number of aircraft with a single threat present is varied in the table.

TABLE 20. SUMMARY OF AVERAGE INTERROGATION RATE, NO THREAT PRESENT

Altitude Rate	Interrogations at Altitudes Between 0 and 5000 ft		Interrogations at Altitudes Between 5000 ft and 10,000 ft	
	Per Epoch	Per Second	Per Epoch	Per Second
$\lvert\dot{h}\rvert < 500$ ft/min	29.62	8.01	19.19	5.19
$500 < \lvert\dot{h}\rvert < 1000$ ft/min	42.81	11.57	28.13	7.61
1000 ft/min $< \lvert\dot{h}\rvert$	52.72	14.25	35.59	9.63

TABLE 21. SUMMARY OF AVERAGE INTERROGATION RATE, WITH A THREAT PRESENT

Altitude Rate	Interrogations at Altitudes Between 0 and 5000 ft		Interrogations at Altitudes Between 5000 ft and 10,000 ft	
	Per Epoch	Per Second	Per Epoch	Per Second
$\lvert\dot{h}\rvert < 500$ ft/min	54.03	14.62	50.51	13.65
$500 < \lvert\dot{h}\rvert < 1000$ ft/min	67.22	18.16	59.45	10.06
1000 ft/min $< \lvert\dot{h}\rvert$	77.13	20.84	66.91	18.08

TABLE 22. EXPECTED AVERAGE INTERROGATION RATE FOR LOS ANGELES BASIN, 1982

Threat Condition	Expected Interrogations per Second		
	Below 5000 ft	Between 5000 ft and 10,000 ft	Average Below 10,000 ft
No threatening aircraft	8.99	5.86	8.27
12.5 percent of aircraft with single threat	9.82	6.93	9.15
25 percent of aircraft with single threat	10.64	7.98	10.03
50 percent of aircraft with single threat	12.29	10.11	11.79
All aircraft with single threat	15.62	14.34	15.33

A computer simulation of the Los Angeles Basin in 1982* prepared by Honeywell (Ref. 3) indicates that 53 percent of the aircraft flying under instrument flight rules (IFR), if equipped with the AVOIDS-I, and 44 percent of the aircraft flying under visual flight rules (VFR), if equipped with the AVOIDS-II, would be in the presence of at least one threat. These simulations assume that the AVOIDS-II had a much smaller threat volume than required by the ANTC-117 specifications so that if all aircraft were equipped with the ANTC-117 logic, an even higher percentage of the aircraft would be expected to be in the presence of a threat.

In the analysis presented in this report, both the AVOIDS-I and AVOIDS-II are assumed to use the same threat logic. The average interrogation rate will therefore be the same for both equipments but will vary with altitude. For aircraft below 10,000 ft near the center of the 1982 Los Angeles traffic model, 77 percent are below 5000 ft, and 23 percent are between 5000 and 10,000 ft. (Less than 5 percent are above 10,000 ft.) The last column of data in Table 22 estimates the average interrogation rate for aircraft below 10,000-ft altitude. The data were obtained by multiplying the interrogation rate for aircraft below 5000 ft by 0.77, the rate between 5000 ft and 10,000 ft by 0.23, and summing the results.

In Ref. 3 the average communication range between two AVOIDS-I aircraft is stated as 17 miles, that between an AVOIDS-I and an AVOIDS-II is stated as 12 miles, and that between two AVOIDS-II aircraft is stated as 8.4 miles. These values are assumed to apply at altitudes below 10,000 ft. The same reference estimates the average communication range for an increase in a transmitter's power of 2 dB as 21.4 miles, 15.1 miles, and 10.6 miles, respectively, for the above combinations

*This report used the MITRE Traffic Model of the 1982 Los Angeles Basin.

of equipment. In Ref. 3, the average communication range between two AVOIDS-II aircraft is deemed adequate, even without the 2 dB increase in power, since the instrumented range is only 25,600 ft, or 4.8 miles. (The gain margin at this range is estimated at 8 dB.) The maximum legal speed for aircraft at altitudes below 10,000 ft is 250 knots, so the maximum encounter speed between two such aircraft would be 500 knots. The maximum range at which a Tau 2 warning could occur, based on ANTC-117 specifications, is 7.36 miles or 44,700 ft. To insure that the alarm is issued at this range, the first detection may have to occur two to three epochs early or at a range of about 10 miles. Due to antenna gain, the loop sensitivity in the forward direction should be 3 dB greater than the average. However, even this seems inadequate for an encounter between two AVOIDS-II aircraft. A 6 dB gain margin in the forward direction is more appropriate, so that an average communication range of at least 14.5 miles should be considered if the maximum range of the ANTC-117 Tau 2 threat logic is to be met. This is the value that will be assumed in estimating fruit rates and transponder blockage for an encounter involving AVOIDS-II aircraft. The stated average communication range of 17 miles for two AVOIDS-I aircraft will be accepted for this analysis.

Reference 3 also reports the results of a computer simulation of AVOIDS-equipped aircraft operating in the projected 1982 Los Angeles Basin Traffic Model. The densest traffic occurred 15 miles east and 10 miles south of the center and only these results will be used in our analysis. Figure 16* shows the communication paths between an AVOIDS-I and surrounding IFR and VFR aircraft in the most dense region. In the Honeywell analysis, all aircraft flying under IFR were assumed to be equipped with AVOIDS-I transponders and all aircraft flying under VFR were assumed to be equipped with AVOIDS-II transponders. Our analysis will use the same assumptions.

* Obtained from Ref. 3.

The top portion of the diagram assumes an average communication range between two AVOIDS-I aircraft of 17 miles and between an AVOIDS-I and an AVOIDS-II of 12 miles. The bottom portion of the diagram assumes the average communication range is increased 2 dB to 21.4 miles and 15.1 miles, respectively. To aid in understanding the diagrams, the paths in the top diagram are labeled "A" and those in the bottom diagram are labeled "B." The subscripts on the paths indicate nodal points of the path. The path between the AVOIDS-I in question and aircraft flying by IFR is marked A_{II} or B_{II} and the path between an AVOIDS-I and an aircraft flying by VFR with an AVOIDS-II transponder is marked A_{IV} and B_{IV}.

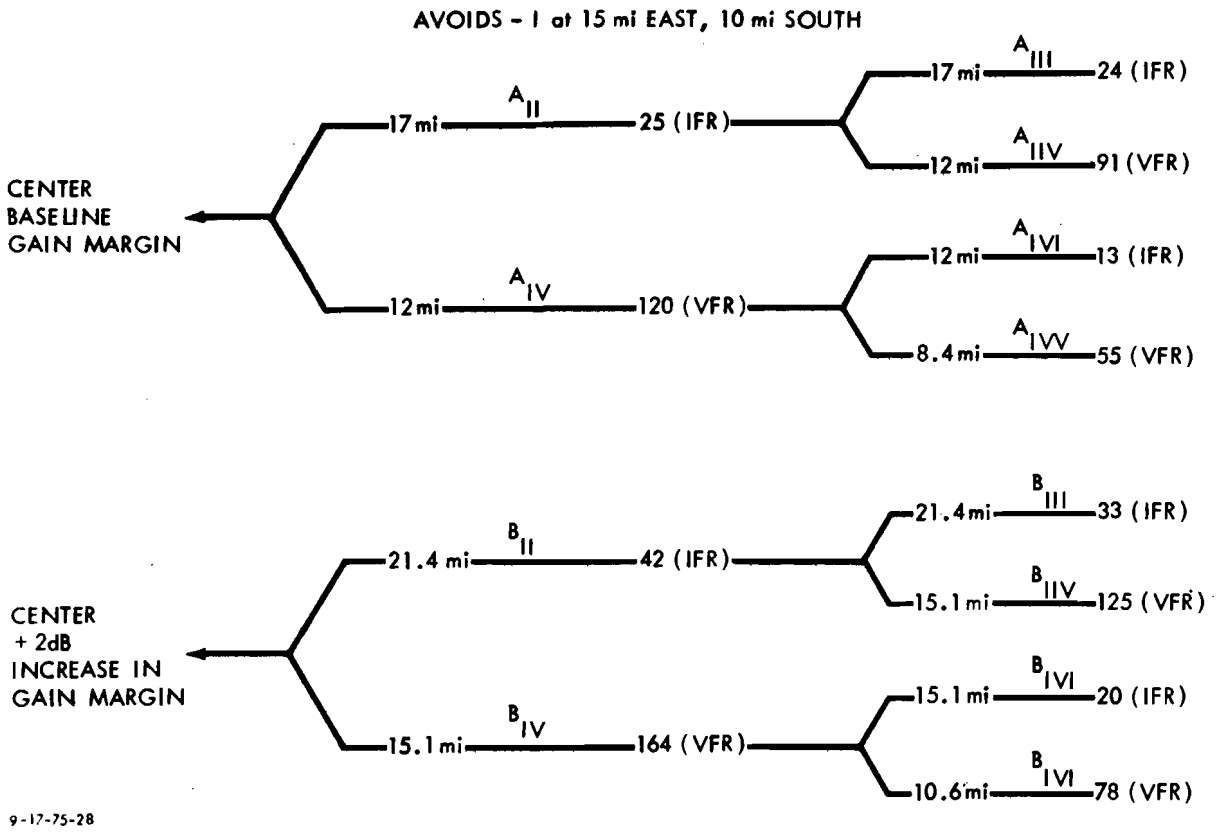

FIGURE 16. Communication Paths to an AVOIDS-I in the Los Angeles Traffic Projected for 1982

Three subscripts are needed to describe the paths involved in fruit generation. Working backward, the AVOIDS-I in question receives replies from IFR aircraft that have been interrogated by other IFR and VFR aircraft. The reply path from the IFR aircraft to the aircraft in question is A_{II}. Based on the data presented in Fig. 16, there are 25 IFR aircraft that would have a path to the central IFR aircraft. The reply rate of these 25 aircraft is dependent on the number of interrogations they receive and the effectiveness of altitude coding in preventing replies to aircraft outside the intended interrogation altitude zone. We will assume that altitude encoding reduces the number of replies to one-fifth the number of received interrogations. The fruit replies received by the AVOIDS-I in the densest part of the Los Angeles Basin projected for 1982 along path A_{II} is then

$$F_{IAII} = 0.2\ A_{II}(A_{III}\ \overline{f}_I + A_{IIV}\ \overline{f}_V) \tag{15}$$

where A_{II}, A_{III}, and A_{IIV} are the number of aircraft in each path and $\overline{f}_I$ and $\overline{f}_V$ represent the average interrogation rate for IFR and VFR aircraft, respectively. Equation 15 gives the fruit generated by the top half of the diagrams of Fig. 16. An AVOIDS-I would also receive fruit along path A_{IV}, so the total fruit replies received would be

$$F_I = 0.2(A_{II}\ A_{III}\ \overline{f}_I + A_{II}\ A_{IIV}\ \overline{f}_V + A_{IV}\ A_{IVI}\ \overline{f}_I + A_{IV}\ A_{IVV}\ \overline{f}_V)\ . \tag{16}$$

If the first subscript in the communication path terms is changed to a V, Eq. 16 gives the fruit rate for an AVOIDS-II.

The communication paths between an AVOIDS-II located in the densest region of the Los Angeles Basin projected for 1982 is shown in Fig. 17*, and equations similar to 15 and 16 can be derived for these diagrams.

*Obtained from Ref. 3.

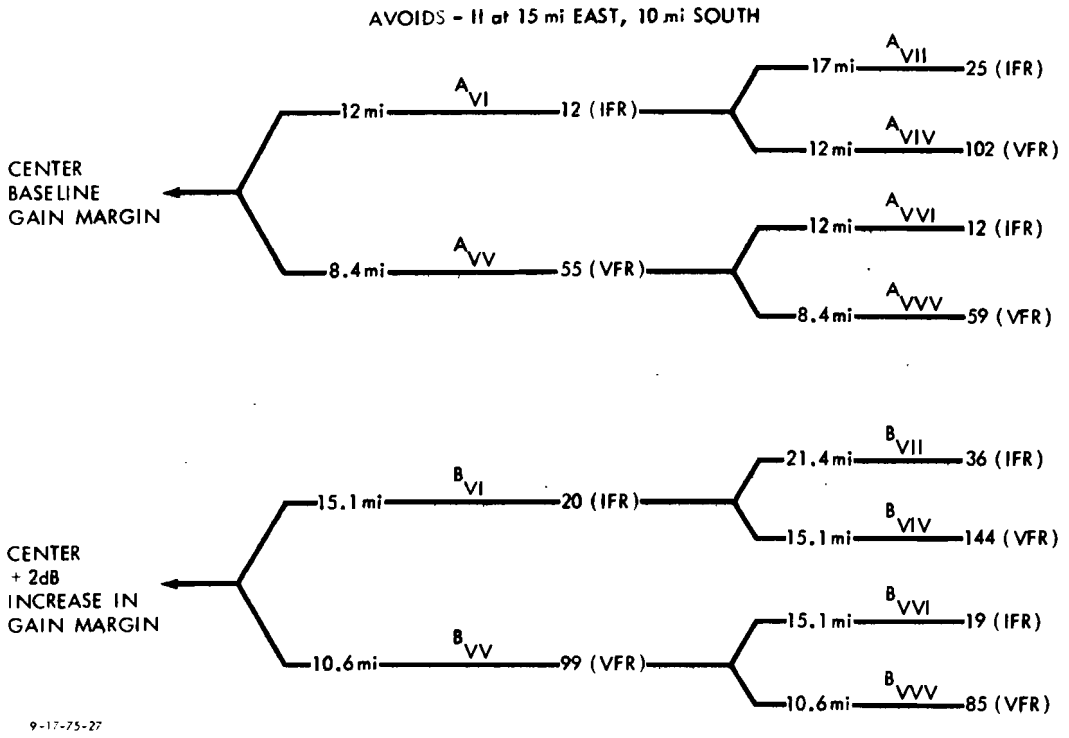

FIGURE 17. Communication Paths to an AVOIDS-II in the Los Angeles Traffic Projected for 1982

The results of the computer simulation presented in Figs. 16 and 17 are useful and informative, but cannot be used directly in our analysis because of the difference in assumed average communications ranges. To estimate the number of reply and interrogation paths for different average communication ranges, the data of Figs. 16 and 17 are plotted vs range in Fig. 18. These data were used to construct Fig. 19, which gives the number of aircraft within communication range for the assumed values of average communication range (i.e., between an AVOIDS-I and another AVOIDS-I of 17 miles and 14.5 miles for other equipment combinations.)

