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REVIEW AND ANALYSIS OF SOME COLLISION AVOIDANCE ALGORITHMS WITH PARTICULAR REFERENCE TO ANTC-117

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16. Abstract

Since 1971, Revision 10 of ANTC Report 117, issued by the Air Transport Association of America, has served as the unofficial standard for cooperative mid-air collision avoidance systems. This paper presents the results of an assessment of those parts of ANTC-117 that deal with threat evaluation and maneuver selection logics. Of primary concern were two questions: (1) whether the threat evaluation logics of ANTC-117 provide sufficient time for performing maneuvers necessary to achieve safe separation and (2) whether in dense traffic the expected number of alarms occur infrequently enough for the CAS to be practical. The answers to both questions were found to be negative. The possibility of modifying the threat-evaluation techniques to achieve safe separation was investigated and found to be feasible. However, attempts to reduce the expected alarm rate to tolerable levels proved unsucessful. It is postulated that existing ATC procedures or modifications thereof can substantially reduce the alarm rates by altering the aircraft range and range rate distributions that were assumed and which are the source of the excessive alarm rate.

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METRIC CONVERSION FACTORS

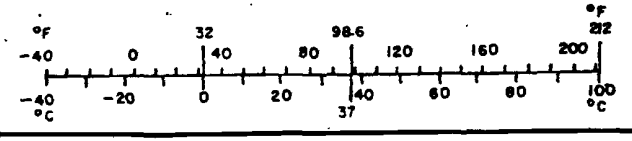
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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I. INTRODUCTION

In 1971, the Air Transport Association of America issued Revision 10 of ANTC Report No. 117 (Ref. 1) on the operational and functional requirements, as well as specifications, for a cooperative mid-air collision avoidance system (CAS). Sponsored by the CAS Technical Working Group of the Airline Air Traffic Control Committee, the original ANTC-117, issued in 1967, and its subsequent revisions, have become an unofficial standard for CAS.

In 1972-73, under contract with the FAA, IDA performed an analysis of three CAS configurations, one of which was proposed by McDonnell Douglas (Ref. 2), one by RCA (Ref. 3), and one by Minneapolis-Honeywell (Ref. 4). During that analysis, it became evident that the requirements of ANTC-117, which were used as a standard for evaluating the systems, had certain deficiencies.

In July 1974, under contract with the FAA, IDA undertook the task of assessing those parts of ANTC-117, Rev. 10, which deal with threat evaluation and maneuver selection logics. This report, in presenting the results of that assessment, is concerned primarily with two questions: whether the threat evaluation logics adopted by ANTC-117 provide sufficient time to perform the maneuvers necessary for achieving safe separation, and whether in dense traffic the expected number of alarms occur infrequently enough for the CAS to be regarded as practical.

Since it was found that the answers to both of these questions are negative, a further investigation was made of the effect of introducing various modifications of the ANTC-117

threat-evaluation techniques in an attempt to improve their adequacy. It was found possible to modify the threat evaluation algorithms to provide sufficient time for encountering aircraft to maneuver to safety, i.e., with an altitude clearance of 150 ft. However, attempts to reduce the expected alarm rate to tolerable levels, even with the assumption that additional data on the relative position and motion of aircraft could be obtained from certain measurements or communications other than those already specified, proved unsuccessful. In accordance with the task statement for this study, bearing and bearing rate were not included among the parameters considered for this purpose.

The 1972 FAA projection of air traffic density over the Los Angeles Basin (LAX) in 1982 (Ref. 5) was the basis for a statistical model of the air traffic environment assumed for the analysis described in this report. It was also assumed that aircraft speeds are normally distributed and that aircraft headings are uniformly distributed random variables.

The negative conclusions concerning the excessive alarm rates predicted for the ANTC-117 threat-evaluation techniques depend on these assumptions and on the selection of associated statistical parameters, which were derived from the FAA 1982 LAX projection. In fact, it was found that some arguable modifications of the traffic density model and the aircraft heading statistics could mitigate the alarm rate problem for dense air traffic. Implementation of the necessary changes in the environmental statistics, however, would undoubtedly depend on air traffic control policy.

Section II provides a description of the ANTC-117 threat-evaluation techniques for CAS. Section III presents the results of a critical analysis of ANTC-117 requirements and specifications. Section IV describes the modified threat-evaluation algorithms necessary to insure that encountering aircraft in a

hazardous situation will have time enough to maneuver to safety. Section V discusses various attempts to reduce the expected alarm rate by introducing additional data on the relative position and motion of encountering aircraft. Section VI summarizes the major implications of the IDA analysis.

A copy of the relevant parts of ANTC-117 is attached as an appendix to this report. References made to ANTC-117 throughout the report are enclosed in brackets.

The other appendices contain the technical details of the analysis from which the report is derived. In particular, the analysis and calculation of the alarm rate appear in Appendix A.

II. SUMMARY OF ANTC-117 EVALUATION AND MANEUVER LOGIC

A. SYSTEM CONCEPT

The specifications given by ANTC-117 for a cooperative airborne collision avoidance system are aimed at insuring the safe separation of aircraft in flight independently of ground control, although ground stations may be required to provide time synchronization for participating aircraft. In order to achieve this goal, several algorithms, involving measured values of the separation distance, separation rate, and altitude difference between two aircraft, as well as the rate of altitude change of the aircraft in which an evaluation of threat is being made, are used to generate warnings and alarms. These, in turn, trigger various cautions and commands to pilots, thereby causing them to perform certain avoidance or escape maneuvers: to stop turning (also known as "rollout"), to limit vertical speeds, to change altitude. Because bearing information is not assumed to be available, escape maneuvers take place in the vertical plane only.

The algorithms which provide alarm criteria are of two types: (1) those based on the separation distance (range), R, and separation rate (range rate), $\dot{R}$; and (2) those based upon the altitude difference Δh between aircraft and altitude rate $\dot{h}$ of an individual aircraft. In principle, since R represents the slant range between aircraft, the R, $\dot{R}$ criteria alone should be sufficient to provide the alarms needed for protection against collision. However, it has been recognized that the use of altitude data is required to reduce the threat volume, and hence the alarm rate, to manageable proportions and also

to determine the proper direction for vertical avoidance maneuvers.

The measurement of R, $\dot{R}$, and Δh is accomplished by means of radio communication between aircraft. The arrival time of a communication signal is used to calculate R, while $\dot{R}$ is obtained from a measurement of the doppler shift, although, recently, the tendency in CAS designs has been to obtain $\dot{R}$ from measurements of R at two different times. Each aircraft measures its own altitude with a barometric altimeter and communicates it in digital steps of 100 ft to other aircraft. Each CAS-equipped aircraft measures its own altitude rate for its own evaluation, but does not communicate this data to others.

ANTC-117 requires, in general, that each aircraft limit its horizontal acceleration to 1/2 G, its vertical acceleration to 1/4 G, and its vertical rate to 5000 ft/min. This implies a maximum relative horizontal acceleration of 1 G and vertical acceleration of 1/2 G, and a maximum vertical separation rate of 10,000 ft/min between two aircraft. [See A-4.b(3),(4) of ANTC-117.]

B. PRIMARY THREAT CRITERIA

In the early days of CAS development, it was proposed that an R, $\dot{R}$ algorithm be based upon a quantity τ, defined by

$$\tau = R/\dot{R} ,$$

representing the time to collision for two aircraft on a non-accelerating collision course. It has since been realized that, because of measurement errors and the possibility that aircraft may be accelerating, a modification of this idea is necessary. Thus, ANTC-117 has adopted two alarm algorithms based on R and $\dot{R}$. These algorithms, sometimes known as modified τ criteria, have the same algebraic form, but different parameters:

$$R + \tau_1 \dot{R} \leq R_{o_1} ,$$

$$R + \tau_2 \dot{R} \leq R_{o_2} ,$$

where τ_1, R_{o_1}, τ_2, R_{o_2} are designated constants.*

In the first algorithm, known as the τ_1 alarm criterion, the designated constants have the values

$$\tau_1 = 25 \text{ sec}$$

$$R_{o_1} = 1/4 \text{ nmi } (1520 \text{ ft}) .$$

In the second algorithm, known as the τ_2 warning criterion, the constants have the values

$$\tau_2 = 40 \text{ sec}$$

$$R_{o_2} = 1.8 \text{ nmi } (10{,}940 \text{ ft}) .$$

If two aircraft are co-altitude (within 600 ft for altitudes below 10,000 ft or 800 ft for altitudes above 10,000 ft), and if measured values of R and $\dot{R}$ satisfy the τ_2 criterion, the pilot is commanded to limit turns (rollout) to a bank angle no greater than 10 deg and to neither climb nor dive. This is supposed to reduce the relative trajectory of the encountering aircraft to an approximately linear horizontal course. Then conditions are assumed to be suitable for the safe use of the more restrictive τ_1 criterion.

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*These algorithms are not actually stated by ANTC-117 in equation form, as is done here, but the equations can be deduced from [B-3.b] and Fig. 2 of that document. Note also that the convention $\dot{R}$ positive corresponding to increasing R is used here rather than the converse used in ANTC-117.

If R and $\dot{R}$ satisfy the τ_1 criterion, vertical maneuver commands are issued, so that the aircraft at the higher altitude climbs while the aircraft at the lower altitude dives until both aircraft are separated by a safe distance. If three aircraft are involved, the aircraft at the intermediate altitude maintains its course, neither climbing nor diving.

The τ_1 criterion has been modified further by the addition of a minimum range criterion, the purpose of which is to protect against the hazardous case of $\dot{R}$ equal to or near zero, since it was felt that with the presence of acceleration and measurement errors, the time available for maneuver might be reduced below an acceptable minimum in this circumstance. Thus, an alarm also occurs with the same commands as for τ_1 alarms if

$$R \leq R_M ,$$

where the minimum range R_M has been designated as 1/2 nmi (3040 ft). The latter alarm will be referred to also as a τ_1 alarm.

C. ALTITUDE CRITERIA

The τ alarms are implemented only if the encountering aircraft are co-altitude, or it is predicted that they may become co-altitude, where "co-altitude" is defined to mean that their vertical separation is less than 600 ft at altitudes below 10,000 ft and 800 ft at altitudes above 10,000 ft. This definition of co-altitude is based upon (1) an altimeter error allowance of $\pm$ 150 ft (3σ) below 10,000 ft and $\pm$ 250 ft (3σ) above 10,000 ft, (2) the assumption that a safe vertical separation between aircraft is 150 ft, and (3) what is intended as an allowance for an undetected vertical drift rate of 500 ft/min.*

*The inadequacy of this allowance for drift rate is discussed in Section III-D and Appendix D.

According to ANTC-117, if an aircraft is climbing or diving at a rate greater than 500 ft/min, it is required to extend its co-altitude protection boundary in the direction of its motion by a predicted co-altitude increment. The predicted co-altitude increment is determined by multiplying the aircraft's own altitude rate $\dot{h}$ by 30 seconds [B-3.a(3)].

In addition to the co-altitude zone wherein the τ alarms become operative, relative altitude buffer zones are established out to ± 3400 ft. In each of these zones vertical altitude rates are limited, from 2000 ft/min in the farthest to successively lower values as the zones get nearer [Fig. 3, 4a, 4b, B-3.e.].

D. LIMITED EQUIPMENTS

ANTC-117 allows the existence of aircraft with limited collision avoidance equipments in order to spare general aviation the expense of a full CAS. Several possible variations are mentioned, but it is implied that unmentioned others may also be permitted.

Of those that are mentioned specifically, there are two types whose properties have an effect on threat evaluation. The first consists of the so-called level one and level two CAS, and the second is the beacon-only equipment.

Since ANTC-117 does not mention that level one and level two CAS-equipped aircraft have the ability to measure altitude rate, it has been assumed here that they do not have this capability. It follows, therefore, that this type of aircraft is unable to use the predicted co-altitude algorithm.

The beacon-only-equipped aircraft has no threat evaluation capability but is required to communicate its altitude. Thus, it can provide the necessary data for threat evaluation to a CAS-equipped aircraft, but is unable to make an avoidance maneuver on command.

III. DEFICIENCIES OF ANTC-117 SPECIFICATIONS

A. GENERAL REMARKS

At present, no one technique for implementing a national standard for CAS has been identified as the best. However, ANTC-117 (Appendix F) presents the thesis that

> The most suitable technique for providing an airborne collision avoidance system at this time is one based on synchronized accurate time permitting range and range rate to be obtained from each transmission as the result of one-way propagation of a radio frequency signal. [A-3.a.(1)]

Since that statement was made, other systems based on a beacon/transponder principle rather than the synchronized time/frequency technique advocated by ANTC-117 have been developed. In some respects, the imposition of ANTC-117 standards on such systems is unnecessarily restrictive. To the extent that the systems adhere to the logic of those standards they must, of course, be adversely affected by its deficiencies.

Moreover, even within the context of the synchronous time/frequency principle, ANTC-117 requirements are much too narrow. They are specifications for a system which has already been designed rather than general standards for a CAS. Such specifications as range and range rate accuracy are not to the point; what should be addressed instead are characteristics such as the reliability of threat detection and avoidance and the false alarm rate, presuming that the logic does not itself generate unnecessary alarms with high frequency (cf. Section III-E).

As a minimum, ANTC-117 avoidance logic assumes that all aircraft involved in threatening encounters must be equipped

with avionics which permit the exchange of altitude information and which permit at least one of two aircraft to measure relative range, range rate, and altitude differences. Aircraft which carry beacons only, and have no means for threat evaluation or parameter measurements are allowed; thus, an exception must be made for the encounter of two beacon-only-equipped aircraft.

In addition, ANTC-117 provides that

> The system shall assure safe vertical separation, even if only one aircraft can or does respond to the maneuver command. [A-2.h.(2)]

and, almost identically,

> The system shall assure safe vertical separation even if only one aircraft can or does obey the climb or descend commands. In the vast majority of all climb/descend avoiding maneuvers involving air carriers or other completely equipped aircraft, both will maneuver cooperatively; however, since exceptional situations may be expected, system design must be predicted (sic) on safe separation should only one aircraft maneuver. [A-4.b.(1)(a)]

Exceptional situations presumably may include: (1) the three-aircraft encounter in which one aircraft may be commanded not to climb or descend [cf. A-4.b.(2)] and (2) one aircraft at such low altitude it would be hazardous to dive. It is evident that in an encounter between a CAS-equipped aircraft which cannot maneuver in such a circumstance and a beacon-only-equipped aircraft the system could not assure safe vertical separation, as required. In fact, it will be shown that cases exist in which ANTC-117 logic will not even protect a fully maneuverable CAS-equipped aircraft in an encounter with a noncooperating aircraft. A similar statement could even be made about present ANTC-117 logic applied to an encounter between two cooperating CAS-equipped aircraft as well; however, if some of the

parameters in the alarm criteria now used were changed (cf. Section IV), that case, at least, could be handled safely by the logic.

B. AMBIGUITIES

The conditions under which two aircraft can encounter each other and result in a τ_1 alarm are not explicitly stated. Presumably, a τ_1 alarm and maneuver command can occur only when the aircraft are in the co-altitude zone [B-3.e., Fig. 3]. Under such conditions, the aircraft are permitted to have vertical rates, since ANTC-117 states that there is

> ...allowance for vertical entry into a threat situation at low altitude rates (e.g., 500 ft/min) which are below the system detection threshold. [A-4.b.(1)(e)]

and, further, that

> Cockpit display of a vertical speed limit within the range of 0-400 ft per minute is acceptable. [B-3.e., Fig. 3, Note 5]

Consequently, it is not clear exactly what residual altitude rates in both aircraft may exist initially when an aircraft receives a τ_1 dive/command after having been in a state of <u>level-off</u>, <u>hold altitude</u>, <u>don't climb</u> (<u>dive</u>), <u>limit vertical speed</u> $\leq$ 500 fpm, $\leq$ 400 fpm, $\leq$ 200 fpm [Fig. 4] or zero. While the CAS altitude rate resolution may be 500 fpm [A-4.b.(1)(e)] and the display may indicate limit rate to 0-400 ft/min, it can be assumed that the pilot can obey a command to level off with an altitude rate approaching zero. However, to show the possible range of problems that this ambiguity allows, some calculations of the probability of inadequate escape maneuvers are given in Appendix D for vertical rates in the range zero to 500 ft/min.

Similarly, the altimeter errors that are given in ANTC-117 are subject to various interpretations. ANTC-117 states that

> The system logic values are based on maximum aircraft altitude system errors of ±250 ft 3 Sigma (Gaussian) at altitudes of 10,000 ft or higher and ±150 [feet] at altitudes of 10,000 ft or lower. [A-3.a.(3)]

and indicates that the altitude is quantized into

> ...100' incremental digitization...[A-4.b.(1)(e)]

One could interpret these statements as implying combined relative errors for two aircraft or the error contribution of a single aircraft. Also, one could suppose that the quantization is either included in the errors of the first reference or is an additional error. Since any specification of an equipment usually addresses the contribution of that specific single equipment, even though it is understood to operate with other mutually compatible equipments, it will be assumed here that the specified errors apply to a single CAS equipment. Further, it will be assumed that systems errors (±150 ft and ±250 ft) include the quantization. It should be noted that if this latter interpretation is correct, all altimeter errors, including those referenced under the detailed equipment standards [e.g., 2.6:2] must be less than 100 ft and 200 ft, 3 sigma, or 33 ft and 66 ft, one sigma. Such low barometric altimeter errors probably do not represent current operational capabilities.

When a τ_1 alarm that requires a mandatory vertical maneuver occurs

> The system shall assure safe vertical separation even if only one aircraft can or does obey the climb or descend commands... [A-4.b.(1)(a)]

ANTC-117 also states that

> Vertical Maneuver--requires a rotation in the direction specified at an [vertical] acceleration of not more than 1/4G, but not less

> than 1/8G up to a terminal climb or descent rate of 2000 feet per minute or more until the maneuver command is terminated. [A-4.a(1)(a)]

In the above "safe vertical separation" is not defined, but a value of 150 ft aircraft center of gravity to aircraft center of gravity is commonly assumed.

The limits of 1/8 G to 1/4 G are usually imposed for passenger comfort rather than because the aircraft has such a limit. Typically, aircraft can accelerate into a climb with higher acceleration without pilot discomfort, but not into a dive without rolling first.

The terminal climb rate of 2000 fpm may not be achieved for many aircraft (e.g., light single aircraft or air carriers at maximum load). However, for the calculations made in Appendix D, the values of 2000 fpm terminal vertical rate and 1/8 G vertical acceleration are used. With such values, about 8 seconds are required to reach the terminal speed, and a single aircraft can change its altitude from level, zero vertical-rate flight by about 330 ft in the assumed minimum available escape time.

In Appendix D, calculations are made of the conditional probability that the safe separation will not be achieved under the following assumptions:

1. Only one aircraft, designated as CAS-equipped, maneuvers. The other, designated as beacon-equipped, does not receive the alarm and continues flight with no change.
2. The CAS-equipped aircraft maneuvers with a vertical acceleration of 1/8 G to a terminal speed of 2000 fpm.
3. There are two conditions of initial vertical rate, zero and 500 fpm, when a τ_1 alarm occurs.

4. If both or one aircraft have initial vertical rates, the worst combination is assumed, e.g., the beacon-equipped aircraft climbing and the CAS-equipped aircraft descending when the CAS-equipped aircraft receives a mandatory climb command. (Note that since the command depends only on altitude separation, it may be more hazardous than that which could be determined if the altitude rates of both were used in the evaluation.)
5. The aircraft have two values of horizontal acceleration which are oriented adversely, i.e., so as to reduce the escape time: 0 and approximately 1/6 G combined acceleration.

Calculations are also made for two aircraft, one beacon-equipped, which have zero vertical rates and for an acceleration of 1/2 G with random orientation.

Although in [A-4.a(1)(c)] there is a statement that the command level-off

> requires decreasing vertical speed to zero in not more than ten seconds,

ANTC-117 does not explicitly provide the assumed response times following alarms. Reasonable values and statistical distributions for response times are assumed in this report when needed, e.g., as in Appendix D.

The diagram in Fig. 1 of ANTC-117 is inconsistent with the text definition [B-3.a.(3)] of predicted co-altitude, since the minimum altitude rate should be 500 ft/min instead of zero, as implied by the diagram. In addition, the text of ANTC-117 [B-3.a(3)] seems to imply that the rules governing commands and advisories for the predicted co-altitude extension are the same as those for the fixed altitude zones, whereas Figs. 3, 4a, and 4b appear to indicate that the command "level off--do not turn" is issued only under conditions which pertain to the predicted co-altitude.

C. CO-ALTITUDE ALARM CRITERIA

In Section IV it will be shown that the τ_1, τ_2 alarm criteria do not always provide enough time for a successful escape maneuver. In accordance with the analysis given there, however, if the values of R_{o_1} and R_{o_2} are changed appropriately, the τ_1, τ_2 criteria can be made to provide complete protection against any co-altitude encounter threat. In fact, this will be true even if the minimum range criterion is dropped; as a consequence, the minimum range criterion is redundant when correct values of R_{o_1} and R_{o_2} are used.

The correct choice of the values of τ_1 and τ_2 depends upon the assumed pilot-aircraft response time (cf. Section IV), which is not specified in ANTC-117. In Ref. 6, a pilot was assumed to respond in 6 sec or less and an aircraft in 2 sec or less with the probability 0.99. If these response times are used in the determination of safe τ values in the alarm criteria with an allowance of 3 sec for the time between measurements, the τ_1 value of 25 sec and the τ_2 value of 40 sec are inappropriate because smaller values could be used for τ_2 at altitudes above and below 10,000 ft while 25 sec is too small for τ_1 at altitudes above 10,000 ft and too large at altitudes below 10,000 ft.

Since the allowance for altimeter error (250 ft) above 10,000 ft is specified as larger than that (150 ft) below 10,000 ft, it is clear, in any case, that the time required for an aircraft to escape a threatening encounter by climbing or descending to a safe altitude will be less for altitudes below 10,000 ft than it will be for altitudes above 10,000 ft. Since Collins Radio has shown (Ref. 7) that the optimum value of τ is the total time required for an escape maneuver after an alarm, it follows that different τ values should be used above and below 10,000 ft.

For non-co-altitude encounters or for co-altitude encounters with adverse residual altitude rate, time must be allowed not only for leveling off if an aircraft is in a vertical motion but also for climbing or diving in a subsequent escape maneuver. Thus, in the non-co-altitude cases the τ values should be increased beyond those appropriate to level flight alarm criteria; e.g., a level flight value of 25 sec for τ_1 would be increased to 30 sec for an aircraft which is climbing or descending with the maximum allowed vertical rate of 5000 ft/min.

D. NON-CO-ALTITUDE ALARM CRITERIA

ANTC-117 provides two kinds of non-co-altitude alarm criteria. The first will be referred to here as NCA and the second, the predicted co-altitude, as PCA.

The NCA criterion defines discrete altitude separation zones with corresponding commands for an ascending or descending aircraft a vertical distance Δh from encountering aircraft to limit its altitude rate as follows* [Fig. 3]:

$800 < \Delta h \leq 1300$ ft , Limit rate $\leq$ 500 fpm
$1300 < \Delta h \leq 1800$ ft , Limit rate $\leq$ 1000 fpm
$1800 < \Delta h \leq 3300$ ft , Limit rate $\leq$ 2000 fpm

In ANTC-117, the boundaries are given as a range of 100 ft, e.g., 800 ft to 900 ft, to indicate the effects of quantizing. Because of quantizing, the indicated altitude might not change when the separation changes by as much as 200 ft, e.g., with indicated altitudes of encountering aircraft of 5100 ft and 6500 ft, neglecting errors, one aircraft may actually be anywhere between 5050 ft and 5150 ft and the other between 6450 ft and 6550 ft, so that the actual separation may be between 1300 ft and 1500 ft. For the aircraft at the higher altitude, the limit command will indicate a descent rate limit but not

*In all cases, Δh is the indicated altitude separation, including the effect of altitude quantizing.

restrict climb rate, while for the other aircraft at the lower altitude, the command will limit the climb rate but not restrict the descent rate.

The PCA criterion defines the altitude separation at which a command "level-off" is given in accordance with the algorithm [B-3.a(3) and Fig. 1]

$$\Delta h = 800 + \dot{h}/2 \ ,$$

in which $\dot{h}$ is the altitude rate of a vertically moving aircraft making an evaluation. The value of 800 ft (closest NCA boundary and minimum indicated vertical separation for $\dot{h} = 0$ in the PCA equation) is changed to 600 ft when the aircraft is below an indicated altitude of 10,100 ft. The commands are displayed to the pilot only if either the τ_1 or τ_2 alarm criteria are satisfied.

