AN ITERATIVE LAYERED ELASTIC COMPUTER PROGRAM
FOR RATIONAL PAVEMENT DESIGN

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for

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The United States Government does not endorse products of manufacturers. Trade or manufacturers' names appear herein solely because they are considered essential to the object of this report.
This study was conducted to develop a simple and easily operated computer program for the rational design of pavements. The program must yield results approximating those computed by the more sophisticated nonlinear finite element program. A linearly layered elastic computer program originally developed by the Chevron Oil Company was expanded and modified to include an iterative procedure for rational pavement design. Pavement response to multiple loads can be predicted using the nonlinear moduli of pavement materials. The new program is designated CHEVIT. The program uses the iterative procedures to determine the nonlinear moduli of pavement materials and then computes the stresses and strains in the pavement under a single wheel load by the Burmister linear layered elastic computational method. The principle of superposition is used to account for multiple wheel loads. For completeness, the CHEVIT program also accounts for linear problems and single loads. Results computed by the iterative layered elastic program were compared with those computed by the nonlinear finite element program. Modifications and improvements to the iterative layered program were made accordingly. Comparisons of computed and measured stresses and displacements are presented in Appendix A. Flow charts and an input guide are presented in Appendix B. Example problems and illustration of the use of the input guide are also presented.
PREFACE

The study described herein was jointly sponsored by the Federal Aviation Administration as a part of Inter-Agency Agreement FA73WAI-377, "New Pavement Design Methodology," and by the Office, Chief of Engineers, U. S. Army, as a part of the Military Construction RDTE Project 4A762719AT04, "Pavements, Soils, and Foundations," and Military Engineering RDTE Project 4A161102B52E, "Research in Military Engineering and Construction."

The study was conducted by the U. S. Army Engineer Waterways Experiment Station (WES), Soils and Pavements Laboratory. Dr. Yu T. Chou, under the general supervision of Messrs. James P. Sale, Richard G. Ahlvin, R. L. Hutchinson, and H. H. Ulery, Jr., was in charge of the study and is the author of this report. The iterative procedure described herein was developed by Dr. Walter R. Barker, who also assisted in the preparation of the computer program.

COL G. H. Hilt, CE, was Director of the WES during the conduct of this study and the preparation of this report. Mr. F. R. Brown was Technical Director.
CONTENTS

PREFACE ................................................. 1
METRIC CONVERSION FACTORS .......................... 5
INTRODUCTION ........................................... 7
  BACKGROUND ........................................ 7
  PURPOSE ............................................ 9
  SCOPE ............................................. 9
ITERATIVE LAYERED ELASTIC COMPUTER PROGRAM (CHEVIT) ........... 10
  THEORETICAL BACKGROUND .......................... 10
  ITERATIVE TECHNIQUE .............................. 13
  MULTIPLE WHEELS ................................... 16
NONLINEAR FINITE ELEMENT COMPUTER PROGRAM ....................... 21
COMPARISONS OF COMPUTED RESULTS, ITERATIVE LAYERED ELASTIC AND NONLINEAR FINITE ELEMENT PROGRAMS ................. 22
DISCUSSION OF THE ITERATIVE LAYERED ELASTIC PROGRAM .......... 27
CONCLUSIONS AND RECOMMENDATION ............................... 28
APPENDIX A: COMPARISONS OF COMPUTED AND MEASURED STRESSES AND DISPLACEMENTS ........................................... 29
APPENDIX B: CHEVIT PROGRAM ................................ 39
  EXAMPLE .......................................... 39
  FLOW CHARTS AND INPUT GUIDES ....................... 39
APPENDIX C: NOTATION ..................................... 67
REFERENCES ............................................. 68

TABLES

1 Comparison of Computed Results, Nonlinear Finite Element and Iterative Layered Elastic (CHEVIT) Programs ........... 23

FIGURES

1 Nonlinear layered elastic pavement system ....................... 11
2 Bilinear representation of the relationship between resilient modulus and deviator stress for fine-grained soil ........... 13
3 Coordinate system of one main gear of C-5A ....................... 18
4 Comparisons of computations by CHEVIT program and nonlinear finite element program ............................ 25
FIGURES (CONTINUED)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Variations of modulus with different stress-strain relations</td>
<td>30</td>
</tr>
<tr>
<td>A2</td>
<td>Comparison of the computed and measured deflections along the load axis of a 30-kip single-wheel load</td>
<td>32</td>
</tr>
<tr>
<td>A3</td>
<td>Comparison of measured and computed vertical stresses along the load axis of a 30-kip single-wheel load</td>
<td>34</td>
</tr>
<tr>
<td>A4</td>
<td>Comparisons of measured and computed deflections along the load axis of two single-wheel loads</td>
<td>35</td>
</tr>
<tr>
<td>A5</td>
<td>Comparisons of measured and computed vertical stresses along the load axis of two single-wheel loads</td>
<td>36</td>
</tr>
<tr>
<td>A6</td>
<td>Comparisons of measured and computed deflection basins along the offset axes of a 30-kip single-wheel load</td>
<td>37</td>
</tr>
<tr>
<td>B1</td>
<td>Pavements illustrating the use of CHEVIT program input guide</td>
<td>40</td>
</tr>
<tr>
<td>B2</td>
<td>Grid pattern for computational points for a Boeing 747 twin-tandem gear assembly</td>
<td>41</td>
</tr>
<tr>
<td>B3</td>
<td>Input guide for CHEVIT program</td>
<td>42</td>
</tr>
<tr>
<td>B4</td>
<td>Input guide for CHEVIT, linear analysis</td>
<td>43</td>
</tr>
<tr>
<td>B5</td>
<td>Input guide for CHEVIT, nonlinear analysis</td>
<td>51</td>
</tr>
<tr>
<td>B6</td>
<td>FORTRAN listing for CHEVIT, linear analysis</td>
<td>62</td>
</tr>
<tr>
<td>B7</td>
<td>FORTRAN listing for CHEVIT, nonlinear analysis</td>
<td>64</td>
</tr>
</tbody>
</table>
### METRIC CONVERSION FACTORS

#### Approximate Conversions to Metric Measures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>When You Know</th>
<th>Multiply by</th>
<th>To Find</th>
<th>Symbol</th>
</tr>
</thead>
</table>

#### LENGTH

| m | inches | 2.5 | centimeters | cm |
| ft | feet | 0.30 | centimeters | cm |
| yd | yards | 0.9 | meters | m |
| mi | miles | 1.6 | kilometers | km |

#### AREA

| in² | square inches | 0.645 | square centimeters | cm² |
| ft² | square feet | 0.093 | square meters | m² |
| yd² | square yards | 0.836 | square meters | m² |
| mi² | square miles | 2.59 | square kilometers | km² |
| acre | acres | 0.4047 | hectares | ha |

#### MASS (weight)

| oz | ounces | 28.35 | grams | g |
| lb | pounds | 0.4536 | kilograms | kg |
| sh.t | short tons | 0.9072 | tons | t |

#### VOLUME

| tsp | teaspoon | 0.5 | milliliters | ml |
| Tbsp | tablespoon | 15 | milliliters | ml |
| fl oz | fluid ounces | 1 | milliliters | ml |
| c | cups | 0.24 | liters | l |
| pt | pints | 0.47 | liters | l |
| qt | quarts | 0.95 | liters | l |
| gal | gallons | 3.8 | liters | l |
| fl oz | fluid ounces | 30 | milliliters | ml |
| yd³ | cubic yards | 0.76 | cubic meters | m³ |
| ft³ | cubic feet | 0.0283168 | cubic meters | m³ |

#### TEMPERATURE (exact)

<table>
<thead>
<tr>
<th>°F</th>
<th>Fahrenheit temperature</th>
<th>5/9 (after subtracting 32)</th>
<th>°C</th>
<th>Celsius temperature</th>
</tr>
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</table>

* 1 in. = 2.54 (exactly). For other exact conversions and more detailed tables, see NBC Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SO Catalog No. C13.10:286.
INTRODUCTION

BACKGROUND

A rational method for the structural design of pavements requires knowledge of the stresses and strains within the layers comprising the pavement structure. It is generally accepted that the fatigue cracking of asphalt pavements is caused by repeated applications of excessive tensile strains at the bottom of the asphalt-bound layer, and that the rutting of pavement surfaces is due mostly to excessive compressive stresses or strains in the subgrade soils. The adequacy of the design may be evaluated by comparing the computed stresses and strains at critical points in a pavement system with the allowable values obtained from laboratory tests on the component materials.