The fruit rate for the Los Angeles Basin of 1982 is estimated in Table 23. The estimates for number of reply pulses received are based on the formulation presented in Eq. 16. The communication paths and number of aircraft are shown in Fig. 19. Equation 16 includes separate terms for the average communication

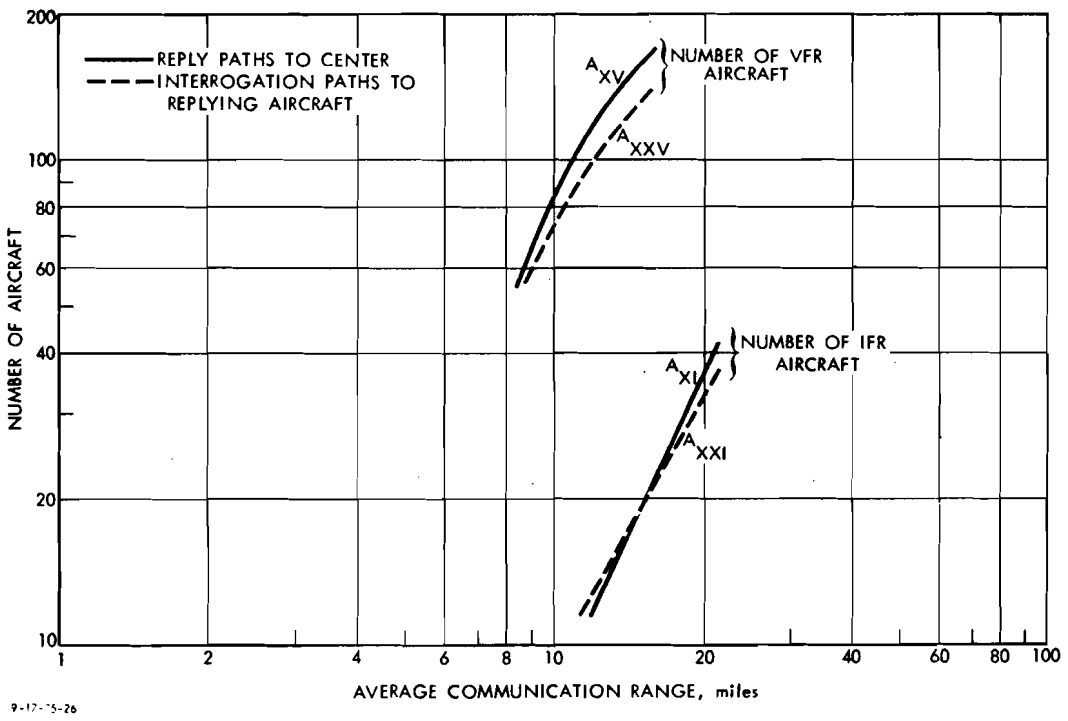

FIGURE 18. Number of Aircraft Within Communication Range

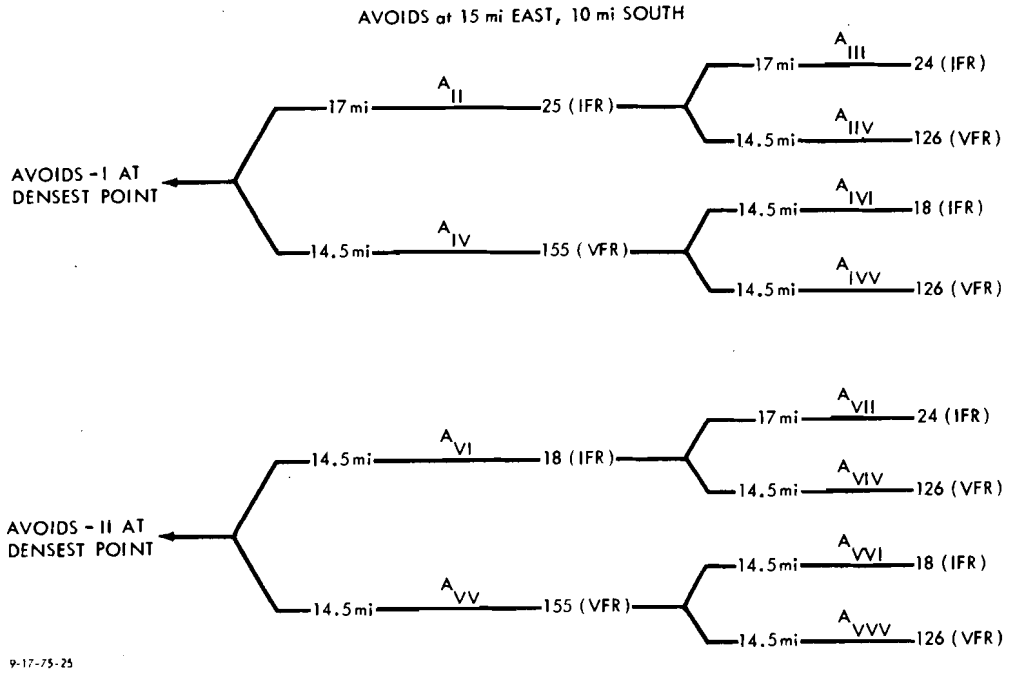

FIGURE 19. Communication Paths and Number of Aircraft Projected for the Los Angeles Basin, 1982

TABLE 23. EXPECTED FRUIT RATE FOR LOS ANGELES BASIN, 1982

Threat Condition	Average Interrogation Rate	Reply Pulses Received Per Second	Interrogations Received		Fruit Pulses Received Per Second
			Waveforms Per Second	Pulses Per Second	
AVOIDS-I					
No Threatening Aircraft	8.27	43,120	1,488	5,952	49,072
12.5 Percent of Aircraft with Single Threat	9.15	47,708	1,647	6,589	54,297
25 Percent of Aircraft with Single Threat	10.03	52,295	1,806	7,222	59,518
50 Percent of Aircraft with Single Threat	11.79	61,473	2,122	8,489	69,962
All Aircraft with Single Threat	15.33	79,930	2,759	11,038	90,969
AVOIDS-II					
No Threatening Aircraft	8.27	41,383	1,430	5,721	47,105
12.5 Percent of Aircraft with Single Threat	9.15	45,786	1,583	6,330	52,116
25 Percent of Aircraft with Single Threat	10.03	50,190	1,735	6,939	57,130
50 Percent of Aircraft with Single Threat	11.79	58,997	2,040	8,159	67,156
All Aircraft with Single Threat	15.33	78,037	2,698	10,791	88,828

rate for an AVOIDS-I and an AVOIDS-II; however, our analysis assumes that both sets of equipment use the same threat logic and therefore experience the same interrogation rate. The interrogation rate used to generate the data presented in Table 23, column 1, is the average below 10,000-ft altitude which is listed in the last column of Table 22. The fruit replies due to received interrogations are four times the interrogation rate (there are four pulses in an interrogation waveform) times the number of aircraft within communication range (i.e., the number of aircraft along the double subscript paths of Fig. 19). Both the received interrogation waveforms and the number of interrogation waveform pulses are listed in Table 23.

Fruit affects transponder blockage by creating false interrogations. To block a transponder requires two fruit pulses with approximately the time separation of an interrogation waveform's first pulse pair. The gate width of the transponder's two-pulse decoder is about 100 nsec, so the false interrogation rate is approximately the fruit rate squared times 10^{-7}. A false interrogation consisting of only the first pulse pair would not require the receiving transponder to respond. If four fruit pulses were received with the correct time separation, a false terrogation could be created which would require a response. This is extremely unlikely and will not be considered.

Table 24 is used to estimate transponder blockage and the probability, P_W, that blockage will not interfere with the reception of range data so as to prevent the detection of a threat. Two epochs of range data are needed for detection of a threat so

$$P_W = (1 - P_B^2)^2 \, (1 - P_B^3)^{14}$$

if the interrogating aircraft transmits two interrogations in the first two sub-epochs and three interrogations in the following fourteen sub-epochs. The term P_B is the probability that the receiving transponder is blocked when an interrogation is received.

TABLE 24. PROBABILITY OF BLOCKAGE AND PROBABILITY OF WARNING FOR AVOIDS EQUIPMENT

Percent of Aircraft With Single Threat	Interrogation Rate			A=Σ (no reply, no multipath)	B=Σ (no reply, multipath)	C=True (reply, no multipath)	D=True (reply, multipath)	$P_{\mu b}$ due to column A	$P_{\mu b}$ due to column B	$P_{\mu b}$ due to column C	$P_{\mu b}$ due to column D	$P_{\mu b}$	P_B	P_W
	True (reply)	True (no reply)	False* (no reply)											
AVOIDS-I														
0	298	1190	406	1117	479	209	89	.9823	.9781	.9960	.9937	.9509	.0491	.9935
12.5	329	1318	498	1271	545	230	99	.9799	.9751	.9956	.9930	.9446	.0554	.9915
25	361	1445	599	1431	613	253	108	.9774	.9720	.9952	.9924	.9383	.0617	.9891
50	424	1698	827	1767	758	297	127	.9721	.9655	.9944	.9911	.9250	.0750	.9830
100	552	2207	1399	2524	1082	386	166	.9604	.9511	.9927	.9883	.8962	.1038	.9634
AVOIDS-II														
0	286	1144	375	1063	456	200	86	.9831	.9791	.9962	.9939	.9530	.0470	.9941
12.5	317	1266	459	1207	518	222	95	.9809	.9763	.9958	.9933	.9472	.0528	.9924
25	347	1388	552	1358	582	243	104	.9785	.9734	.9954	.9927	.9412	.0588	.9903
50	408	1632	762	1676	718	286	122	.9735	.9673	.9946	.9914	.9285	.0715	.9847
100	540	2158	1333	2444	1047	378	162	.9617	.9527	.9928	.9886	.8992	.1008	.9658

* Assuming fruit rate increased by 30 percent due to multipath.

The procedure for estimating P_B in Table 24 is as follows. One-fifth of the received true interrogation waveforms listed in Table 23 are assumed to require the receiving transponder to respond. The number of these interrogations which would be received in a second is given in the second column of Table 24. The third column lists the true interrogation rate for interrogations which do not require a response, so the sum of columns 2 and 3 of Table 24 is equal to column 4 of Table 23. The fourth column of Table 24 is the expected number of false interrogations; these are assumed not to require a reply. The false interrogations are caused by fruit, as explained earlier. To account for multipath, the expected received fruit rate listed in Table 23 was increased by 30 percent in computing the false interrogation rate of Table 24.

Columns 5 through 8 of Table 24 separate the received interrogations (both true and false) into whether or not a reply is to be made and whether or not multipath is received. The data presented assume multipath is present 30 percent of the time. Columns 9 through 12 give the probability that the transponder is not blocked (see Eq. 10) for each type of received interrogation listed in columns 5 through 8. These data are based on the blockage times discussed at the beginning of the section. The last three columns of Table 24 give the overall probability that the transponder is unblocked, the probability that it is blocked, and finally, the probability that 16 sub-epochs of range data can be collected in order to detect a threatening intruder. The results of Table 24 are presented for both the AVOIDS-I and AVOIDS-II and for the various threat conditions discussed earlier.

C. DETECTION PROBABILITY FOR ACCELERATING TARGETS

The AVOIDS Tau filter has two range gate patterns which check for threatening values of range rate skip. These patterns are referred to as the high- and low-acceleration patterns and

are described in Chapter III. For zero range rate skip, the low-acceleration patterns are symmetrical about the A and H sub-epoch end points and allow a bulge in the range track for the inner sub-epochs of between 50 ft and 100 ft, depending on the position of the target in sub-epochs A and H. A target track with a ±1-G acceleration will bulge ±48 ft in sub-epochs C and D, so, excluding noise measurement effects, the low-acceleration patterns can handle these targets for zero values of range rate skip. The high-acceleration pattern is asymmetrical and allows the range track to bulge between 100 and 150 ft in the direction of increasing range and only 0 to 50 ft in the other direction. As a result, this pattern is capable of detecting targets with higher negative values of acceleration than the low-acceleration pattern.

The movement of the range bin patterns between sub-epochs is discussed in Chapter III and is quantized in 50-ft steps. As the clock phase of the range gate movement changes, the amount of bulge which can be tolerated in the range track is affected. To show this, the low-acceleration range rate patterns have been plotted in Fig. 20 with the average range rate removed so that the A and H sub-epoch range gates are in line. All seven clock phases are shown. (In the critical C and D sub-epochs, where most of the bulge occurs, the high-acceleration pattern extends an additional 50 ft to the right so that targets with more than 1 G of deceleration can be detected.) As shown in the figure, targets with a range acceleration of +1 G will not always be detected, even without considering noise, since the range gates in the C and D sub-epochs extend only 42.9 ft closer in range for clock phases 2 and 5.

Geometric effects as well as target maneuvers can cause apparent target acceleration and result in a bulge in the range track. The acceleration due to target maneuvers is expected to be limited to within ±1 G and the detection of a target which exhibits a deceleration in measured range is of more concern than one in which the rate of change in range

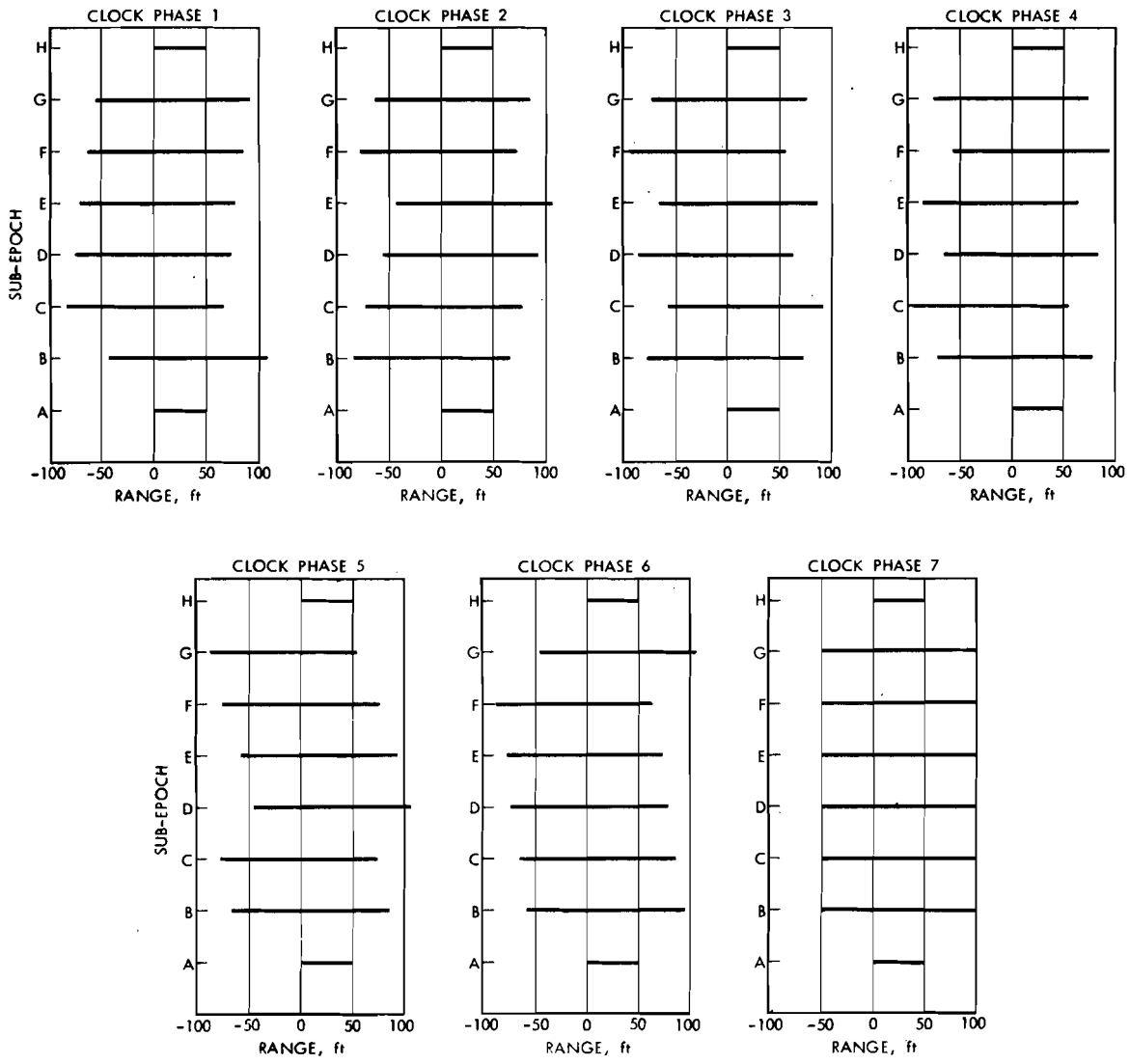

FIGURE 20. Low-Acceleration Range Gate Patterns with Average Velocity Removed

rate is positive. The apparent acceleration due to changes in geometry can be much larger. For example, a family of curves is plotted in Fig. 21 which shows the range-rate-vs-range relationship for two aircraft approaching each other at 2000 ft/sec. Also shown are the Tau 1 and Tau 2 boundaries. Each curve represents a different miss distance at closest approach and the tick marks on the curves give the time before closest approach in seconds and the rate of change in range rate in ft/sec^2. The highest acceleration occurs at closest approach and is equal to

$$A_{max} = (V_C)^2/R_{min} \tag{17}$$

where V_C is the closing velocity between the two aircraft and R_{min} is the miss distance at closest approach. The acceleration in the range data can be very high at closest approach. For a miss distance of 40,000 ft, the maximum acceleration is 100 ft/sec^2. However, according to ANTC-117 logic, such an aircraft is outside the Tau 2 boundary and need not be tracked since it does not represent a threat. Higher accelerations will be experienced for encounters in which the miss distance is less, and it will be impossible for the AVOIDS tracker to follow the target's movements and continue an alarm in accordance with ANTC-117 logic. The consequences of the equipment's inability to follow ANTC-117 logic do not appear to be serious, since the time at which a range track is dropped due to a high rate of change in the range rate occurs closer to the point of minimum miss distance as the miss distance decreases. Also, the equipment is built so that an alarm will only be dropped after the target has been missed for two consecutive epochs. The time separation between epochs is 3.7 seconds, on the average, and if the track is maintained through half of the second epoch, the alarm or warning will be displayed for about 5.5 seconds after the Tau filter is unable to follow the target. For encounters with a near-zero miss distance, the high