As previously noted, there is some ambiguity in ANTC-117 with respect to the onset of the PCA criterion. Specifically, ANTC-117 refers to the PCA criterion as being

> extended from the outer edge [800 or 600 ft] of the co-altitude band in the direction of altitude rate when the rate is greater than 500 feet/minute. The height of this band shall be equal to "own" altitude rate times 30 seconds [1/2 minute]. [B-3.a(3)]

Figure 1 of ANTC-117 shows the boundary of the PCA zone which is a straight line with a slope of 30 sec intersecting the co-altitude zone at 800 ft or 600 ft, but this figure does not indicate the altitude rate at the intersection. Also, Fig. 2 of ANTC-117 shows that the PCA criterion results in a different command level-off, do not turn, than does the co-altitude zone dive/climb, do not turn (τ_1) or don't climb/descend, do not turn (τ_2) and therefore is not a true "extension" of the co-altitude zone. The problem in interpretation of the PCA revolves around the words "...extended from the outer edge of the

co-altitude band..." and "...when the rate is greater than 500 feet/min...". When $\dot{h}$ = 500 fpm is inserted in the equation for PCA, one obtains an altitude separation Δh = 800 ft + 250 = 1050 ft (1000 or 1100 indicated), not the 800 ft suggested by "extended." If it were intended that the PCA criteria only apply for vertical rates in excess of 500 ft/min, then the PCA line in Fig. 1 should terminate at 500 ft/min, corresponding to Δh = 1050 ft for aircraft above 10,000 ft or 850 ft (800 or 900 indicated) below 10,000 ft.

Also, after a PCA alarm, when the altitude rate has been reduced below the value satisfying the PCA criterion, the indicated alarm will presumably change to that governed by the corresponding NCA altitude zone; e.g., an aircraft above 10,000 ft that initially has a PCA alarm and reduces the altitude rate below 800 ft/min by the time the altitude separation is 800 + 800/2 = 1200 ft would experience a command change from <u>level off</u> to <u>limit altitude rate to less than 500 fpm</u>.

In general, ANTC-117 is not very explicit about allowances made for response times and other delays following an alarm. However, it does state

> Level off--requires decreasing vertical speed to zero in not more than ten seconds. [A-4.a.(1)(c)]

and

> If vertical speed exceeds the restriction, it must be reduced in not more than 10 seconds. [A-4.a.(1)(e)]

"Level off," however, permits a maximum vertical rate of 400 ft/min [Note 5 to Fig. 3, B-3.e]. Beyond the references given above to altitude response times, time allowances for maneuvers have not been explicitly given in ANTC-117 [A-5]; however, it is stated that

> ...it is essential that the pilot maneuver without delay when commanded. [A-5.a]

and

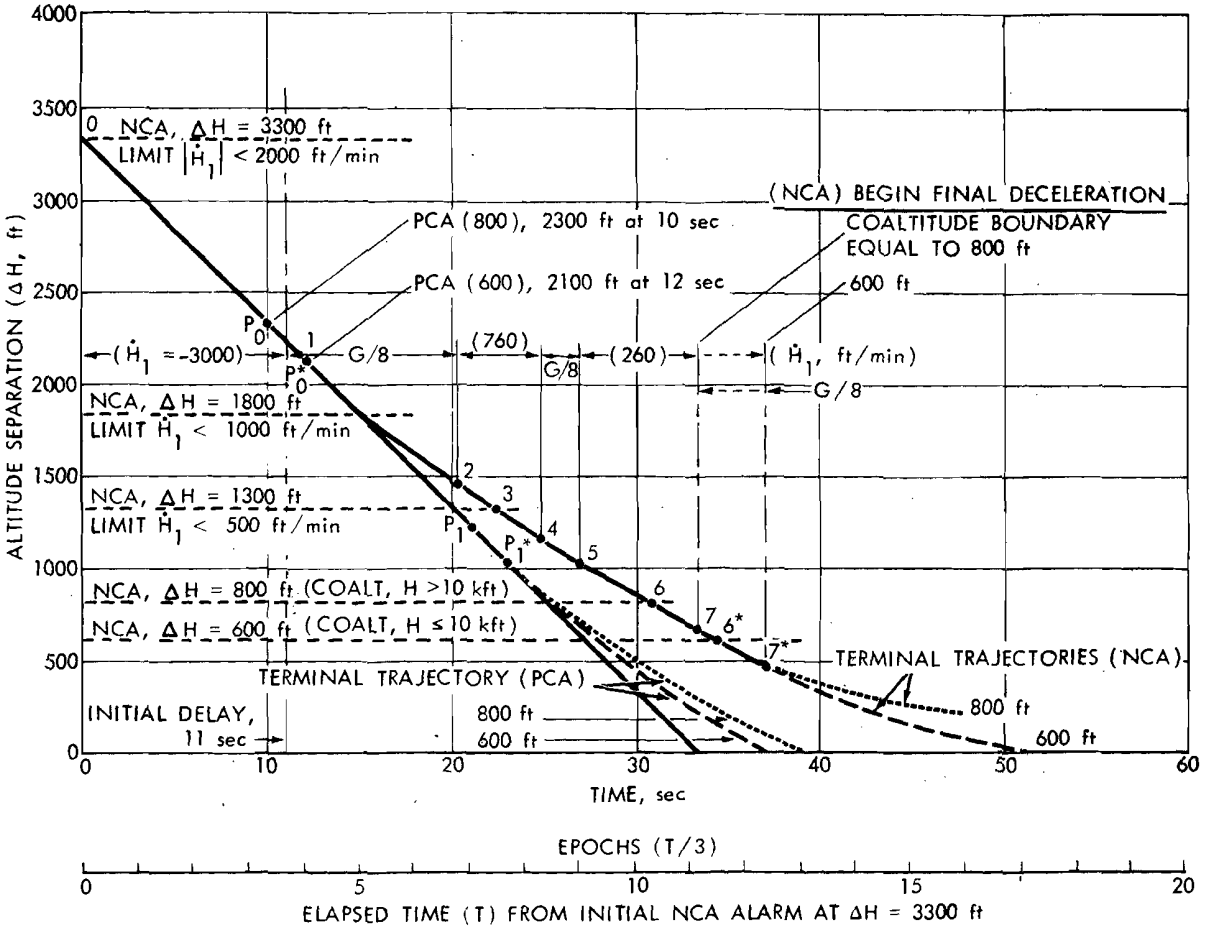
> Additional seconds allowed and added for pilot reaction time, movement of the aircraft controls, the cycling of the CAS system, and CAS error, should be kept to a minimum since all have the effect of increasing the [necessary] protected volume of airspace around each aircraft. [A-5.b.]

Such time responses seem inconsistent with the specified minimum vertical deceleration of 1/8 G and expected pilot/aircraft response times. Examples of allowed vertical rates for given NCA and PCA commands suggest that a 10-second allowance is inadequate, even if the pilot/aircraft response were immediate; e.g., the maximum allowed vertical rate of 5000 ft/min can be reduced with 1/8 G to 2000 ft/min (NCA) in 12.4 sec and to 0 ft/min (PCA, level off) in 20.7 sec; in other words, the increment in vertical rate is reduced at 1/8 G deceleration by about 240 ft/min/sec. Only if the incremental reduction is less than 2400 ft/min can it be accommodated in less than 10 seconds.

When pilot/aircraft response times are included, even smaller incremental reductions in altitude rate are possible in 10 seconds. In this report (cf. Appendix D), combined pilot/aircraft response times are assumed to be normally distributed with mean 3.88 sec and standard deviation 1.95 sec. In addition, the alarm may not be displayed until after a delay of 3 sec (epoch time of the CAS) or longer if the logic of the CAS requires the alarm to persist for more than one epoch period before the alarm is displayed to the pilot. If average values of these times are combined (the average delay due to the epoch interval is 1.5 sec), then 5.38 seconds elapse after the alarm condition occurs (3.88 sec after the alarm is displayed) before the aircraft begins decelerating. Therefore, on the average, only 4.62 sec (6.12 sec measured from alarm display) of the allowed 10 sec is available for deceleration and the maximum

incremental rate reduction is only about 1100 ft/min (or 1500 ft/min) with 1/8 G. If one is concerned with statistically possible events with a probability less than 0.01, the pilot/aircraft response will exceed 8 sec (11 sec including epoch delay), allowing less than 2 sec for deceleration, which accommodates less than 500 ft/min change in altitude rate; the latter is generally the least severe maneuver required.

During an encounter with another aircraft which is not completely equipped, e.g., a beacon-equipped aircraft, or with another equipped aircraft that is not responding to commands, the NCA and PCA may both fail to give adequately early alarms. Figure 1 shows an altitude-separation-versus-time plot for two aircraft, one climbing at 3000 ft/min and one descending at 3000 ft/min. Various changes in conditions are indicated by points keyed to the tabulation on Fig. 1. It is assumed that only the descending aircraft is responding to the alarms. The descending aircraft receives an initial NCA alarm and command to limit descent rate $\leq$ 2000 ft/min at an altitude separation of 3300 ft and responds 11 seconds later with a deceleration of 1/8 G, which lasts for about 9 seconds. At that time, the separation is reduced to about 1400 ft and the altitude rate is reduced to less than 1000 ft/min. During the deceleration, the altitude separation passes through the 1800 ft boundary, and the alarm command is changed to limit rate $\leq$ 1000 ft/min. At this point, the aircraft reduces its deceleration, but about 2 sec later receives another change in command as it passes through the 1300 ft separation zone boundary to limit rate $\leq$ 500 ft/min. It responds to the latter command in about 2.5 seconds for an additional two seconds, bringing its rate below 500 ft/min. About 3 sec later, if the aircraft is within the τ_1 zone above 10,000 ft, it receives a climb command to which it responds in about 2.5 sec and, as shown in Fig. 1, passes the other aircraft with a vertical separation of only about 200 ft. If the aircraft is below



CONDITIONS ALONG TRAJECTORIES

POINT	ΔH, ft	$\dot{H}_1$ ft/min	NOTES
0	3300	-3000	NCA ALARM
1	2200	-3000	BEGIN DECELERATION
2	1450	-760	END DECELERATION
3	1300	-760	NCA ALARM
4	1140	-760	BEGIN DECELERATION
5	1020	-260	END DECELERATION
6	800	-260	NCA ALARM, $H_1 > 10{,}000$
7	670	-260	BEGIN DECELERATION
IF $H_1 \leq 10{,}000$ ft then following in place of 6 and 7			
6*	600	-260	NCA ALARM
7*	470	-260	BEGIN DECELERATION
ALTERNATE PCA CONDITIONS			
P_1	1200	-3000	BEGIN DECELERATION, $H_1 > 10{,}000$
P_1*	1000	-3000	BEGIN DECELERATION, $H_1 \leq 10{,}000$

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FIGURE 1. Exemplary Altitude Separation vs. Time Plot for Hazardous Encounter with two Aircraft with Initial Vertical Rates of 3,000 feet per minute, one Climbing and one Descending.

10,000 ft, the final maneuver command will not be received until the separation reaches 600 ft and the two aircraft will pass each other with zero altitude separation.

Figure 1 shows also that the PCA criterion would sound an alarm initially at an altitude separation of 2300 ft above an altitude of 10,000 ft (2100 ft below an altitude of 10,000 ft), and, if it were the only alarm to be received, the aircraft would begin decelerating only after the separation was less than 1200 ft (1000 ft) if the total response time were 11 seconds after PCA alarm. However, it can be assumed that since the aircraft has already received an NCA alarm, its total response would not be altered from that described for the NCA alarm at 3300 ft; only the indicated alarm would change. Since the aircraft decelerates with 1/8 G at a rate of about 240 ft/min/sec, the PCA alarm criterion is changing at a rate of about 120 ft/sec and the PCA alarm would change back to the NCA alarm at just about the time that the aircraft passes through a separation of 1800 ft. At this point, the NCA alarm would change to limit rate $\leq$ 1000 ft/min. The PCA alarm would not occur again, provided the aircraft limits its altitude rate, as it does in Fig. 1, below that commanded for the NCA zone, i.e., $\leq$ 1000 ft/min above 1300 ft and $\leq$ 500 ft/min above 800 (or 600) ft. In a sense, the PCA is completely redundant and contributes little over and above the NCA for non-co-altitude encounters.

Encounters involving aircraft at initial altitude rates higher than 3000 ft/min, with only one responding, will result in the aircraft passing at zero altitude separation before the final τ_1 or τ_2 maneuver is completed. Shortly after these aircraft pass through zero altitude separation, the command in the responding aircraft would change to the complementary commands; i.e., for a descending aircraft the command would switch from <u>dive</u>, <u>do not turn</u> to <u>climb</u>, <u>do not turn</u> (τ_1) or <u>don't climb</u>, <u>do not turn</u> to <u>don't descend</u>, <u>do not turn</u> (τ_2). Such an occurrence may lead to confusion and anxiety in the cockpit. Clearly,

neither the PCA nor NCA lead to the desideratum of avoiding close altitude separation in cases involving a nonresponding aircraft and high-altitude rates.

For lower initial altitude rates, the encounters with only one responding will be similar to that of Fig. 1, in that one of the aircraft will have a rate $\leq$ 500 ft/min as the 800 (600) ft boundary is crossed and the τ_1 or τ_2 alarm command is given. In these cases, the final altitude separation will increase monotonically to 800 (600) ft as the initial rates are reduced. (Appendix E shows the final separation for such cases.)

E. ALARM RATE

In Appendix A, a calculation is made of the expected alarm rate due to a τ alarm criterion for an aircraft at the center of the terminal area. This calculation is based on a statistical model given in Ref. 4 for the traffic distribution of the FAA 1982 LAX projection as well as a model from Ref. 4 for the distribution of aircraft separation rates.

The result predicts an excessive alarm rate for any modified τ criterion with parameters similar to those required by ANTC-117. In fact, even if no acceleration or measurement error is allowed for, the case in which the parameter R_{o_1} is zero* and the moderate assumption is made that 150 aircraft are in the co-altitude band (at all ranges within the LAX 1982 model), with 6 seconds between measurements, the alarm rate for a τ_1 criterion will be about 5 per minute. Of course, the rate will be much worse for a τ_2 alarm.

As a mitigating factor, it is possible that too large a spread in range rates may have been assumed since the original figure of a 325 ft/sec standard deviation (σ_o) for the range rates at altitudes below 10,000 ft was based upon the assumption of a uniform random distribution of aircraft headings. If

*cf. Section IV-A

it is assumed that all or most aircraft in a co-altitude band travel in the same direction, the value of the standard deviation may be reduced from this figure by as much as an order of magnitude. However, if measurement errors alone are taken into account (R_{o_1} = ∿3000 ft for ANTC-117 specifications*), a simple calculation based on the curve in Fig. A-1a or A-1b of Appendix A shows that the alarm rate will still be at least 3 every 2 minutes, even if the standard deviation of range rates is taken to be zero.

The alarm rate predicted for modified τ_1 criteria used to determine co-altitude threats in the FAA-projected LAX 1982 traffic distribution seems unacceptable by any reasonable standard. It would also appear that this conclusion can be altered only by assuming that a more favorable traffic distribution would occur relative to an aircraft located at the terminal center. The necessary change in the traffic distribution might be brought about by an Air Traffic Control policy of maintaining a more stringent minimum separation distance between aircraft in the terminal area than is predicted by the FAA projection.

In order to provide a model for estimating roughly the effect of maintaining a minimum separation between aircraft in this manner, the double Rayleigh traffic distribution model assumed for the original alarm rate calculation was modified by inserting a hole in the center; e.g., it was assumed that if the aircraft of the example is located at the center, the intruding traffic density would be zero at radial distances smaller than 3000 ft, which is the minimum range criterion specified by ANTC-117. Alarm probability curves based on the resulting traffic distribution, analogous to those given in Fig. A-1b, are given in Fig. A-2.

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*cf. Section IV-A.

For σ_o = 325 ft/sec, i.e., $\sigma_o\tau \sim 8000$ for a τ_1 alarm, the alarm rate is still very high, even when R_o is zero. However, if σ_o is assumed to be an order of magnitude smaller, so that $\sigma_o\tau \sim 800$, then for $R_o = 0$, for an aircraft in the most dense traffic the alarm rate due to 150 intruding co-altitude aircraft, over all ranges, will be less than one in 100 hours when the time between measurements is 6 sec.*

However, if R_o = 3200 ft, to account for measurement errors, then 150 co-altitude aircraft will still cause an alarm rate of about 18 per hour. If, by improving the measurements, R_o is reduced to 2000 ft, the rate will drop to less than 3 per hour, and if R_o is only 1000 ft, the rate will be less than one in 10 hours. Thus, it is apparent that even if the traffic distribution is altered by the imposition of a minimum 3000-ft separation between aircraft, the measurement errors specified in ANTC-117 will permit excessive alarm rates; however, by reducing measurement errors, tolerable alarm rates may be achieved in this case.

Figures A-3 and A-4 provide similar curves for holes in the traffic distribution with radii equal to 1.0 and to 1.5 nmi. As expected, the curves shift to the right as the radius of the hole increases, thereby decreasing the alarm rate for a given value of $\sigma_o\tau$. For none of these cases, nor even for the case of a 3-nmi hole for that matter, will the alarm rate be reasonably low, however, if $\sigma_o\tau$ is as much as 8000, which is the value for a τ_1 alarm based on the assumption of completely random aircraft speeds and headings.

*To obtain the alarm rate in alarms/hr from Fig. A-2 for the stated conditions, multiply the ordinate by 9×10^4.

IV. NONHAZARDOUS ALARM ALGORITHMS

A. CO-ALTITUDE (τ-ALARM) ALGORITHMS

Despite the impression given in previous studies (Ref. 6) that the problem of studying CAS warning criteria is too complicated for an analytical approach, as opposed to an approach based on a computer simulation, the analysis given by Collins Radio (Ref. 7) is applicable to this area. Both Refs. 6 and 7 are cited in ANTC-117, although the influence of the McDonnell Douglas report seems dominant, in that the ANTC-117 τ_1, τ_2 warning criteria are consistent with McDonnell Douglas' recommendations. When the analysis of Ref. 7 is applied to the determination of warning criteria, however, in view of other specifications in ANTC-117, the parameters chosen for the τ_1, τ_2 warnings do not seem adequate.

The Collins analysis shows that for co-altitude aircraft with zero-altitude rates, the only region in the plane from which an intruder x can threaten another CAS-protected aircraft a in a time interval of duration t is given by a circle whose radius is $1/2\ Ut^2$, where U is an absolute bound on the relative acceleration of x and a, and whose center is a distance Vt from the protected aircraft. Figure 2 shows the relative geometry of such an encounter for the time interval of duration τ allowed for escape. In Fig. 2, V is the magnitude of the relative velocity vector of the two aircraft (a and x) which, in the absence of any acceleration U, would result in a miss distance C.

ANTC-117 specifies the measurements that should be made in the collision plane to be relative range R and range rate $\dot{R}$.

The range rate is given by $\dot{R} = -V \cos \theta$ in Fig. 2. Range rate is specified to be measured by the relative doppler shift of the signals transmitted by the aircraft, but it can also be derived from successive range measurements separated by a known time interval as has been done in other proposed CAS (Honeywell, RCA, and Sierra).

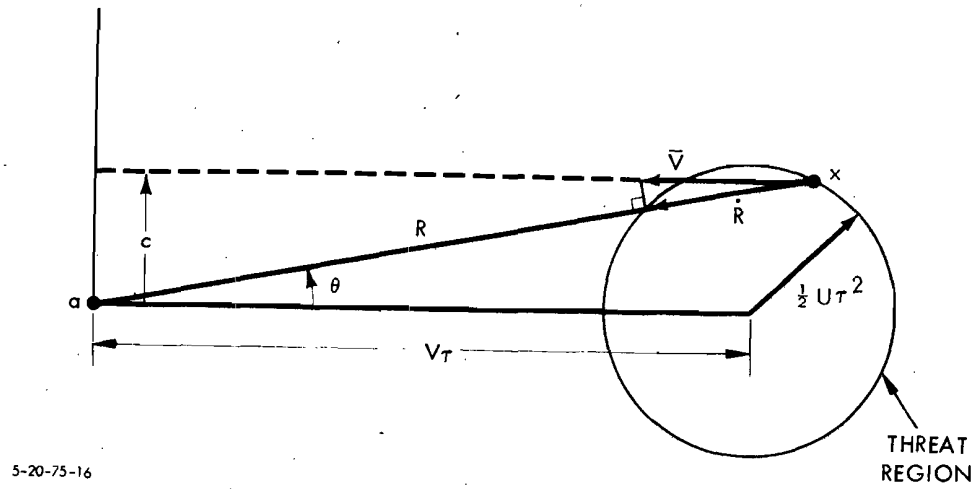

FIGURE 2. Relative Geometry of Encounter Showing Threat Region for Given U, V

Collins Radio has shown that the modified τ criterion

$$R + \tau\dot{R} \leq R_o \ , \qquad (1)$$

where R is the measured distance between two aircraft and $\dot{R}$ is the rate at which that separation distance is increasing, will guarantee a warning in sufficient time to permit the completion of a vertical escape maneuver before collision can occur if appropriate values of the constants τ, the escape time, and R_o, the offset range allowance for acceleration and measurement error, are chosen. It was also shown that, given only the measured values of R and $\dot{R}$, Eq. (1) is the most efficient criterion possible in the sense that if the condition it imposes on R and

$\dot{R}$ is not met, a collision could not occur within the time τ allowed for escape after an alarm, but if the condition is met, there will exist a trajectory for which, if it is not changed by some maneuver, a collision will occur before τ seconds have elapsed.

The result depends upon the existence of an absolute bound U for the relative acceleration between the aircraft and a bound ε on the total measurement error. If ε_R is a bound on the error in the measurement of R and $\varepsilon_{\dot{R}}$ a bound on the error in the measurement of $\dot{R}$, then a bound on the total error would be given by

$$\varepsilon = \tau\varepsilon_{\dot{R}} + \varepsilon_R \ . \tag{2}$$

Since measurement errors are random variables, these bounds can only be specified in terms of confidence limits or, equivalently, in terms of the standard deviation σ of the error distribution.

Collins Radio showed that an appropriate value of the parameter τ in (1) is the total time required to execute the escape maneuver. This time is the sum

$$\tau = t_m + t_r + t_c \ , \tag{3}$$

where t_m is the maximum time between measurements, i.e., the epoch interval, t_r is the combined pilot and aircraft reaction time, and t_c is the time it takes an aircraft to climb or dive to a safe altitude.

The parameter R_o is given by

$$R_o = \tfrac{1}{2}U\tau^2 + \varepsilon \ , \tag{4}$$

unless aircraft are required to roll out, i.e., reduce horizontal acceleration to zero, before the climb or dive escape

maneuver is executed. If rollout is required, then (4) is replaced by

$$R_o = \frac{1}{2}U\left(\tau^2 - t_c^2\right) + \varepsilon \ . \tag{5}$$

Note that (1), with parameters given by (3), and either (4) or (5) takes into account acceleration and constitutes a single criterion. If the two-alarm τ_2, τ_1 warning procedure specified by ANTC-117, with rollout to zero acceleration at a τ_2 alarm, is desired, the earlier, or τ_2, alarm condition would be given by (1), (3), and (5), with t_c replaced by τ_1. If the τ_2 alarm required rollout to a bank angle of 10 deg or less, as actually specified, however, (5) would be replaced by

$$R_o = \frac{1}{2}U\left(\tau_2^2 - \tau_1^2\right) + \frac{1}{2}U_{10}\ \tau_1^2 + \varepsilon \ , \tag{6}$$

where $U_{10} = 2\ G \tan(10\ \text{deg}) = 11.4\ \text{ft/sec}^2$.

Figure 3 shows a representative contour for a Collins-modified τ criterion with rollout, neglecting errors, as well as one for the ANTC-117 τ_1 criterion superimposed on the threat region previously defined for a given value of V* in Fig. 1. The safe Collins criterion is tangent to the threat region at the maximum range along $V\tau$, whereas, generally, except for the minimum range circle, the ANTC-117 τ_1 criterion lies inside the Collins criterion and intersects the threat region. To the extent that the ANTC-117 criterion does not include all of the threat region, it is unsafe for the conditions shown. If only the measured quantities R and $\dot{R}$ are used in the alarm criteria, the alarm volume must, by necessity, be larger than the actual hazard volume, as a comparison of the Collins criterion and hazard contours indicate in Fig. 3.

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*The ANTC-117 circle of radius R_{MIN} shown in Fig. 3 as an addition to the protected region is intended to protect against small R and V. The Collins criterion protects against all R and V without the need for such an addition.