The use of elastic theory for determining stresses and displacements in a homogeneous linear elastic medium was first developed by Timoshenko and Goodier and later extended to layered systems by Burmister and Mehta and Veletos for multiple layers. Since the advent of high-speed electronic computers, the finite element technique has gained wide recognition in pavement structure analysis. Both linear and nonlinear stress-strain relations of pavement materials have been used. Boussinesq's homogeneous and Burmister's layered elastic solutions have been widely used in pavement analysis because of their simplicity. The basic assumptions behind the theory relating to the nature of pavement problems, however, are always subject to questions and criticisms. For instance, Sowers and Vesic and Vesic questioned the validity of applying Burmister's layered theories to flexible pavements. Their experimental results showed that normal stresses tended to exceed those computed by the layered theory. The recorded results usually were in the range of, or higher than, the homogeneous analysis of Boussinesq. Test data obtained at the U. S. Army Engineer Waterways Experiment Station (WES) support Sowers' and Vesic's conclusion.

Burmister's layered theories assume that the materials in each layer are homogeneous, isotropic, and linearly elastic. Vesic concluded that anisotropy of the paving materials and deformation moduli
variation in compression and tension were the main factors contributing to the observed differences between the measured and computed values. Linearity implies the applicability of the superposition principle. The material constants must be invariant relative to the state of the stresses. Since the stress-strain relation is linear, a doubling of the load doubles the deformation. Even though the superposition principle provides the backbone for the mathematical theory of linear elasticity, it is this principle itself that so drastically limits the application of the theory to the field of pavement design. Generally, most paving materials do not exhibit linear stress-strain behavior and their elastic moduli vary with the state of the stresses.

The finite element method incorporated with nonlinear material characterization is generally considered the most promising method for solving pavement problems. This method has the potential to overcome most of the inherent difficulties of Burmister's linear layered elastic theory. A study was conducted to compare the stresses and displacements computed by the nonlinear finite element program with the measurements from one of the full-scale test pavements constructed at the WES. Predicted and actual measurements were very similar, and the results are presented in Appendix A. More information on the comparisons can be found in Reference 8.

Although the nonlinear finite element program is considered to be the best current program for pavement design and analysis, the use of the program requires a large computer and long computer time. Understanding the input form also requires basic knowledge of the finite element method. Apparently the nonlinear finite element computer program can be best used for pavement research and analysis. For routine pavement design work, however, engineers need a workable program which is as simple in use as the linear layered elastic program but which yields results as good as those from the available nonlinear finite element program.* A program developed by the Chevron Oil Company,9 which is based on Burmister's linear layered elastic solution,2 was used

* The available nonlinear finite element program is explained on page 19.
to develop the initial design procedure for flexible airport pavements in a study jointly sponsored by Office, Chief of Engineers (OCE), and Federal Aviation Administration (FAA). The iterative layered elastic program developed in this study and described herein will be used in future revision of the procedure by FAA.

PURPOSE

The purpose of the study reported below was to develop a workable computer program for the rational design of pavements. The program should incorporate the stress-dependent moduli of pavement materials, and be easy and economical to operate.

SCOPE

The existing Chevron program, which was developed from the Burmister linear layered elastic solution, was expanded and modified to incorporate the stress-dependent moduli of pavement materials through the use of iterative procedures. For completeness, the program also accounts for linear problems. Comparisons were made between the computed results of the nonlinear finite element program and the iterative layered elastic program (the program developed in this study). Locations of offset distances were determined from these comparisons for averaging the stress intensities to determine the stress-dependent moduli, and the accuracy of the iterative layered elastic program was evaluated and determined. A superposition subroutine was added to the program to accommodate multiple wheel loads. The developed program is called CHEVIT. The limitations of the CHEVIT program are discussed below. The flow chart and input guidance are included in Appendix B with example problems and illustrations of the use of the input guide.
THEORETICAL BACKGROUND

As stated previously, the iterative layered elastic program (CHEVIT) is a modified version of the Chevron program,\textsuperscript{9} with the inclusion of iterative procedures to determine the stress-dependent moduli of pavement materials and a superposition subroutine for multiple-wheel loads. The program is similar in principle to the Chev5L\textsuperscript{11} developed for dual wheels. Also, it will be recalled that the Chevron program was based on the Burmister linear elastic theory.\textsuperscript{2} The basic assumptions behind Burmister's theory in relation to pavement structures are:

a. Single-wheel loads are vertically and uniformly distributed to the pavement over a circular area.

b. Pavement materials in each layer are homogeneous, isotropic, and linearly elastic. A constant modulus and Poisson's ratio represent the material property for the whole layer.

c. There is continuous contact between each interface of the layered pavement system.

Since the moduli of resilient deformation of pavement materials are stress dependent, and the stresses and strains are determined by the modulus values, a compatible solution requires successive approximations. Because the stresses in the pavement induced by the moving loads may vary appreciably both vertically and horizontally, the resilient modulus within the pavement may vary considerably from place to place.

Modulus variation in the vertical direction can be approximated by subdividing the pavement into many horizontal layers, each layer having a constant modulus throughout its thickness. Since variations along a horizontal plane cannot be considered in linear elastic layered theory, it was necessary to determine a single modulus value to represent the entire layer. The value is based on an effective state of stress determined by averaging the stresses at several points within the layer.

Figure 1 shows a layered elastic pavement system. Each layer of
Figure 1. Nonlinear layered elastic pavement system
granular materials is divided into layers normally 6 to 10 in.* in thickness. When subgrade soil is considered to be nonlinear, the thicknesses of divided layers may be increased at greater depths. For simplicity, however, the lower portion of subgrade may be assumed to be linear because stress intensities are very small in that region. Since it is known that stress varies greatly with depth in the base course layer, and stresses at midpoint of the layer do not truly represent the average stress conditions of the entire layer along the depth, three or four points along the depth should be used to determine the average stress (see Figure 1). Since the stress does not vary much with depth in the subbase and subgrade soils as compared with that in the base, stresses at midpoint may sufficiently represent the average stress conditions of the entire layer (see Figure 1).

The computational procedure used for analysis was as follows:

a. The initial modulus of each nonlinear layer was estimated and the stresses at preselected depths along the load axis and at offset points in each layer were computed. For layers in which computations were made at more than one depth, the stresses at middepth of each layer at each offset were approximated by averaging the stresses computed at all the preselected depths. The stresses at middepth along the entire horizontal plane of each layer were then approximated by averaging the stresses at the preselected offsets. The resilient moduli $M_R$** were computed by the following equations:

For unbound granular materials:

$$M_R = K_1 \theta_2$$

(1)

For fine-grained soils:

$$M_R = K_2 + (K_1 - \sigma_d)K_3$$, for $\sigma_d < K_1$ \hspace{1cm} (2a)

$$M_R = K_2 + (\sigma_d - K_1)K_4$$, for $\sigma_d > K_1$ \hspace{1cm} (2b)

* A table of metric conversion factors is presented on page 5.
** For convenience, symbols are listed and defined in the Notation (Appendix C).
where

\[ M_R = \text{resilient modulus} \]

\[ K_1, K_2, K_3, \text{ and } K_4 = \text{constants} \]

\[ \theta = \sigma_x + \sigma_y + \sigma_z = \text{bulk stress} \]

\[ \sigma_d = \text{deviator stress} \]

b. The moduli corresponding to the stresses determined in Step a were computed from Equation 1 or 2 for each nonlinear layer.

c. The moduli obtained in Step b were compared with those estimated in Step a. If they were different by an amount greater than a specified tolerance, the whole procedure was repeated until the computed moduli of two consecutive iterations were within the specified tolerance. A two percent criterion for closure was used in this study.

d. The stresses, strains, and displacements in the pavement at any location were computed using the last computed moduli.