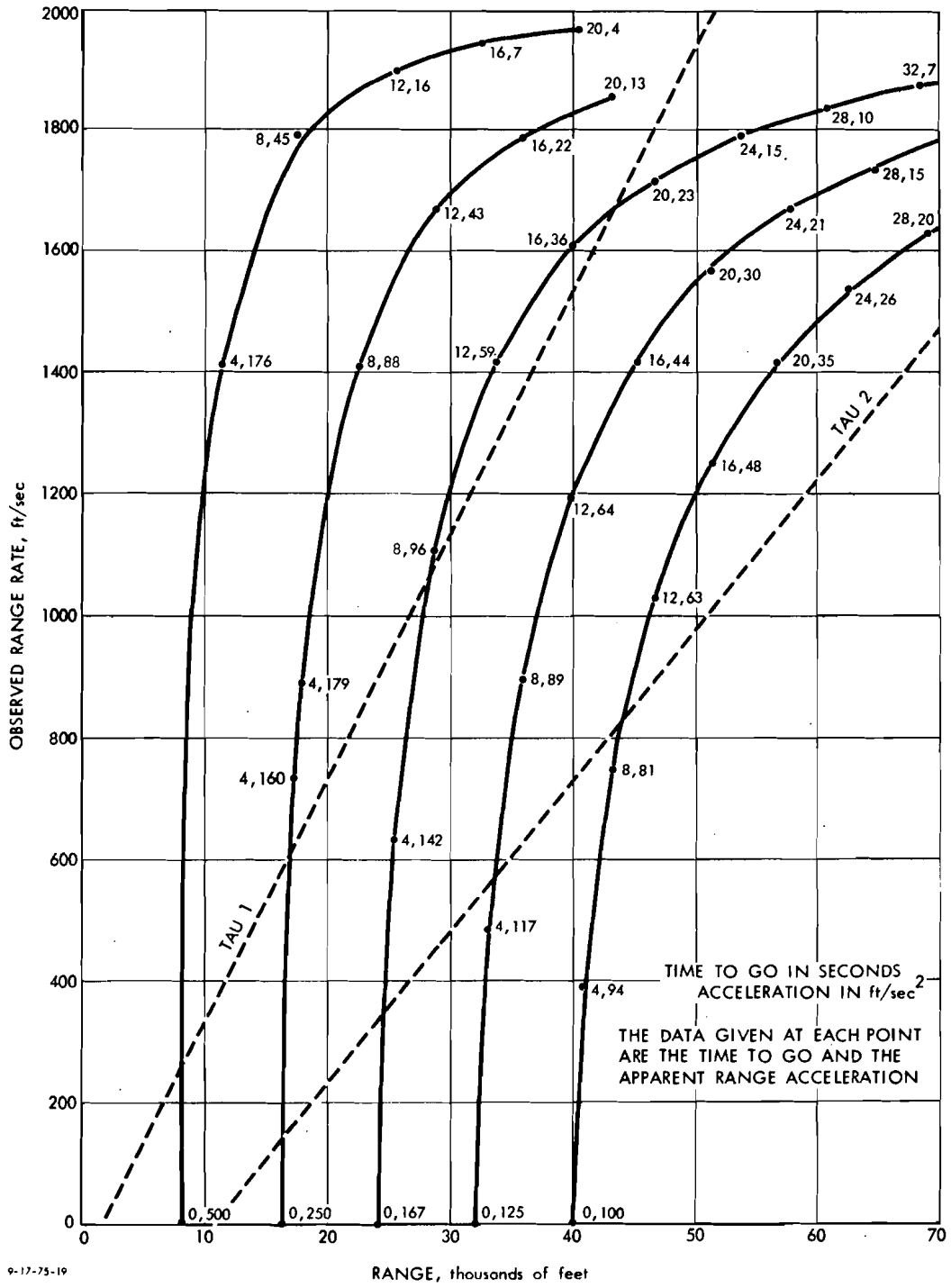

FIGURE 21. Tau Diagram for a Closing Velocity of 2000 ft/sec

acceleration occurs close enough to the time of closest approach so that the alarm will be maintained long enough to meet the ANTC-117 requirements.

The capability of the equipment to track accelerating targets was determined from the computer simulation program discussed in Chapter IV. Only threatening values of range rate were used to generate the range bin data and these were selected randomly so that it was equally likely that the track would be a threat in any one of the seven possible clock phases. Gaussian noise with a standard deviation of 5 ft was added to the sub-epoch range data for these simulation runs. Fruit replies were not included, and the threatening aircraft were spaced so that their range tracks would not interfere with one another. The results are presented in Table 25. As shown, the detection probability of a target with a rate of change rate of -1 G was found to be 1.0 and that for +1 G was 0.98. To estimate its detection probability for each value of acceleration listed in the table, a total of 9,000 simulated threatening aircraft were used. The locations of the threatening track were randomly distributed in range.

TABLE 25. PROBABILITY OF DETECTION FOR ACCELERATING TARGETS

$\ddot{R}$ (ft/sec^2)	Bulge in Range Track (ft)	P_d
-32.2	48	1.00
0	0	1.00
32.2	48	0.98
40.0	60	0.90
46.7	70	0.75
53.3	80	0.54
60.0	90	0.32
66.7	100	0.15
73.3	110	0.04

The acceleration vs time is plotted in Fig. 22 for the 2000 ft/sec encounter and the time at which a threat advisory is dropped as a result of target acceleration is estimated as follows. The single-epoch detection probability is presented on the right side of Fig. 22. This probability is based on the detection probabilities from Table 25 and an assumed blockage of 0.07. This blockage corresponds to the condition in which 50 percent of the aircraft have a single threat within coverage. Two time scales are provided at the bottom of Fig. 22. The upper scale shows the time to closest approach and the lower scale the time at which an advisory is dropped, assuming a 5.5 sec delay. The cases of primary concern are for small miss distances at closest approach. The acceleration increases rapidly for these conditions, so that the probability of detection is controlled by the probability of miss on the first epoch For example, the probability of single-epoch detection for a miss distance of 12,000 ft is 0.87 at 10.5 sec before closest approach. This corresponds to an acceleration of 40 ft/sec^2. In the following epoch (i.e., 3.7 sec later), the acceleration exceeds 100 ft/sec and the single-epoch detection probability is close to zero. The overall probability of dropping the alarm at 4.5 seconds (i.e., scale T_2) is therefore about 0.87. This is about the time the target recrosses the Tau 1 boundary (see Fig. 21) so only the Tau 2 advisory, as dictated by ANTC-117 logic, will be missing.

The estimated time at which a warning or advisory will be dropped is shown in Fig. 23. As stated earlier, this effect is noted as a deviation from ANTC-117 threat logic and does not appear to be a serious problem since it does not occur on collision encounters.

The acceleration in range measurements caused by changes in geometry is affected by the velocity of the encounter. The AVOIDS equipment is not designed to handle encounters between supersonic aircraft, but if it were, the inability of the Tau

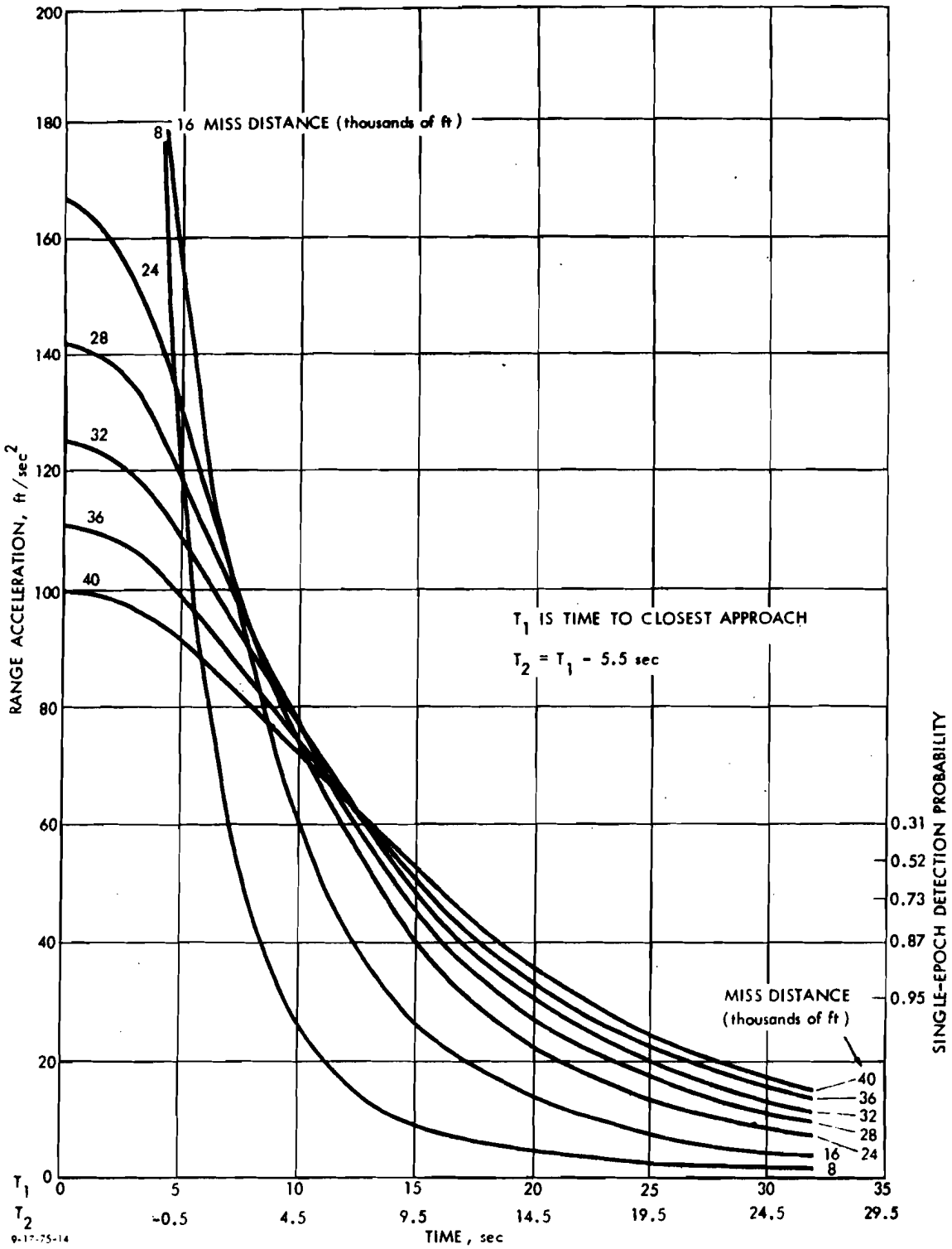

FIGURE 22. Acceleration Profile for a 2000 ft/sec Encounter

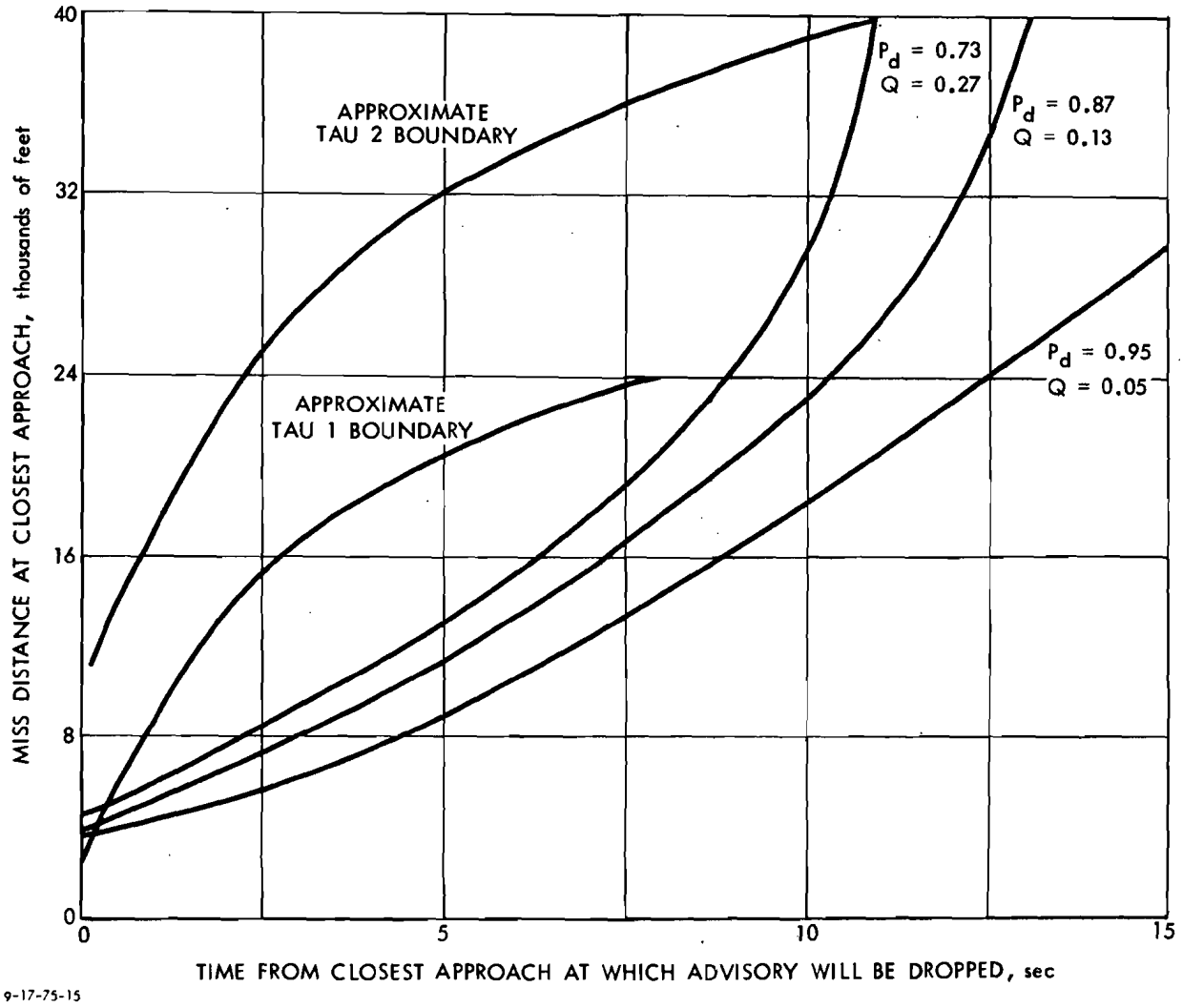

FIGURE 23. Time at Which an Advisory Will Be Dropped for a 2000 ft/sec Encounter

filter to track a target near the point of closest approach would be more pronounced. As the velocity of the encounter decreases, the capability of the equipment improves. This is demonstrated in Figs. 24 through 26 which provide data for an encounter of 900 ft/sec.