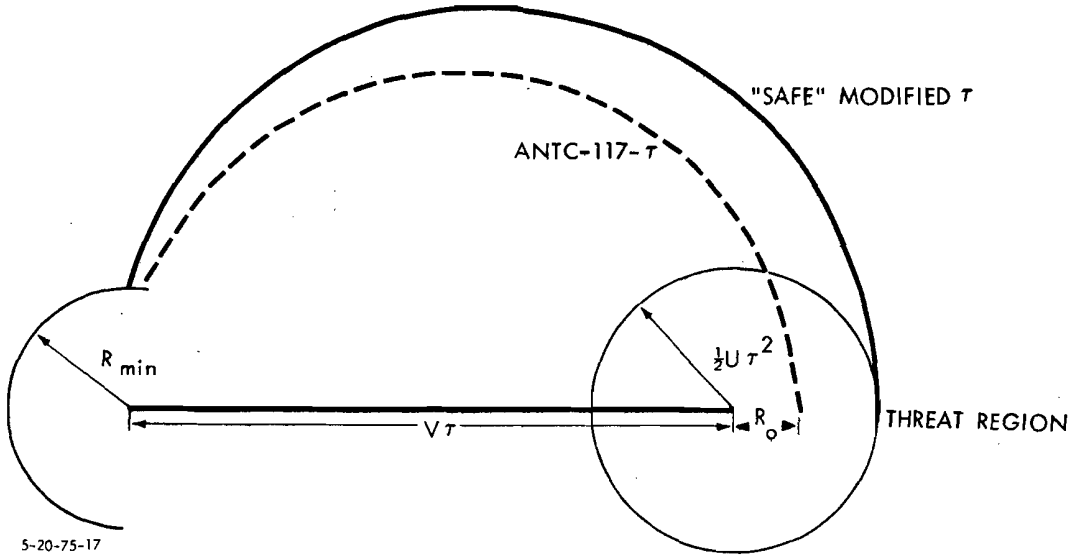

FIGURE 3. Typical Contours for ANTC-117 and Collins Modified τ

The τ_1 alarm allows sufficient escape time if both aircraft have limited their accelerations to bank angle turns of 10 deg by the time the alarm occurs and if

$$R_{o_1} = 1/2\ U_{10}\ \tau_1^2 = 3550 \text{ ft} \quad ,$$

where

$$U_{10} = 11.4 \text{ ft/sec}^2 \ .$$

Assuming both aircraft respond to τ_2 warnings and reduce horizontal acceleration from 1/2 G prior to the warning to that corresponding to a 10 deg bank angle (11.4 ft/sec^2) prior to the τ_1 alarm, the τ_2 warning would allow sufficient escape time if

$$R_{o_2} = \frac{1}{2}U\left(\tau_2^2 - \tau_1^2\right) + R_{o_1} = 19{,}250 \text{ ft} \ ,$$

where

$$U_2 = 32.2 \text{ ft/sec}^2 .$$

In the above refined calculations of R_{o_1} and R_{o_2}, no allowance has been made for possible measurement error. A previous IDA study (Ref. 2) shows that the measurement errors specified by ANTC-117 allow combined errors of about 1060 ft, one sigma. A relatively safe rule of thumb would be to increase R_{o_1} by the 3-sigma error or about 3200 ft. Other proposed CAS designs may have different combined measurement errors and the values of R_{o_1} and R_{o_2} should be changed accordingly.

Clearly, the value of R_{o_1} = 1520 ft given in ANTC-117 does not allow for the worst combinations of allowed horizontal acceleration and measurement errors, the combined effects of which would imply that R_{o_1} = 6750 ft. While the minimum range criterion with R_{min} = 3040 ft does provide better protection at low closure rates (it applies rather than the τ_1 criterion for closure rates less than about 60 ft/sec), it is marginal in that it almost accommodates either the effect of acceleration (3550 ft) or the 3-sigma measurement error (3200 ft), but not both.

The two parameters τ and R_o which appear in (1) determine the modified τ warning criterion. These, in turn, are given by (3) and (4), (5), or (6). Thus, the system parameters which affect the safe warning criteria, and which determine the alarm rate (cf. Appendix A), are the time between measurements or epoch interval t_m, the pilot-aircraft reaction time t_r, the climbing time t_c of the aircraft, the bound U on acceleration, and the total measurement error ε.

In the simulation of Ref. 6, the worst case (99 percent confidence) pilot reaction time was about 6 sec and the worst case aircraft response time about 2 sec, for a total of 8 sec.

The epoch interval chosen for the type of system on which ANTC-117 is based is 3 sec.*

In Ref. 7, an estimate of 400 ft was made for the minimum altitude change a single aircraft would have to make to guarantee the success of a vertical escape maneuver. This estimate was based on the assumption that the minimum desired vertical separation between aircraft is 150 ft in a warning situation, which implies a total altitude measurement error of 250 ft, including both the quantization and random measurement errors. ANTC-117 specifies ± 250 ft as the altitude error above 10,000 ft and ± 150 ft below 10,000 ft. If the escaping aircraft has the minimum vertical acceleration of 1/8 G specified by ANTC-117 and accelerates until it has a vertical speed of 2000 ft/min as required, it will take about 16 sec of climbing (or diving) time to complete the maneuver (400 ft altitude change) above 10,000 ft and about 13 sec (300 ft altitude change) below 10,000 ft.

In summary, then:

$t_m = 3$ sec, $t_r = 8$ sec, $t_c = 16$ sec above 10,000 ft and

$t_c = 13$ sec below 10,000 ft.

The total escape time above 10,000 ft is, therefore, 27 sec, which is greater than the specified value of 25 sec for τ_1, and, below 10,000 ft, 24 sec, which is less than the specified value of τ_1.

For τ_2, the epoch interval t_m and the pilot-aircraft reaction time t_r also contribute. However, since the only action required by the τ_2 warning is rollout in time to respond to a

*Actually, ANTC-117 permits a 6-sec epoch if two antennas, an upper and a lower, are used alternately, and one antenna happens to be obscured by the aircraft fuselage or wings.

τ_1 warning, t_c should be replaced by τ_1 in calculating τ_2. Thus,

$$\tau_2 = t_m + t_r + \tau_1 = 3 + 8 + 27 = 38 \text{ sec above } 10{,}000 \text{ ft} ,$$

instead of the 40 sec now specified by ANTC-117. Below 10,000 ft, τ_2 = 35 sec. If τ_2 is set equal to 38 sec or 35 sec instead of 40 sec, the warning rate will obviously be reduced.

In order to determine the offset distance parameter R_o, the acceleration bound U specified in ANTC-117 as 1 G and the 3-sigma total measurement error ε, which was estimated in Ref. 2 to be about 3200 ft for the McDonnell Douglas CAS, are needed. The values of R_o, suitable for altitudes above 10,000 ft, given by (4), (5), and (6) are then

$$R_o = 10{,}800 \text{ ft, } 14{,}900 \text{ ft, and } 18{,}800 \text{ ft, respectively} .$$

The first two values are based on a single τ of 27 sec, with and without rollout, and the third on τ_1 = 27 sec and τ_2 = 38 sec. Similar values of R_o suitable for altitudes below 10,000 ft (τ_1 = 24 sec and τ_2 = 35 sec) are

$$R_o = 9750 \text{ ft, } 12{,}500 \text{ ft, } 16{,}900 \text{ ft} .$$

B. NON-CO-ALTITUDE ALARM (PCA) CRITERIA

When aircraft are not co-altitude but are climbing or descending, ANTC-117 provides other rules and warning criteria in addition to the τ criteria in order to reduce the excessive alarm rate which would result if it were necessary to consider aircraft at all altitudes. The escape maneuver in this case is to level off in time to maintain safe altitude separation between aircraft. However, the time allowed for this purpose seems inconsistent with that allowed for vertical escape maneuvers in the co-altitude τ warning criteria.

If the epoch interval of 3 sec and the pilot-aircraft response time of 8 sec are assumed for a total time of 11 sec before a level-off deceleration maneuver can begin, then a projection of an aircraft's altitude band when it levels off could be made by adding, if the aircraft is climbing, or subtracting, if the aircraft is descending,

$$11\,\dot{h} + \frac{1}{2}\dot{h}t_d\ ,$$

where $\dot{h}$ is the aircraft's own vertical speed before deceleration and t_d is the time it takes to decelerate.* For the minimum allowable deceleration of 1/8 G

$$t_d = \frac{8\dot{h}}{G}\ ;$$

therefore, the projected altitude separation would be $11\,\dot{h} + \frac{\dot{h}^2}{8}$ instead of 30 $\dot{h}$ as specified by ANTC-117. In addition, a projection should be made for the other aircraft based upon its maximum possible vertical speed of 5000 ft/min. This additional separation would be $917 + 83.3\,(8\,\frac{\dot{h}}{32}) = 917 + 20.5\,\dot{h}$ ft. The specified safe altitude separation of 800 ft after leveling off could then be added, making a total of

$$31.5\,\dot{h} + \frac{\dot{h}^2}{8} + 1720$$

for the vertical hazard region. The maximum altitude separation allowance would be 5200 ft and the minimum separation allowance, based on the minimum detectable altitude rate of 500 ft/min, would be 2000 ft. These numbers refer to aircraft above 10,000 ft. Below 10,000 ft, they can be reduced by 200 ft, since the safe altitude separation in that case is specified as 600 ft.

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*The factor of 11 will, of course, be higher if an epoch interval longer than 3 sec is used.

In addition to those altitude projections for climbing and descending aircraft, an allowance should be made in their τ criterion escape time for the time required to level off since, for example, a climbing aircraft might find it necessary to level off and then dive to complete an escape maneuver. Thus, an additional quantity t_d should be added to τ in this case, so that

$$\tau = t_r + t_m + t_c + \frac{8\dot{h}}{G} \sim t_r + t_m + t_c + \frac{\dot{h}}{4} .$$

For example, with $\dot{h}$ = 2000 ft/min, t_d = 8.3 sec.

V. TECHNIQUES FOR REDUCING THE THREAT VOLUME

A. THREAT VOLUME REDUCTION

It was pointed out in Section III that τ alarm criteria, even for co-altitude encounters only, will produce unacceptably high alarm rates. Thus, it seems necessary to look for additional criteria in order to reduce the threat volume. Some possibilities are discussed in this section.

While it does not appear feasible to measure the relative acceleration between two aircraft because of the inherent sensitivity of acceleration to measurement error, as was observed by Collins Radio in Ref. 7, each aircraft should be able to measure its own absolute acceleration with little difficulty or additional expense. It should also be relatively easy for an aircraft to communicate such a measurement to other aircraft. In addition, a bound on the relative cross range component of velocity between two aircraft can be estimated from three range or range rate measurements made at three different times on a trajectory.

The possible application of measurement and communication of acceleration, as well as the measurement of cross range velocity, to the reduction of the threat volume is discussed in this section. The effect on the alarm rate when a cross range velocity criterion is used in addition to the τ criterion is analyzed in Appendix B.

B. ACCELERATION CONSTRAINTS

In the Collins analysis, the direction of the absolute accelerations A_x and A_a, due to two encountering aircraft, x and a, were unconstrained. The acceleration of an aircraft will

usually be larger perpendicular to its velocity vector than along its velocity vector. It was thought that if the acceleration of an aircraft were constrained to be perpendicular to its velocity vector, i.e., to a turning acceleration, the threat region against which the CAS must protect might be reduced. A general analysis of this question will be found in Appendix C. However, the following, more specialized discussion suffices to indicate why such a constraint will not reduce the threat region significantly.

Figure 4 shows the relative velocity geometry for two aircraft, a and x, with velocities V_a and V_x for two different encounter conditions: one in which the aircraft are nearly head-on and one in which aircraft x is overtaking aircraft a. In Fig. 4a, the time to collision can be reduced significantly if the aircraft are turning so as to increase the angle ϕ between their velocity vectors, i.e., so as to increase the relative velocity vector $\overline{V}$; whereas, in Fig. 4b, the time to collision cannot be reduced as much, since once $\phi = \pi$, any further turning by either or both aircraft will reduce the magnitude of $\overline{V}$ from its maximum of $|V_a| + |V_x|$. Thus, the threat boundary

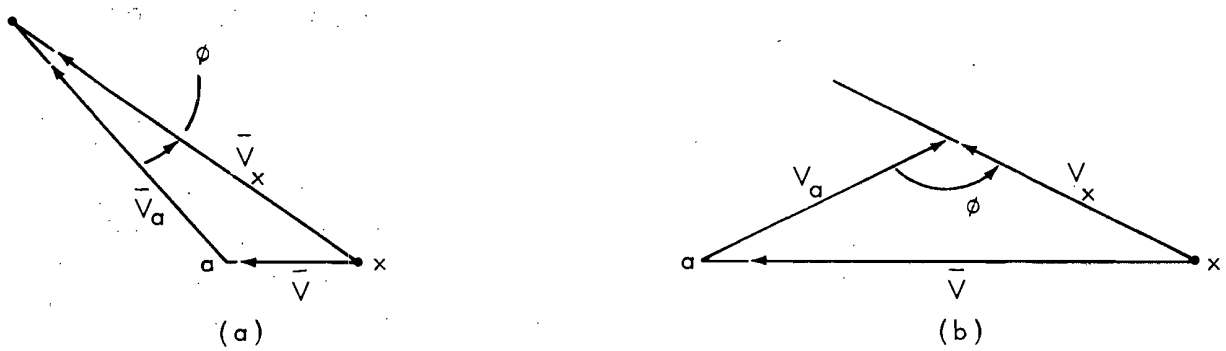

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FIGURE 4. Relative Velocity Geometry of Two Encounters

given by Collins will be approached if ϕ is increased by the aircraft turns. Further, this boundary, a circle of radius $U\tau^2/2$, will be most closely approximated if the velocity vectors are large relative to the turning rates for a given acceleration. Under these conditions, the relative acceleration can lie approximately along the direction of increasing separation and thus make the maximum contribution to $\dot{R}$. For example, assume that the two aircraft have the maximum speed allowed below 10,000-ft altitude (i.e., $V_a = V_x = 250$ knots or 422 ft/sec) and are flying along constant altitude parallel paths in the same direction, such that $\phi = 0$ and $\overline{V} = 0$. If measurement error and epoch measurement time are ignored, one would then calculate, for the maximum allowed acceleration of A = 1/2 G for each aircraft, in the absence of any alarm or warning, that the safe alarm distance for $\tau = 40$ sec is

$$D' = 1/2\ U\tau^2 = 25{,}760 \text{ ft} .$$

If the accelerations were constrained to be perpendicular to the velocity vectors of each aircraft, one should use:

$$D = D' \left[\frac{\sin(\omega\tau/2}{\omega\tau/2)}\right]^2 = D'f^*$$

where

$$\omega = A/V_a .$$

In the limit of $\omega\tau/2 \to 0$, the factor f multiplying D′ approaches unity. For a given acceleration in both aircraft and a given τ, $\omega\tau/2 \to 0$ corresponds to $V_a = V_x \to \infty$. For the assumptions previously used of $V_a = V_x = 422$ ft/sec and $A_a = A_x = 1/2$ G, $\omega/2 = 0.019$ radius/sec. Thus, $\omega\tau/2$ has a minimum value of 0.76 for $\tau = \tau_2 = 40$ sec and f = 0.82. For $\tau = \tau_1 = 25$ sec, $\omega\tau/2$ has a minimum value of 0.48 and f = 0.93.

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*cf. Appendix C.

In summary, for 25 sec and 40 sec, one would calculate the following safe distances to which the aircraft could approach if both aircraft were allowed 1/2 G maneuvers.

Time-to-go (sec)	$D' = U\tau^2/2$ (ft)	$D = D'f$ (ft)
$\tau = \tau_2 = 40$	25,760	21,130
$\tau = \tau_1 = 25$	10,060	9,320

In the above calculations, it is assumed that neither aircraft has limited its horizontal acceleration below 1/2 G at either the τ_2 or τ_1 boundary. In ANTC-117, it is implicitly assumed that both aircraft receiving τ_2 warnings will reduce horizontal accelerations before the τ_1 alarm boundary is reached. ANTC-117 allows each aircraft to have an acceleration corresponding to a 10 deg bank angle turn (5.68 ft/sec^2, approximately 1/6 G) in the τ_1 region. For such an acceleration, $\omega/2$ in the above expressions equals 0.0067 and the factor f for $t = \tau_1 = 25$ sec is 1.000.

Thus, for $A_a = A_x = 5.68$ ft/sec^2, and $V_a = V_x = 422$ ft/sec

	D' (ft)	D (ft)
$\tau = \tau_1 = 25$	3550	3550

Clearly, for this case, the constrained and unconstrained aircraft horizontal accelerations give identical results.

For aircraft speeds V_a and V_x slower than 422 ft/sec, the factor f can be less than unity at $\tau = \tau_1$, e.g., for $\omega\tau/2 = 0.81$, $f = 0.8$ and corresponds to $V_a = V_x = 88$ ft/sec, which is probably close to the lower limit on aircraft speed. Thus, for τ_1 and the allowed 10 deg bank angle turns, a reasonable and safe approximation at all speeds is to use $D' = U\tau_1^2/2$ rather than the value D for constrained accelerations, since, within the guidance provided by ANTC-117, the aircraft involved generally would have insufficient information on both aircraft velocities to take advantage of the reduction implied by $f < 1.0$.

For accelerations constrained to be perpendicular to the aircraft velocity vectors, the value of R_{o_2} corresponding to the maximum allowable aircraft speed of 422 ft/sec below an altitude of 10,000 ft is

$$R_{o_2} = 2R_1[1 - \cos\theta_1] + 2R_2[\cos\theta_1 - \cos\theta_2] ,$$

where

$$R_1 = V_a^2/A_1$$

$$R_2 = V_a^2/A_2$$

$$\theta_1 = (A_1/V_a)(\tau_2 - \tau_1)$$

$$\theta_2 = (A_2/V_a)\ \tau_1 + \theta_1 ,$$

with

$V_a = V_x = 422$ ft/sec

$A_1 = 1/2$ G, the acceleration between τ_2 and τ_1

$A_2 = 1/6$ G (10 deg bank), the acceleration after τ_1 .

On substituting the parametric values, it is found that $R_{o_2} = 17{,}690$ ft. Thus, the value of R_{o_2} for unconstrained acceleration is conservative in that it is larger than the constrained value given in Section IV by 1560 ft, or about two seconds more for the maximum allowable value of $\dot{R} = 844$ ft/sec below 10,000 ft. Above an altitude of 10,000 ft where the aircraft velocities can exceed 422 ft/sec, the unconstrained value of $R_{o_2} = 19{,}200$ ft should be used. It should be reiterated that this calculation of R_{o_2} does not include measurement errors and must be increased accordingly.

C. EFFECT OF KNOWN HORIZONTAL ACCELERATION ON TAU PARAMETERS

In Section IV, it was shown that the magnitudes of R_{o_1} and R_{o_2} that should be used are larger than the corresponding values used in ANTC-117. Also, the values of R_{o_1} and R_{o_2} were calculated, assuming that both aircraft respond to alarms and limit the magnitude of their accelerations to the specified values: 1/2 G before τ_2 and approximately 1/6 G during τ_1. No adjustment was made in R_{o_1} or R_{o_2} should the CAS of either aircraft have available knowledge of the magnitude of the horizontal acceleration of either or both aircraft. Since the sum of the magnitudes of the allowable accelerations ($U = |A_a| + |A_x|$) is used in calculating R_{o_1} and R_{o_2}, the values of R_{o_1} and R_{o_2} will generally be larger than necessary, given complete or even partial knowledge of the accelerations. On the other hand, it is possible, according to ANTC-117, that only one of the aircraft will respond to the τ_2 alarm by limiting its horizontal acceleration to approximately 1/6 G, and that the other will continue to turn at 1/2 G. In such a case, the values of R_{o_1} and R_{o_2} must be increased beyond those calculated in Section IV.

The values that should be chosen for the R_{o_1} and R_{o_2} then depend on the state of knowledge of the acceleration of each aircraft and whether or not both aircraft respond to the alarms. Inasmuch as the difference between the constrained and unconstrained cases is insignificant for τ warnings, only values for the constrained cases will be calculated here. The usual relationships for R_{o_1} and R_{o_2} should be modified as follows:

$$R_{o_1} = (A_{a1} + A_{x1})\ \tau_1^2/2 \ ,$$

where

A_{a1} = the acceleration allowed for aircraft a during τ_1 ,

A_{x1} = the acceleration allowed for aircraft x during τ_1 ,

and

$$R_{o_2} = (A_{a2} + A_{x2})(\tau_2^2 - \tau_1^2) + R_{o_1} ,$$

where

A_{a2} = the acceleration allowed for aircraft a during $\tau_2 - \tau_1$,

A_{x2} = the acceleration allowed for aircraft x during $\tau_2 - \tau_1$.

If the actual horizontal acceleration of neither aircraft is used, one should set $A_{a2} = A_{x2} = 1/2$ G. Also, if both aircraft must respond to the τ_2 warning and reduce their respective accelerations to the value required by ANTC-117, then one can set $A_{a1} = A_{a2} = 5.68$ ft/sec^2. If, however, one of the aircraft is allowed to ignore a τ_2 warning and not reduce its horizontal acceleration, one should use $A_{a1} = 5.68$ ft/sec for the responding aircraft acceleration and $a_{x1} = 1/2$ G for the aircraft not responding. If any aircraft knows and uses its own acceleration A_a but not that of the other, then it should use the values given above (A_{x1} and A_{x2}), depending on the assumed state of the other's acceleration.

If the other aircraft communicates its actual acceleration, then an aircraft can use that value and its known acceleration in the calculations of R_{o_1} and R_{o_2}. Table 1 summarizes these cases. Note that if both aircraft are in unaccelerated horizontal flight, R_{o_1} and $R_{o_2} \rightarrow 0$ if one ignores measurement errors.

By inspection of Table 1, which does not include the measurement error, one can observe that the value of R_{o_1} and R_{o_2} given by ANTC-117, which assumes no knowledge of horizontal acceleration, is not large enough to accommodate allowed horizontal accelerations of both aircraft even if both respond. Even if partial knowledge of horizontal acceleration were

TABLE 1. PARAMETRIC VALUES OF R_{o_1} AND R_{o_2} FOR GIVEN STATE OF KNOWLEDGE OF AIRCRAFT HORIZONTAL ACCELERATIONS

Parameter	ANTC-117	Modified Tau with Acceleration Information,* Both Responding			Modified Tau with Acceleration Information,* Only One Responding		
		(None)	(Partial)	(Full)	(None)	(Partial)	(Full)
R_{o_1} (ft)	1,520	3,550	1,775- 3,550	0- 3,550	6,810	5,920- 6,810	0- 6,810
R_{o_2} (ft)	10,940	19,250	9,620-19,250	0-19,250	22,500	12,880-22,500	0-22,500

*Acceleration Information:

"None" means that neither aircraft horizontal acceleration is known or used in calculating R_{o_1} and R_{o_2}. Therefore, the maximum allowable accelerations of each aircraft are used, i.e.,

$$A_{a1} = A_{x1} = 5.68 \text{ ft/sec (10-degree bank) and } A_{a2} = A_{x2} = 1/2 \text{ G}$$

with both responding; otherwise, A_{x1} = 1/2 G if aircraft x does not respond.

"Partial" means that the known horizontal acceleration of one aircraft, a, is used; i.e., $A_{a1} \leq 5.68$ ft/sec ($\leq$ 10-degree bank) and $A_{a2} \leq 1/2$ G and A_{x1} and A_{x2} are defined under "none".

"Full" means that the horizontal accelerations of both aircraft are known and used.

assumed and both aircraft respond, the R_{o_2} value of ANTC-117 is inadequately small unless the one known acceleration, A_{a2}, is nearly zero, and the R_{o_1} value is too small for any value of A_{a1}. While partial knowledge of horizontal acceleration can reduce the lower limits of R_{o_1} and R_{o_2} by a factor of about one-half and full knowledge can reduce the lower limit to zero, the upper limit for both partial and full knowledge of horizontal acceleration is the same as that for none. Therefore, the effectiveness of knowledge of horizontal acceleration is a function of the occurrence of acceleration. According to the MITRE Los Angeles Basin 1982 Model, approximately 10 percent of the aircraft have horizontal accelerations, i.e., are in turns, so that knowledge of acceleration should be effective in reducing the values of R_{o_1} and R_{o_2} in at least 80 percent of the cases.

The reduction of R_{o_1} and R_{o_2} causes a reduction in the effective alarm volumes.[1] The ANTC-117 volumes are smaller (but not safe) than those needed if maximum accelerations must be accommodated, but are somewhat larger than those needed if complete knowledge of horizontal acceleration is available in the 80 percent of cases in which horizontal acceleration is zero. As observed in Section III, however, the alarm rates cannot be significantly reduced below those of ANTC-117, even with full knowledge of acceleration, unless the projected 1982 LAX traffic statistics are altered and the estimated distribution of range rates can be modified.

D. COMMUNICATION OF HORIZONTAL ACCELERATION AND ALTITUDE RATE

As has been previously suggested, horizontal acceleration and altitude rate information on both aircraft can be used to refine the alarm criteria and, at least when the aircraft have no horizontal accelerations or altitude rates, result in a reduction of protected alarm volumes. Some of the proposed CAS can provide communications modes between aircraft (e.g., the

RCA system and the ANTC-117 "full" CAS), while others cannot (e.g., ANTC-117 lower-level CAS, beacons, and the Honeywell system).

There is probably no simple modification of the signal structure to provide communication in any of the CAS that do not already have this capability. However, if a single modified τ logic criterion is deemed acceptable (there are other arguments in favor of using a single τ rather than the two τ criterion of ANTC-117, particularly if one aircraft is permitted to ignore warnings), the signal structure could be modified at least to the extent of communicating horizontal acceleration. Since the contribution of each aircraft is additive in the term $U\tau^2/2$ of R_o in a modified single τ criterion, each aircraft could delay its transmission by a precise amount proportional to its horizontal acceleration to increase the measured value of R by an amount equivalent to adding the correct acceleration term to R_o.

It would be necessary, of course, for each aircraft to be instrumented with acceleration sensors. Since some aircraft may not have adequate accelerometers, these aircraft could delay their transmissions by a fixed amount corresponding to the maximum allowed acceleration. The latter compromise, of course, is less desirable.