A bilinear representation of the functions is shown in Figure 2.

Figure 2. Bilinear representation of the relationship between resilient modulus and deviator stress for fine-grained soil

ITERATIVE TECHNIQUE

When using Burmister's layered elastic solution to solve pavement problems, tensile stresses usually are developed near the bottom of the upper layers. The bulk stress, \( \sigma_x + \sigma_y + \sigma_z \), computed at the middepth
of some layers, consequently, becomes either very small or negative (a negative stress is a stress causing extension), which results in small or undefined values when Equation 1 is used. The situation is worse when strong, well-graded, dense base materials are placed directly on weak subgrade soils. Since the modulus of a negative bulk stress is not defined, a small positive modulus (or minimum modulus) is assigned to the layer when the computed bulk stress becomes negative in the CHEVIT program. Under this situation, the convergence of solution becomes rather difficult because of "overrelaxation", this is explained below.

In the first analysis, large tensile stresses were developed in the bottom layers and minimum moduli (modulus that may be less than the supporting subgrade modulus) were thus assigned to these layers. Reasonable moduli were then obtained in the upper layers. In the second iteration, positive moduli (much greater than the subgrade modulus) were obtained in the bottom layers but minimum moduli were generated in the upper layers; the poorly supported underlying layers have excessively low moduli generated from the previous iteration. The results of the third iteration were very similar to those of the first one; i.e., minimum moduli were obtained in the bottom layers while large moduli were obtained in the upper layers. The results of the fourth iteration were then very similar to those of the second one; i.e., minimum moduli were obtained in the upper layers and large moduli in the bottom layers. This oscillation continued with slow or no convergence of solution.

Dehlen also reported the absence of convergence after six iterations working with a homogeneous sand. Using the underrelaxation technique, Dehlen modified the iterative solution so that instead of letting (new) moduli \( \left( \frac{M_R}{M_R} \right)_{i+1} \) in the \((i + 1)\)th iteration equal computed \( \left( \frac{M_R}{M_R} \right)_i \) which were computed from the stresses of the \(i\)th iteration, the change was damped as follows:

\[
\left( \frac{M_R}{M_R} \right)_{i+1} = \frac{\left( \frac{M_R}{M_R} \right)_i + \left( \frac{M_R}{M_R} \right)'_i}{2}
\]  

(3)
where

\[
\begin{align*}
(M_R)_{i+1} &= \text{new } M_R \text{ used in the } (i + 1)\text{th iteration} \\
(M_R)_i &= \text{new } M_R \text{ used in the } i\text{th iteration} \\
(M'_R)_i &= \text{computed } M_R \text{ in the } i\text{th iteration}
\end{align*}
\]

In the development of the CHEVIT computer program, Equation 3 was modified as

\[
(M_R)_{i+1} = \frac{(M_R)_i + f(M'_R)_i}{1 + f}
\]

where \( f \) is a constant determining the rate of relaxation. Equations 3 and 4 are identical when \( f = 1 \). The value of \( f \) is evaluated by the procedure described below.

a. In the first iteration, \( f \) is always equal to unity; i.e., new \( M_R \) is the average of computed \( M_R \) and initial \( M_R \). If tensile bulk stress developed in the layer, instead of reducing the \( M_R \) drastically from its initial value to the minimum values, nearly one-half of this value is used (as determined by Equation 4).

b. In the second iteration, if the difference between the computed and initial \( M_R \) follows the same direction as that of the first iteration, the new \( f \) will be twice the old \( f \) to speed up the relaxation. If the difference follows the reverse direction, the new \( f \) will be one-half of the old \( f \) to slow down the relaxation.

c. The same principle applies to the remaining iterations until the convergence criterion is met.

The following numerical examples illustrate the procedures:

Example 1:

Iteration 1. Initial \( M_R = 20,000 \text{ psi} \), computed \( M_R = 2,000 \text{ psi} \), \( f = 1 \), and

new \( M_R = \frac{(20,000 + 2,000)}{2} = 11,000 \text{ psi} \)
Iteration 2. Initial $M_R = 11,000$ psi, computed $M_R = 5,000$ psi; hence $f = 1 \times 2 = 2$, and
new $M_R = \frac{[11,000 + (2 \times 5,000)]}{(1 + 2)} = 7,000$ psi

Iteration 3. Initial $M_R = 7000$ psi, computed $M_R = 4000$ psi, hence $f = 2 \times 2 = 4$, and
$[7000 + (4 \times 4000)]/(1 + 4) = 4600$ psi

Example 2:

Iteration 1. Initial $M_R = 20,000$ psi, computed $M_R = 2,000$ psi, $f = 1$, and
new $M_R = (20,000 + 2,000)/2 = 11,000$ psi

Iteration 2. Initial $M_R = 11,000$ psi, computed $M_R = 15,000$ psi; hence $f = 1/2 = 0.5$, and
new $M_R = \frac{[11,000 + (0.5 \times 15,000)]}{(1 + 0.5)} = 12,330$ psi

Iteration 3. Initial $M_R = 12,300$ psi, computed $M_R = 8,000$ psi; hence $f = 0.5/2 = 0.25$, and
new $M_R = \frac{[12,300 + (0.25 \times 8,000)]}{(1 + 0.25)} = 11,400$ psi

Iteration 4. Initial $M_R = 11,400$ psi, computed $M_R = 8,000$ psi; hence $f = 0.25 \times 2 = 0.5$, and
new $M_R = \frac{[11,400 + (0.5 \times 8,000)]}{(1 + 0.5)} = 10,270$ psi

In Equation 4, with variable $f$, convergence is much faster than in Equation 3 ($f = 1$). In a problem in which convergence could not be obtained with the original Chev5L program even after 20 iterations, with the same program modified by replacing Equation 3 with Equation 4, convergence occurred on the fifth iteration. Averaging the stresses of several depths in one layer, when the stresses vary greatly along the depth, also assists in the convergence of solution.

MULTIPLE WHEELS

The Chevron computer program, from which the CHEVIT program is
derived, deals only with a single-wheel load. A superposition sub-
routine was added to the iterative layered elastic program to handle
multiple-wheel loads. It should be noted that the multiple-wheel
stresses are not used in determining the modulus of nonlinear layers.
In the use of the superposition principle, the stresses, strains, and
displacements were first computed at nine offset points: 0, 0.5, 1,
1.5, 2, 3, 4, 8, and 12 radii. The values at desired offset distances
for the multiple wheels were then interpolated from the known values of
the nine points. The Gregory-Newton Interpolation Formula with three
points of equal intervals was used in this study, as explained in the
following paragraphs.

An interpolating parabola was expressed in a second-order poly-
nomial \( p(x) \) through the three points. It should be noted that equal
intervals exist in the following sets of three points.