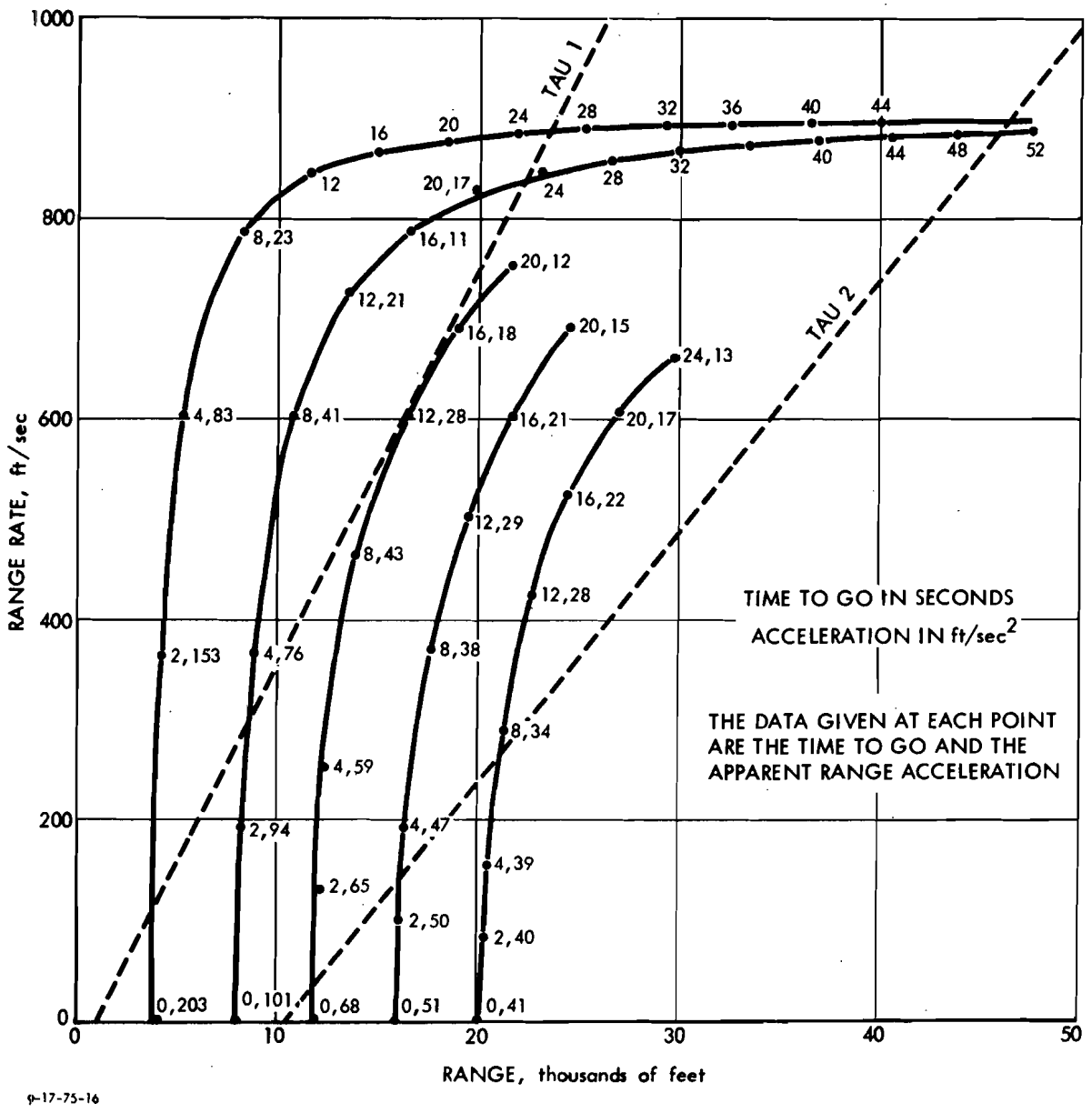

FIGURE 24. Tau Diagram for a 900 ft/sec Encounter

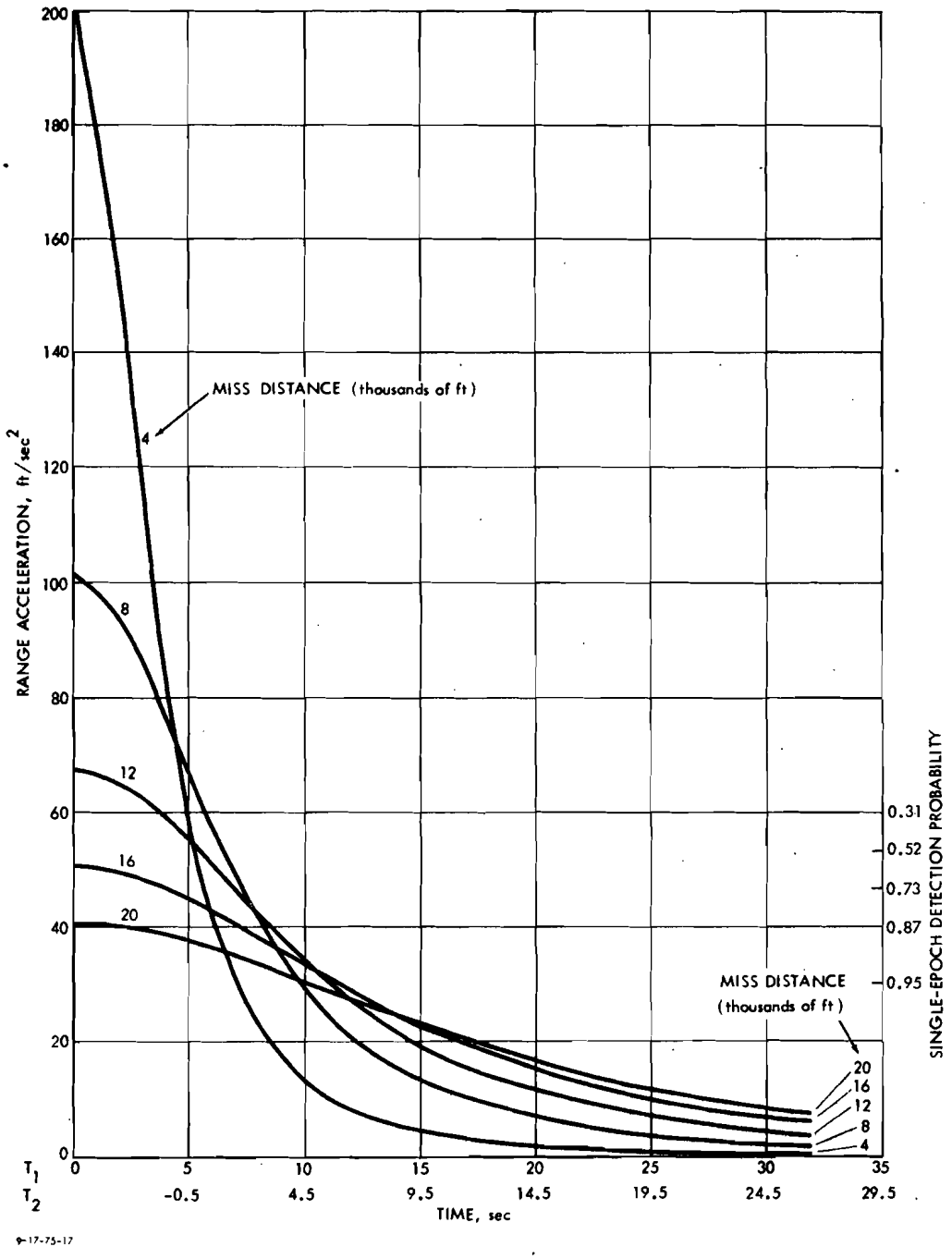

FIGURE 25. Acceleration Profile for a 900 ft/sec Encounter

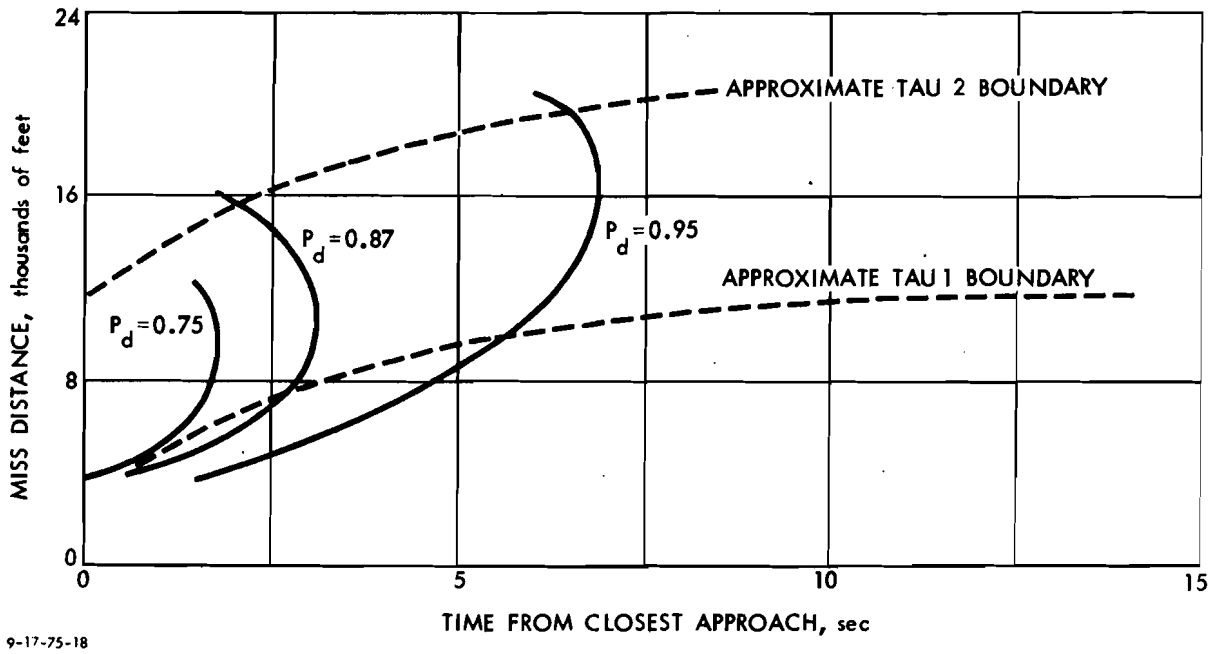

FIGURE 26. Time at Which an Advisory Will Be Dropped for a 900 ft/sec Encounter

D. POWER BUDGET CONSIDERATIONS

Five pulses must be transmitted and received during an interrogation/reply sequence for a single range measurement to be made. As a result, round reliability, P_I, is given by

$$P_I = P_d^5 \quad , \tag{18}$$

where P_d is the detection probability of receiving a transmitted pulse. In IDA's earlier analysis of the Honeywell system (Ref. 1), the round reliability was shown to change rapidly as a function of range. This is due to the use of a high threshold (i.e., 20 to 23 dB signal-to-noise ratio) and the functional form of P_I as expressed in Eq. 18. In fact, in Ref. 1 the round reliability for the communication between two AVOIDS-II aircraft was shown to change from $P_I = 0.9$ to $P_I = 0.1$, with a change of range from 16.83 miles to 18.24 miles. (The latter is the range at

which the signal-to-noise ratio is equal to the threshold-to-noise ratio.) The change in range is only 1.41 miles and represents a change in received signal strength of less than 1 dB. Such a change in signal strength is less than the margins which must be included for variations in threshold, antenna lobing, transmitter power, receiver gain, etc.

Based on the data presented in Ref. 3, Honeywell has adjusted the AVOIDS systems parameter to provide a minimum gain margin of 6.5 dB at the stated required range and discusses the possible increase in transmitter power of 2 dB, which would provide a gain margin of 8.5 dB. The data in Ref. 3 that lead to these suppositions are summarized in Tables 26 and 27. For comparison, Table 25 also includes the power budget parameters included in Ref. 1 and some revised parameters provided by Honeywell after Ref. 3 was issued.

These data suggest the following:

1. The loop sensitivity between two AVOIDS-1 aircraft below 10,000 ft is currently given as 129 dB by Honeywell. This is the same as the baseline case given in Ref. 3, and is 3 dB less than IDA's estimate presented in Ref. 1. Based on a Tau 2 requirement of 40 sec and a possible warning delay of 2 to 3 epochs, the required detection range at altitudes lower than 10,000 ft is about 10 miles. This range gives a path loss of -121 dB, so that the gain margin is 8 dB for an antenna with zero dB gain. The antennas should provide between 1.5 dB and 3 dB gain in the forward direction so that the gain margin for a head-on encounter is somewhere between 11 dB and 14 dB. With reasonable equipment specification, these margins should be adequate to provide a signal-to-noise ratio which is several dB above the threshold-to-noise ratio.

TABLE 26. SUMMARY OF AVOIDS POWER BUDGET PARAMETERS

	Reference 1		Reference 2		Revised Parameters (Spring 1975)	
	Above 10,000 ft	Below 10,000 ft	Baseline	Baseline +2 dB	Above 10,000 ft	Below 10,000 ft
AVOIDS-I Transmitter Power	+62 dBm	+58 dBm	58 dBm	60 dBm	53 dBm	53 dBm
AVOIDS-I Receiver Sensitivity	-74 dBm	-74 dBm	-71 dBm	-71 dBm	-80 dBm	-76 dBm
AVOIDS-II Transmitter Power	-	58 dBm	55 dBm	57 dBm	-	48 dBm
AVOIDS-II Receiver Sensitivity	-	-71 dBm	-68 dBm	-68 dBm	-	-73.5 dBm
LOOP SENSITIVITY						
a. AVOIDS-I to AVOIDS-II	134 dB	132 dB	129 dB	131 dB	133 dB	129 dB
b. AVOIDS-I to AVOIDS-II	-	129 dB	126 dB	128 dB	-	126.5 dB
c. AVOIDS-II to AVOIDS-I	-	132 dB	126 dB	128 dB	-	124 dB
d. AVOIDS-II to AVOIDS-II	-	129 dB	123 dB	125 dB	-	121.5 dB

TABLE 27. COMMUNICATION LINK PARAMETERS FOR AVOIDS EQUIPMENT, BASED ON REF. 2

Transmitter	Receiver	Required Range (ft)	Path Loss (dB)	Loop Sensitivity (dB)	Gain Margin (dB)	Average Communication Range (m)
Baseline Case						
AVOIDS-I	AVOIDS-I	52,800	121.0	129	8.0	17
AVOIDS-I	AVOIDS-II	43,500	119.5	126	6.5	12
AVOIDS-II	AVOIDS-I	43,500	119.5	126	6.5	12
AVOIDS-II	AVOIDS-II	25,600	115.0	123	8.0	8.4
2 dB Increase in Transmitter Power						
AVOIDS-I	AVOIDS-I	52,800	121.0	131	10.0	21.4
AVOIDS-I	AVOIDS-I	43,500	119.5	128	8.5	15.1
AVOIDS-II	AVOIDS-I	43,500	119.5	128	8.5	15.1
AVOIDS-II	AVOIDS-II	25,600	115.0	125	10.0	10.6

2. The loop sensitivity between two AVOIDS-II aircraft is not adequate to support the ANTC-117 threat logic. The current loop sensitivity is estimated at 121.5 dB, which is only 0.5 dB above the path loss for a Tau 2 detection. For a minimum gain margin of 6 dB, a minimum loop sensitivity of 127 dB is required. The current estimated loop sensitivity between an AVOIDS-I transmitter and an AVOIDS-II receiver is within 0.5 dB of this value. However, the reverse combination is deficient by 3 dB.

3. The AVOIDS-I equipments are not designed to handle supersonic aircraft. At altitudes above 9,700 ft, the Tau filter can handle encounters with a 2000 ft/sec closing velocity. The detection range needed to support such an encounter is 168,000 ft; however, the memory of the tracking filter only has storage for ranges out to 102,000 feet. At the maximum tracking range provided by the memory, the path loss is 126.5 dB. Based on Honeywell's current loop sensitivity estimate of 133 dB, the gain margin is 6.5 dB without considering antenna gains. For a head-on encounter, the gain margin should exceed 10 dB.

E. SUMMARY OF DETECTION PROBABILITY ANALYSES

Based on current Honeywell estimates for loop sensitivity, the AVOIDS-II does not have sufficient round reliability to meet ANTC-117 Tau 2 threat specifications. If the system is to meet these specifications, the loop sensitivity will have to be increased and the threat logic and data storage must be changed to provide essentially the same capability as an AVOIDS-I operated below 10,000 ft. If this is done, the blockage rate--the time fraction that the transponder is blocked--in the densest region of the 1982 Los Angeles Basin Traffic Model would be about 0.075 for an AVOIDS-I and 0.072 for an

AVOIDS-II. This small difference is due to an assumed average communication range of 17 miles for an AVOIDS-I and 14.5 miles for an AVOIDS-II. These blockage rates will limit round reliability to between 0.925 and 0.929, i.e., the probability that a given interrogation results in a successfully received reply. These values for round reliability lead to a probability of warning, P_W in Table 24, of about 0.98.

In IDA's Phase I analysis of the Honeywell system, Ref. 1, round reliability due to signal-to-noise considerations was shown to change rapidly near the range at which the signal-to-noise ratio was equal to the threshold-to-noise ratio. A decrease in range of less than 8 percent results in an improvement in signal-to-noise ratio of only 0.7 dB, yet changes the conditional round reliability from 0.1 to 0.9. This is due to the high threshold-to-noise ratio and the need to successfully transmit and receive five pulses in an interrogation reply sequence. The gain margin in loop sensitivity needed to accommodate variations in threshold, antenna lobing, transmit power, receiver gain, etc., was not estimated in this report but should be greater than that required to insure good detectability by overcoming the interference effects of receiver noise. Once the signal-to-noise ratio equals the threshold-to-noise ratio, only a small increase in signal level (i.e., 1 to 2 dB) is needed to provide good detection. As a result, a system designed to handle variations in equipment parameters and antenna gain need only be provided with a few extra dB of signal margins to insure that the probability of Tau 2 warning will be limited by transponder blockage rather than loop gain. This assumes detection is not limited by target dynamics or equipment reliability.

A detection probability less than unity may result in a delay in warning. The probability, P_W, in Table 24 is for a required Tau threat detection in two successive epochs. A

probability of 0.98 for two epochs corresponds to a single-epoch detection probability of 0.99. The single-epoch detection probability will be denoted as P_W'. The probability of a one-epoch delay in warning is $(1 - P_W')P_W$ which is about 0.01 and the probability of a two-epoch delay is the same as a one-epoch delay. The probability of a three-epoch delay in detection is equal to $P_3 = (1 - P_W')(1 - P_W)P_W$ and for the AVOIDS-I is about 0.0002 and can be ignored.

Once a warning is issued, it will not be dropped until the threat is missed for two successive epochs, so the probability that round reliability will cause a warning to be prematurely dropped is equal to $(1 - P_W')^2$ and for the AVOIDS-I is equal to 0.0001.