The method of implementing altitude rate communication in a simple manner is not as apparent for all CAS designs. Again, it would depend on what is available in the aircraft. However, there is a simple scheme to accomplish this for the particular ANTC-117 specified CAS design. In that system, a 200 usec ranging pulse is followed by a 25 μsec pulse which is position coded to designate altitude. The 25 μsec altitude pulse could be single side band modulated with a frequency offset proportional to the altitude rate. The peak frequency deviation should be made sufficiently large to reduce the effect of the

doppler shift induced by the range rate; alternatively, this error could be removed by differencing with the 200 μsec doppler pulse. Frequency discriminators in the existing ANTC-117 CAS design may be suitable for this communication of altitude rate. For aircraft not equipped with altitude rate meters, a fixed frequency shift proportional to the maximum allowed altitude rate could be used, but it would be necessary to indicate the sign of the rate, a requirement which could probably be met without too much difficulty. It should be observed that the resolution of the quantized altitude is probably inadequate for the purpose of obtaining altitude rate, since the resolution for a 3-second epoch time is only about 2000 ft/min.

E. CROSS-RANGE VELOCITY

In Appendix B, the estimate of a bound on the relative cross-range velocity between two aircraft and its use as a supplementary alarm criterion is discussed. If neither aircraft is accelerating and the measurement errors are zero, the cross range velocity criterion reduces the alarm rate to zero according to equations (B-16) and (B-13) of Appendix B. The effect is to reduce a three-dimensional threat region to a two-dimensional one.

The effectiveness of this criterion depends crucially on the range and range rate measurement errors. Those specified or implied in ANTC-117 are too high for the cross-range velocity criterion to be useful; however, Minneapolis Honeywell claims much smaller measurement errors for their system. Therefore, it may be feasible for some CAS designs to incorporate the cross range velocity criterion, thereby significantly reducing the threat volume and hence the alarm rate.

VI. SUMMARY AND CONCLUSIONS

The following observations can be made concerning ANTC-117 as a standard for CAS.

1. Its requirements are much too specific; they pertain to a single example of a particular system concept, i.e., one version of the synchronized time-frequency type of CAS. Thus, ANTC-117 limits unnecessarily the flexibility of competitive system designs.
2. It proposes threat criteria which do not allow sufficient time for the required escape maneuvers in all circumstances. In particular, the criteria do not provide adequate protection for encounters between a beacon-equipped aircraft and one equipped with the full recommended CAS.
3. The alarm criteria specified by ANTC-117 for threat evaluation will probably result in alarm rates which are intolerable by any standards, i.e., several τ_1 alarms per minute for an aircraft in dense traffic at the center of the 1982 LAX basin.
4. The use of two modified τ criteria, one to provide an early warning (τ_2), to be followed by a reduction of acceleration, and the other an alarm (τ_1), to be followed by evasive maneuvers, is ineffective against beacon-only-equipped-aircraft and will increase the alarm rate unnecessarily. A single modified τ alarm criterion could provide as much protection in encounters among CAS-equipped aircraft and superior protection in encounters between a CAS-equipped aircraft and a beacon-only-equipped aircraft. Such a single τ

criterion would have a smaller alarm rate than that resulting from the τ_2 warning now specified by ANTC-117.

5. If range and range rate errors are specified at all (a national standard for CAS should be concerned with system performance in terms of characteristics such as escape time and alarm rate rather than details pertaining to the techniques used in the system concept), the tolerances should probably be considerably smaller than those permitted by ANTC-117. Otherwise, the hazard region defined by the available threat criteria will be much too large.
6. ANTC-117 specifications of range, range rate, and altimeter errors and the allowed residual vertical drift rate of aircraft are ambiguous.
7. ANTC-117 does not define safe separation between aircraft, nor does it specify the pilot-aircraft reaction time.*
8. The predicted co-altitude threat criterion of ANTC-117 is inadequate and should be replaced by a better algorithm (cf. Section IV-B). The present algorithm, for example, does not guard against a noncooperating aircraft such as one equipped only with a beacon.

A reasonable definition of a safe alarm criterion, consistent with the conventional idea of a safe vertical separation distance between aircraft, is one that allows sufficient time after an alarm occurs to achieve a vertical separation of at least 150 ft. The modified τ alarm criterion can be made safe in this sense for all encounters involving a CAS-equipped aircraft by increasing the offset range parameter R_o sufficiently.

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*For these factors, this report uses the following values which have been used by other evaluators in the past: 150 ft vertical separation and 8 seconds maximum pilot-aircraft reaction time.

Safe τ criteria which produce the smallest hazard regions should be different above and below 10,000 ft, i.e., both τ and R_o should have different values in high- and low-altitude bands. The value of τ should also increase for an ascending or descending aircraft to allow time for leveling off as well as for the escape maneuver.

Some techniques, other than those prescribed by ANTC-117, which should not be too difficult to implement, can be used to reduce the alarm rate. These include (1) having each aircraft measure and communicate its own acceleration, which would permit the use of smaller values of the range offset R_o in the τ criteria and thereby reduce the hazard region whenever an aircraft has less acceleration than that normally allowed for; (2) communication of altitude rate, which would permit the use of a more accurate predicted co-altitude algorithm and thereby reduce the hazard region for a climbing or descending aircraft as well as the residual errors due to the existence of finite vertical rates for co-altitude aircraft. It is also possible to estimate a bound on the relative cross-range velocity between two aircraft from a sequence of several range or range rate measurements (cf. Appendix B). This bound could be used to formulate a supplementary alarm criterion to reduce the hazard region if the range and range rate measurements were sufficiently accurate.

However, even with the aid of all of these techniques, it is doubtful that the alarm rate can be made tolerable in the densest region of the traffic predicted by the FAA 1982 LAX projection. Without altering the assumed traffic distribution, the best one could expect would be a reduction of the alarm rate from several per minute to about one every few minutes.

The statistical model assumed herein for the distribution of range rates, which is based on the assumption that aircraft headings are uniformly distributed, might be made more realistic

by taking into account the fact that co-altitude aircraft tend to have the same heading. This effect should produce a decrease in the range rate standard deviation, perhaps by as much as an order of magnitude.

In addition, it might be assumed, as a modification of the LAX 1982 traffic projection, that air traffic control will maintain a minimum separation distance, perhaps somewhere in the range 1/2 to 1-1/2 nmi, between aircraft in the terminal area. If both the range and range rate distributions are modified as suggested, a reasonable improvement in the range and range rate measurement accuracy over that specified by ANTC-117, e.g., 30 to 50 percent, would permit a reduction in the value of the R_o used in the τ criteria sufficient to reduce the alarm rate to less than one every 10 hours.

In fact, if it is valid to assume that a minimum separation distance will be maintained between aircraft in the terminal area, then for sufficiently small values of the range rate standard deviation, the alarm probability will be very sensitive to changes in R_o. For larger values of the standard deviation, the alarm probability will remain quite insensitive to changes in R_o. This suggests that the effectiveness of CAS threat evaluation techniques which depend exclusively on range, range rate, altitude, and altitude rate measurements depends critically on how large aircraft separation rates are and how small aircraft separations are permitted to get.

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APPENDIX A

ALARM RATES FOR ANTC-117 LOGIC

APPENDIX A

ALARM RATES FOR ANTC-117 LOGIC

A. ALARM PROBABILITY

This appendix will treat the probability of alarm due to a modified τ criterion, and the results are independent of the particular CAS employed. The calculation will be made for a centrally located aircraft in an altitude band below 5000 ft where the traffic density is highest. Only the case in which no aircraft are climbing or diving will be considered so that all alarms are due to co-altitude traffic.

It will be assumed that the probability distribution of aircraft about the given centrally located aircraft is independent of direction and has the double Rayleigh form discussed in Ref. A.1 as a model for the FAA projected 1982 Los Angeles traffic pattern. It will also be assumed that the distribution of aircraft velocities is independent of position and obeys the Gaussian distribution law given in Ref. A.1. The joint probability density function $P(R, \dot{R})$ of the distance R between the given aircraft and other aircraft in the traffic and the separation rate $\dot{R}$ for those aircraft will then have the form

$$P(R, \dot{R}) = P_R(R)\ P_V(\dot{R}) \ ,$$

where

$$P_R(R) = \frac{R}{2\pi}\left(\frac{\gamma}{\sigma_1^2}\, e^{\frac{-R^2}{2\sigma_1^2}} + \frac{(1-\gamma)}{\sigma_2^2}\, e^{\frac{-R^2}{2\sigma_2^2}}\right) \qquad \text{(A-1)}$$

and

$$P_V(\dot{R}) = \frac{1}{\sigma_o\sqrt{\pi}} e^{\frac{-\dot{R}^2}{2\sigma_o^2}} .$$

According to Ref. A.1

$$\gamma = 0.36$$

$$\sigma_1 = 7.5 \text{ nmi}$$

$$\sigma_2 = 20 \text{ nmi}$$

for altitudes below 5000 ft, and

$$\sigma_o = 325 \text{ ft/sec} .$$

The value of σ_o is that given for aircraft below 10,000 ft. It may depend more specifically on altitude, in which case 325 ft/sec may be too large for aircraft in an altitude band with the highest traffic density.*

The probability p of an alarm due to a single aircraft chosen at random (ignoring altitude criteria) from the population defined by (A-1) can be expressed as

$$p = 2\pi \int_{-\infty}^{\frac{R_o}{\tau}} P_V(\dot{R}) \int_0^{(R_o - \tau\dot{R})} P_R(R) dR d\dot{R} , \qquad \text{(A-2)}$$

where it has been assumed that the alarm criterion is

$$R + \tau\dot{R} \leq R_o .$$

*This value for σ_o also depends on the assumption that aircraft have uniformly distributed random headings. If, instead, co-altitude aircraft tend to have the same heading, then 325 ft/sec may also be too large for that reason.

It should be noted that no minimum range criterion is assumed. If a minimum range criterion were added, the value of p, of course, would be larger than that given by (A-2).

If p_h is the probability that an aircraft chosen at random from the total number N of aircraft in the terminal area is in the given altitude band centered at h, then the probability P_n that n aircraft will be in this altitude band is given by

$$P_n = \binom{N}{n} p_h^n (1 - p_h)^{N-n} . \tag{A-3}$$

The probability that these aircraft produce no alarms is $P_n(1- p)^n$; thus, the probability P_h that at least one alarm is produced by the co-altitude aircraft is given by

$$P_h = 1 - \sum_{n=0}^{N} P_n (1 - p)^n . \tag{A-4}$$

If (A-3) is substituted into (A-4), the result is

$$\begin{aligned} P_h &= 1 - \sum_{n-0}^{N} \binom{N}{n} [p_h(1-p)]^n (1-p_h)^{N-n} \\ &= 1 - [p_h(1-p) + (1-p_h)]^N = 1 - (1 - pp_h)^N . \end{aligned} \tag{A-5}$$

It may be anticipated that p is so small that

$$(pp_h)^2 N << 1 .$$

Then a good approximation to (A-5) will be given by

$$P_h \sim 1 - e^{-Np_h p} = 1 - e^{-pN_h} , \tag{A-6}$$

where N_h is the expected number of aircraft in the altitude band centered at h.

Every measurement, constituting a so-called "epoch", can be regarded as an independent sample from the same population. Then P_h is the probability that a given epoch produces an alarm. For small values, P_h is approximately the same as the expected number of alarms $N_h p$ per epoch.

When the expressions in (A-1) are substituted into (A-2), the expression for p becomes

$$p = \frac{1}{\sigma_o\sqrt{2\pi}} \int_{-\infty}^{\hat{R}_o} e^{\frac{-\dot{R}^2}{2\sigma_o^2}} \left\{ 1 - \left[\gamma e^{\frac{-(\hat{R}_o-\dot{R})^2}{2\hat{\sigma}_1^2}} + (1-\gamma) e^{\frac{-(\hat{R}_o-\dot{R})^2}{2\hat{\sigma}_2^2}} \right] \right\} d\dot{R}$$

$$= G\left(\frac{\hat{R}_o}{\sigma_o}\right) - \frac{\gamma\,\hat{\sigma}_1 \varepsilon^{\frac{-\hat{R}_o^2}{2(\sigma_o^2+\hat{\sigma}_1^2)}}}{\sqrt{\sigma_o^2 + \hat{\sigma}_1^2}}\, G\left(\frac{\hat{\sigma}_1 \hat{R}_o}{\sigma_o\sqrt{\sigma_o^2 + \hat{\sigma}_1^2}}\right) \qquad \text{(A-7)}$$

$$- \frac{(1-\gamma)\hat{\sigma}_2 e^{\frac{-\hat{R}_o^2}{2(\sigma_o^2+\hat{\sigma}_2^2)}}}{\sqrt{\sigma_o^2 + \hat{\sigma}_2^2}}\, G\left(\frac{\hat{\sigma}_2 \hat{R}_o}{\sigma_o\sqrt{\sigma_o^2 + \hat{\sigma}_2^2}}\right)$$

where

$$\hat{R}_o = \frac{R_o}{\tau} \quad ,$$

$$\hat{\sigma}_1 = \frac{\sigma_1}{\tau} \quad ,$$

$$\hat{\sigma}_2 = \frac{\sigma_2}{\tau} \quad ,$$

and

$$G(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy .$$

For the values of τ of interest

$$\frac{\sigma_o^2}{\hat{\sigma}_1^2} << 1$$

and

$$\frac{\sigma_o^2}{\hat{\sigma}_2^2} << 1 .$$

Thus, a reasonable approximation to the expression for p in (A-7) is given by

$$p \sim G\left(\frac{R_o}{\tau\sigma_o}\right)\left[1 - \gamma\left(1 - \frac{\tau^2\sigma_o^2}{2\sigma_1^2}\right)\left(1 - \frac{R_o^2}{2\sigma_1^2}\right)\right. \tag{A-8}$$

$$\left. - (1 - \gamma)\left(1 - \frac{\tau^2\sigma_o^2}{2\sigma_2^2}\right)\left(1 - \frac{R_o^2}{2\sigma_2^2}\right)\right].$$

It may be observed that τ and σ_o only occur in (A-7) and (A-8) in the product $\sigma_o\tau$. In accordance with this observation, curves showing p as a function of $\sigma_o\tau$ for various values of R_o are given in Figs. A-1a (on page A-8) and A-1b (on page A-14). The values of γ, σ_1, and σ_2 which are used for that purpose are, respectively, 0.36, 45,600 ft, and 121,600 ft, corresponding to the radial traffic distribution model derived in Ref. A.1 for the FAA 1982 LAX projection.

In (A-8) $\sigma_o\tau$ and R_o appear symmetrically, except in the G function factor which is slowly varying for most of the range

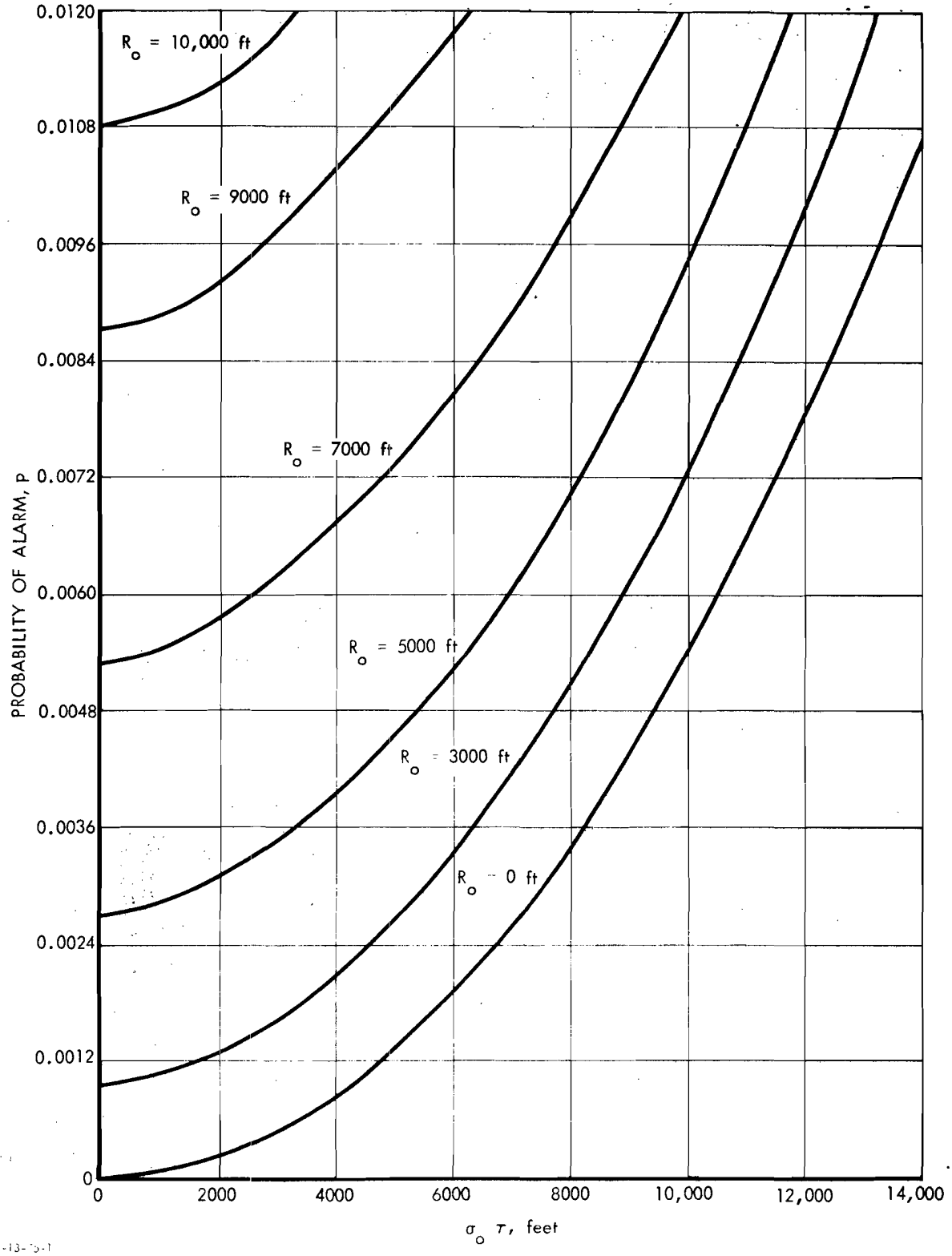


FIGURE A-1a. Probability of Alarm per Altitude Aircraft vs. $\sigma_0\tau$ for LAX 1982 Projected Traffic (Linear Plot)

of its argument. Thus, generally, p depends upon R_o in the same way that it depends upon $\sigma_o\tau$. In addition, both appear divided either by σ_1, or by σ_2.

From these observations, some conclusions can be drawn about the dependence of p on various parameters. One of these is that a change in σ_o will have the same effect on p as a change in τ if R_o is fixed, as it would be in the case of nonaccelerating aircraft. If there is a nonzero relative acceleration allowed for in the τ criterion, decreasing τ will also cause R_o to decrease and will, therefore, be more effective than decreasing σ_o. Increasing σ_1 and σ_2 has essentially the same effect as decreasing R_o and $\sigma_o\tau$.

The effect on the single epoch probability of alarm P_h is essentially the same because (A-6) gives the approximate relation

$$P_h \sim N_h p \quad . \tag{A-9}$$

It can also be seen from (A-9) that decreasing the single co-altitude band traffic density results in a proportionate decrease in P_h.

From Figs. A-1a and A-1b and these remarks, it may be concluded that P_h is more sensitive to radial distribution changes in traffic density for low aircraft speed environments than it is to altitude distribution changes and that the opposite is true for high aircraft speed environments.

The alarm rate observed by an aircraft in the terminal area depends upon the radial and vertical distribution of traffic about the terminal as well as the distribution of aircraft velocities. It also depends upon the aircraft's location in the traffic. In this appendix, the probability of alarm has been calculated for the worst case of an aircraft located in the center of the traffic. The traffic distribution radially is assumed to be that used as a model in Ref. A.1, i.e., a sum

of two Rayleigh distributions. If the aircraft is located a distance x from the center, the probability of alarm would differ from the result of that calculation by a factor of the order $\exp\left(-x^2/2\sigma_1^2\right)$ or less, and thus would decrease very rapidly outside of a region of radius σ_1, which is 7.5 nmi in the model.

B. ALARM RATES

The probability P_h depends upon N_h, the expected number of co-altitude aircraft, and p, the probability that a co-altitude aircraft selected at random will produce an alarm during the epoch. If t_m is the epoch length, or time between measurements of R and $\dot{R}$, the actual alarm rate W_h will be given by

$$W_h = \frac{N_h p}{t_m} \quad , \tag{A-10}$$

but the rate W_h' of alarmed epochs will be given by

$$W_h' = \frac{P_h}{t_m} \ .$$

If R_o is zero, it can be seen from Figs. A-1a and A-1b or from (A-8) that the alarm rate grows monotonically with increasing τ and approaches zero with τ. If R_o is not zero, however, there will be a nonzero optimum value of τ for which the alarm rate W_h is a minimum, even though the probability P_h of alarm during an epoch is larger for that value of τ than it is for smaller values.

As an example, consider the case of a co-altitude band below 10,000 ft, where the expected number of aircraft N_h is 150, which is possible according to the FAA 1982 LAX projection and even probable according to the MITRE projection. If complete rollout is required in conjunction with a single τ criterion, so that (IV-5) should be used to calculate R_o, then for protection against a 1-G relative acceleration, a minimum of 9750 ft for R_o and 24 sec for τ will be necessary, assuming

the total 3 sigma measurement error of 3200 ft. The probability p of an alarm due to a single aircraft chosen at random is then given by

$$p = 0.015 ,$$

and the probability P_h of at least one alarm during an epoch is given, according to (A-6), by

$$P_h = 0.895 .$$

Since this is a case in which the epoch interval t_m is assumed to be 3 sec, the alarm rate W_h is given, according to (A-10), by

$$W_h = 0.750 \text{ alarms/sec} ,$$

or 45 alarms/min, which is undoubtedly an unacceptable number. The rate of alarmed epochs is given by

$$W_h' = 0.298/\text{sec} ,$$

or almost 18 per min, which is also unacceptable.

The epoch interval of 3 sec is not an optimum in this case; however, some numerical experimenting shows that the minimum alarm rate is achieved for an epoch interval between 8 and 9 sec, for which the alarm rate is given by

$$W_h = 0.533 \text{ alarms/sec}$$

and the alarmed epoch rate by

$$W_h' = 0.123/\text{sec} .$$

This is nearly 32 alarms/min and more than 7 alarmed epochs/min, which is still unacceptable.

Elsewhere in this report it is suggested that if each aircraft measures its own acceleration and communicates the value of its acceleration to all other aircraft by one means or another, the value of R_o required for most cases would be greatly reduced. If it is known that the acceleration is zero, then R_o reduces to the measurement error of 3200 ft. In that case, when the epoch interval is 3 sec, the alarm rate is given by

$$W_h = 0.254 \text{ alarms/sec}$$

and the alarmed epoch rate by

$$W_h' = 0.178/\text{sec} ,$$

or more than 15 alarms/min and nearly 11 alarmed epochs/min. If the optimum epoch interval, which, again, is between 8 and 9 secs, is used, the alarm rate is given by

$$W_h = 0.12 \text{ alarms/sec}$$

and the alarmed epoch rate by

$$W_h' = 0.077/\text{sec} .$$

C. ATC MODIFIED TRAFFIC MODEL

The double Rayleigh distribution traffic model used to calculate the alarm probability here was derived in Ref. A.1 from the FAA 1982 LAX projected distribution. It seems entirely possible that Air Traffic Control would not permit co-altitude aircraft in the terminal area to become as closely spaced as the projection implies. Thus, if, as assumed here, an aircraft is at the center of the distribution, other traffic may be inhibited from appearing in the immediate neighborhood of the center.

To obtain some idea of the possible effect of such a change in the traffic pattern, a rough model of the resulting distribution can be constructed by inserting a hole in the double Rayleigh distribution. For a hole of radius x, the equation (A-2) for the alarm probability p will become

$$p = \frac{2\pi}{\nu(x)} \int_{-\infty}^{\frac{R_o - x}{\tau}} P_V(\dot{R}) \int_{x}^{(R_o - \tau\dot{R})} P(R)\,dR\,d\dot{R} \,. \qquad \text{(A-11)}$$

where

$$\nu(x) = \gamma e^{-\frac{x^2}{2\sigma_1^2}} + (1 - \gamma)\, e^{-\frac{x^2}{2\sigma_2^2}} \,.$$

Figures A-2, A-3, and A-4 give p as a function of $\sigma_o\tau$ for several values of R_o in the cases where x is 3000 ft, 1 nmi, and 1.5 nmi. It can be seen that values of p, especially those corresponding to the lower values of $\sigma_o\tau$, are considerably smaller than the values shown in Figs. A-1a and A-1b.

The effect of adding a hole at the center of the traffic distribution is very pronounced. However, the change in p due to increasing the radius of the hole appears to be less and to grow smaller as the radius is made larger. Thus, it would seem that the high alarm probability is due primarily to traffic located near the center, as might be expected.