0, 0.5, and 1 radii
1, 1.5, and 2 radii
2, 3, and 4 radii
4, 8, and 12 radii

Assuming the known values at the three points in each set are \( f(x_i) \),
i equals 1, 2, and 3, and the interval between two points is \( h \). The
polynomial has the form

\[
p(u) = f(x_1) + \Delta f(x_1)u + \frac{\Delta^2 f(x_1)}{2} u(u - 1)
\]

(5)

where

\[
u = \frac{x - x_1}{h}
\]

\[
\Delta f(x_1) = f(x_1 + h) - f(x_1)
\]

\[
\Delta^2 f(x_1) = f(x + 2h) - 2f(x_1 + h) + f(x_1)
\]

The following numerical example illustrates the use of Equation 5.
The coordinate system for a C-5A 12-wheel gear assembly is shown in Figure 3. Each wheel has an equivalent radius of 9.54 in. The surface deflections under one wheel load of C-5A computed by the Chevron program at the nine offset points 0, 0.5, 1, 1.5, 2, 3, 4, 8, and 12 radii are 0.102, 0.1, 0.098, 0.092, 0.08, 0.05, 0.04, 0.019, and 0.005 in., respectively. The surface deflection at the point \( x = 0 \) and \( y = 0 \) induced by the wheel load at location \( x = 34 \) in. and \( y = 0 \) is needed.

Figure 3. Coordinate system of one main gear of C-5A
SOLUTION

The computer would first determine that a point at a distance $R = 34$ in. lies within the interval of offset points of 2, 3, and 4 radii. $R$ is the radial distance and equal to $\sqrt{x^2 + y^2}$. The following values for deflections at offset points of 2, 3, and 4 radii are assigned to the computer.

a. $f(x_1) = f(x = 19.1) = 0.08$ in.
   $f(x_2) = f(x_1 + h) = f(x = 28.7) = 0.05$ in.
   $f(x_3) = f(x_1 + 2h) = f(x = 38.2) = 0.04$ in.

b. The following values are computed.
   $\mu = \frac{x - x_1}{h} = \frac{34 - 19.1}{9.54} = 1.58$
   $\Delta f(x_1) = f(x_1 + h) - f(x_1) = -0.03$
   $\Delta^2 f(x_1) = f(x_1 + 2h) - 2f(x_1 + h) + f(x_1) = 0.04 - 2 \times 0.05 + 0.08 = 0.02$

c. The surface deflection $p(x = 34)$ can be computed from Equation 5.
   $p(x = 34) = f(x_1) + \Delta f(x_1)\mu + \frac{\Delta^2 f(x_1)}{2} \mu(\mu - 1)$
   $= 0.08 - 0.03(1.58) + 0.01 \times 1.58(0.58)$
   $= 0.042$ in.

Deflections induced by other wheels are interpolated similarly, and the total displacement for the C-5A 12-wheel gear assembly (shown in Figure 3) is the linear summation of individual wheel load deflections.

The principle of superposition is the method of obtaining the actual resultant effect by adding or combining independent partial effects. The method is applicable only if a linear relationship exists between the loads and the effect they produce. Since granular materials are highly nonlinear, the use of the superposition principle to
obtain solutions for multiple-wheel loads is theoretically unsound. Unfortunately, there is no computer program now available capable of obtaining solutions for multiple-wheel loads, which can be economically operated without the use of the superposition principle. Therefore, the results for the multiple-wheel loads computed by the CHEVIT are subject to the common criticism of the superposition principle.
The nonlinear finite element program used for comparison purposes was obtained from the University of California at Berkeley. The program is suitable only for analysis of axisymmetric solids. The pavement structure is first idealized as an assemblage of a finite number of discrete structural elements interconnected at a finite number of joints or nodal points. The sizes of the elements are chosen to vary in accordance with the anticipated stress gradients. The elements are actually complete rings in the horizontal direction, and the nodal points are circular lines in plan view. There is a boundary on which the nodal points are fixed and a vertical boundary on which the nodal points are constrained from moving radially.

The load at the surface is circular and is assumed to be applied in steps so that the nonlinear stress-strain behavior and the modulus-stress dependency of the material can be included in the analysis. Therefore, the accuracy of the solution is a function of the number of load increments used, with greater accuracy being associated with smaller load increments. The accuracy can also be improved by increasing the number of elements into which the pavement structure is divided. In this study, ten load increments were generally used.

Equations 1 and 2 were used in computing the resilient moduli of nonlinear materials and are the same as those employed in the CHEVIT program.
COMPARISONS OF COMPUTED RESULTS, ITERATIVE LAYERED ELASTIC AND NONLINEAR FINITE ELEMENT PROGRAMS

As stated above, the purpose of this study was to develop a computer program that is simple and economical to use and will yield results approaching those of the nonlinear finite element program. The CHEVIT program can be simply and economically used, and can incorporate the stress-dependent moduli of pavement materials in the computations. To determine whether CHEVIT yields results close to those of the nonlinear finite element program, computations were made on many selected pavement structures to find the differences between results computed by the two programs. Modifications to improve the CHEVIT program were made accordingly.

Three types of pavements were used in the computations: (a) a 3-in. asphaltic concrete surface and 21-in. high-quality, well-compacted crushed stone base, (b) a 9-in. asphaltic concrete surface and 15-in. gravelly sand subbase, and (c) a 3-in. asphaltic concrete surface, a 6-in. high-quality well-compacted crushed stone base, and gravelly sand subbase with thicknesses varying from 6 to 32 in. These were typical test sections constructed at the WES. For these pavements, the loads were varied from 15 to 75 kips and the subgrades were varied from 2 to 16 CBR to provide a wide range of conditions. A total of 39 pavements were used in the computations, as given in Table 1.

For each pavement, both the nonlinear finite element program and the CHEVIT program were used to compute the stresses and strains in the pavements, and the results were compared. In the computations, a modulus of 100,000 psi was assigned to the asphaltic concrete. Equation 1 was used to characterize the nonlinear behavior of the granular materials with $K_1$ and $K_2$ equal to 8300 and 0.71, respectively, for base materials, and 2900 and 0.47, respectively, for subbase materials. The relation of $M_R = 1500$ CBR was used to determine the subgrade modulus values.

Because a bottom boundary on which the nodal points are constrained from moving vertically was used in the nonlinear finite element
## Table 1
Comparison of Computed Results, Nonlinear Finite Element and Iterative Layered Elastic (CHEVIT) Programs

<table>
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<th>Pavement</th>
<th>Load kips</th>
<th>Thickness, in.</th>
<th>Sub-</th>
<th>Sub-</th>
<th>Nonlinear Finite</th>
<th>Iterative Layered</th>
<th>Percent Difference</th>
<th>Difference</th>
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program, a cutoff was used in the CHEVIT program at a depth about 300 in. beneath the surface of the subgrade soil. The exact depth varies with the magnitude and contact area of the wheel load. A high modulus of 555,555 psi was arbitrarily assigned to the cutoff layer. Higher moduli would not change the computed results noticeably, but may create some problem in the computations because of the extremely small modulus ratio between the subgrade and the cutoff layer.

In the use of the CHEVIT program, computations were first made with moduli determined by the stresses only along the load axis. It was found, however, that the computed values of stress and strain were much larger than those computed by the nonlinear finite element program. This can be logically explained because the stresses along the load axis are generally greater than those at offset points. Since the response of pavement structures does not depend solely upon the materials along the load axis but also upon the materials at offset distances, average modulus values determined from the points at the load axis and at offset points should be used in the computation. In so doing, the computed values were reduced as the number of offset points was increased. Computed values of the asphaltic concrete surface deflection and vertical strain at the subgrade surface for the 39 pavements computed by the two different programs are compared in Table 1. In the CHEVIT program, the moduli in the granular layers were computed based on the average stress intensities at offset distances of 0, 2, 4, and 8 radii. For purpose of illustration, the comparison procedures are shown in Figure 4.