Target acceleration causes a bulge in the sub-epoch range data. The range gate pattern which is used for threat evaluation allows only a deviation from linearity of 43 ft in an intruder's range track for some clock phases. This is not sufficient to insure the detection of a threatening aircraft which has an acceleration of +1 G. An intruder with a negative rate of change in $\dot{R}$ is of more concern since the time to go before closest approach is decreasing. A high acceleration range gate pattern is provided which insures that an acceleration of -1 G can be processed by the threat evaluation logic. A range gate pattern to insure the detection of a threat with an acceleration of +1 G would increase the system's false alarm rate and equipment complexity and is not provided. Based on a computer simulation of the threat evaluation logic, an aircraft with a +1 G acceleration has a 2 percent chance of being rejected during any one epoch. This may cause a delay in warning. Considering both acceleration and blockage effect, the single-epoch probability of detecting an intruder with a +1 G acceleration is 0.97. The probability of a one- or two-epoch delay is 0.056 and the probability of a three-epoch delay is about 0.002. The probability of dropping an alarm on a

threatening intruder is about 0.001 if the intruder initiates a maneuver which causes a +1 G range acceleration.

The alarm or advisory on a noncollision threat may be dropped earlier than the ANTC-117 threat logic specifies because of the rapid change in range rate which occurs as aircraft near the time of closest approach. This effect is most pronounced at high altitudes where large values of closing velocity can be expected and is not affected by traffic density. The phenomenon does not occur in collision encounters and does not appear to present a serious problem since the alarm or warning is dropped, at most, 5.5 seconds earlier than specified by ANTC-117. This leaves little time for the aircraft to maneuver after the alarm has been dropped so the effect does not allow time for a hazardous condition to develop.

REFERENCES

1. IDA, A Review and Analysis of the Honeywell Collision Avoidance System, Phase I, Vols. 1 and 2, Study S-424, October 1973.

2. IDA, Review and Analyses of Some Collision Avoidance Algorithms with Particular Reference to ANTC-117, James J. Bagnall, Jr., Study S-450, unpublished.

3. Customer Engineering Letter from Honeywell to James J. Bagnall, Jr. of IDA, dated 26 Dec. 1974.

4. Honeywell Government and Aeronautical Products Division, Honeywell Midair Collision Prevention Systems -A Status Report, R. J. Follen, C. P. Harman, and D. B. Bennett, 3 April 1972.

5. Alexander M. Mood and Franklin A. Graybill, Introduction to the Theory of Statistics, McGraw-Hill Book Co., Inc., 1963, pp. 36 and 37.

APPENDIX A

STATEMENT OF WORK

APPENDIX A

STATEMENT OF WORK

SCHEDULE

ARTICLE I. STATEMENT OF WORK

The contractor shall provide the necessary qualified personnel, facilities, material, and services to perform the effort described herein:

TASK A. INDEPENDENT ASSESSMENT OF THE ADEQUACY OF THE THREAT EVALUATION AND MANEUVER SELECTION LOGICS DESCRIBED IN DOCUMENT ANTC-117, REVISION 10.

Conduct an independent assessment of the adequacy of the threat evaluation and maneuver selection logics as described in Document ANTC-117, Revision 10, for the aircraft performance parameters specified (horizontal closure rates to 3600 knots, combined horitzontal accelerations to 1G, vertical closure rates to 10,000 feet per minute and combined vertical accelerations to 1/2G). Assumptions to be made during the analysis are as follows:

(a) The only information exchanged between participating aircraft is that required to determine range, closure rate and relative altitude (no bearing or bearing derivation information is exchanged or measured);

(b) Threatening encounters existing simultaneously between more than three aircraft are non-existent;

Specific attention should be paid to the minimum range (R_m), offset range (Ro_1, Ro_2) and tau (T_1, T_2) parameters, their values and derivations (see various references listed in ANTC-117). Recommendations for improvements, if discovered, shall be made and discussed (an example could be sensing own aircraft's horizontal acceleration and reducing/expanding the alarm volume width accordingly), including the trade-offs involved (i.e., increased expense, larger system cycle time, additional power requirements, etc.). Improvements such as a small reduction in system performance requirements resulting in a correspondingly large reduction in system cost can also be considered. While it is not the purpose of this task to analyze or redesign any proposed ACAS, attention should be paid to the ANTC-117 logics as they specifically relate to the RCA SECANT VECAS and SECANT VECAS GA/M, Honeywell AVOIDS I and II and McDonnell Douglas EROS II and Mini-CAS Model 2007 (i.e., are the logics attainable with the system designs?; if not, what are the effects?, etc.).

TASK B. INDEPENDENT ANALYSIS OF THE COMMUNICATIONS CAPABILITY OF THE ACAS'S

Conduct an independent analysis of the communication capacity capability of the three ACAS's analyzed under TASK A above, but with all design changes, improvements, etc., accounted for under each system through March 1, 1974. Consideration shall be given, but not limited, to: (1) communication reliability as a result of transmitter power, receiver sensitivity, antenna gain and shadowing, signal processing, such as sensitivity time control, etc.; (2) both synchronous and asynchronous garble (including interweaved and overlapped pulse trains); and (3) multipath. The traffic model to be used for evaluation purposes is that furnished you for the previous analysis. Obvious improvements to the system shall be recommended (although it is not the purpose of this task to do any substantial redesign effort) and inherent limitations in the system, which will preclude satisfactory operations in the designated environment, will be pointed out (due consideration will be given to conclusions reached as a result of TASK A.).

ARTICLE I. STATEMENT OF WORK (Continued)

TASK C. INDEPENDENT ASSESSMENT OF THE "DME COLLISION AVOIDANCE SYSTEM" DESCRIBED IN DOCUMENT TR-1115

Conduct an independent assessment of the "DME Collision Avoidance System" developed by Sierra Research Corporation as described in their report TR-1115 dated December 1973. Particular emphasis should be placed on the relationship between traffic density and missed alarms (accepting the hypothesis that the intruder is not a hazard when in fact he is, in accordance with the threat evaluation criteria), and false alarms (accepting the hypothesis that the intruder is a hazard when in fact he is not, again in accordance with the threat evaluation criteria). The analysis performed under this task shall consist in part of a review of work already accomplished in this area in order to validate or invalidate previous conclusions reached, and of original analysis where previous work is non-existant, incomplete or invalid. Consideration in the analysis shall be given, but not limited, to: (1) communication reliability as a result of transmitter power, receiver sensitivity, antenna gain and shadowing, signal processing, such as sensitivity time control, etc.; (2) both synchronous and asynchronous garble (including interweaved and overlapped pulse trains); and (3) multipath. The traffic model to be used for evaluation purposes is that furnished you for the previous analysis. Obvious improvements to the system shall be recommended (although it is not the purpose of this task to do any substantial redesign effort) and inherent limitations in the system, which will preclude satisfactory operations in the designated environment, will be pointed out (due consideration will be given to conclusions reached as a result of TASKS A and B).

APPENDIX B

SUMMARY OF ANTC-117 EVALUATION AND MANEUVER LOGIC

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SUMMARY OF ANTC-117 EVALUATION AND MANEUVER LOGIC

A. SYSTEM CONCEPT

The specifications given by ANTC-117 for a cooperative airborne collision avoidance system are aimed at insuring the safe separation of aircraft in flight independently of ground control, although ground stations may be required to provide time synchronization for participating aircraft. In order to achieve this goal, several algorithms, involving measured values of the separation distance, separation rate, and altitude difference between two aircraft, as well as the rate of altitude change of the aircraft in which an evaluation of threat is being made, are used to generate warnings and alarms. These, in turn, trigger various cautions and commands to pilots, thereby causing them to perform certain avoidance or escape maneuvers: to stop turning (also known as "rollout"), to limit vertical speeds, to change altitude. Because bearing information is not assumed to be available, escape maneuvers take place in the vertical plane only.

The algorithms which provide alarm criteria are of two types: (1) those based on the separation distance (range), R, and separation rate (range rate), $\dot{R}$; and (2) those based upon the altitude difference Δh between aircraft and altitude rate $\dot{h}$ of an individual aircraft. In principle, since R represents the slant range between aircraft, the R, $\dot{R}$ criteria alone should be sufficient to provide the alarms needed for protection against collision. However, it has been recognized that the use of altitude data is required to reduce the threat volume, and hence the alarm rate, to manageable proportions and also

to determine the proper direction for vertical avoidance maneuvers.

The measurement of R, $\dot{R}$, and Δh is accomplished by means of radio communication between aircraft. The arrival time of a communication signal is used to calculate R, while $\dot{R}$ is obtained from a measurement of the doppler shift, although, recently, the tendency in CAS designs has been to obtain $\dot{R}$ from measurements of R at two different times. Each aircraft measures its own altitude with a barometric altimeter and communicates it in digital steps of 100 ft to other aircraft. Each CAS-equipped aircraft measures its own altitude rate for its own evaluation, but does not communicate this data to others.

ANTC-117 requires, in general, that each aircraft limit its horizontal acceleration to 1/2 G, its vertical acceleration to 1/4 G, and its vertical rate to 5000 ft/min. This implies a maximum relative horizontal acceleration of 1 G and vertical acceleration of 1/2 G, and a maximum vertical separation rate of 10,000 ft/min between two aircraft. [See A-4.b(3),(4) of ANTC-117.]

B. PRIMARY THREAT CRITERIA

In the early days of CAS development, it was proposed that an R, $\dot{R}$ algorithm be based upon a quantity τ, defined by

$$\tau = R/\dot{R} ,$$

representing the time to collision for two aircraft on a non-accelerating collision course. It has since been realized that, because of measurement errors and the possibility that aircraft may be accelerating, a modification of this idea is necessary. Thus, ANTC-117 has adopted two alarms algorithms based on R and $\dot{R}$. These algorithms, sometimes known as modified τ criteria, have the same algebraic form, but different parameters:

$$R + \tau_1 \dot{R} \leq R_{o_1} ,$$

$$R + \tau_2 \dot{R} \leq R_{o_2} ,$$

where τ_1, R_{o_1}, τ_2, R_{o_2} are designated constants.*

In the first algorithm, known as the τ_1 alarm criterion, the designated constants have the values

$$\tau_1 = 25 \text{ sec}$$

$$R_{o_1} = 1/4 \text{ nmi } (1520 \text{ ft}) .$$

In the second algorithm, known as the τ_2 warning criterion, the constants have the values

$$\tau_2 = 40 \text{ sec}$$

$$R_{o_2} = 1.8 \text{ nmi } (10{,}940 \text{ ft}) .$$

If two aircraft are co-altitude (within 600 ft for altitudes below 10,000 ft or 800 ft for altitudes above 10,000 ft), and if measured values of R and $\dot{R}$ satisfy the τ_2 criterion, the pilot is commanded to limit turns (rollout) to a bank angle no greater than 10 deg and to neither climb nor dive. This is supposed to reduce the relative trajectory of the encountering aircraft to an approximately linear horizontal course. Then conditions are assumed to be suitable for the safe use of the more restrictive τ_1 criterion.

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*These algorithms are not actually stated by ANTC-117 in equation form, as is done here, but the equations can be deduced from [B-3.b] and Fig. 2 of that document. Note also that the convention $\dot{R}$ positive corresponding to increasing R is used here rather than the converse used in ANTC-117.

If R and $\dot{R}$ satisfy the τ_1 criterion, vertical maneuver commands are issued, so that the aircraft at the higher altitude climbs while the aircraft at the lower altitude dives until both aircraft are separated by a safe distance. If three aircraft are involved, the aircraft at the intermediate altitude maintains its course, neither climbing nor diving.

The τ_1 criterion has been modified further by the addition of a minimum range criterion, the purpose of which is to protect against the hazardous case of $\dot{R}$ equal to or near zero, since it was felt that with the presence of acceleration and measurement errors, the time available for maneuver might be reduced below an acceptable minimum in this circumstance. Thus, an alarm also occurs with the same commands as for τ_1 alarms if

$$R \leq R_M ,$$

where the minimum range R_M has been designated as 1/2 nmi (3040 ft). The latter alarm will be referred to also as a τ_1 alarm.

C. ALTITUDE CRITERIA

The τ alarms are implemented only if the encountering aircraft are co-altitude, or it is predicted that they may become co-altitude, where "co-altitude" is defined to mean that their vertical separation is less than 600 ft at altitudes below 10,000 ft and 800 ft at altitudes above 10,000 ft. This definition of co-altitude is based upon (1) an altimeter error allowance of $\pm$ 150 ft (3σ) velow 10,000 ft and $\pm$ 250 ft (3σ) above 10,000 ft, (2) the assumption that a safe vertical separation between aircraft is 150 ft, and (3) what is intended as an allowance for an undetected vertical drift rate of 500 ft/min.

According to ANTC-117, if an aircraft is climbing or diving at a rate greater than 500 ft/min, it is required to extend its co-altitude protection boundary in the direction of its motion by a predicted co-altitude increment. The predicted co-altitude increment is determined by multiplying the aircraft's own altitude rate $\dot{h}$ by 30 seconds [B-3.a(3)].

In addition to the co-altitude zone wherein the τ alarms become operative, relative altitude buffer zones are established out to ± 3400 ft. In each of these zones, vertical altitude rates are limited from 2000 ft/min in the farthest to successively lower values as the zones get narrower (Fig. 4, 4a, 4b, B-3.e].

D. LIMITED EQUIPMENTS

ANTC-117 allows the existence of aircraft with limited collision avoidance equipments in order to spare general aviation the expense of a full CAS. Several possible variations are mentioned, but it is implied that unmentioned others may also be permitted.

Of those that are mentioned specifically, there are two types whose properties have an effect on threat evaluation. The first consists of the so-called level one and level two CAS, and the second is the beacon-only equipment.

Since ANTC-117 does not mention that level one and level two CAS-equipped aircraft have the ability to measure altitude rate, it has been assumed here that they do not have this capability. It follows, therefore, that this type of aircraft is unable to use the predicted co-altitude algorithm.

The beacon-only-equipped aircraft has no threat evaluation capability but is required to communicate its altitude. Thus, it can provide the necessary data for threat evaluation to a CAS-equipped aircraft, but is unable to make an avoidance maneuver on command.

APPENDIX C

DIAGRAMS FOR COMPUTING THE CONDITIONAL PROBABILITY OF A FALSE TRACK FOR NEAR-THREATENING INTRUDERS

APPENDIX C

DIAGRAMS FOR COMPUTING THE CONDITIONAL PROBABILITY OF A FALSE TRACK FOR NEAR-THREATENING INTRUDERS

The procedure for estimating the false track probability for a near-threatening aircraft was described in the body of this report. Basically, the range gate pattern in question is plotted against a background of sloping lines which form so-called range rate corridors. An idealized range track from an aircraft with the nonthreatening range rate in question would fall entirely within one of the range rate corridors. For a range track in one of the range rate corridors, the conditional probability of a false track can be obtained by noting the range gates which are not occupied. That is, when the range track is in one of the range rate corridors, the conditional probability of false track is equal to the probability that another nonthreatening aircraft and/or fruit replies fall in the unoccupied range gates.

The AVOIDS Tau filter changes the relative position between the intruder's range track and the range gate pattern during a threat evolution sequence so that the overall conditional probability of a false track is found by summing the results obtained by alternately considering the range track in each of the range rate corridors.

The range rate which is considered threatening varies with range; however, the critical parameter which determines the false warning probability is the difference in the range bin skip between the range rate of the intruder and the range gate tracking pattern. Only one value of range need be considered in the

analysis and has arbitrarily been set at 20,000 ft. At this range, a range bin skip of 15 is considered threatening, whereas 14 is not.

All altitude bands require a false track to be present in two successive epochs to produce a false warning; however, some altitude bands require range correlation between the false tracks while others do not. Without range correlation, the false tracks can occur anywhere in range and the false warning probability is equal to the square of the single-epoch false track probability. Only single-epoch diagrams are needed for this analysis. Two epochs must be considered to analyze the likelihood of a false warning when range correlation between the successive false tracks is checked. The second false track must occur within one range bin outbound to nine range bins inbound of the last position of the first false track. The highest conditional false warning probability occurs when the second range gate pattern is as far inbound as possible so only this alignment is a consideration in the analysis.

The single-epoch diagrams used to compute the conditional probability of a false track in the altitude bands without range correlation between epochs are shown in Figs C-1 through C-4. Two values of nonthreatening range rate are considered. Range rate corridors in Figs. C-1 and C-2 slope at a range rate of -175 ft/sec and those in Figs. C-3 and C-4 at a range rate of -150 ft/sec. Both the low- and the high-acceleration patterns are presented. The low pattern is shown in Figs. C-1 and C-3 and the high pattern in Figs. C-2 and C-4.