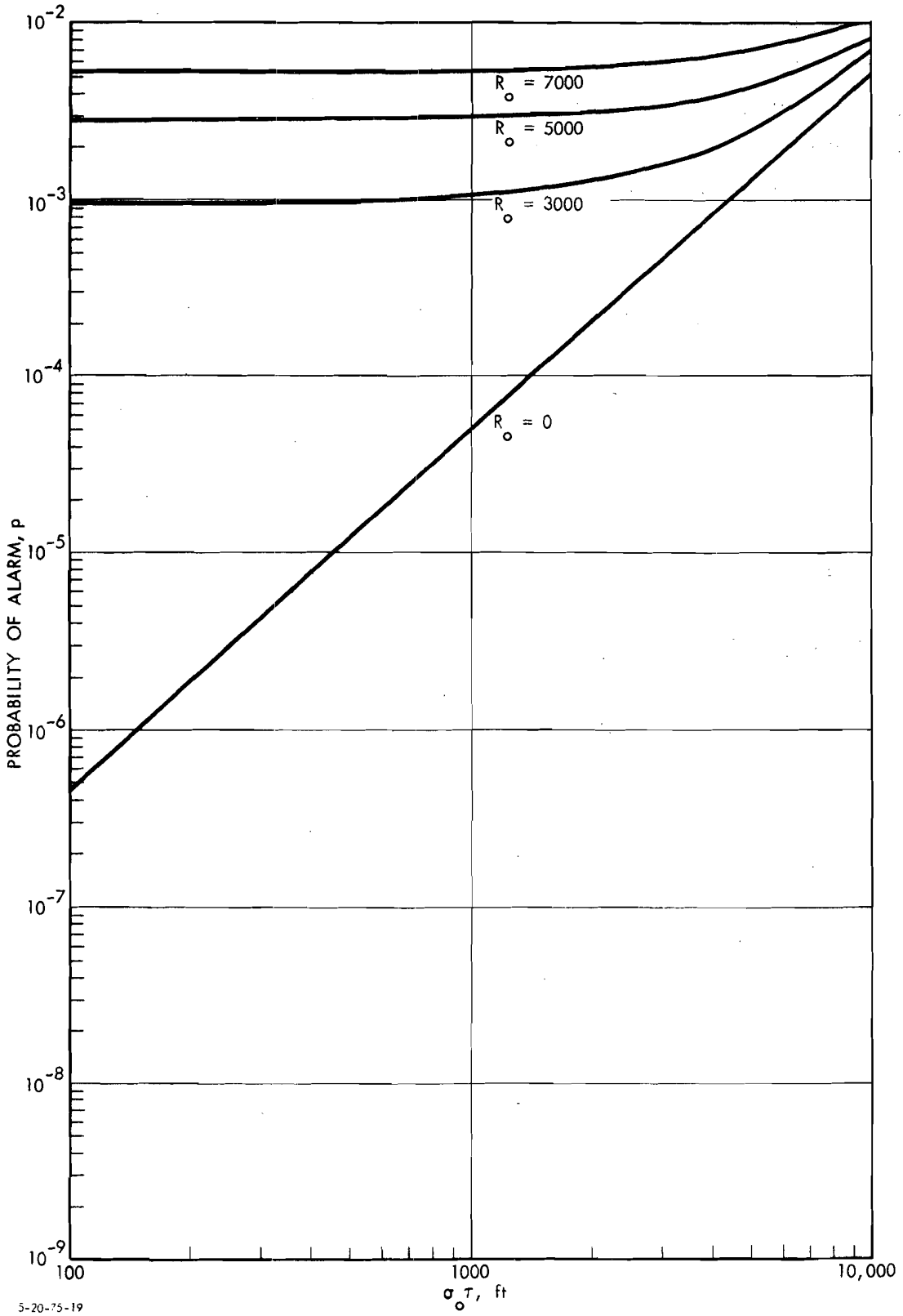


FIGURE A-1b. Probability of Alarm Per Altitude Aircraft vs. $\sigma_o\tau$ for LAX 1982 Projected Traffic (Logarithmic Plot)

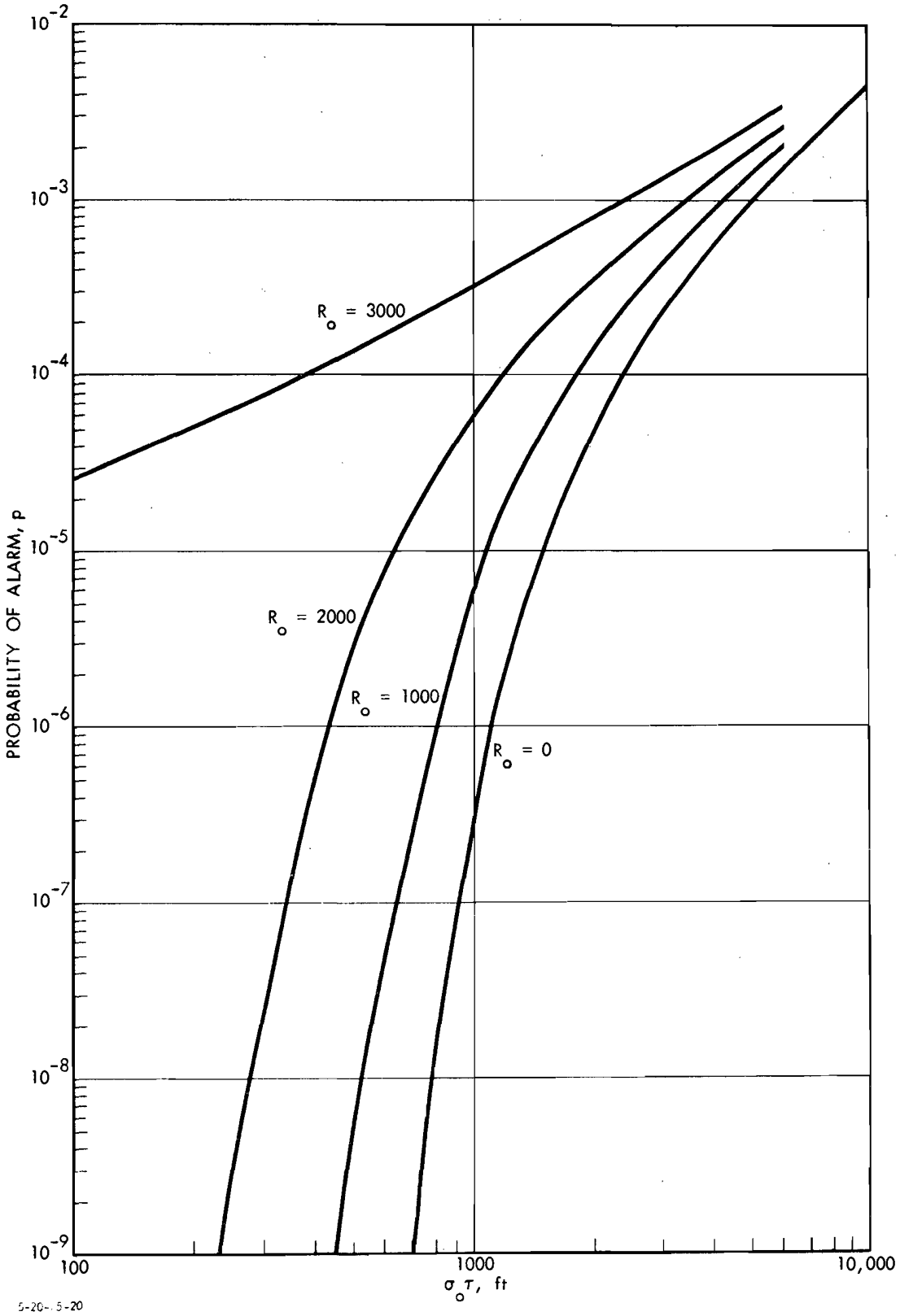


FIGURE A-2. Probability of Alarm Per Altitude Aircraft vs. $\sigma_0\tau$ for Traffic Hole of Radius 3,000 feet

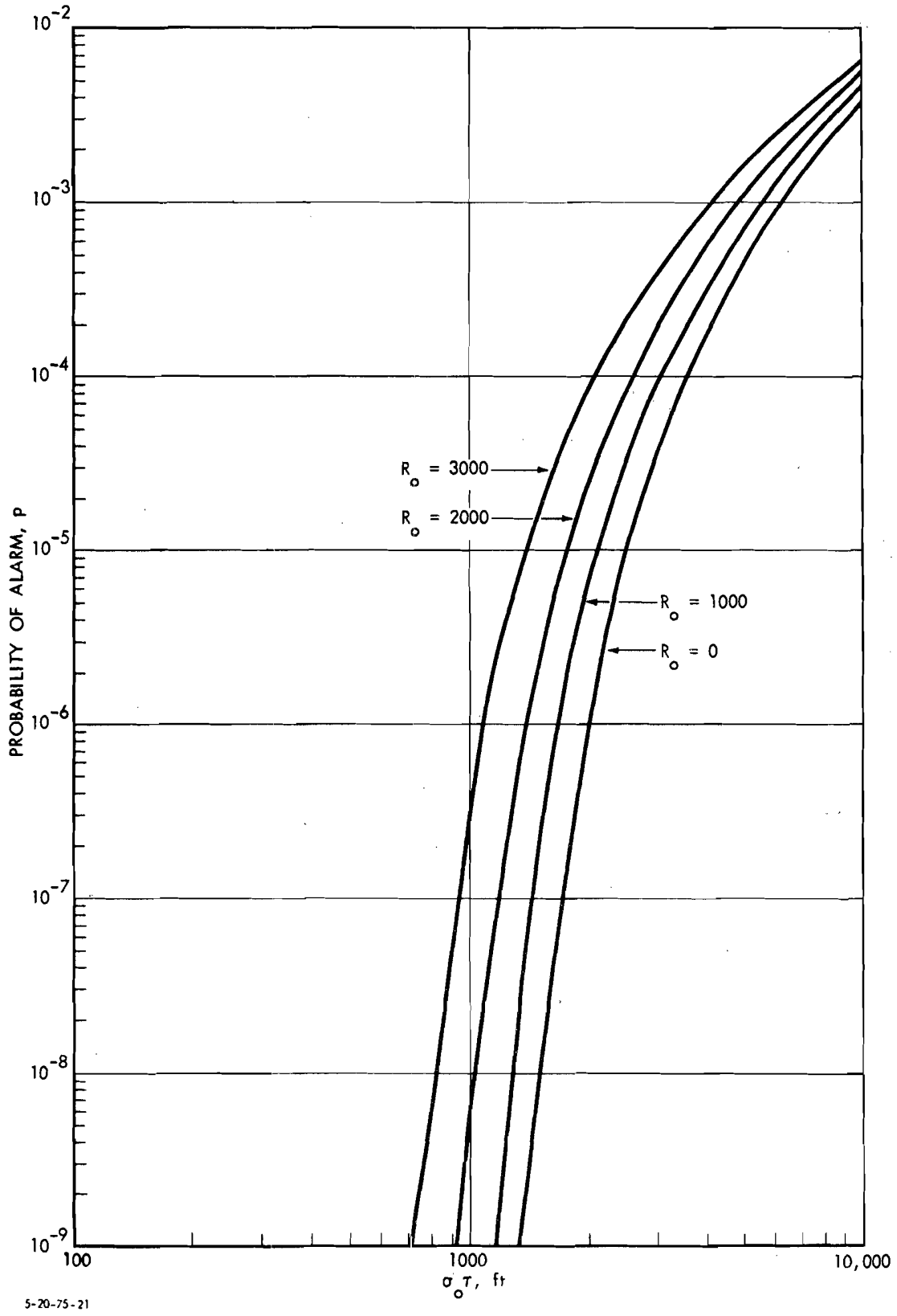


FIGURE A-3. Probability of Alarm Per Altitude Aircraft vs. $\sigma_0 \tau$ for Traffic Hole of Radius 1 nmi

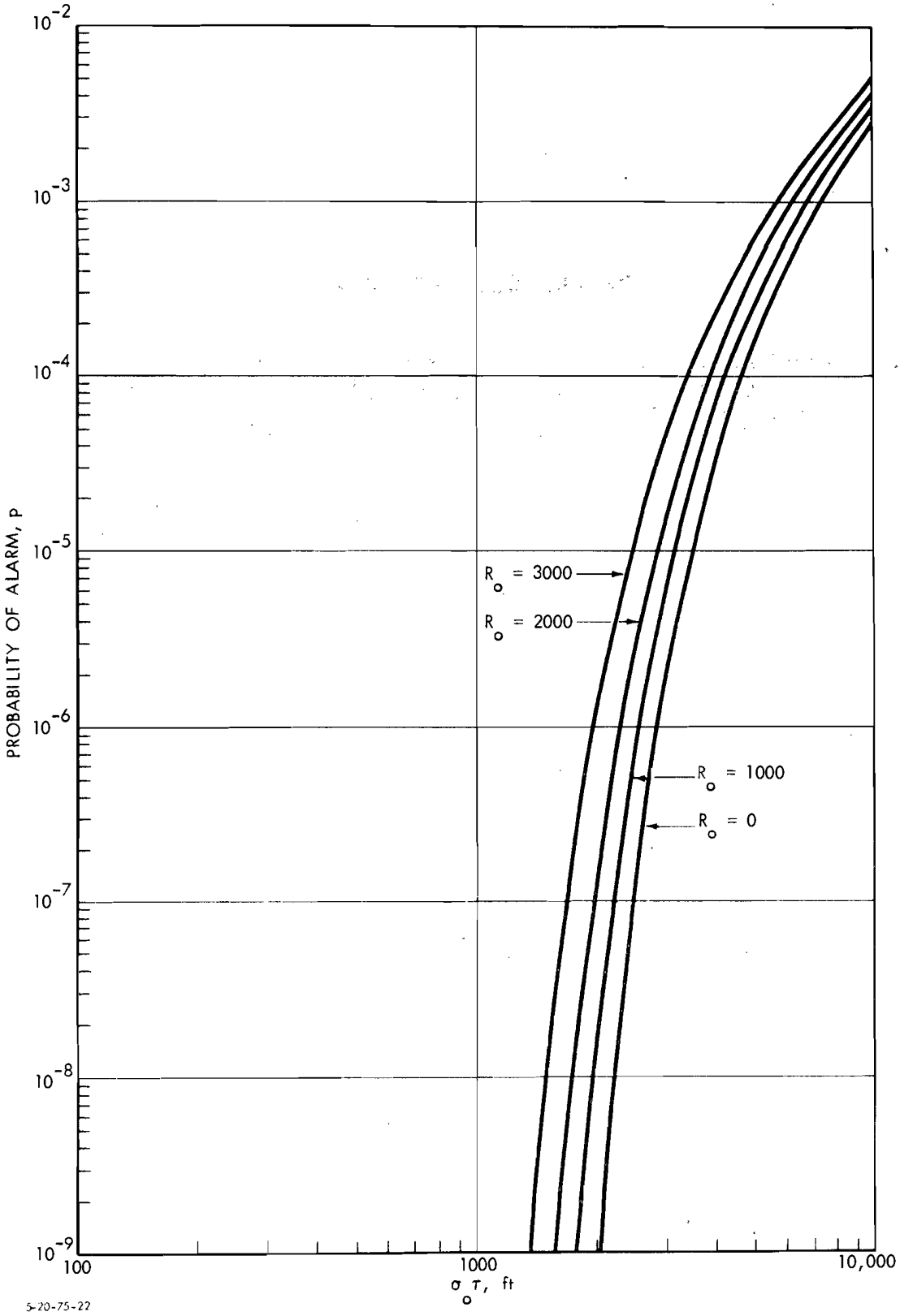


FIGURE A-4. Probability of Alarm Per Altitude Aircraft vs. $\sigma_0\tau$ for Traffic Hole of Radius 1.5 nmi

REFERENCE, APPENDIX A

A.1. Bagnall, J.J., et al., "A Review and Analysis of the RCA Collision Avoidance System," IDA Study S-426, September 1973.

APPENDIX B

CROSS RANGE VELOCITY HAZARD CRITERION

APPENDIX B

CROSS RANGE VELOCITY HAZARD CRITERION

A. ERRORS IN THE MEASUREMENT OF CROSS-RANGE VELOCITY

One approach to the reduction of the CAS alarm rate is to supplement the τ criterion with a second criterion based on an estimate of the cross-range velocity, V_θ. Unlike the separation distance R between two aircraft and their separation rate $\dot{R}$, the cross-range velocity V_θ cannot be measured directly. However, in principle, a bound on V_θ can be obtained from measurements of R and $\dot{R}$ at various times during the course of the relative trajectory of the aircraft. This bound must include the effect of the unknown relative acceleration vector $\underset{\sim}{A}$ of the aircraft as well as errors in the measurement of R and $\dot{R}$.

The relative position vector $\underset{\sim}{P}(t)$ of one aircraft seen from another is given as a function of time by

$$\underset{\sim}{P}(t) = \underset{\sim}{R} + \dot{\underset{\sim}{R}}t + \int_0^t \int_0^s \underset{\sim}{A}(r)\, dr\, ds \ , \qquad \text{(B-1)}$$

where $\underset{\sim}{R}$ and $\dot{\underset{\sim}{R}}$ are the initial relative position and velocity vectors at the time t = 0. It is assumed that the magnitude of $\underset{\sim}{A}(t)$ is bounded at all times by a known constant U, i.e.,

$$|\underset{\sim}{A}| < U \ . \qquad \text{(B-2)}$$

It is useful to define

$$P(t) = |\underset{\sim}{P}(t)|$$

$$R = |\underset{\sim}{R}| = P(o)$$

$$V = |\dot{\underset{\sim}{R}}|$$

$$\dot{R} = \dot{P}(o) = (\underset{\sim}{R} \cdot \dot{\underset{\sim}{R}})/R \ .$$

Then the cross-range velocity V_θ satisfies

$$V_\theta^2 = V^2 - \dot{R}^2 \ . \tag{B-3}$$

Also,

$$P^2 = (R + \dot{R}t)^2 + t^2 \, V_\theta^2 + E(t) \ , \tag{B-4}$$

where

$$E(t) = 2(\underset{\sim}{R} + \dot{\underset{\sim}{R}}t) \cdot \int_0^t \int_0^s \underset{\sim}{A}(r)drds + \left| \int_0^t \int_0^s \underset{\sim}{A}(r)drds \right|^2 .$$

Two relations are available for the calculation of V_θ. The first is (B-4) and leads to

$$V_\theta^2 = \frac{P^2(t)}{t^2} - \frac{(R + \dot{R}t)^2}{t^2} - \frac{E(t)}{t^2} \ . \tag{B-5}$$

The second, which is obtained by differentiating (B-4), leads to

$$V_\theta^2 = \frac{P(t)\dot{P}(t)}{t} - \dot{R}\left(\frac{R}{t} + \dot{R}\right) - \frac{\dot{E}(t)}{2t} \ , \tag{B-6}$$

where the dots refer, as usual, to derivatives with respect to time. Corresponding to these are inequalities for the cross-range velocity to be used in formulating alarm criteria:

$$V_\theta^2 \leq \frac{P^2(t)}{t^2} - \frac{(R + \dot{R}t)^2}{t^2} + \varepsilon_1 \ , \tag{B-7}$$

and

$$V_\theta^2 \leq \frac{P(t)\dot{P}(t)}{t} - \dot{R}\left(\frac{R}{t} + \dot{R}\right) + \varepsilon_2 \ , \qquad \text{(B-8)}$$

where ε_1 and ε_2 are bounds on the terms involving E in (B-5) and $\dot{E}$ in (B-6) added to bounds on the errors contributed by the measurement of R, $\dot{R}$, P(t), and $\dot{P}(t)$. The quantities R and $\dot{R}$ are obtained from measurements at the time t = 0 while P(t) and $\dot{P}(t)$ are obtained from the same measurements made at a given time t other than zero.

The error bounds ε_1 and ε_2 can be obtained directly from (B-5) and (B-6) in terms of the maximum allowable errors in the measurement of P and $\dot{P}$ at any time. It will be assumed that a measurement of P has a total 3σ, or other confidence level, error ε_P consisting of a bias error which has a maximum value b_P and which does not change in time, i.e., the bias error is the same for R and P(t), and a 3σ, or other confidence level, fluctuating error ε_P. It will also be assumed that such a total error $\varepsilon_{\dot{P}}$ consisting of a bias error with a maximum value $b_{\dot{P}}$ and a fluctuating error $\varepsilon_{\dot{P}}$, exists for measurements of $\dot{P}$.

From the definition of $\underset{\sim}{E}(t)$, it follows that

$$\left|\frac{\underset{\sim}{E}(t)}{t^2}\right| \leq \frac{\left|\int_0^t\int_0^s \underset{\sim}{A}(r)drds\right|^2}{t^2} + 2\left(\frac{R}{t} + V_m\right)\frac{\left|\int_0^t\int_0^s \underset{\sim}{A}(r)drds\right|}{t}$$

$$\leq \frac{\left\{\int_0^t\int_0^s U\ drds\right\}^2}{t^2} + 2\left(\frac{R}{t} + V_m\right)\frac{\left\{\int_0^t\int_0^s U\ drds\right\}}{t}$$

$$= \frac{U^2t^2}{4} + RU + V_m\ Ut \ , \qquad \text{(B-9)}$$

where V_m is an assumed maximum allowable magnitude for the relative velocity of the aircraft. From a similar argument applied to (B-1), it follows that

$$P \leq \frac{1}{2} Ut^2 + V_m t + R \ . \tag{B-10}$$

By substituting $P(t) + \varepsilon_R$ for $P(t)$, $R + \varepsilon_R$ for R, and $\dot{R} + \varepsilon_{\dot{R}}$ for $\dot{R}$ in (B-5) and using (B-7), it can be seen that, within the assumed confidence levels,

$$\begin{aligned} \varepsilon_1 \leq \frac{|E|}{t^2} &+ \frac{2\left(P\varepsilon_R + R\varepsilon_R + \varepsilon_R^2 - b_R^2\right)}{t^2} \\ &+ \frac{2\left(\varepsilon_R|\dot{R}| + \varepsilon_{\dot{R}}R\right)}{t} + 2\varepsilon_{\dot{R}}|\dot{R}| + \varepsilon_{\dot{R}}^2 \ . \end{aligned} \tag{B-11}$$

If (B-9) and (B-10) are used in (B-11), it will be found that ε_1 has the form

$$\varepsilon_1 \leq A_1|\dot{R}| + B_1R + C_1 \ , \tag{B-12}$$

where

$$A_1 = 2\left(\varepsilon_{\dot{R}} + \frac{\varepsilon_R}{t}\right) ,$$

$$B_1 = 2\left(\frac{2\varepsilon_R}{t^2} + \frac{\varepsilon_{\dot{R}}}{t}\right) + U \ ,$$

and

$$C_1 = \frac{U^2}{4} t^2 + V_m Ut + \varepsilon_{\dot{R}}^2 + U\varepsilon_R + \frac{2V_m\varepsilon_R}{t} + \frac{2\left(\varepsilon_R^2 - b_R^2\right)}{t^2} \ .$$

Similar arguments applied to (B-6) instead of (B-5), with the additional help of the inequality

$$\dot{P} \leq Ut + V_m ,$$

lead to the result that

$$\varepsilon_2 \leq A_2|\dot{R}| + B_2|R| + C_2 , \qquad \text{(B-13)}$$

where

$$A_2 = \frac{\varepsilon_R}{t} + 2\varepsilon_{\dot{R}} ,$$

$$B_2 = \frac{2\varepsilon_{\dot{R}}}{t} + U ,$$

and

$$C_2 = \frac{U^2}{2} t^2 + \frac{U}{2} (3V_m + \varepsilon_{\dot{R}}) t + (V_m + \varepsilon_{\dot{R}}) \varepsilon_{\dot{R}} + U\varepsilon_R$$

$$+ \frac{2(\varepsilon_R\varepsilon_{\dot{R}} - b_R b_{\dot{R}}) + V_m\varepsilon_R}{t} .$$

It should be noted that while the parameters A and B decrease with increasing measurement time t, this is not necessarily true of C, unless the acceleration is known to be zero. In that case, all parameters can be made smaller by delaying the time interval between measurements at 0 and t. In fact, B can be made arbitrarily small while A and C will be limited by the error $\varepsilon_{\dot{R}}$ in the separation rate measurement.

B. CROSS RANGE VELOCITY ALARM CRITERIA AND THE PROBABILITY OF ALARM

In Ref. B.1, the modified τ criterion

$$R + \tau\dot{R} \leq R_o$$

was derived along with a cross range velocity criterion

$$|V_\theta| \leq \frac{R_o}{\tau} ,$$

where, in this case, R_o is a term used to allow for the maximum relative acceleration, but does not include the effect of measurement error. It was found that a suitable value for R_o, if aircraft are not required to roll out when performing a vertical escape maneuver, is given by

$$R_o = \frac{1}{2} U\tau^2 ,$$

where τ is the total time required after an alarm to escape to a safe altitude. If τ_c is that part of τ actually spent in climbing or diving, and rollout is required after an alarm, then a smaller value of R_o, namely,

$$R_o = \frac{1}{2} U\left(\tau^2 - \tau_c^2\right) ,$$

can be assumed.

Since measurement errors are significant in applying the alarm criteria, the τ criterion should be

$$R + \tau\dot{R} \leq R_o + \varepsilon ,$$

where ε is the 3σ total error in $R + \tau\dot{R}$, if R and $\dot{R}$ are measured values. The corresponding cross range velocity criterion will then be

$$\hat{V}_{\theta 1}^2 \leq \frac{R_o^2}{\tau^2} + \varepsilon_1 = \frac{R_o^2}{\tau^2} + A_1|\dot{R}| + B_1 R + C_1 ,$$

or (B-14)

$$\hat{V}_{\theta 2}^2 \leq \frac{R_o^2}{\tau^2} + \varepsilon_2 = \frac{R_o^2}{\tau^2} + A_2|\dot{R}| + B_2 R + C_2 ,$$

where $\hat{V}_\theta^2$ is the "measured value" of V_θ^2 given by

$$\hat{V}_{\theta 1}^2 = \frac{P^2(t)}{t^2} - \frac{(R + \dot{R}t)^2}{t^2}$$

or

$$\hat{V}_{\theta 2}^2 = \frac{P(t)\dot{P}(t)}{t} - \dot{R}\left(\frac{R}{t} + \dot{R}\right) ,$$

in accordance with (B-5) and (B-6).