Table 1 indicates that the differences between the values computed by the two programs are rather small for the majority of pavements. The computation time required by the CHEVIT program in most cases was about 1/20 of that required by the nonlinear finite element program. In the percent difference column in Table 1, the plus sign indicates that the computed value by the CHEVIT program was greater than that computed by the nonlinear finite element program; the minus sign indicates that the reverse was true. Concerning the comparisons, the following points should be noted:

a. The $K_1$ and $K_2$ values used in both computer programs do
Figure 4. Comparisons by CHEVIT program and nonlinear finite element program
not significantly affect the comparisons. In other words, the same conclusions would be derived if different $K_1$ and $K_2$ values (within reasonable range) were used in the computations. The determination of $K_1$ and $K_2$ values used in this study is explained in Reference 14.

b. The nonlinear properties of subgrade soils were not considered in the computations for the pavements shown in Table 1 because the information on the $K$ values (Equation 2) for different CBR subgrade soils is not available.
DISCUSSION OF THE ITERATIVE LAYERED ELASTIC PROGRAM

Incorporating nonlinear characteristics of materials in the CHEVIT program is a great step forward in the improvement of the linear layered elastic program. However, the CHEVIT program still possesses many pitfalls which make the program far from perfect. These are the assumptions of (a) isotropy of materials, (b) identical deformation modulus in tension and compression, (c) continuous contact along each interface, and (d) existence of tensile stresses in the granular layers.

Since the CHEVIT program is basically a linear layered elastic program, the materials in each layer are homogeneous, isotropic, and linearly elastic during each iteration. Excessive amounts of tensile stresses are developed frequently at bottom layers of the granular materials. This is, of course, physically impossible, as granular materials cannot take large tensile stress. Therefore, the use of the CHEVIT program to evaluate the radial stresses in granular layers should be used cautiously. Despite this abnormality, however, the responses, such as stress, strains, and displacements, computed by the CHEVIT program were generally very close to those of the nonlinear finite element program, in which tensile stresses usually do not develop provided the moduli of granular materials are not excessively high. Therefore, the CHEVIT program can be used for rational pavement designs.
CONCLUSIONS AND RECOMMENDATION

The iterative layered elastic program (CHEVIT), developed for rational pavement design, has the capability of incorporating the stress-dependent characteristics of pavement materials. Pavement responses computed by CHEVIT yield results comparable to those computed by the nonlinear finite element program; furthermore, the CHEVIT program is much easier and more economical to operate. It is recommended, therefore, that the CHEVIT program be adopted in the rational design of flexible pavements.
In this portion of the study, the vertical stresses and displacements measured in item 3 of the multiple-wheel heavy gear load test section at the U. S. Army Engineer Waterways Experiment Station (WES) were compared with those computed by the nonlinear-finite element method incorporated with various nonlinear stress-strain relations of the pavement materials. The multiple-wheel heavy gear load test section was composed of a 3-in. asphaltic concrete (AC) surface course, a 6-in. well-compacted limestone base, a 24-in. sand and gravel subbase, and a 4-CBR buckshot clay subgrade soil. Four-inch WES pressure cells and linear variable differential transducers (LVDT) were installed at various depths in the test section to measure stresses and displacements under various loading conditions. Details of the instrumentation, construction, and testing of the test section are presented in Reference 7, Volume III.

The meshes of the finite element grid are shown in Figure A-1. Since the thickness of the AC surface course was only 3 in., a constant modulus was considered to be appropriate and was used in the computations. Pavement temperature during measurement was about 90°F, and a modulus of 100,000 psi was chosen. Three different computations were made with different stress-strain relations for the granular materials and subgrade soil:

a. Stress-strain relation No. 1.

(1) Granular materials:

\[ M_R = K_1 \sigma_1^2 \]

\[ K_2 \]

and \( \nu = 0.3 \). For base materials, \( K_1 = 13,126 \) and \( K_2 = 0.55 \); for subbase materials, \( K_1 = 7,650 \) and \( K_2 = 0.59 \).
### Figure A-1. Variations of modulus with different stress-strain relations

Note: Values within each element are moduli corresponding to stress-strain relations No. 1, 2, and 3, respectively.

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NOTE: Values within each element are moduli corresponding to stress-strain relations No. 1, 2, and 3, respectively.
(2) **Subgrade soil:**

\[
E_t = \left[ 1 - \frac{R_f (1 - \sin \phi) (\sigma_1 - \sigma_3)}{2C \cos \phi + 2\sigma_3 \sin \phi} \right]^2 E_i \tag{A-2}
\]

and \( \nu = 0.4 \).

**b. Stress-strain relation No. 2.**

(1) **Granular materials:**

\[ M_R = K_3 \nu \]  

(1 bis)

and \( \nu = 0.48 \). For base materials, \( K_3 = 8300 \) and \( K_4 = 0.71 \); for subbase materials, \( K_3 = 2900 \) and \( K_4 = 0.47 \).

(2) **Subgrade soil:**

Equation A-2, where \( \nu = 0.4 \).

**c. Stress-strain relation No. 3.**

(1) **Granular materials:**

Same as in stress-strain relation No. 2.

(2) **Subgrade soil:**

\[ M_R = K_2 + (K_1 - \sigma_d)K_3, \text{ for } \sigma_d < K_1 \]  

(2a bis)

\[ M_R = K_2 + (\sigma_d - K_1)K_4, \text{ for } \sigma_d > K_1 \]  

(2a bis)

and \( \nu = 0.4 \).

The three numbers shown in Figure A-1 within each element are moduli computed by stress-strain relations Nos. 1, 2, and 3, respectively.

A large value of Poisson's ratio was used in stress-strain relations Nos. 2 and 3. It is believed that a large value for Poisson's ratio represents field conditions better than the small value that is used in stress-strain relation No. 1. Both static and resilient moduli for subgrade soil were determined from laboratory tests on undisturbed samples. The determination of values for \( K_1 \) to \( K_4 \) were described in Reference 13.

Figure A-2 compares computed and measured deflections along the
load axis of a 30-kip single-wheel load. Using stress-strain relation No. 1, the computed values were much higher than the measured. This was anticipated because of the inadequate values used for the granular materials and subgrade soil. Equation A-1 assumes that the elastic moduli of granular materials depend solely on the minor principle stress $\sigma_3$ (confining pressure). When tensile stresses were developed at the bottom of the granular layers, the elastic moduli reduced drastically as the load increment increased, as indicated in the computer outputs.

Also, the use of static moduli for the subgrade soil was inadequate because the measured deflections were elastic, i.e., the rebound portion of the curve. Resilient moduli of subgrade soil should be used in the computations, which would result in a reduction of deflections since the resilient moduli are generally much greater than the static ones.

The use of Equation 1 can greatly increase the elastic moduli of
granular materials. The major principle stresses $\sigma_1$ are generally in compression and are very much greater than the intermediate and minor principle stresses $\sigma_2$ and $\sigma_3$, which are in tension with load at the bottom of the layers. Hence, in most cases, the first stress invariant $\Theta$, the sum of the principal stresses, is positive in magnitude. The computed deflections based on Equation A-1 are shown in curve 2 of Figure A-2. The computed deflections, however, were still much greater than the measured. This was also anticipated because static moduli were used in the computations. Nevertheless, it is clearly illustrated that the use of the first stress invariant $\Theta$ is superior to the use of the minor principle stress $\sigma_3$ in characterizing granular materials.

When the static moduli of subgrade soil were replaced by the resilient moduli in the computations. The resulting deflections were plotted in curve 3 of Figure A-2. The deflections were greatly reduced and agreed very well with the measured deflections. It is interesting to note that the portion of curve 3 above the subgrade is parallel to curve 2. This is due to the material characterizations for materials above the subgrade soil being the same for these two computations.

Figure A-3 compares measured and computed vertical stresses for a 30-kip single-wheel load. Curves 1 and 2 were computed with static moduli of the subgrade soil; therefore, the computed stresses at the surface of the subgrade soil were much too low. Since resilient moduli of the subgrade soil were used (providing better subgrade support), the computed vertical stress at the subgrade surface increased and approached the measured value (curve 3). For comparison, the curve for Boussinesq's solution is also presented in Figure A-3.