The single-epoch conditional probability as a function of reply density, μ, which was derived from Figs. C-1 through C-4, is presented in Table C-1. These results must be squared to obtain the conditional probability of false warning.

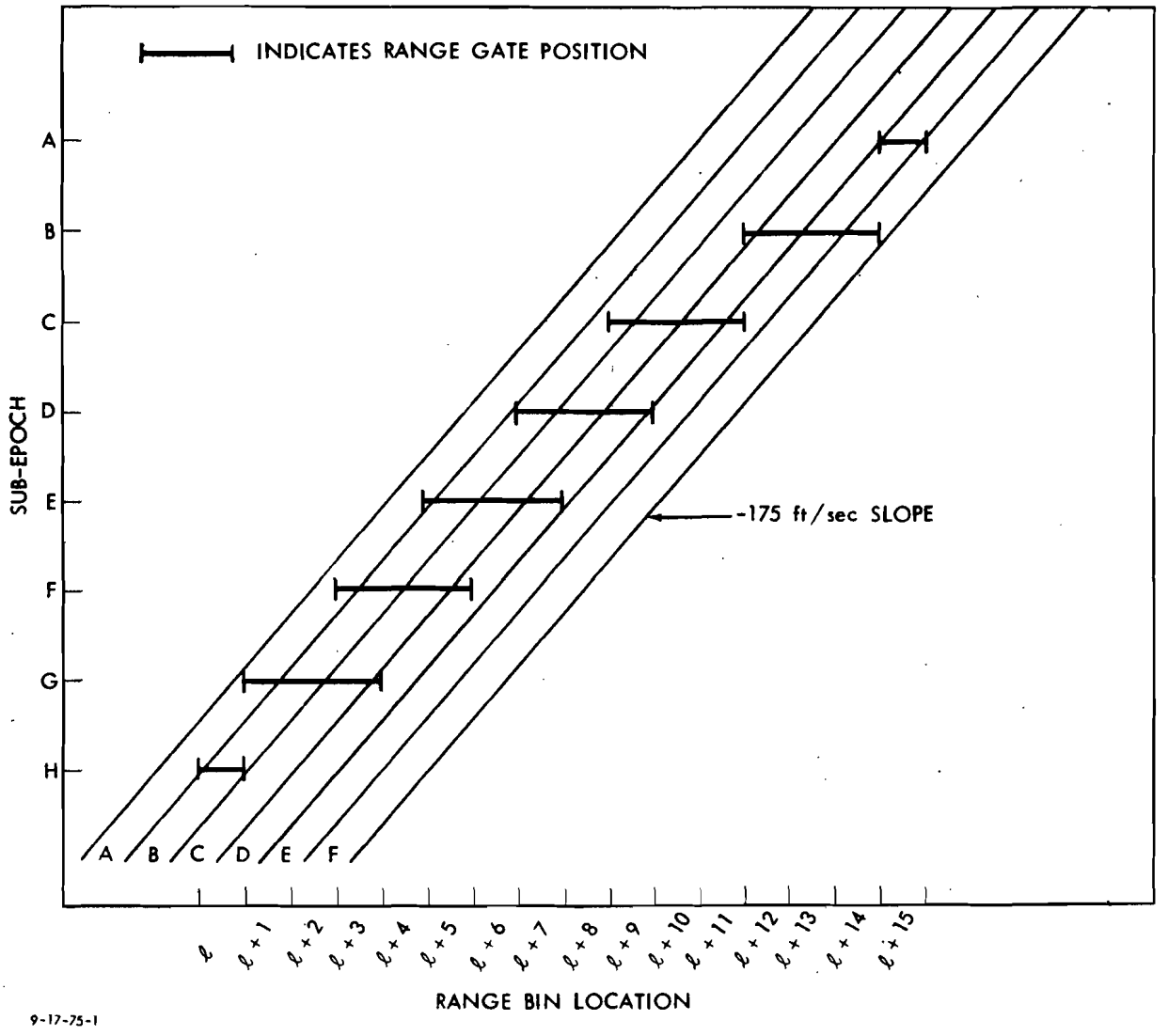

FIGURE C-1. Single-Epoch Low-Acceleration Pattern for a Range Rate of -175 ft/sec

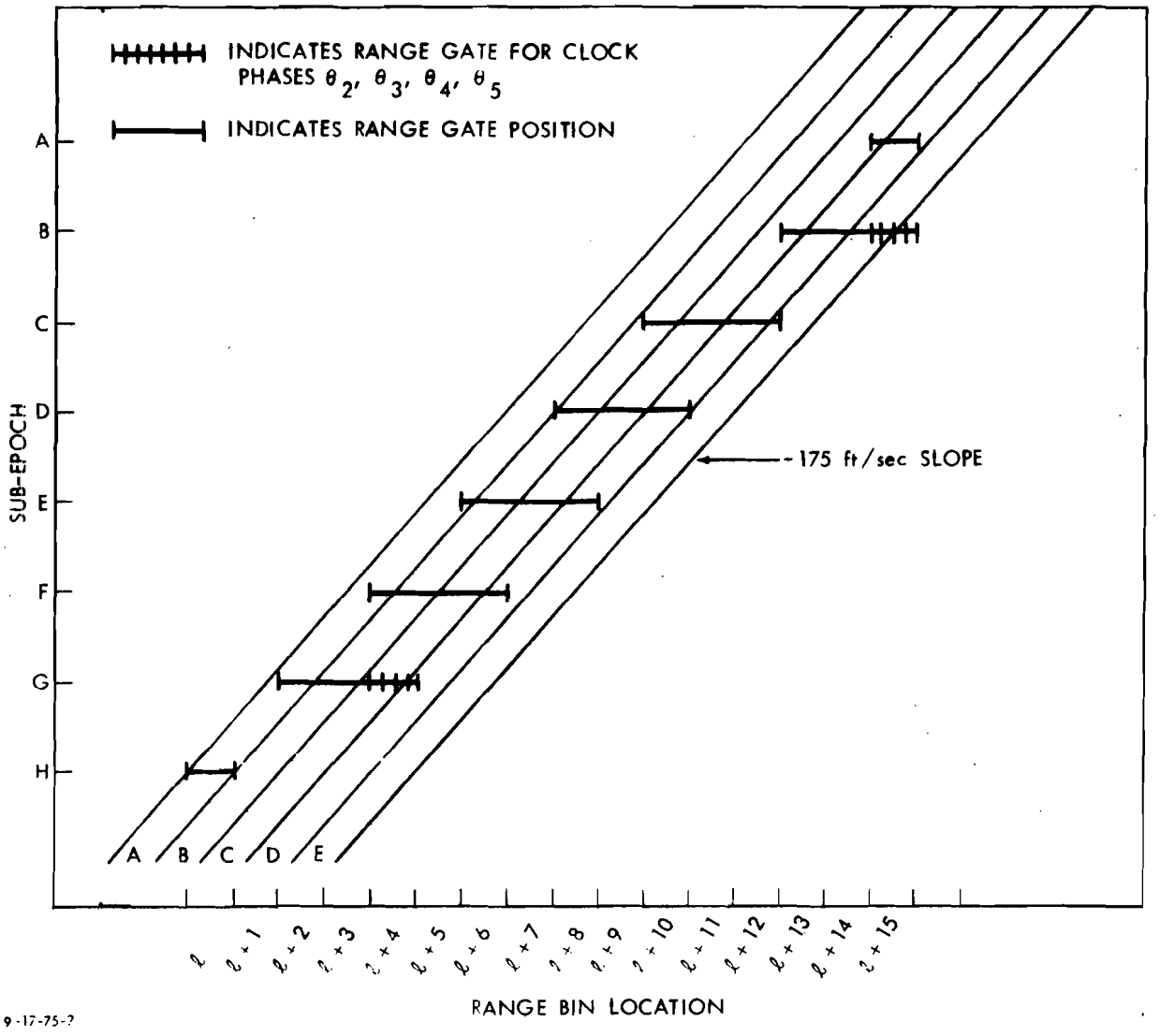

FIGURE C-2. Single-Epoch High-Acceleration Pattern for a Range Rate of -175 ft/sec

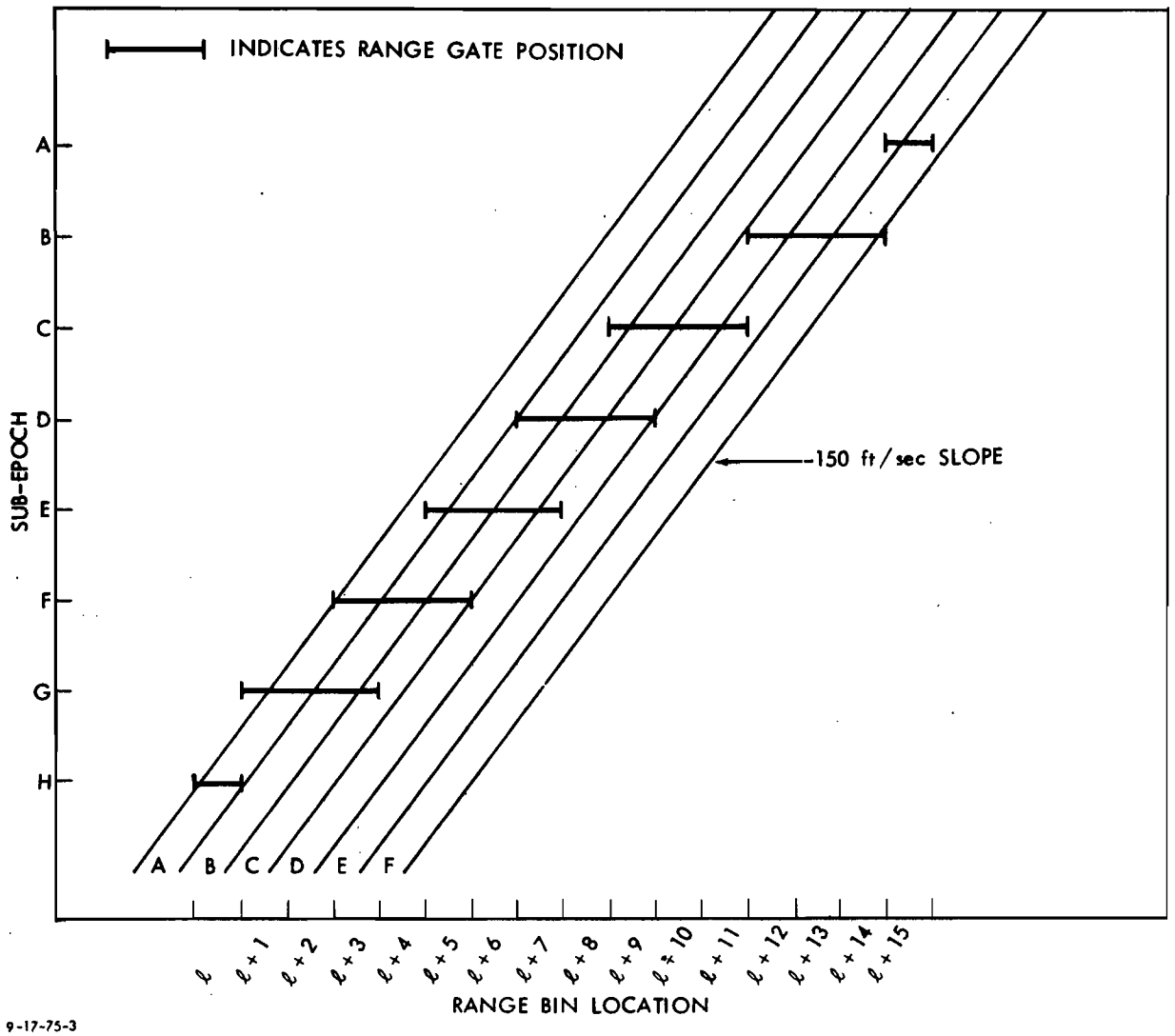

FIGURE C-3. Single-Epoch Low-Acceleration Pattern for a Range Rate of -150 ft/sec

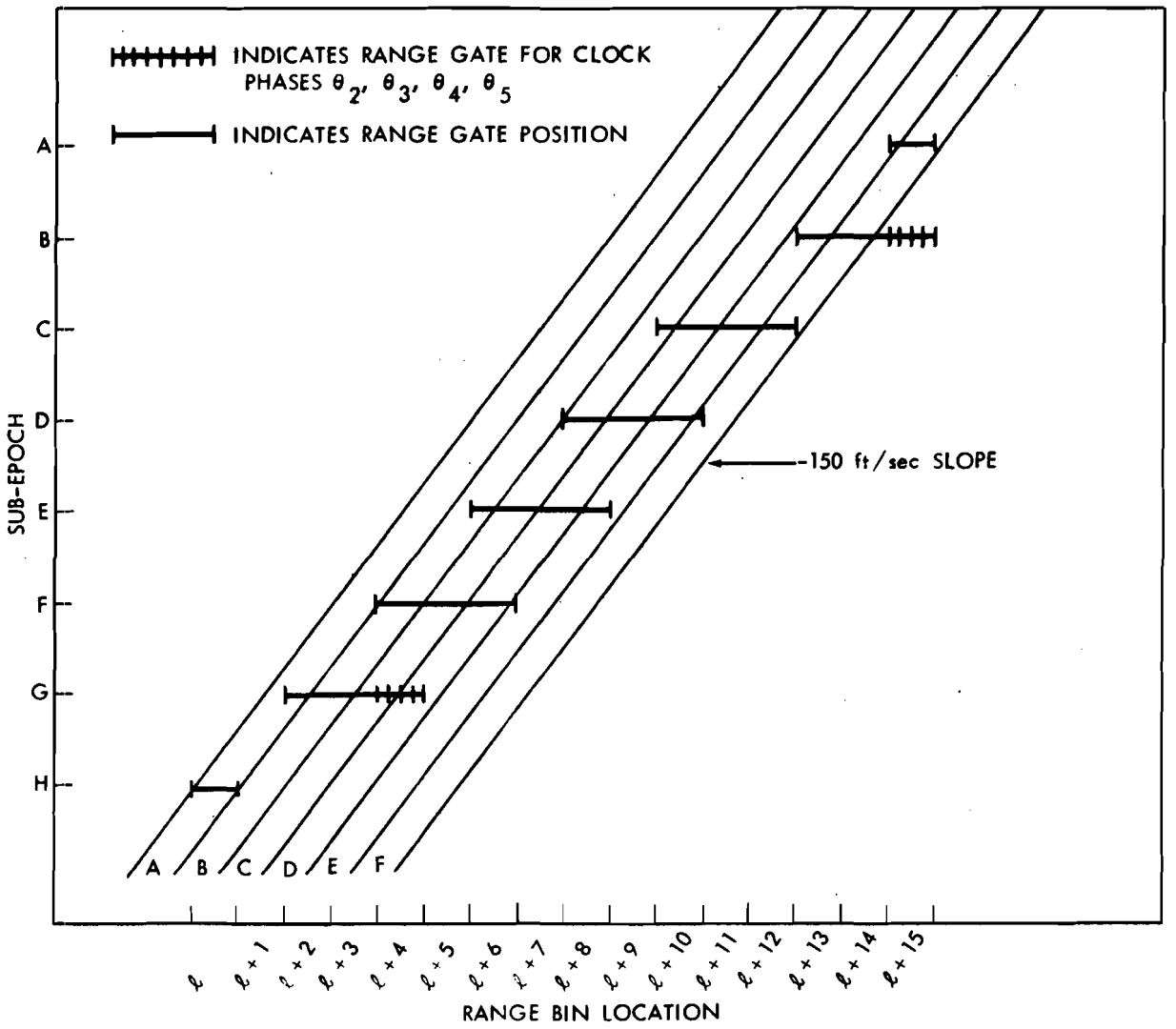

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FIGURE C-4. Single-Epoch High-Acceleration Pattern for a Range Rate of -150 ft/sec

TABLE C-1. SINGLE-EPOCH CONDITIONAL FALSE TRACK PROBABILITY AS A FUNCTION OF REPLY DENSITY

Acceleration Pattern	Conditional Probability
Near-Threatening Intruders With $\dot{R}$ = -175 ft/sec	
Low	$3\mu^2 + 3\mu^3 + 9\mu^4$
High	$0.14\mu + 1.64\mu^2 + 5.6\mu^3$
Near-Threatening Intruders With $\dot{R}$ = -150 ft/sec	
Low	$3\mu^3 + 27\mu^4$
High	$1.7\mu^3 + 18.7\mu^4 + 35\mu^5$

The two-epoch diagrams for analyzing the altitude bands in which range correlation between epochs is checked are shown in Figs. C-5 through C-8 for a nonthreatening range rate of -175 ft/sec and in Figs. C-9 through C-12 for a nonthreatening range rate of -150 ft/sec. Four diagrams are needed for each value of range rate in order to analyze all combinations of the high- and low-acceleration range gate patterns. The conditional probability of false warning as a function of reply density (i.e., μ) which was derived from the two-epoch diagrams is presented in Table C-2.