It will be assumed here that $\hat{V}_\theta$ has the same probability density that has been assumed for aircraft velocities (cf. Appendix A), i.e.,

$$P_{\hat{V}\theta} = P_V(\hat{V}_\theta) = \frac{1}{\sigma_o\sqrt{2\pi}}\, e^{\frac{-\hat{V}_\theta^2}{2\sigma_o^2}} . \qquad \text{(B-15)}$$

The probability p of an alarm at the center of the aircraft distribution about the terminal from a single aircraft chosen at random, when both the τ criterion and the cross range velocity criterion are required for an alarm, is a modified form of the expression (A-2) in Appendix A, namely,

$$p = 2\pi \int_{-\infty}^{\frac{R_o+\varepsilon}{\tau}} P_V(\dot{R}) \int_{-(R_o+\varepsilon-\tau\dot{R})}^{(R_o+\varepsilon-\tau\dot{R})} P(R) \int_{-\sqrt{(R_o/\tau)^2+C+BR+A|\dot{R}|}}^{\sqrt{(R_o/\tau)^2+C+BR+A|\dot{R}|}} P_V(\hat{V}_\theta)\, d\hat{V}_\theta\, dR\, d\dot{R} , \qquad \text{(B-16)}$$

where here the error term ε has been shown explicitly added to R_o, although in Appendix A it was understood to be included in R_o.

The integral over R in (B-16) can be eliminated by integrating by parts, beginning with an integration of P(R) which is a linear combination of Rayleigh distribution functions. The result is

$$p = \int_{-\infty}^{\frac{R_o+\varepsilon}{\tau}} P_V(\dot{R}) \Bigg\{ - \Bigg[\gamma e^{\frac{-(R_o+\varepsilon-\tau\dot{R})^2}{2\sigma_1^2}} + (1-\gamma) e^{\frac{-(R_o+\varepsilon-\tau\dot{R})^2}{2\sigma_2^2}} \Bigg] I(R_o + \varepsilon - \tau\dot{R}, R) \tag{B-17}$$

$$+ I(0,\dot{R}) + \int_0^{R_o+\varepsilon-\tau\dot{R}} \Bigg[\gamma e^{\frac{-R^2}{2\sigma_1^2}} + (1-\gamma) e^{\frac{-R^2}{2\sigma_2^2}} \Bigg] \frac{\partial I}{\partial R}\, dR \Bigg\} d\dot{R} \ ,$$

where

$$I(R,\dot{R}) = 2 \int_0^{\sqrt{(R_o/\tau)^2 + C + BR + A|\dot{R}|}} P_V(V)\, dV \ ,$$

and

$$\frac{\partial I}{\partial R} = \frac{B}{\sigma_o\sqrt{2\pi}} \; \frac{e^{-\frac{\left[(R_o/\tau)^2+C+BR+A|\dot{R}|\right]}{2\sigma_o^2}}}{\sqrt{(R_o/\tau)^2+C+BR+A|\dot{R}|}} \; .$$

C. EFFECTIVENESS OF THE CROSS RANGE VELOCITY ALARM CRITERION

It is clear from (B-14) that the cross range velocity criterion can provide no information whatever unless there exist values of R and $\dot{R}$ such that

$$V_m^2 - \dot{R}^2 > A|\dot{R}| + BR + C + \frac{R_o^2}{\tau^2} \; , \qquad \text{(B-18)}$$

since, because of (B-3), no value of V_θ^2 can exceed $V_m^2 - \dot{R}^2$. In order to observe the dependence of the region in $(\dot{R},R)$ space determined by (B-18) on the parameters involved, it is convenient to write the inequality in the form

$$R < \frac{\left(\frac{A}{2}\right)^2 + \left(V_m^2 - C - R_o^2/\tau^2\right)}{B} - \frac{1}{B}\left(|\dot{R}| + \frac{A}{2}\right)^2 \; . \qquad \text{(B-19)}$$

The region of effectiveness of the cross range velocity criterion, then, is determined by (B-19) and is consequently bounded by two parabolic arcs joining at the line $\dot{R} = 0$ which is an axis of symmetry for the region.

For an example to illustrate what conditions may be required for an effective cross range velocity criterion, the following conditions are assumed:

$$\tau = 25 \text{ sec}$$

$$\tau_c = 14 \text{ sec}$$

$$R_o = 6907 \text{ ft } (U = 32.2 \text{ ft/sec}^2)$$

$$R_o + \varepsilon = 7664 \text{ ft}$$

$$b_R = 50 \text{ ft}$$

$$\varepsilon_R = 110 \text{ ft}$$

$$b_{\dot{R}} = 0^*$$

$$\varepsilon_{\dot{R}} = 30 \text{ ft/sec} .$$

In addition, the traffic density parameters are assumed to have the values given in Appendix A:

$$\sigma_o = 325 \text{ ft/sec}$$

$$\sigma_1 = 7.5 \text{ nmi}$$

$$\sigma_2 = 20 \text{ nmi}$$

$$\gamma = 0.36 .$$

For the case where the maximum allowance U for acceleration is 1 G and the measurement time t is 3 sec after zero, in accordance with (B-12), the values of A_1, B_1, and C_1 are:

$$A_1 = 133.3 \text{ ft/sec}$$

$$B_1 = 10.1. \text{ ft/sec}^2$$

$$C_1 = 150{,}200 \text{ ft}^2/\text{sec}^2 .$$

Calculations of p based on (B-17) and on (A-2) from Appendix A show that p is about 0.025 for a τ alarm alone and that the effect of adding the cross range velocity criterion is negligible.

---

*This value of $b_{\dot{R}}$ would result if $\dot{R}$ were derived from the difference between two R measurements made at different times.

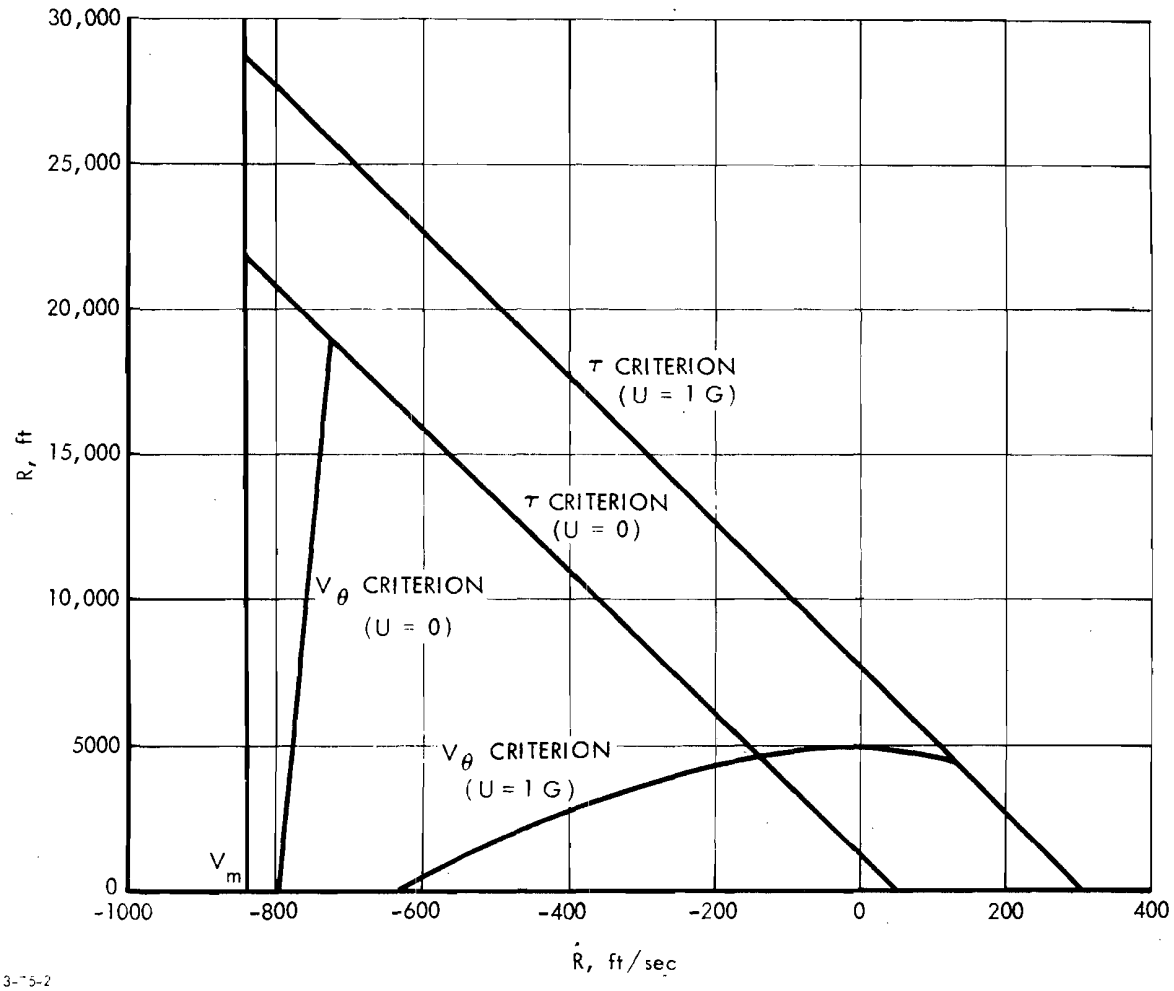
For the case in which it is known that the acceleration is zero and the measurement time interval is 15 sec,* the values of A_1, B_1, and C_1 given by (B-12) are:

$$A_1 = 74.67 \text{ ft/sec}$$
$$B_1 = 5.96 \text{ ft/sec}^2$$
$$C_1 = 13{,}364 \text{ ft}^2/\text{sec}^2 .$$

In this case, the value of p for the τ criterion alone is 0.0040, and the value of p if the cross range velocity criterion is added is 0.0027, a reduction of about 32 percent.

The alarm regions in $(\dot{R},R)$ space for the τ criterion, bounded by a triangle, and the effectiveness regions for the cross range velocity criterion, bounded by parabolic arcs, within the corresponding τ criterion alarm regions for both the numerical examples considered are shown in Fig. B-1. The points under the parabolic arcs bounding the cross range velocity alarm regions are $(\dot{R},R)$ pairs which satisfy a τ-alarm criterion and which might fail to satisfy the cross range velocity criterion. Points above that boundary correspond to $(\dot{R},R)$ values which will always trigger a cross range velocity alarm, so that for such points the alarm conveys no information about the existence of a hazard.

*This implies that data must be taken over several epochs.



2-13-75-2

FIGURE B-1. Alarm Regions for τ and Cross Range Velocity Criteria

REFERENCE, APPENDIX B

B.1. Bagnall, J.J., and Dausin, L.R., "A Review and Analysis of the McDonnell Douglas Collision Avoidance System," IDA Study S-425, October 1973.

APPENDIX C

THE EFFECT ON HAZARD REGIONS OF RESTRICTING AIRCRAFT ACCELERATIONS TO TURNS

APPENDIX C

THE EFFECT ON HAZARD REGIONS OF RESTRICTING AIRCRAFT ACCELERATIONS TO TURNS

If no aircraft can accelerate except in turning, it is natural to ask whether this restriction reduces significantly the hazard region for mid-air collisions. A positive answer could lead to a useful modification of CAS alarm criteria resulting in a reduction of alarm rates. This appendix will demonstrate, however, that the answer is negative.

Collins Radio (Ref. C.1) showed that if a collision between two aircraft occurs at time t for any relative acceleration $\underset{\sim}{A}(t)$ whose magnitude remains bounded less than some quantity U at all times between 0 and t, then there exists a constant acceleration $\underset{\sim}{A}'$, also bounded less than U, for which a collision will occur at time t. In fact, a more general result was obtained, namely, that for any $\underset{\sim}{A}(t)$ for which

$$|\underset{\sim}{A}(s)| \leq U, \quad 0 \leq s < t ,$$

and a relative position trajectory $\underset{\sim}{P}(t)$ given by

$$\underset{\sim}{P}(t) = \underset{\sim}{R} + \dot{\underset{\sim}{R}}t + \int_0^t \int_0^s \underset{\sim}{A}(r)drds , \qquad \text{(C-1)}$$

there exists a constant vector $\underset{\sim}{A}'$ with

$$|\underset{\sim}{A}'| \leq U$$

such that

$$\underset{\sim}{P}(t) = \underset{\sim}{R} + \dot{\underset{\sim}{R}}t + \int_0^t \int_0^s \underset{\sim}{A}' \, dr ds$$

$$= \underset{\sim}{R} + \dot{\underset{\sim}{R}}t + \frac{1}{2} \underset{\sim}{A}' t^2 \ . \tag{C-2}$$

The turning acceleration condition is as follows: if $\underset{\sim}{V}_i(t)$ is the absolute velocity of a single aircraft i and $\underset{\sim}{A}_i(t) = \dot{\underset{\sim}{V}}_i(t)$ is its acceleration, then

$$\underset{\sim}{V}_i(t) \cdot \underset{\sim}{A}_i(t) = 0 \ . \tag{C-3}$$

A canonical form for an acceleration $\underset{\sim}{A}_i(t)$ which satisfies (C-3) is given by*

$$\underset{\sim}{A}_i(t) = a_i[\cos(\omega_i t + \theta_i), \sin(\omega_i t + \theta_i)] \ . \tag{C-4}$$

Since the relative acceleration between two aircraft with absolute accelerations $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ is $\underset{\sim}{A}_1 - \underset{\sim}{A}_2$, the relative acceleration $\ddot{\underset{\sim}{P}}(t)$ can, in this case, be written in the form

$$\ddot{\underset{\sim}{P}}(t) = a_1(\cos \omega_1 t, \sin \omega_1 t)$$

$$- a_2[\cos(\omega_2 t + \theta), \sin(\omega_2 t + \omega)] \ . \tag{C-5}$$

In order to satisfy the condition (C-3), then, $\dot{\underset{\sim}{P}}(t)$ must be given by

$$\dot{\underset{\sim}{P}}(t) = \frac{a_1}{\omega_1} (\sin \omega_1 t, - \cos \omega_1 t)$$

$$+ \frac{a_2}{\omega_2} [-\sin(\omega_2 t + \theta), \cos(\omega_2 t + \theta)] \tag{C-6}$$

---

*More generally, the argument $\omega_i t + \theta_i$ of the trigonometric functions can be replaced by an arbitrary function $\phi_i(t)$. It is assumed here that any variation of $\phi_i(t)$ over the time of interest is small enough to permit approximation by a linear function of t.

and $\underset{\sim}{P}(t)$ by

$$\underset{\sim}{P}(t) = \underset{\sim}{R} + \frac{a_1}{\omega_1^2}(1 - \cos\omega_1 t, -\sin\omega_1 t)$$
$$+ \frac{a_2}{\omega_2^2}[\cos(\omega_2 t + \theta) - \cos\theta, \sin(\omega_2 t + \theta) - \sin\theta] . \tag{C-7}$$

The expression (C-7) for $\underset{\sim}{P}(t)$ can be used in (C-2) to relate the turning accelerations to the constant vector $\underset{\sim}{A}'$. In order to identify $\dot{R}$ for this purpose, 0 can be substituted for t in (C-6). The result is

$$\dot{\underset{\sim}{R}} = \left(-\frac{a_2}{\omega_2}\sin\theta, -\frac{a_1}{\omega_1} + \frac{a_2}{\omega_2}\cos\theta\right) . \tag{C-8}$$

Then the desired relation is

$$a_1 \underset{\sim}{u}_1 + a_2 \underset{\sim}{u}_2 = 1/2\ \underset{\sim}{A}' , \tag{C-9}$$

where

$$\underset{\sim}{u}_1 = \left(\frac{1 - \cos\omega_1 t}{\omega_1^2 t^2}, \frac{\omega_1 t - \sin\omega_1 t}{\omega_1^2 t^2}\right)$$

and

$$\underset{\sim}{u}_2 = \left[\frac{(\cos\omega_2 t - 1)\cos\theta + (\omega_2 t - \sin\omega_2 t)\sin\theta}{\omega_2^2 t^2},\right.$$
$$\left.\frac{(\sin\omega_2 t - \omega_2 t)\cos\theta + (\cos\omega_2 t - 1)\sin\theta)}{\omega_2^2 t^2}\right] .$$

If both sides of (C-9) are squared, the resulting equation has the form

$$\sum_{i,j=1}^{2} Q_{ij}\ a_i a_j = 1/4\ |\underset{\sim}{A}'|^2 = 1/4\ A'^2\ , \qquad \text{(C-10)}$$

where the Q_{ij} are elements of the positive semidefinite symmetric matrix

$$Q = \begin{pmatrix} |\underset{\sim}{u}_1|^2\ , & \underset{\sim}{u}_1 \cdot \underset{\sim}{u}_2 \\ \\ \underset{\sim}{u}_1 \cdot \underset{\sim}{u}_2\ , & |\underset{\sim}{u}_2|^2 \end{pmatrix} . \qquad \text{(C-11)}$$

If λ_1, and λ_2 are the eigenvalues of Q, then (C-10) can be written in the form

$$\lambda_1\ \alpha_1^2 + \lambda_2\ \alpha_2^2 = 1/4\ A'^2\ , \qquad \text{(C-12)}$$

where α_1 and α_2 are components of the vector (a_1, a_2) along the eigenvectors of Q corresponding to λ_1, λ_2. Thus,

$$\alpha_1^2 + \alpha_2^2 = a_1^2 + a_2^2\ . \qquad \text{(C-13)}$$

The trace T of Q is given by

$$T = |\underset{\sim}{u}_1|^2 + |\underset{\sim}{u}_2|^2\ . \qquad \text{(C-14)}$$

In addition,

$$T = \lambda_1 + \lambda_2\ . \qquad \text{(C-15)}$$

If one of the eigenvalues, e.g., λ_2 is zero, then according to (C-15) the other, λ_1, must be equal to T. If this occurs, λ_1 will have its largest possible value, and if the vector (a_1, a_2) is proportional to the eigenvector corresponding

to λ_1, the left side of (C-12) will then have the largest possible value for any given values of a_1 and a_2. Then (C-12) will have the form

$$\left(a_1^2 + a_2^2\right)\left(|\underset{\sim}{u}_1|^2 + |\underset{\sim}{u}_2|^2\right) = 1/4\ A^{-2} . \tag{C-16}$$

The condition that an eigenvalue of Q be zero is that the vector $\underset{\sim}{u}_2$ be proportional to $\underset{\sim}{u}_1$. This condition can be obtained for any ω_1, ω_2 by setting

$$\theta = \tan^{-1}\left[\frac{(\omega_1 t-\sin\omega_1 t)(1-\cos\omega_2 t) - (\omega_2 t-\sin\omega_2 t)(1-\cos\omega_1 t)}{(\omega_1 t-\sin\omega_1 t)(\omega_2 t-\sin\omega_2 t) + (1-\cos\omega_1 t)(1-\cos\omega_2 t)}\right] . \tag{C-17}$$

A straightforward calculation shows that $|\underset{\sim}{u}_1|^2 + |\underset{\sim}{u}_2^2|$ is independent of θ. In fact, for any θ

$$|\underset{\sim}{u}_1^2| + |\underset{\sim}{u}_2|^2 = \frac{(1 - \cos\omega_1 t)^2 + (\omega_1 t - \sin\omega_1 t)^2}{\omega_1^4\ t^4} + \frac{(1 - \cos\omega_2 t)^2 + (\omega_2 t - \sin\omega_2 t)^2}{\omega_2^4\ t^4} . \tag{C-18}$$

The accelerations a_1 and a_2 must satisfy the condition

$$a_1 \leq \frac{U}{2} , \quad a_2 \leq \frac{U}{2} , \tag{C-19}$$

where U is the maximum relative acceleration between two aircraft. It will be assumed that U is 1 G or 32.2 ft/sec^2 and that t is the τ_1 escape time of 25 sec.

In addition, a_1 and a_2 must also be bounded because of a maximum single aircraft velocity v_m, which will be assumed to

be 422 ft/sec. The magnitudes of the aircraft velocities corresponding to a_1 and a_2 are a_1/ω_1 and a_2/ω_2. Thus,

$$\frac{a_1}{\omega_1} \leq v_m \ , \ \frac{a_2}{\omega_2} \leq v_m \ . \tag{C-20}$$

It can be seen, e.g., numerically, that the function $|\underset{\sim}{u}_1|^2$ has a maximum at $\omega_1 = 0$, where it has the value 1/4, and is monotonically decreasing. Since $|\underset{\sim}{u}_2|^2$ is the same function of ω_2, it has a maximum at $\omega_2 = 0$ and is also monotonically decreasing. Thus, for given positive values of a_1 and a_2, the left side of (C-16) will attain its greatest value for ω_1 and ω_2 as close as possible to zero, i.e., in accordance with (C-20), when

$$\omega_1 = \frac{a_1}{v_m} \text{ and } \omega_2 = \frac{a_2}{v_m} \ . \tag{C-21}$$

The left side of (C-16) will therefore reach its largest admissible value, taking into account the restrictions of (C-19), for some ω_1 and ω_2 in the region

$$0 \leq \omega_1 \leq \frac{U}{2v_m} \ , \quad 0 \leq \omega_2 \leq \frac{U}{2v_m} \ . \tag{C-22}$$

A little numerical experimenting makes it evident that this occurs when

$$\omega_1 = \omega_2 = \frac{U}{2v_m} \ . \tag{C-23}$$

In accordance with (C-19), the largest values of a_1 and a_2 are given by

$$a_1 = a_2 = \frac{U}{2} \ . \tag{C-24}$$

Thus, from (C-16), with the use of (C-18), (C-23), and (C-24) and the assumed values of U and v_m, it can be seen that the largest value A' can attain is 31.4 ft/sec^2. This differs from the maximum allowable relative acceleration between two aircraft, i.e., 1 G, by only 2.5 percent. It would, therefore, appear that restricting accelerations to turns will have little effect on reducing the acceleration range which must be allowed for in CAS warning criteria.

REFERENCE, APPENDIX C

C.1. Collins Radio Co., Third Interim Report #RD68-34, Computer/Simulation Study of Air-Derived Separation Assurance Systems in Multiple Aircraft Environment," July 1968.

APPENDIX D

THE IMPACT OF REACTION TIME, MEASUREMENT ERRORS, AND INITIAL CONDITIONS ON THE SUCCESS OF ESCAPE MANEUVERS

APPENDIX D

THE IMPACT OF REACTION TIME, MEASUREMENT ERRORS, AND INITIAL CONDITIONS ON THE SUCCESS OF ESCAPE MANEUVERS

A. GENERAL REQUIREMENTS FOR CAS PERFORMANCE

ANTC-117 describes the logic and specific implementation (time/frequency technique) for a collision-avoidance system (CAS) that is intended to provide sufficient warning time to allow for pilot reaction, aircraft reaction, and system delay times followed by a vertical escape maneuver in one or both aircraft. Typically, the tolerance specified for each system parameter is given in terms of the 3 sigma confidence level or a level corresponding to the probability 0.001 that the tolerance is not exceeded. If all parameter values were determined properly to such tolerances, the probability of not avoiding a collision would be vanishingly small. However, as has been noted in Sections III and IV, some values of the principal parameters R_o and τ appear to have been selected improperly. Presumably, such values resulted from simulations that were conducted primarily for the purpose of demonstrating their validity.

ANTC-117 gives the impression that if only one aircraft responds to the alarm, a successful escape is assured. This particular case may be a very important and frequent occurrence if a large fraction of the aircraft are equipped with beacons or with CAS the logic of which deviates significantly from that specified in ANTC-117. This appendix is concerned with estimating the probability of successful escape for such cases.

B. ASSUMPTIONS

1. Escape Maneuvers

According to ANTC-117:

> The system shall assure safe vertical separation even if only one aircraft can or does obey the climb of descend commands. [A-4.b (1)(a)]

The safe vertical separation is a vertical clearance of 150 ft.

ANTC-117 further states:

> Vertical Maneuver--requires a rotation in the direction specified at an acceleration of not more than 1/4G, but not less than 1/8G up to a terminal climb or descent rate of 2000 feet per minute or more until the maneuver command is terminated. [A-4.a(1)(a)]

Most aircraft can provide at least 1/8 G vertical acceleration by converting dynamic horizontal energy into vertical energy, provided other aerodynamic limits, such as stall, are not approached. However, under some conditions of flight, such as that of a light single-engine aircraft near 10,000-ft altitude or a heavily loaded aircraft, the minimum climb rate may not reach 2000 ft/min, although a dive may be possible at 2000 ft/min.* The vertical acceleration is limited to 1/8 G to 1/4 G so as not to discomfort passengers, although most aircraft are stressed to accommodate much higher acceleration limits. Higher climb accelerations can be tolerated, but higher dive accelerations cannot.