Figure A-4 compares measured and computed deflections along the load axes of 15- and 50-kip single-wheel loads. The computations were made with stress-strain relation No. 3. It can be seen that the computed values were quite close to the measured ones. Figure A-5 is similar to Figure A-4 except that it shows the vertical stresses. The computations here were also in good agreement with the measurements.

Figure A-6 compares measured and computed deflection basins at various depths under a 30-kip single-wheel load. The computations were
Figure A-3. Comparison of measured and computed vertical stresses along the load axis of a 30-kip single-wheel load.
Figure A-4. Comparisons of measured and computed deflections along the load axis of two single-wheel loads.
Figure A-5. Comparisons of measured and computed vertical stresses along the load axis of two single-wheel loads.
Figure A-6. Comparisons of measured and computed deflection basins along the offset axes of a 30-kip single-wheel load.
made with stress-strain relation No. 3. Unfortunately, the agreement between the computations and the measurements along the offset axes was not as good as that along the load axis.
APPENDIX B: CHEVIT PROGRAM

EXAMPLE

To illustrate the use of the input guide for the CHEVIT program, the pavement structure shown in Figure B-1 was used as an example to compute the stresses, strains, and displacements in the pavement. A single-wheel load of 30,000 lb with 105-psi contact pressure and a Boeing 747 twin-tandem load of 240,000 lb with 210-psi contact pressure were used in the computations. In the nonlinear case, the moduli shown in Figure B-1b were the initial values; the true values were determined by the computer program with the following equations:

For base materials: \[ M_R = 6000(\sigma_x + \sigma_y + \sigma_z)^{0.61} \] (1 bis)

For subbase material: \[ M_R = 2900(\sigma_x + \sigma_y + \sigma_z)^{0.47} \] (1 bis)

For fine-grained soils: \[ M_R = 4500 + (5 - \sigma_d) \times 700 \]

\[ \text{for } \sigma_d < 5 \] (2a bis)

\[ M_R = 4500 + (\sigma_d - 5) \times (-400) \text{ for } \sigma_d > 5 \] (2b bis)

where the constants were chosen arbitrarily.

For single-wheel load, the computed values were at offset distances 0, 9.54, and 19.08 in. and at depths 0, 3, 38, 50, 70, 100, 150, 200, 250, and 300 in. For multiple-wheel loads, the computed values were at depths 0 and 38 in.; the offset distances are shown in Figure B-2 by the points on the grid pattern. The coordinate system for the Boeing 747 twin-tandem is also shown in Figure B-2.

FLOW CHART AND INPUT GUIDES

The flow chart and input guides for linear and nonlinear analyses are given in Figures B-3 through B-7.
### Pavement with Linear Material Properties

<table>
<thead>
<tr>
<th>Layer</th>
<th>Modulus of Elasticity (MPa)</th>
<th>Poisson's Ratio</th>
<th>Thickness (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>183000</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>Subbase</td>
<td>14000</td>
<td>0.4</td>
<td>10</td>
</tr>
<tr>
<td>Nonlinear Layers</td>
<td>6000</td>
<td>0.4</td>
<td>20</td>
</tr>
<tr>
<td>Nonlinear Layers</td>
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<td>0.4</td>
<td>30</td>
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<tr>
<td>Nonlinear Layers</td>
<td>6000</td>
<td>0.4</td>
<td>40</td>
</tr>
<tr>
<td>Subbase</td>
<td>11500</td>
<td>0.4</td>
<td>7</td>
</tr>
<tr>
<td>Base</td>
<td>20000</td>
<td>0.4</td>
<td>7</td>
</tr>
<tr>
<td>Subbase</td>
<td>14000</td>
<td>0.4</td>
<td>7</td>
</tr>
<tr>
<td>Nonlinear Layers</td>
<td>6000</td>
<td>0.4</td>
<td>10</td>
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<td>20</td>
</tr>
<tr>
<td>Nonlinear Layers</td>
<td>6000</td>
<td>0.4</td>
<td>30</td>
</tr>
<tr>
<td>Subbase</td>
<td>11500</td>
<td>0.4</td>
<td>7</td>
</tr>
<tr>
<td>Base</td>
<td>183000</td>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>Subbase</td>
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<td>0.4</td>
<td>10</td>
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</tbody>
</table>

### Pavement with Nonlinear Material Properties

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<th>Layer</th>
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<th>Poisson's Ratio</th>
<th>Thickness (in)</th>
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</thead>
<tbody>
<tr>
<td>Base</td>
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<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>Subbase</td>
<td>14000</td>
<td>0.4</td>
<td>10</td>
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<tr>
<td>Nonlinear Layers</td>
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<td>20</td>
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<tr>
<td>Nonlinear Layers</td>
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<td>30</td>
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<tr>
<td>Subbase</td>
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<tr>
<td>Base</td>
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<tr>
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<tr>
<td>Subbase</td>
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<td>0.4</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure B-1. Pavements illustrating the use of CHEVIT program input guide.
Figure B-2. Grid pattern for computational points for a Boeing 747 twin-tandem gear assembly.
Figure B-3. Input guide for CHEVIT program
**GENERAL PURPOSE DATA FORM**

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
<th>PAGE</th>
<th>OF</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>Option Card (A5 or A3)</td>
<td>In case of start and continuation of input, columns 1-5 should read:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S.T.A.B.</td>
<td>In case of termination of input, columns 1-3 should read:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E.N.D.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 2</td>
<td>Identification Card (20A4)</td>
<td>Problem description (title) can be described in columns 1-50.</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Figure B-4. Input guide for CHEVIT, linear analysis (sheet 1 of 8)
**GENERAL PURPOSE DATA FORM**

<table>
<thead>
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<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
<th>PAGE OF</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 3 Wheel Card (FP10.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 1-10</td>
<td>WGT = wheel load, lb</td>
<td>11-20</td>
<td>psi = tire contact pressure, psi</td>
<td>21-30</td>
<td>Zero</td>
</tr>
<tr>
<td></td>
<td>31-40</td>
<td>Zero</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| No. 4 Layer Card (FP10.0) | | | | | |
| Columns 1-10 | XNS = number of layers |

| No. 5 Layer Properties Card (FP10.0) | | | | | |
| Columns 1-10 | E(1) = modulus of elasticity for layer 1 |
| | 11-20 | v(1) = Poisson's ratio for layer 1 |
| | 21-30 | E(2) = modulus of elasticity for layer 2 |
| | 31-40 | v(2) = Poisson's ratio for layer 2 |
| | 41-50 | E(3) = modulus of elasticity for layer 3 |
| | 51-60 | v(3) = Poisson's ratio for layer 3 |
| | 61-70 | E(4) = modulus of elasticity for layer 4 (if needed) |
| | 71-80 | v(4) = Poisson's ratio for layer 4 |

**Figure B-4. (sheet 2 of 8)**
GENERAL PURPOSE DATA FORM

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
<th>PAGE</th>
<th>OF</th>
</tr>
</thead>
</table>

If the number of layers is greater than 4, continue to next data card using same format until the number of layers is satisfied.

No. 6 Layer Thickness Card (8F10.0)

Columns 1-10 HM(1) = thickness of layer 1, in.
11-20 HM(2) = thickness of layer 2, in.
21-30 HM(3) = thickness of layer 3, in.
31-40 HM(4) = thickness of layer 4, in.
41-50 HM(5) = thickness of layer 5, in.
51-60 HM(6) = thickness of layer 6, in.
61-70 HM(7) = thickness of layer 7, in.
71-80 HM(8) = thickness of layer 8, in. (if needed)

If the number of layers is greater than 8, continue to next data card using same format until the number of layers is satisfied.