TABLE C-2. FALSE WARNING ESTIMATES AS A FUNCTION OF REPLY DENSITY FOR DIFFERENT COMBINATIONS OF ACCELERATION RANGE GATE PATTERNS

Acceleration Pattern Combination	Conditional Probability
Near-Threatening Intruder With $\dot{R}$ = -175 ft/sec	
Low-Low	$P_{FLL}(T) \mid_{-175,\ 15} = 0.75\mu^4 + 3.75\mu^5$
Low-High	$P_{FLH}(T) \mid_{-175,\ 15} = 7.7\mu^6 + 15.4\mu^7$
High-Low	$P_{FHL}(T) \mid_{-175,\ 15} = 0.14\mu^3 + 0.57\mu^4 + 3.0\mu^5$
High-High	$P_{FHH}(T) \mid_{-175,\ 15} = 2.1\mu^4 + 1.7\mu^5 + 20.6\mu^6$
Near-Threatening Intruder With $\dot{R}$ = -150 ft/sec	
Low-Low	$P_{FLL}(T) \mid_{-150,\ 15} = 104\mu^9 + 1145\mu^{10}$
Low-High	$P_{FLH}(T) \mid_{-150,\ 15} = 38.6\mu^7 + 162\mu^8$
High-Low	$P_{FHL}(T) \mid_{-150,\ 15} = 69.7\mu^8 + 347\mu^9$
High-High	$P_{FHH}(T) \mid_{-150,\ 15} = 122\mu^8 + 608\mu^9$

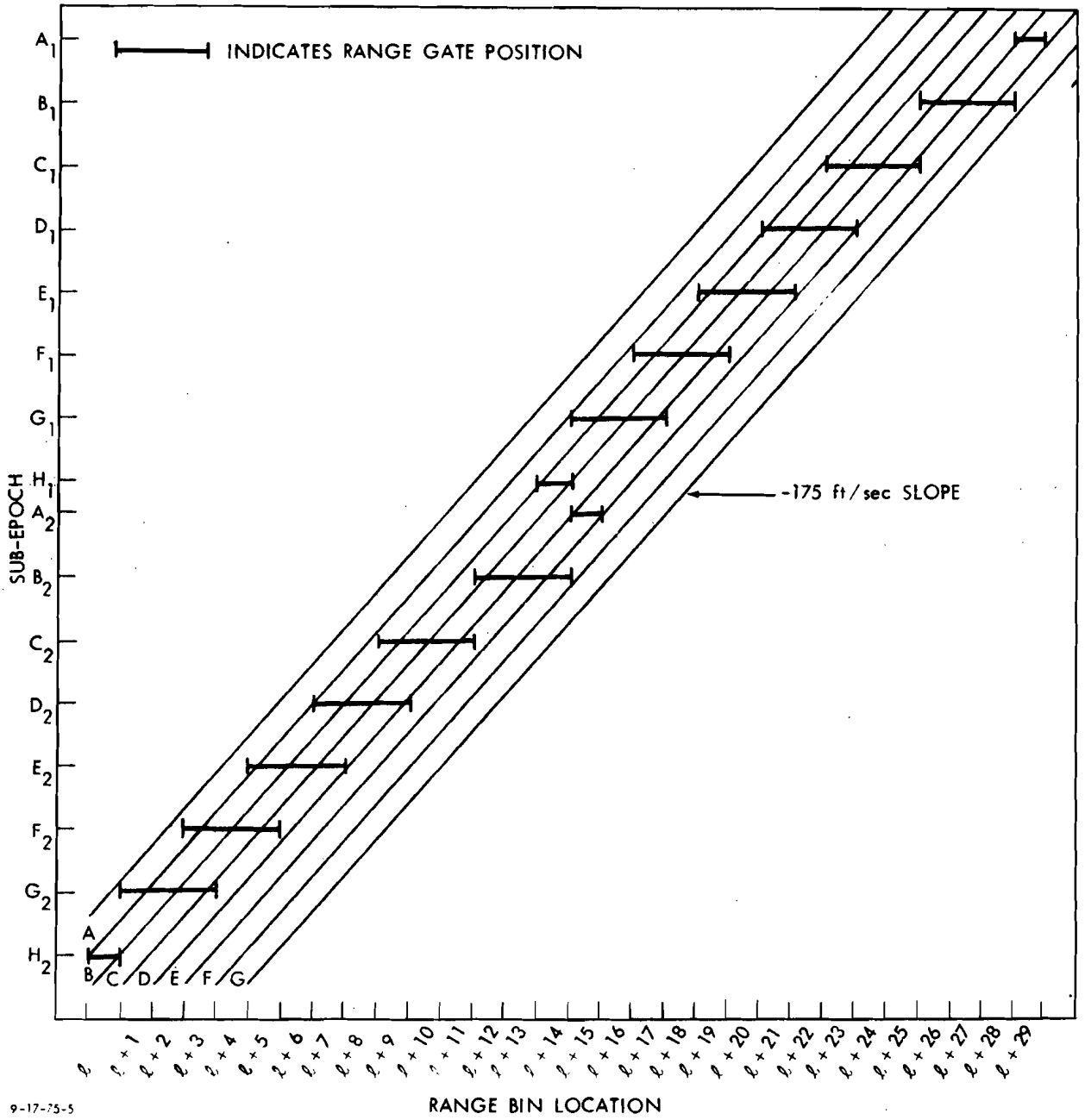

FIGURE C-5. Two-Epoch Low- and Low-Acceleration Patterns for a Range Rate of -175 ft/sec

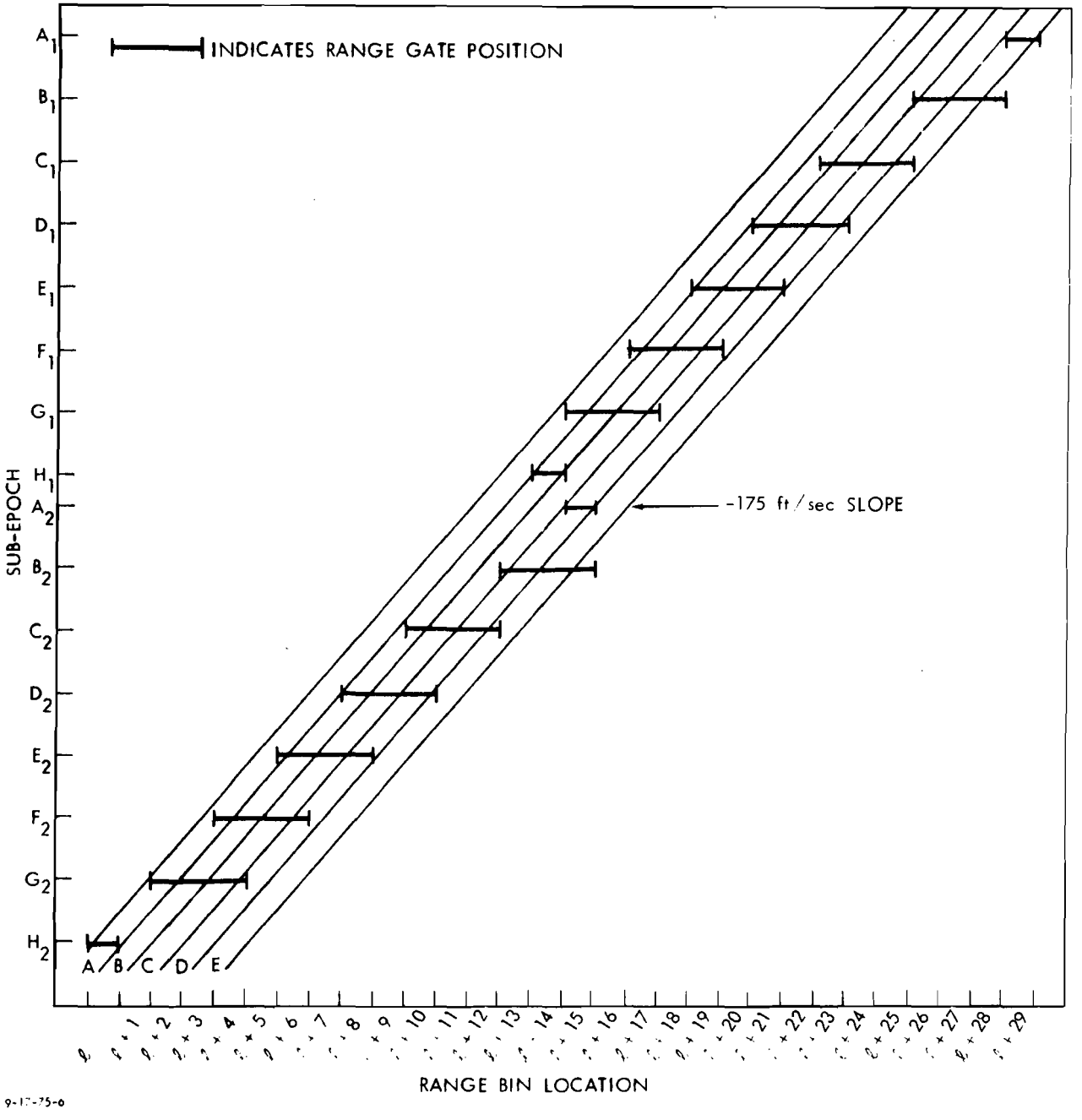

FIGURE C-6. Two-Epoch High- and Low- Acceleration Patterns for a Range Rate of -175 ft/sec

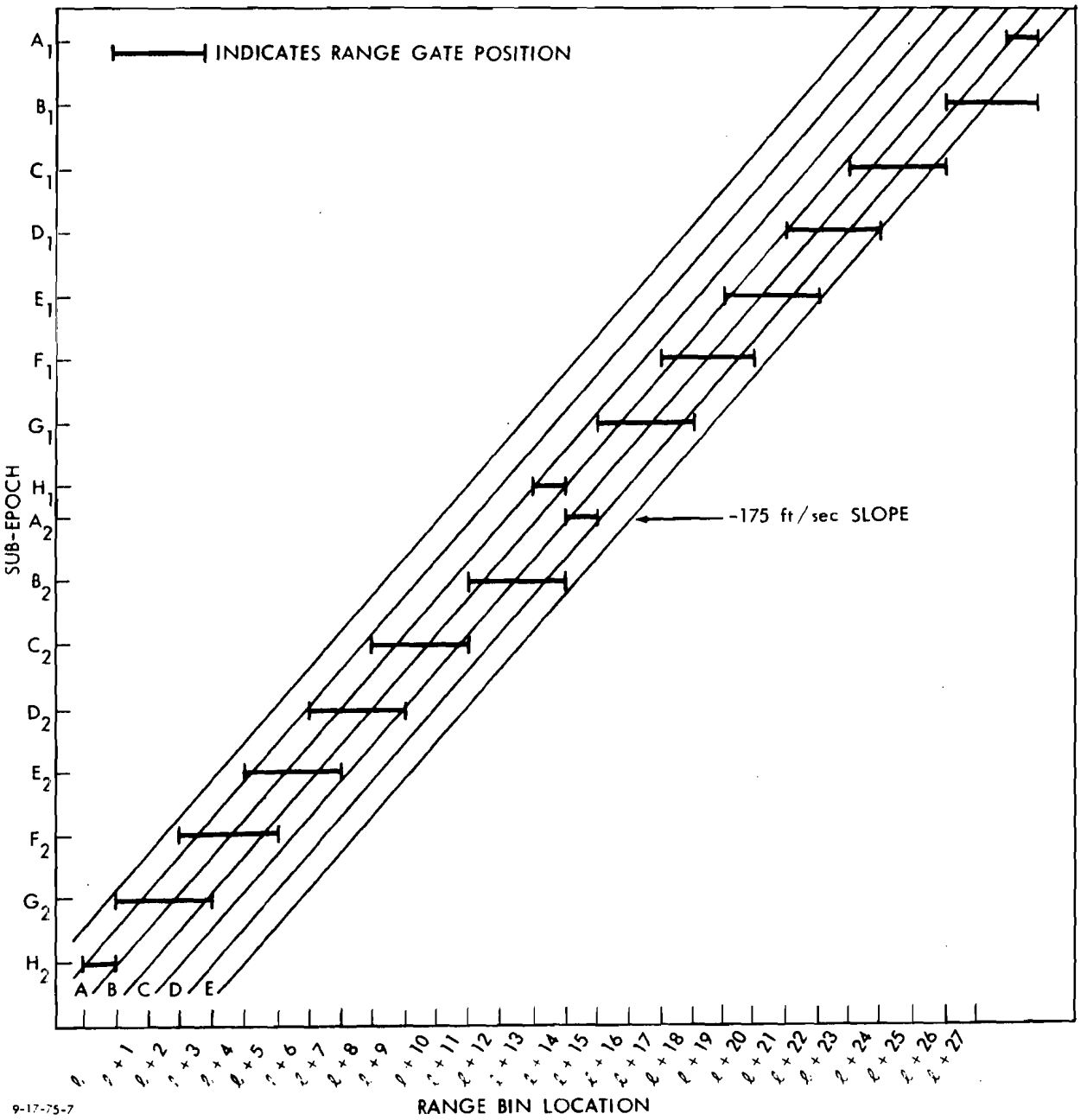

FIGURE C-7. Two-Epoch Low- and High-Acceleration Patterns for a Range Rate of -175 ft/sec

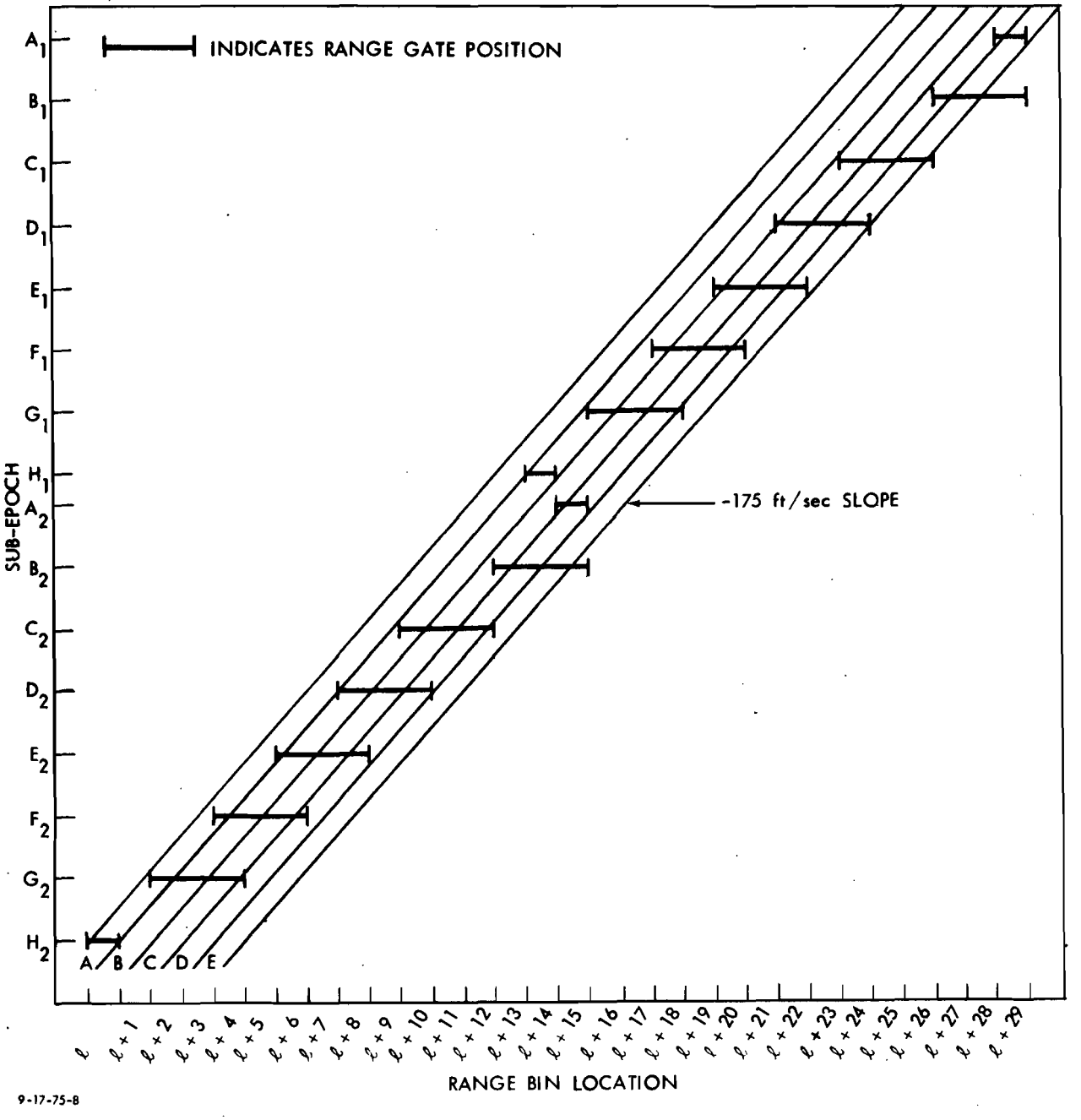

FIGURE C-8. Two-Epoch High- and High-Acceleration Patterns for a Range Rate of -175 ft/sec