The escape maneuver is a vertical acceleration of 1/8 G to a vertical rate of 2000 ft/min. From level flight, the displacement from such a maneuver is given by

---

*Typical sustained terminal climb speeds for light single-engine aircraft are 100-200 ft/min at 10,000 ft and 600-800 ft/min at sea level and for light twin-engine aircraft are 600-700 ft/min at 10,000 ft and 1300-1400 ft/min at sea level.

$$\Delta hm = -138 + 33.3\ t_e\ (\text{ft})\ ,\quad t_e \leq 8.28\ \text{sec}\ ,$$

in which t_e is the elapsed escape time from the beginning of the maneuver. The time to reach a rate of 2000 ft/min (33.3 ft/sec) with an acceleration of 1/8 G is 8.28 sec.

2. Required Maneuvers

When two aircraft encounter one another in a τ_1-alarm situation, altimeter errors and any residual altitude rates that either aircraft might have may introduce a significant error in the magnitude, and possibly the direction, of the required maneuver to escape.

ANTC-117 allows for

> ...maximum aircraft altitude system errors of ± 250 ft 3 Sigma (Gaussian) at altitudes of 10,000 ft or higher and ± 150 [ft] at altitudes of 10,000 ft or lower. [A-3.a.(3)]

and

> ...100 ft incremental digitization of altitude... [A-4.b.(1)(e)]

Such statements are subject to interpretation, but since ANTC-117 presumably describes a particular aircraft installation it is reasonable to assume that the referenced error and quantization apply to a single equipment. Whether the quantization error is included in the random error is not clear, but it will be assumed that it is.

Since it is assumed that a safe vertical separation between aircraft is 150 ft, for two aircraft in indicated co-altitude level of flight, the required escape maneuver must result in an indicated separation

$$\Delta h_R = 150 + \varepsilon_h\ ,$$

where ε_h is the combined altimetry error.

The combined relative altimetry error is normally distributed with a mean $\mu = 0$, and standard deviation $\sigma = \sqrt{2}\ 250/3 = 117.9$ ft above 10,000 ft and $\sigma = \sqrt{2}\ 150/3 = 70.7$ ft below.

In a co-altitude encounter, the system must provide

> ...allowance for vertical entry into a threat situation at low altitude rates (e.g., 500 ft/min) which are below the system detection threshold. [A-4.b.(1)(e)]

and

> Cockpit display of a vertical speed limit within the range of 0-400 per minute is acceptable. [B-3.e, Figs. 3, 4, and 5, Note 5]

There is no provision in ANTC-117 for communicating vertical rate. If the communicated altitude is used to estimate another aircraft's rate, the quantization of 100 ft and epoch time of 3 sec imply a resolution for such an estimate no better than 2000 ft/min in one epoch; this can be improved to 500 ft/min by averaging over four epochs (12 sec). However, such estimates can be grossly in error if aircraft are accelerating vertically with as little as 1/8 G. Consequently, any encounter may be accompanied with individual aircraft having nondetected altitude rates as large as 500 ft/min. Use of the non-co-altitude and predicted co-altitude criteria should serve to limit the altitude rates of each aircraft in a co-altitude encounter to less than 500 ft/min.

The worst combination of altitude rates and relative altitude is the case in which the aircraft that responds initially has an altitude rate with a sign opposite to that of the other aircraft but then receives a command that requires it to reverse its altitude rate. For the case in which the indicated altitudes are equal, assuming that the altitude rates of each are random, such an occurrence would be expected 25 percent of the time. It is possible, if both aircraft have relatively

high rates (e.g., 500 ft/min each or 1000 ft/min combined relative rate), that the pilot who receives an alarm may subsequently receive a change in the commanded maneuver, but such an occurrence would be delayed because of the altitude quantization and epoch time by as much as 100/16.7 + 3 = 9 seconds. This would probably degrade substantially the response time of the pilot beyond that to be assumed here; e.g., the mean time* for such an occurrence is 2 + 1.5 = 3.5 sec which is only slightly less time (4.14 sec) than it takes the responding aircraft to reverse a 500 ft/min climb into a 500 ft/min dive with 1/8 G. For initial combined relative altitude rates lower than 1000 ft/min, the delay will be even longer.

3. Maneuver Excess

The difference between the magnitude of the escape maneuver relative displacement and the required indicated separation after the maneuver is the maneuver excess.

For aircraft in level flight, the maneuver excess is

$$\Delta h = \Delta h_m - \Delta h_R = -288 + 33.3\, t_e - \varepsilon_h \; .$$

For aircraft with initial altitude rates $-\dot{h}_1$ and $\dot{h}_2$ and for the first aircraft responding with an acceleration $a = 1/8$ G to a terminal rate of $\dot{h} = 2000$ ft/min, the above expression is modified to the following:

$$\Delta h = 1/2a \left(\frac{\dot{h}-\dot{h}_1}{a}\right)^2 + \left(t_e - \frac{\dot{h}+\dot{h}_1}{a}\right) \dot{h} - t_e \dot{h}_2 - 150 - \varepsilon_h \; . \tag{D-1}$$

*This assumes that the aircraft altitudes are each uniformly distributed over the 100 ft quantization level and therefore that their joint distribution is triangular. The mean of a triangular distribution with a maximum of 100 ft is one-third of the maximum or 33.3 ft; thus, for a 1000 ft/min relative rate, the mean time delay is 2 sec.

The parameter $\dot{h}_1$ and $\dot{h}_2$ could be treated as random variables, but this would be difficult without resorting to a cumbersome numerical calculation, since they occur nonlinearly in the relation for Δh. The random variables in the relation are then Δh, t_e and ε_h.

4. Pilot and Aircraft Reaction Time

Pilot and aircraft reaction times are not given explicitly by ANTC-117, which simply states that they will be kept to a minimum.

> The maneuvers specified by A-4.a are acceptable for routing passenger operations. Having specified such nominal avoiding maneuvers, it is essential that the pilot maneuver without delay when commanded. [A-5.a]
>
> Additional seconds allowed and added for pilot reaction time, movement of the aircraft controls, the cycling of the CAS system, and CAS error, should be kept to a minimum since all have the effect of increasing the protected volume of airspace around each aircraft. [A-5.b]
>
> A detailed analysis of warning times have been conducted by Collins Radio and is reported by Reference i. A computer analysis of threat logic was conducted by McDonnell Douglas Corp. The results of this analysis (Reference aa) have permitted the selection of the most suitable warning times and protection envelopes. [A-5.d]

The analysis conducted by the McDonnell Douglas Corp. that is cited above provides the following pilot and aircraft reaction times. Pilot reaction time is 1.0 sec fixed delay plus a variable delay that has a Rayleigh distribution with a parameter, $\sigma_p = 1.5$ sec. Such a distribution has a mean $\mu_p = \sqrt{\pi/2}\ \sigma_p = 1.88$ sec.

For ease of mathematical manipulation here the pilot reaction time distribution is approximated by a normal distribution with a mean $\mu_p = 1 + 1.88 = 2.88$ and a standard deviation

of $\sigma_p = 1.93$ sec. These parameters were estimated by matching the normal and Rayleigh distributions at the probability level at $p = 0.99$.

Aircraft reaction time is normally distributed with a mean $\mu_A = 1.0$ sec and standard deviation $\sigma_A = 0.25$ sec.

Therefore, the pilot/aircraft response time is assumed to be a combined normal distribution with parameters: $\mu=3.88$, $\sigma=1.95$ sec.

5. System Delays

The principal system delay is the epoch time of 3.0 sec. Under some conditions, i.e., communications supported by just one of the antennas of an aircraft equipped with top and bottom fuselage-mounted antennas, the epoch time interval would be 6.0 sec. Since the alarm can occur with equal probability at any time within the epoch interval, its random occurrence is modeled as uniformly distributed with a mean of 1.5 sec and a maximum deviation of ± 1.5 sec.

Again, for mathematical convenience, the epoch delay is approximated here by a normal distribution with mean $\mu_E = 1.5$ sec, and sigma $1.5/\sqrt{3} = 0.866$ sec.

The three contributions to the delay time can be combined into a single normally distributed delay time (t_D) with a mean $\mu_D = 5.38$ sec, and sigma $\sigma_D = 2.13$ sec.

The separation between two aircraft at the occurrence of a τ_1-alarm (the actual alarm may be delayed by as much as the epoch time, normally 3 sec) can be represented by:

$$R = R_{o_1} + \dot{R}\tau + \varepsilon_R ,$$

in which $\dot{R}$ is the closing range rate*, and ε_R is the combined

*Here the convention has been adopted that $\dot{R}$ represents the closing rate, which means that $\dot{R}$ is positive when the separation R between aircraft is decreasing. Elsewhere in this report, $\dot{R}$, representing the separation rate, would be negative in that case.

equivalent error in range. In Ref. D.1, an estimate of ε_R shows that this error within tolerances allowed by ANTC-117 is normally distributed, with a mean zero and standard deviation σ_R = 1000 ft.

For two nonmaneuvering aircraft on a collision course $\dot{R}$ is constant and the time to collision is

$$t = R_{o_1}/\dot{R} + \tau_1 + \varepsilon_R/\dot{R} .$$

The escape time is given by

$$t_e = R_{o_1}/\dot{R} + \tau_1 + \varepsilon_R/\dot{R} - t_D . \qquad \text{(D-2)}$$

From ANTC-117, the parameters R_{o_1} and τ_1 are, respectively:

$$R_{o_1} = 1500 \text{ ft}$$

$$\tau_1 = 25 \text{ sec} .$$

The alarm time t has an interesting behavior which becomes clear if one performs the following manipulation. Define f by

$$f = \frac{t - \tau_1}{R_o/\dot{R}} = \frac{R_o/\dot{R} + \varepsilon_R/\dot{R}}{R_o/\dot{R}} .$$

Then

$$f = 1 + \varepsilon_R/R_o .$$

Since ε_R is normally distributed, with zero mean and standard deviation σ_R, f is likewise normally distributed with a mean of unity and standard deviation σ_R/R_o = 2/3 for ANTC-117 parametric values. When $f \leq 0$, then $t \leq \tau_1$, which occurs for the distribution of f with frequency 0.07. That is, the

time at τ_1-alarm will be less than τ_1 = 25 sec 7 percent of the time, independently of the closure rate $\dot{R}$. Any system that has better measurement accuracy than that given by ANTC-117 can reduce this effect; e.g., if the σ_R were 500 ft, $\sigma_R/R_o = 1/3$, and the occurrence would have probability equal to 0.0013.

Effectively, the degree by which R_{o_1} exceeds σ_R, given by the probability that $f \leq 0$, is a measure of the protection that R_{o_1} gives for measurement errors ε_R. As has been shown elsewhere, R_{o_1} = 1500 is inadequate to protect against residual aircraft horizontal maneuvers, and the actual escape time t_e will be less than that given in (D-2) by an amount $(1/2U\tau_1^2/R)$.

C. CALCULATIONS OF PROBABILITIES OF ESCAPE MANEUVER DEFICIENCY

Calculations of the probability of maneuver deficiency have been made, with the aid of (D-1) and (D-2), using the estimated distribution of delay times, measurement errors, and the following parametric values

$$\dot{R} = 60 \text{ to } 600 \text{ ft/sec}$$

$$\dot{h}_1 = 0 \text{ and } 500 \text{ ft/min}$$

$$\dot{h}_2 = 0 \text{ and } 500 \text{ ft/min} .$$

It was assumed that only one aircraft maneuvers. Curves based on the results are presented in Figs. D-1 and D-2. The parameters associated with each curve represent the vertical rates in ft/min of the two aircraft. Note that the lower limit of 60 ft/sec corresponds to the minimum $\dot{R}$ at which the minimum range (R_M = 3000 ft) criterion would apply, i.e., such that $R_M = R_{o_1} + \tau\dot{R}$.

If the combined residual horizontal acceleration of both aircraft were chosen such that U = 4.8 ft/sec^2, i.e., $R_{o_1} = \frac{1}{2} U\tau_1^2 = 1500$, which corresponds to a single aircraft in

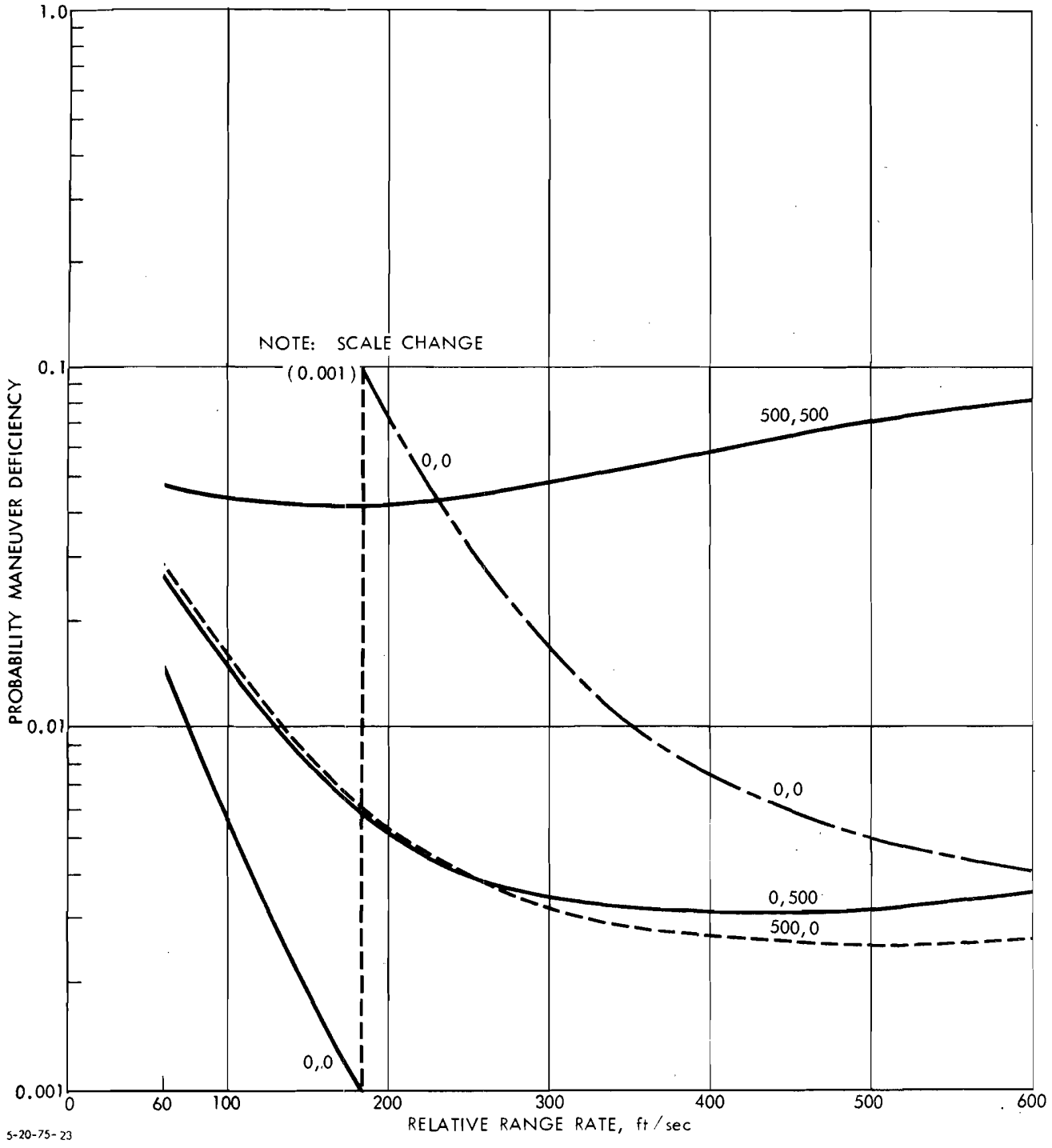


FIGURE D-1. Probability Maneuver Deficiency vs. Range Rate with Altitude Rate a Parameter, $h \leq 10{,}000$ ft, U = 0

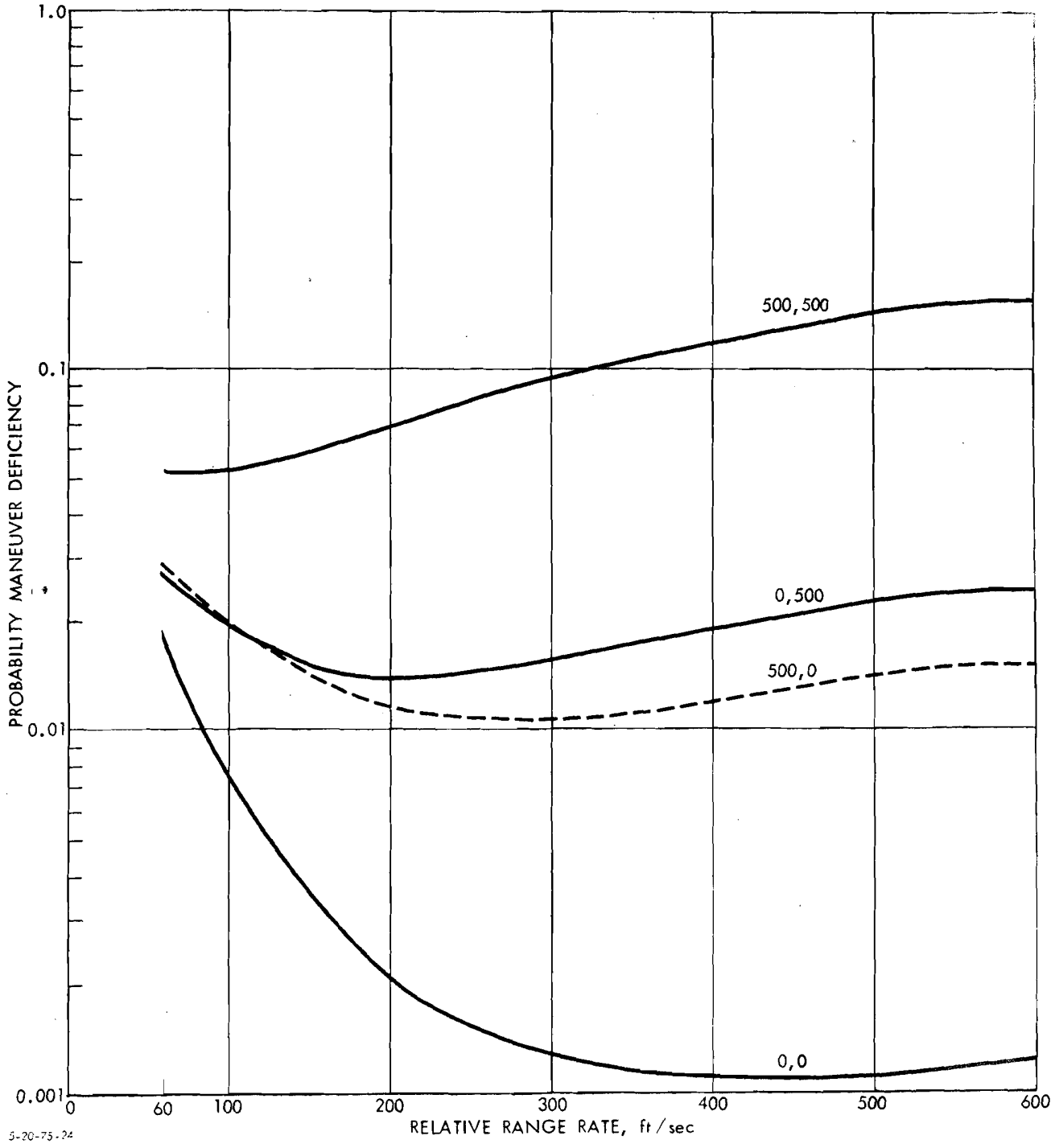


FIGURE D-2. Probability Maneuver Deficiency vs. Range Rate with Altitude Rate a Parameter, h > 10,000 ft, U = 0

an 8.5 deg bank turn or each in a 4.3 deg bank turn, and if the aircraft were maneuvering so as to collide, then an additional term $R_{o_1}/\dot{R}$, must be subtracted from the expression for t_e. Curves for the probability of maneuver deficiency resulting from a calculation based on that assumption are presented in Figs. D-3 and D-4. Note that in such a case the factor f given above would have a very large variance compared to its mean and the probability that $t \leq \tau_1$ would be equal to 0.5. For this case, the value chosen for R_{o_1} gives very little protection against the measurement error, since it is used up by the horizontal maneuvers.

Figures D-1 through D-4 show that the probability that the vertical clearance does not exceed the safe clearance (150 ft) is highly sensitive to the initial conditions of vertical rates and horizontal accelerations assumed. In most cases, except for U = 0 and $\dot{h}_1 = \dot{h}_2 = 500$ ft/min, the probability is greatest for lower range rates. If both high vertical rates and horizontal acceleration are present in the worst combination, the probability is about 0.3 averaged over range rate at all altitudes. Note, however, that by considering the likelihood that the vertical rates have the worst sign and the altitude difference is such that it forces a random choice, this probability is reduced by 1/4. Even with U = 0, this results for the relatively high vertical rates in p = 0.1/4 = 0.025 above 10,000 ft, and p = 0.013 below 10,000 ft.

One can observe also from the figures by comparing the curves for $\dot{h}_1 = 0$, $\dot{h}_2 = 500$ ft/min and $\dot{h}_1 = 500$ ft/min, $\dot{h}_2 = 0$ that the effect of the vertical rate in either aircraft is approximately the same.

A recent estimate of the Los Angeles traffic in 1982 (Ref. D.2) notes that approximately 10 percent of the traffic is turning at any one time. If a large portion of this traffic is equipped with beacon only, the traffic encountered by a

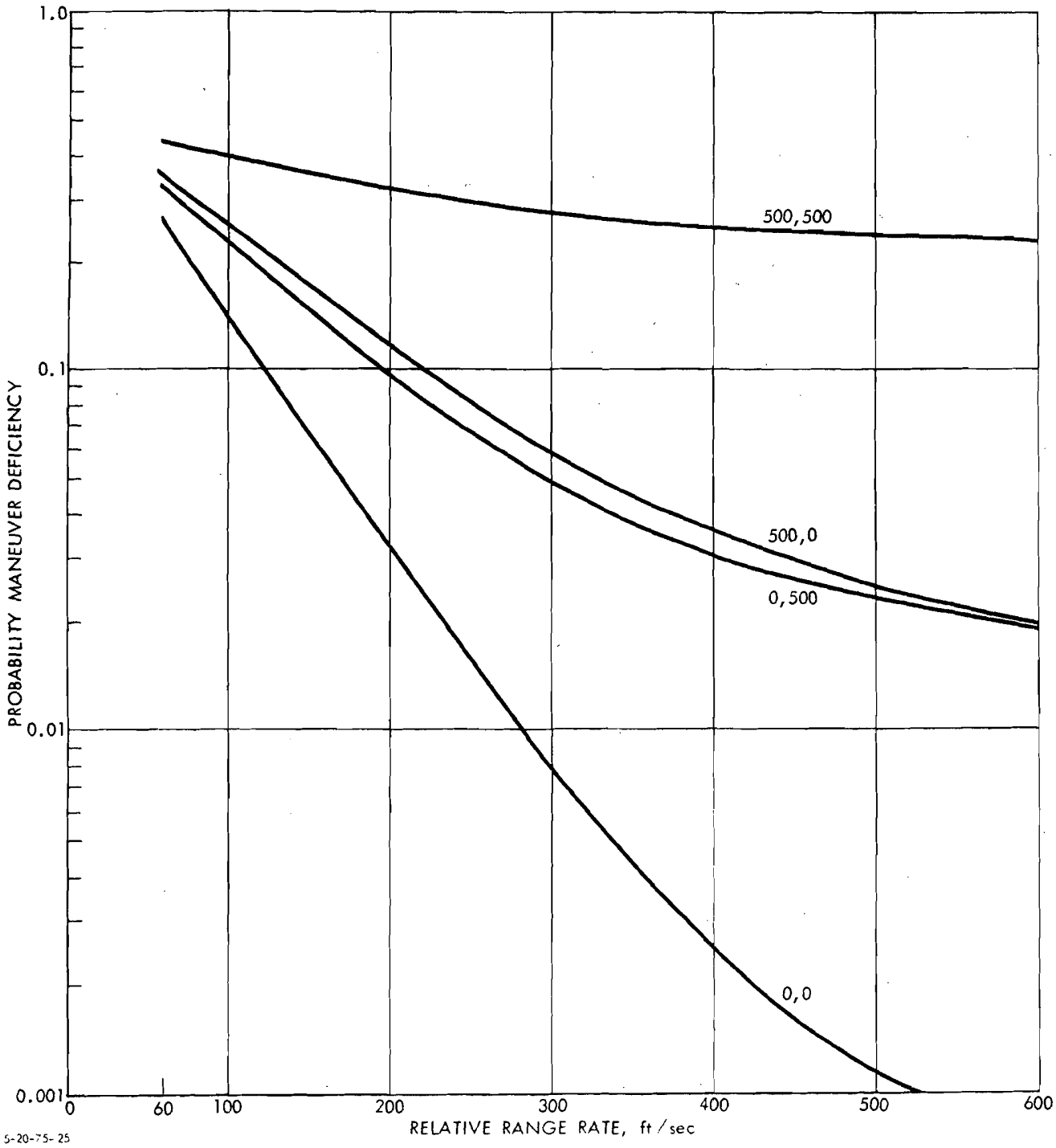


FIGURE D-3. Probability Maneuver Deficiency vs. Range Rate with Altitude Rate a Parameter, $h \leq 10{,}000$ ft, $\dot{U} = 4.8$ ft/sec^2