Figure B-4. (sheet 3 of 8)
### GENERAL PURPOSE DATA FORM

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<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
<th>PAGE</th>
<th>OP</th>
</tr>
</thead>
</table>

**No. 7 Offset Card (F10.0)**

<table>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>X1R</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Number of offsets computations made.</td>
</tr>
</tbody>
</table>

This card is needed only for single-wheel load. For multiple wheels, set X1R = 0 and branch out to No. 9.

**No. 8 Offset Distances Card (8F10.0)** Not applicable for multiple wheels

<table>
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<th>3</th>
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<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>RR(1)</th>
<th>RR(2)</th>
<th>RR(3)</th>
<th>RR(4)</th>
<th>RR(5)</th>
<th>RR(6)</th>
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<td></td>
</tr>
</tbody>
</table>

Columns 1-10 RR(1) = distance to first offset, in.

11-20 RR(2) = distance to second offset, in.

21-30 RR(3) = distance to third offset, in.

31-40 RR(4) = distance to fourth offset, in.

41-50 RR(5) = distance to fifth offset, in.

51-60 RR(6) = distance to sixth offset, in.

61-70 RR(7) = distance to seventh offset, in.

71-80 RR(8) = distance to eighth offset, in. (if needed)

If the number of offsets is greater than 8, continue to next data card using same format until the number of offsets is satisfied.

---

Figure B-4. (sheet 4 of 8)
## GENERAL PURPOSE DATA FORM

### No. 9 Computational Depths Card (SP10.0)

Columns 1-10 XiZ = number of depths to computational points

### No. 10 Depths to Computational Points Card (SP10.0)

Columns 1-10 ZZ(1) = first computational depth, in.
11-20 ZZ(2) = second computational depth, in.
21-30 ZZ(3) = third computational depth, in.
31-40 ZZ(4) = fourth computational depth, in.
41-50 ZZ(5) = fifth computational depth, in.
51-60 ZZ(6) = sixth computational depth, in.
61-70 ZZ(7) = seventh computational depth, in.
71-80 ZZ(8) = eighth computational depth, in. (if needed)

If the number of depths is greater than 8, continue to next data card using same format until the number of depths is satisfied.

Note: In the main program of CHEVIT, ZZ(1) has a dimension of 19. If ZZ falls on the interface, two computational points are generated, one in each layer. Subroutine WHEELS is dimensioned for 15 points only. Therefore, if subroutine WHEELS is utilized, the maximum number of ZZ's is 15. However, it should be remembered that at each interface depth, ZZ(1) represents 2 ZZ's in storage.

---

Figure B-4. (sheet 5 of 8)
For problems involving only a single-wheel load, a blank card should be placed after No. 10 input. Thus ends the input for the main program. For problems involving multiple wheels, the input continues.

---

**No. 11 Multiple Wheels Card (F10.0)**

Columns 1-10 XNW = number of wheels on multiple-wheel gear assembly

---

**No. 12 Coordinates Card (F210.0)**

Columns 1-10 XW(1) = X-coordinate of first wheel, in; 11-20 YW(1) = Y-coordinate of first wheel, in.

---

Figure B-4. (sheet 6 of 8)
**GENERAL PURPOSE DATA FORM**

<table>
<thead>
<tr>
<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
<th>PAGE</th>
<th>OF</th>
</tr>
</thead>
</table>

This format is repeated until the number of wheels XFW is satisfied.

<table>
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<tr>
<th></th>
<th>XW(2)</th>
<th>YV(2)</th>
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<tr>
<td></td>
<td>XW(3)</td>
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</tr>
<tr>
<td></td>
<td>XW(4)</td>
<td>YV(4)</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>XW(5)</td>
<td>YV(5)</td>
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</tr>
<tr>
<td></td>
<td>XW(6)</td>
<td>YV(6)</td>
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**No. 13 Grid Card (F10.0)**

Columns 1-10 XGRID = number of grid generator cards to follow.

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<tbody>
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<td>XGRID</td>
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</tbody>
</table>

**No. 14 Grid Generator Card (6F10.0)**

Columns 1-10 XI = initial X-coordinate of grid
11-20 YI = initial Y-coordinate of grid
21-30 SX = step size in X-direction
31-40 SY = step size in Y-direction
41-50 XNUM = number of lines in X-direction
51-60 YNUM = number of lines in Y-direction

(Repeat this card for each XGRID)

<table>
<thead>
<tr>
<th>XI</th>
<th>YI</th>
<th>SX</th>
<th>SY</th>
<th>XNUM</th>
<th>YNUM</th>
</tr>
</thead>
<tbody>
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<td></td>
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</tbody>
</table>

(WES FORM 1233 EDITION OF SEP 82 WILL BE USED UNTIL EXHAUSTED)

Figure B-4. (sheet 7 of 8)
This completes the input for one problem. If another problem follows, return to No. 1 and use the START option. If no additional problems are desired, use the END option.
**Figure B-5. Input guide for CHEVIT, nonlinear analysis (sheet 1 of 11)**
### GENERAL PURPOSE DATA FORM

<table>
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<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
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<th>PAGE OF</th>
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<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Columns 21-30**
- XNL = maximum number of iterations
- TOL = tolerance, percent

**Columns 31-40**
- W.G.T. = other
- R.e. = other
- X.N.L.T.H. = other
- T.O.L. = other

**No. 4 Layer Card (F10.0)**
- Columns 1-10 XNS = number of layers

**No. 5 Layer Properties Card (F10.0)**
- Columns 1-10
  - E(1) = modulus of elasticity for layer 1
  - v(1) = Poisson's ratio for layer 1
  - E(2) = modulus of elasticity for layer 2
  - v(2) = Poisson's ratio for layer 2
  - E(3) = modulus of elasticity for layer 3
  - v(3) = Poisson's ratio for layer 3
  - E(4) = modulus of elasticity for layer 4
  - v(4) = Poisson's ratio for layer 4

**Figure B-5.** (sheet 2 of 11)
## GENERAL PURPOSE DATA FORM

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<tr>
<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

If the number of layers is greater than 4, continue to next data card using same format until the number of layers is satisfied.

---

### No. 6 Layer Thickness Card (SF10.0)

Columns 1-10: **H1(1)** = thickness of layer 1, in.
11-20: **H1(2)** = thickness of layer 2, in.
21-30: **H1(3)** = thickness of layer 3, in.
31-40: **H1(4)** = thickness of layer 4, in.
41-50: **H1(5)** = thickness of layer 5, in.
51-60: **H1(6)** = thickness of layer 6, in.
61-70: **H1(7)** = thickness of layer 7, in.
71-80: **H1(8)** = thickness of layer 8, in. (if needed)

---

If the number of layers is greater than 8, continue to next data card using same format until the number of layers is satisfied.

---

Figure B-5. (sheet 3 of 11)
### GENERAL PURPOSE DATA FORM

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<th>PROGRAM</th>
<th>REQUESTED BY</th>
<th>PREPARED BY</th>
<th>CHECKED BY</th>
<th>DATE</th>
</tr>
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</table>

No. 7.1 Nonlinear Offset Card (F10.0)

Columns 1-10 XR = number of offsets to use for computing reference stresses to determine the average modulus of the nonlinear layers.

<table>
<thead>
<tr>
<th>XR</th>
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</thead>
</table>

No. 7.2 Nonlinear Offset Distance Card (8F10.0)

Columns 1-10 RR(1) = distance to first reference stress, in.
11-20 RR(2) = distance to second reference stress, in.
21-30 RR(3) = distance to third reference stress, in.
31-40 RR(4) = distance to fourth reference stress, in.
41-50 RR(5) = distance to fifth reference stress, in.
51-60 RR(6) = distance to sixth reference stress, in.
61-70 RR(7) = distance to seventh reference stress, in.
71-80 RR(8) = distance to eighth reference stress, in. (if needed)

<table>
<thead>
<tr>
<th>RR(1)</th>
<th>RR(2)</th>
<th>RR(3)</th>
<th>RR(4)</th>
<th>RR(5)</th>
<th>RR(6)</th>
<th>RR(7)</th>
<th>RR(8)</th>
</tr>
</thead>
</table>

If the number of offsets is greater than 8, continue to next data card using same format until the number of offsets is satisfied.