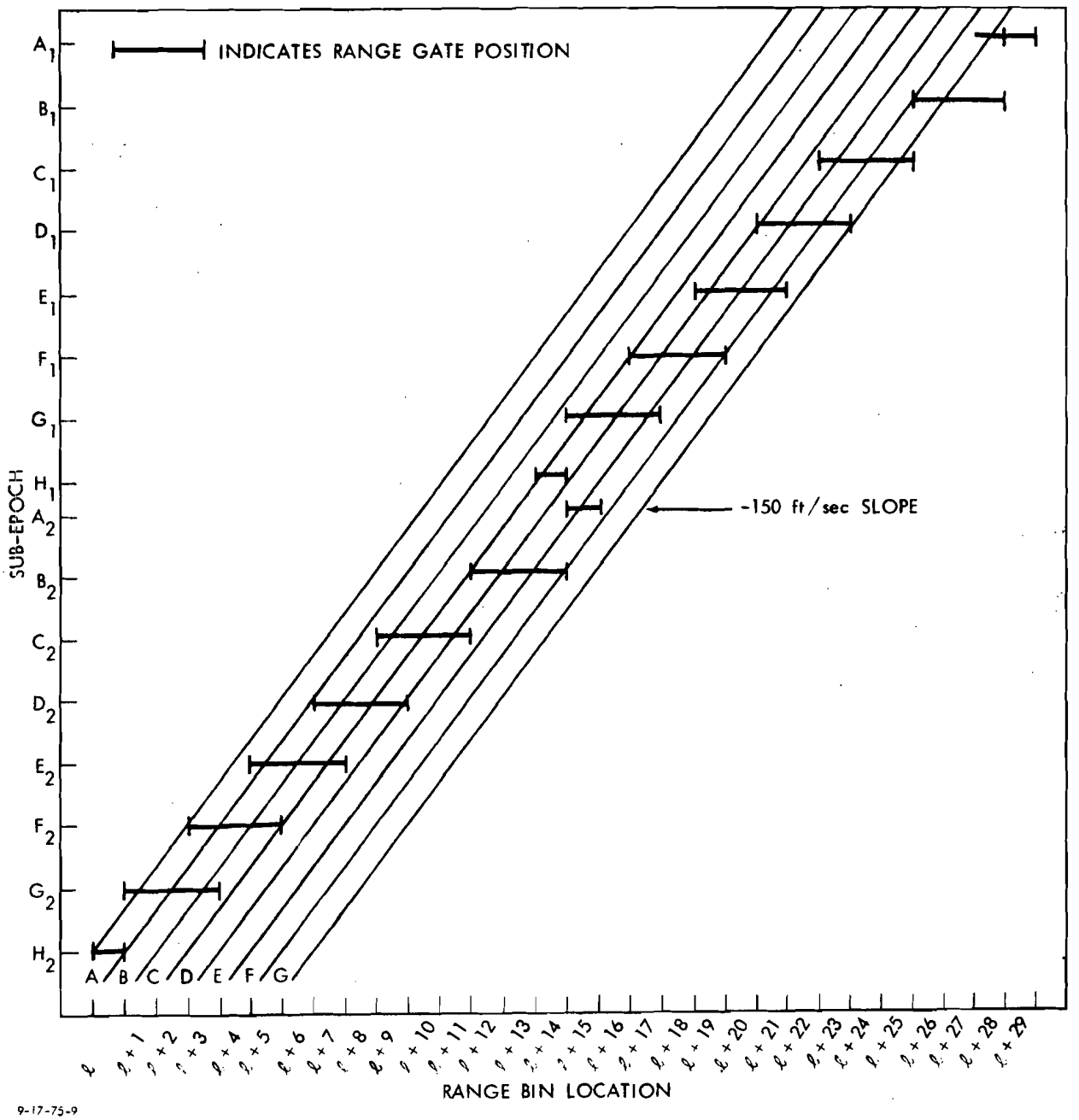

FIGURE C-9. Two-Epoch Low- and Low-Acceleration Patterns for a Range Rate of -150 ft/sec

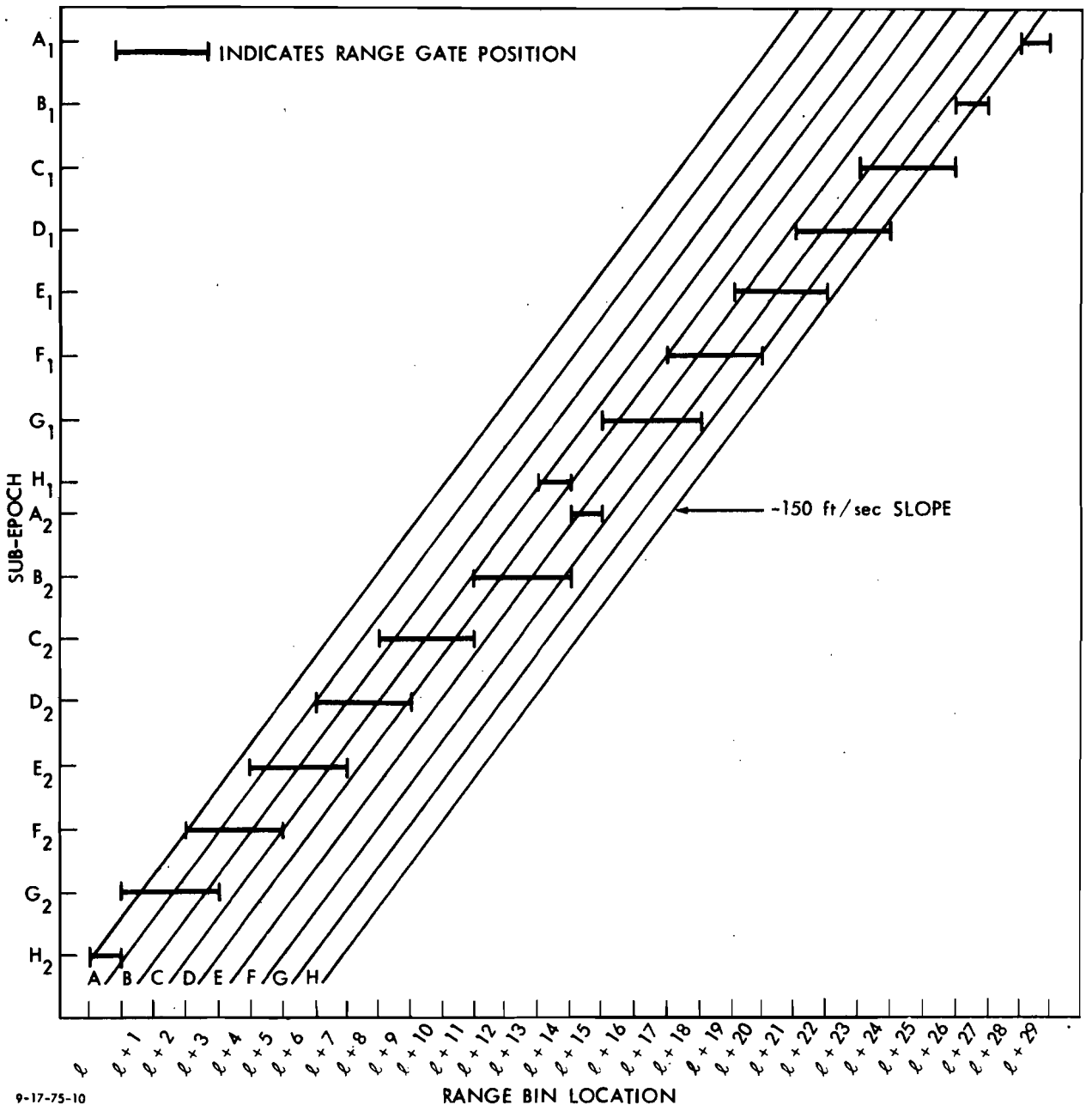

FIGURE C-10. Two-Epoch Low- and High-Acceleration Patterns for a Range Rate of -150 ft/sec

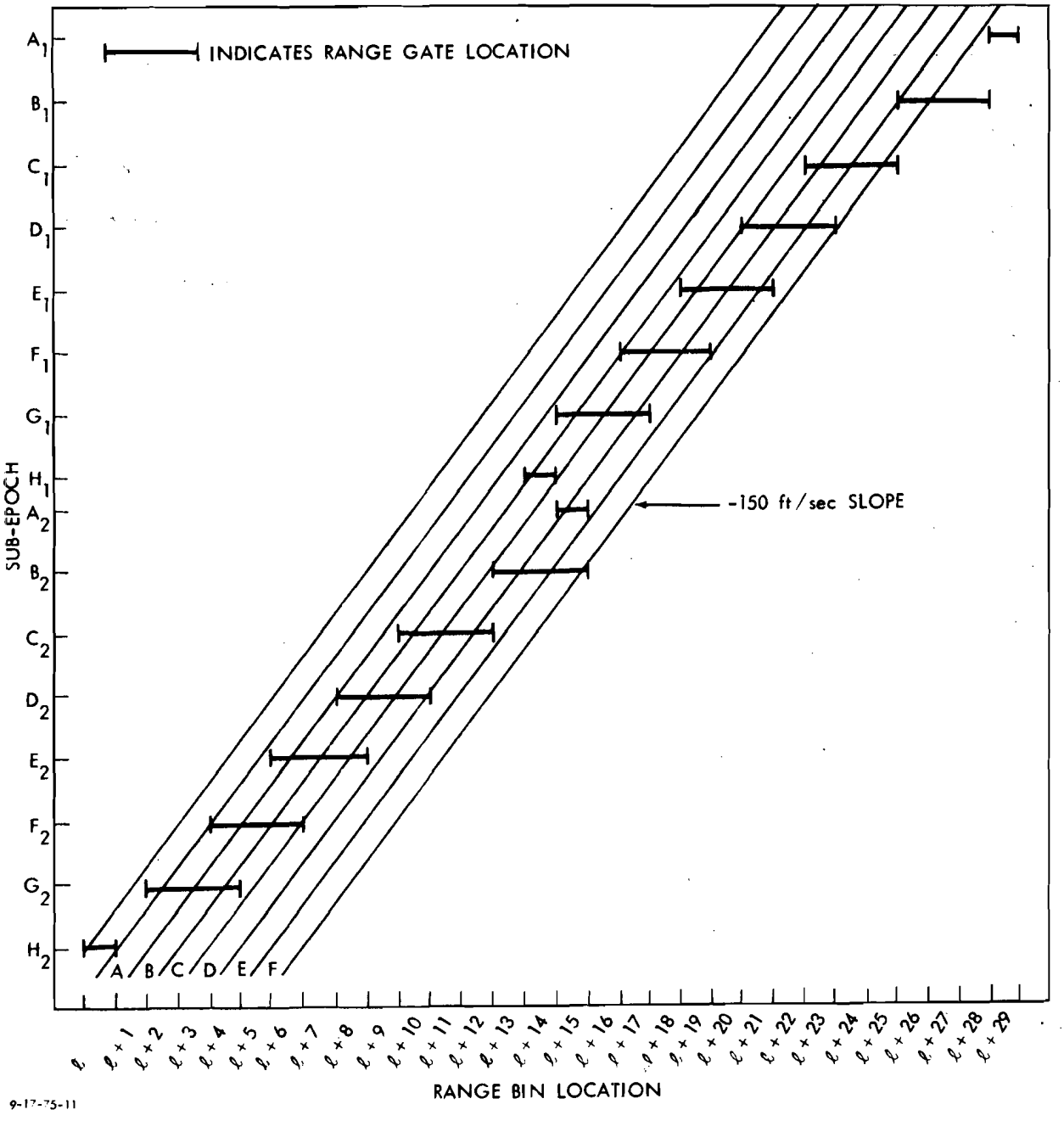

FIGURE C-11. Two-Epoch High- and Low-Acceleration Patterns for a Range Rate of -150 ft/sec

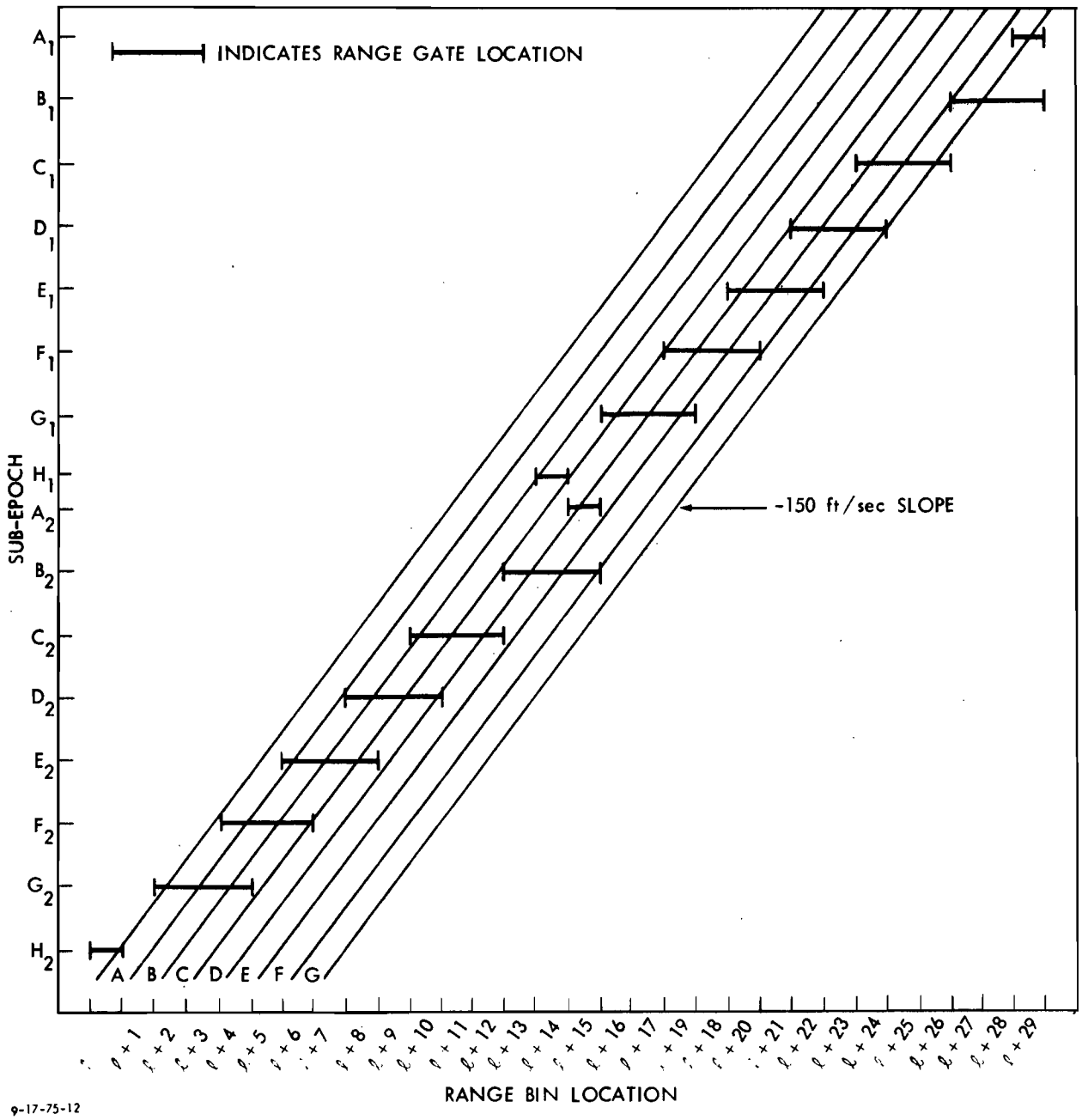

FIGURE C-12. Two-Epoch High- and High-Acceleration Patterns for a Range Rate of -150 ft/sec