<table>
<thead>
<tr>
<th>RR(9)</th>
<th>RR(10)</th>
<th>RR(11)</th>
<th>RR(12)</th>
<th>RR(13)</th>
<th>RR(14)</th>
<th>RR(15)</th>
<th>RR(16)</th>
</tr>
</thead>
</table>

Figure B-5. (sheet 4 of 11)
GENERAL PURPOSE DATA FORM

<table>
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<tr>
<th>PROGRAM</th>
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<tbody>
<tr>
<td>No. 7.3 Nonlinear Layer Card (F10.0)</td>
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<tr>
<td>Columns 1-10</td>
<td>X1Z = number of nonlinear layers. In general an asphaltic concrete layer and chemically stabilized layers are considered to be linear.</td>
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<td>X1Z =</td>
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<td>No. 7.4 Nonlinear Layer Characterisation Card (B10.0)</td>
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<tr>
<td>Columns 1-10</td>
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<tr>
<td>11-50 X1Z(i) = layer number</td>
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<td>21-30 X1Z2(i) = number of computational levels depthwise for reference stress at that layer</td>
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<tr>
<td>31-40 XKarter(1,1) = characterisation constants</td>
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<tr>
<td>41-50 XKarter(1,2) = characterisation constants</td>
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<tr>
<td>51-60 XKarter(1,3) = characterisation constants</td>
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<tr>
<td>61-70 XKarter(1,4) = characterisation constants</td>
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<tr>
<td>71-80 XKarter(1,5) = characterisation constants</td>
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<tr>
<td>X1Z(i) =</td>
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<tr>
<td>Note: One card for each layer (i = X1Z)</td>
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</tbody>
</table>

Figure B-5. (sheet 5 of 11)
### GENERAL PURPOSE DATA FORM

**Columns 1-10**  
ZZ(1) = depths to first computational level for first reference stress, in.

**Columns 11-20**  
ZZ(2) = depths to second computational level for first reference stress, in.

**Columns 21-30**  
ZZ(3) = depths to third computational level for first reference stress, in.

**Columns 31-40**  
ZZ(4) = depths to fourth computational level for first reference stress, in.

**Columns 41-50**  
ZZ(5) = depths to fifth computational level for first reference stress, in.

**Columns 51-60**  
ZZ(6) = depths to sixth computational level for first reference stress, in.

**Columns 61-70**  
ZZ(7) = depths to seventh computational level for first reference stress, in.

**Columns 71-80**  
ZZ(8) = depths to eighth computational level for first reference stress, in.

If the number of nonlinear depths is greater than 8, continue to next card using same format until:

\[
1 = \sum_{j=1}^{X_{12}} X_{12}(j)
\]

where X_{12} is from card No. 7.3 and X_{12} is from card No. 7.4.

---

**Figure B-5. (sheet 6 of 11)**
### GENERAL PURPOSE DATA FORM

#### No. 8 Offset Card (FlO.0)

Columns 1-10 \( XIR \) = number of offset computations made.

This card is needed only for single-wheel load. For multiple wheels,
set \( XIR = 0 \) and branch out to No. 10.

#### No. 9 Offset Distances Card (FlO.0). Not applicable for multiple wheels.

Columns 1-10 \( RN(1) \) = distance to first offset, in.
11-20 \( RN(2) \) = distance to second offset, in.
12-30 \( RN(3) \) = distance to third offset, in.
31-40 \( RN(4) \) = distance to fourth offset, in.
41-50 \( RN(5) \) = distance to fifth offset, in.
51-60 \( RN(6) \) = distance to sixth offset, in.
61-70 \( RN(7) \) = distance to seventh offset, in.
71-80 \( RN(8) \) = distance to eighth offset, in. (if needed)

If the number of offsets is greater than 8, continue to next data card using same format until the number of offsets is satisfied.

---

Figure B-5. (sheet 7 of 11)
### GENERAL PURPOSE DATA FORM

<table>
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<th>PROGRAM</th>
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<td>No. 10</td>
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</table>

#### No. 10 Computational Depths Card (F10.0)

<table>
<thead>
<tr>
<th>Columns 1-10</th>
<th>XIZ = number of depths to computational points</th>
</tr>
</thead>
</table>

#### Notes:

- Columns 1-10 XIZ = number of depths to computational points
- ZZ(1) = first computational depth, in.
- 11-20 ZZ(2) = second computational depth, in.
- 21-30 ZZ(3) = third computational depth, in.
- 31-40 ZZ(4) = fourth computational depth, in.
- 41-50 ZZ(5) = fifth computational depth, in.
- 51-60 ZZ(6) = sixth computational depth, in.
- 61-70 ZZ(7) = seventh computational depth, in.
- 71-80 ZZ(8) = eighth computational depth, in. (if needed)

If the number of depths is greater than 8, continue to next data card using same format until the number of depths is satisfied.

Note: In the main program of CEXVIT, ZZ(1) has a dimension of 99. If ZZ falls on the interface, two computational points are generated, one in each layer. Subroutine WHEELS is dimensioned for 15 points only. Therefore, if subroutine WHEELS is used, the maximum number of ZZ's is 15. However, it should be remembered that at each interface depth, ZZ(1) represents 2 ZZ's in storage.

---

Figure B-5. (sheet 8 of 11)
For problems involving only single-wheel loads, a blank card should be placed after No. 11 input. Thus ends the input into the main program.

For problems involving multiple wheels, the input continues.

No. 12 Multiple Wheels Card (F10.0)
Columns 1-10 XW = number of wheels of multiple-wheel gear assembly.

No. 13 Coordinates Card (F10.0)
Columns 1-10 XW(1) = X-coordinate of first wheel, in.
11-20 YW(1) = Y-coordinate of first wheel, in.

Figure B-5. (sheet 9 of 11)
This format is repeated until the number of wheels XNW is satisfied.

No. 14 Grid Card (F10.0)
Columns 1-10 XNGRID = number of grid generator cards to follow

No. 15 Grid Generator Card (6F10.0)
Columns 1-10 XI = initial X-coordinate of grid
11-20 YI = initial Y-coordinate of grid
21-30 SX = step size in X-direction
31-40 SY = step size in Y-direction
41-50 XNUM = number of lines in X-direction
51-60 YNUM = number of lines in Y-coordinate of grid

(Repeat this card for each XNGRID)
This completes the input for one problem. If another problem follows, return to No. 1 and use the START option. If no additional problems are desired, use the END option.
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Figure B-6. FORTRAN listing for CHEVIT, linear analysis (sheet 1 of 2)
### Figure B-6. (sheet 2 of 2)
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Figure B-7. FORTRAN listing for CHRVIT, nonlinear analysis (sheet 1 of 3)
## GENERAL PURPOSE DATA FORM

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Figure B-7. (sheet 2 of 3)
Figure B-7. (sheet 3 of 3)
APPENDIX C: NOTATION

\( f \) = constant determining rate of relaxation
\( h \) = interval between two points
\( K_1, K_2, K_3, K_4 \) = constants
\( M_R \) = resilient modulus
\( (M_R)_i \) = new \( M_R \) used in the \( i^{th} \) iteration
\( (M_R)_{i+1} \) = new \( M_R \) used in the \( (i+1)^{th} \) iteration
\( p(x) \) = second order polynomial
\( W \) = surface deflection
\( \varepsilon_z \) = subgrade strain
\( \Theta \) = bulk stress
\( \mu = \frac{x - x_1}{h} \)
\( \sigma_d \) = deviator stress
\( \sigma_x + \sigma_y + \sigma_z \) = bulk stress
REFERENCES