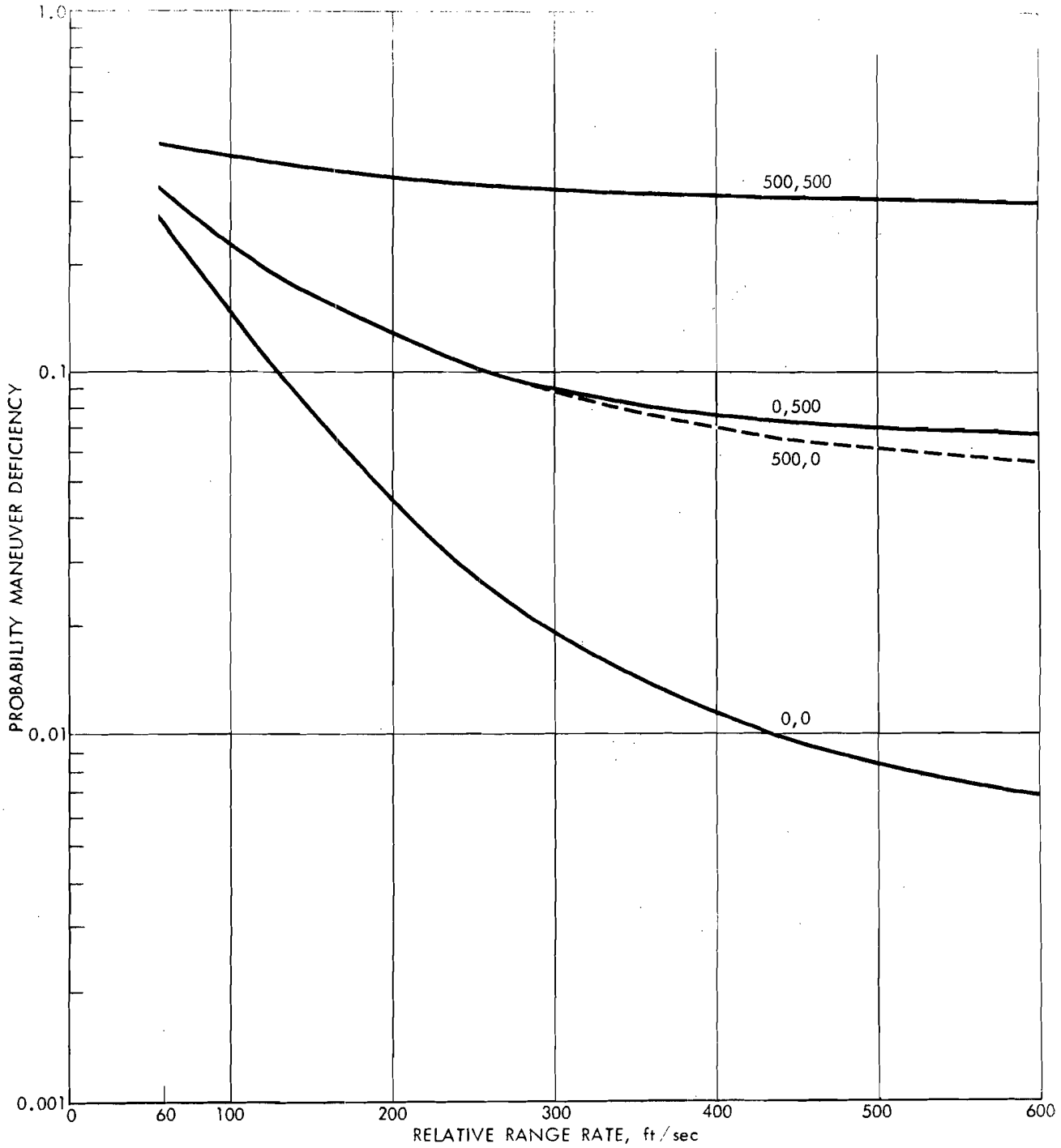


FIGURE D-4. Probability Maneuver Deficiency vs. Range Rate with Altitude Rate a Parameter, h > 10,000 ft, U = 4.8 ft/sec^2

CAS-equipped aircraft may be turning with a horizontal acceleration of 1/2 G, despite the occurrence of τ_2 warnings. Assuming that this is the case, and that at the time of an encounter the acceleration is uniformly distributed in heading relative to the line of sight between the two aircraft, i.e., relative to the range direction, the acceleration a along the relative range will be distributed in accordance with the following probability density function over the range $-A \leq a \leq A$ (the case of two-aircraft acceleration is ignored):

$$p(a) = \frac{1}{\pi} \frac{1}{\sqrt{A^2-a^2}} ,$$

in which A = 1/2 G.

A numerical calculation of the probability of maneuver deficiency was made using the change in warning time corresponding to the acceleration a, weighted by p(a). The results are given in Fig. D-5 for $\dot{h}_1 = \dot{h}_2 = 0$.

It must be remembered that when the probabilities of unsafe escape are calculated, the result must be reduced by the probability that acceleration is present (0.10), to which one should add the probability that no acceleration exists (0.9) times the probability of unsafe escape in the absence of acceleration. When the curves given in Fig. D-5 are modified in accordance with this prescription, Fig. D-6 results, and the probability of deficiency in maneuver is then seen to be less than 0.034 at low closing rates and less than 0.002 at the higher closing rates.

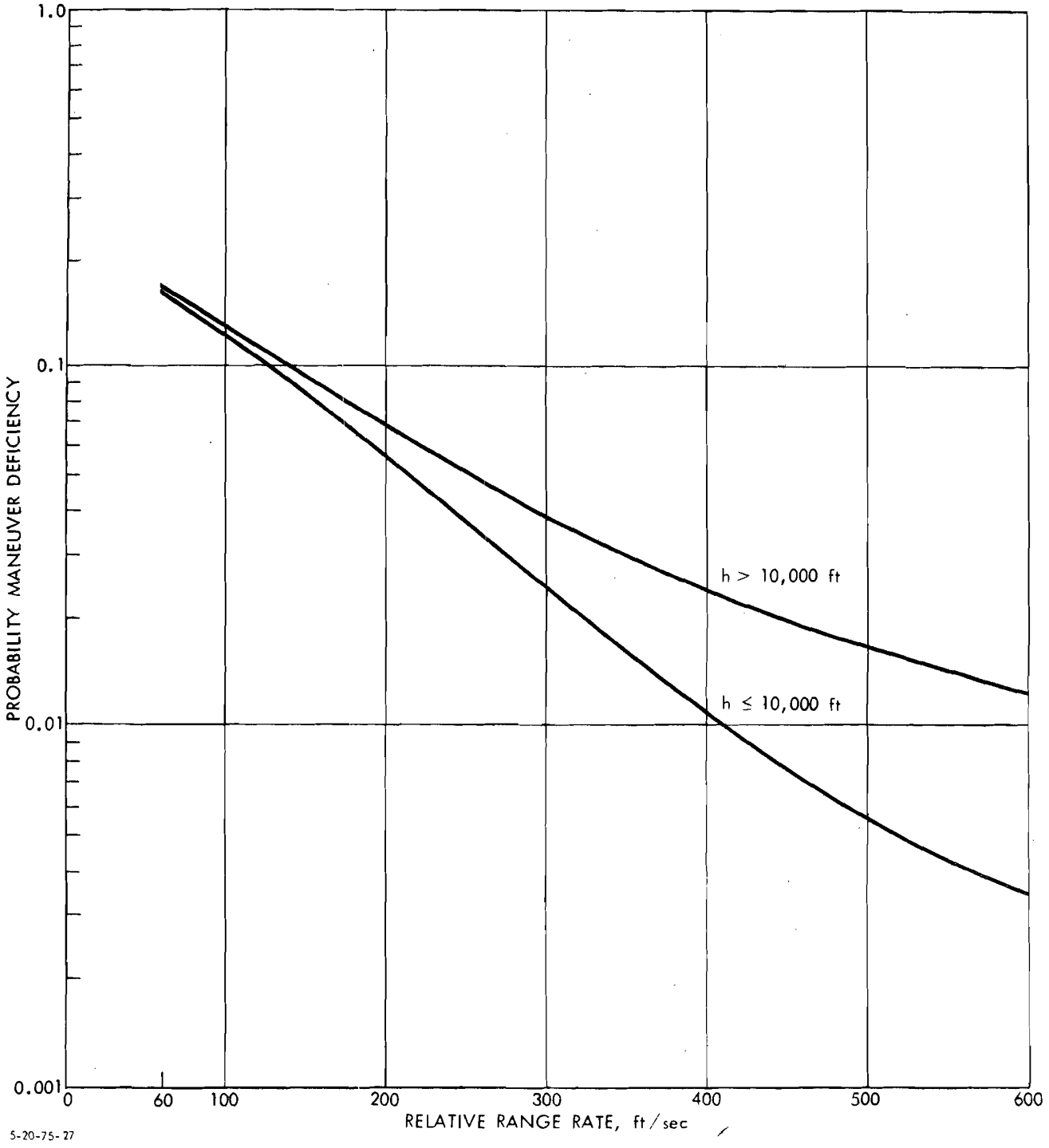


FIGURE D-5. Probability Maneuver Deficiency vs. Range Rate with Horizontal Acceleration of ½ G, Randomly Oriented and $\dot{h}_1 = \dot{h}_2 = 0$

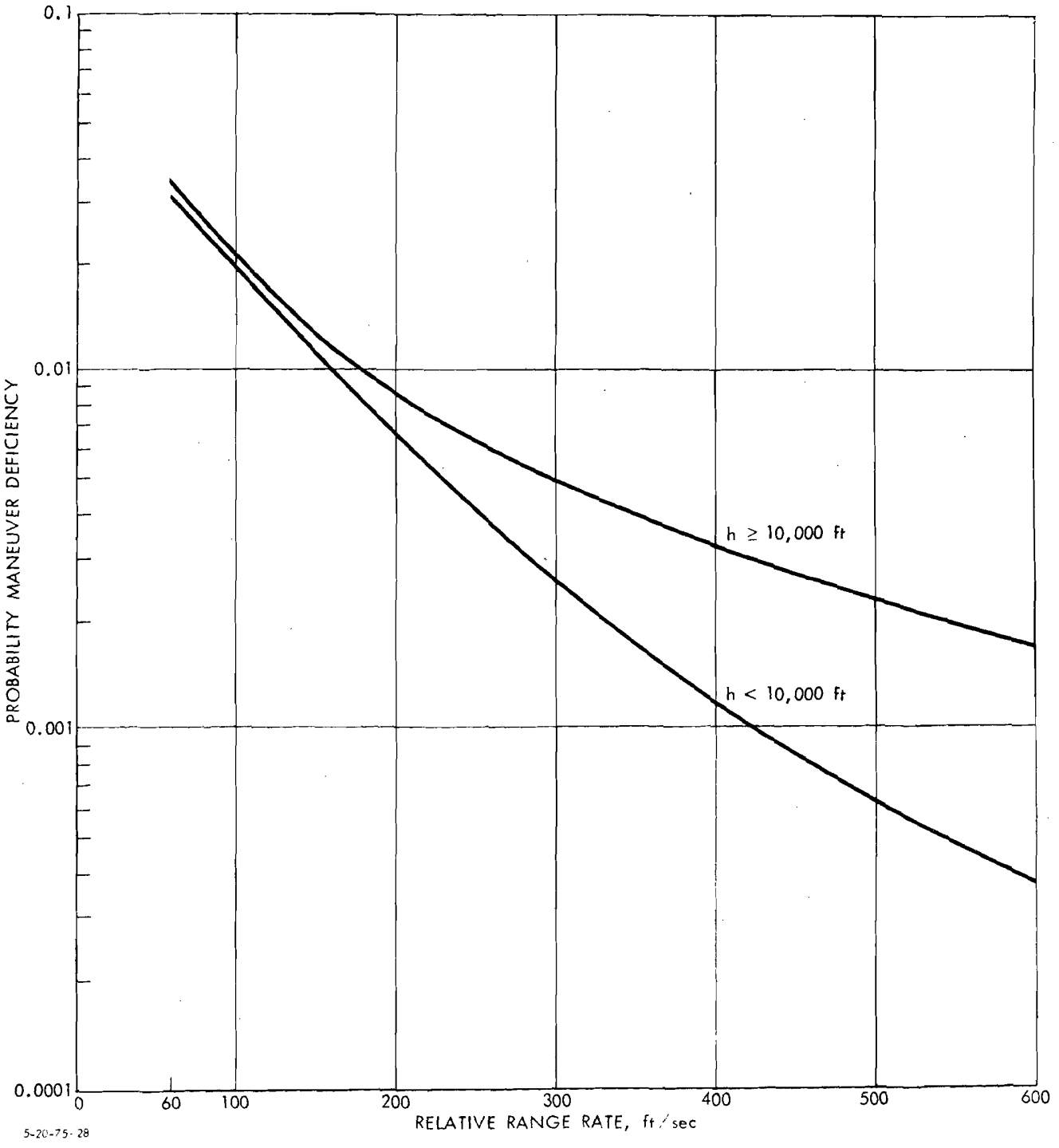


FIGURE D-6. Probability Maneuver Deficiency vs. Range Rate with Horizontal Acceleration of ½ G, Randomly Oriented and $\dot{h}_1 = \dot{h}_2 = 0$, Assuming that 10 percent of the Population is Turning

REFERENCES, APPENDIX D

D.1. Bagnall, J.J., and Dausin, L.R., "A Review and Analysis of the McDonnell Douglas Collision Avoidance System," IDA Study S-425, October 1973.

D.2. Mitre Corp., Report #MTR-6387, "Statistical Summary of the 1982 Los Angeles Basin Standard Traffic Model," April 1973.

APPENDIX E

CALCULATIONS OF MINIMUM SEPARATION FOLLOWING NCA ALARMS

APPENDIX E

CALCULATIONS OF MINIMUM SEPARATION FOLLOWING NCA ALARMS

As indicated in Section III-D, the non-co-altitude (NCA) and predicted co-altitude (PCA) criterion provided inadequate separations following commanded maneuvers in most cases involving two aircraft, one of which does not or cannot respond, e.g., a beacon-equipped aircraft. This appendix provides the results of calculations for the following assumptions:

1. One aircraft responds to alarms with an altitude rate deceleration of 1/8 G.
2. There is an initial time delay of 11 seconds following an alarm for a separation of 3300 ft. (This time delay arises from an epoch time of 3 seconds and combined pilot/aircraft response of 8 seconds.)
3. There is a time delay (t_d) of 2.5 seconds following subsequent alarms (average epoch delay of 1.5 seconds plus an assumed pilot/aircraft response of 1.0 second).
4. If a pilot is executing rate deceleration when the alarm changes but continues to indicate deceleration, no additional delay is involved.
5. Time delay terminating rate deceleration is ignored; this generally results in the aircraft coasting at a given limit rate, e.g., 1000 ft/min, until the next NCA boundary is crossed. (Note in Section III-D, Fig. 1, a delay of 1 second was used for deceleration termination.)

Results are presented for two cases: (1) Fig. E-1, in which the nonresponding aircraft has a climb rate of 2000 ft/min, and (2) Fig. E-2, in which the nonresponding aircraft has a

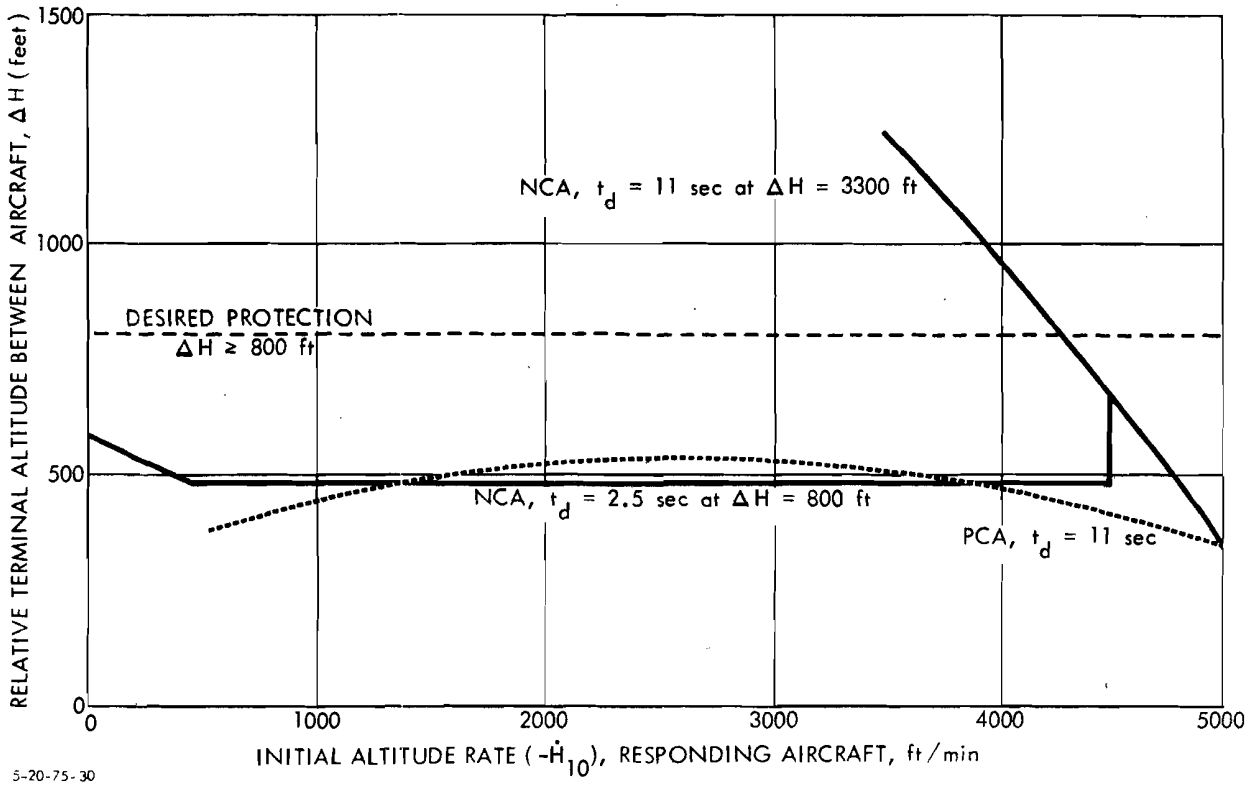


FIGURE E-1. Relative Altitude Between Maneuvering and Nonmaneuvering Aircraft at end of Maneuver with $\dot{H}_2 - \dot{H}_1 = 0$, τ_1 Alarm. $\dot{H}_2$ = 2,000 ft/min

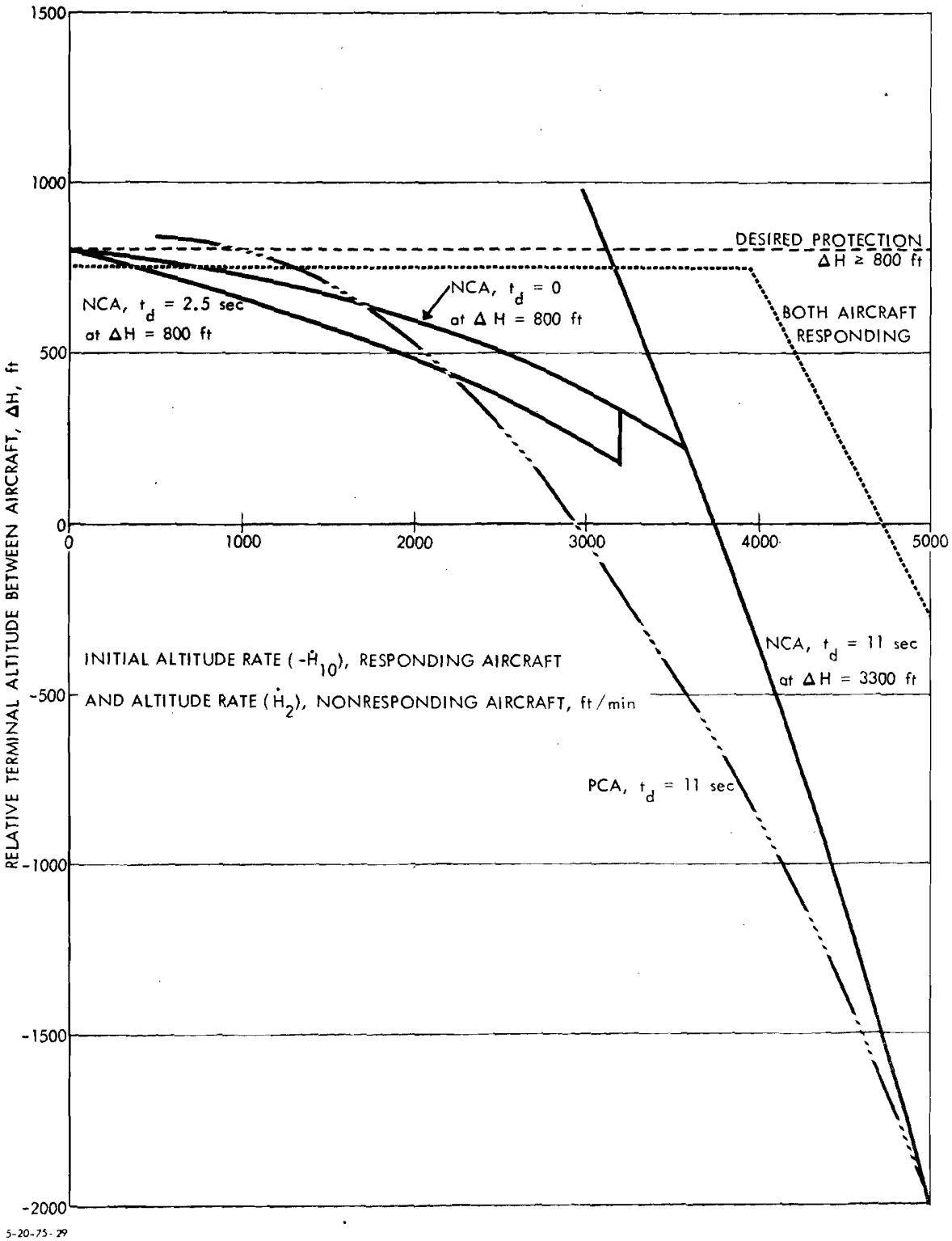


FIGURE E-2. Relative Altitude Between Maneuvering and Nonmaneuvering Aircraft at end of Maneuver with $\dot{H}_2 - \dot{H}_1 = 0$, τ_1 Alarm. Initial Rates: $\dot{H}_{20} = -\dot{H}_{10}$, variable

climb rate equal to the dive rate of the responding aircraft. The curves presented in the figures give the separation of the aircraft at the end of the terminating τ_1 commanded climb maneuvers of the responding aircraft. A negative separation in Fig. E-2 implies that the responding aircraft has passed through the altitude of the nonresponding aircraft. Under some conditions of high initial rates, $-\dot{H}_{10}$ greater than about 4500 ft/min in Fig. E-1 and $-\dot{H}_{10} = \dot{H}_2$ greater than about 3600 ft/min in Fig. E-2, the responding aircraft is in deceleration when the co-altitude boundary is crossed and the result corresponds to deceleration all the way from the initial NCA command. This condition also occurs, as shown in Fig. E-2, at about 3200 ft/min but with the responding aircraft having achieved a previous limit command with a short interruption of the deceleration, so that a $t_d = 0$ curve is also given. Although Fig. E-2 assumes that the responding aircraft would continue its maneuver, it would receive a new command to dive when it passes through the altitude of the other aircraft.

For comparison, the altitude separation for PCA is also included. In the latter case, the earlier incidence of NCA alarms has been ignored. Generally, the PCA command will appear only momentarily and only after initial NCA commands.

APPENDIX F

PARTS A AND B OF ANTC REPORT NO. 117

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PART A

AIRLINE POLICY AND REQUIREMENTS

A-1 Airline Policy

a. The airlines support the development and installation of an airborne collision avoidance system (CAS).

b. The CAS is not intended to be a substitute for a ground-based air traffic control system, nor should that system be designed to rely on the airborne CAS for separation of aircraft. The best solution to the problem of potential mid-air collisions is a positive, fail-safe air traffic control system under the jurisdiction of a central agency exercising control from gorund facilities. The role of an airborne CAS is to provide safe separation of aircraft when, for any reason, the ATC system does not fulfill its intended role. The CAS must not be dependent upon the ATC system.

c. A cooperative CAS, which requires both protected and intruding aircraft to be equipped with the CAS before the protected aircraft can reap the benefits of the system, is less desirable than a non-cooperative system which gives CAS benefits to equipped aircraft without requiring the intruding aircraft to be equipped. Since ample evidence is provided (by references a and b), and elsewhere, that a cooperative system is essential for an effective CAS at this time, the airlines support the development of a cooperative CAS.

d. The airlines oppose incompatible cooperative collision avoidance systems. A common system CAS is an essential airline requirement.

A-2 Basic Requirements

a. Functions Performed by CAS-

(1) The airborne CAS must:

- Detect all aircraft which represent a potential danger

A-2 a.

- Evaluate the hazards presented by the detected aircraft.
- Determine if a maneuver is required.
- Indicate the evasive maneuver required in time to maintain safe operation.

b. Accommodating All Airspace Users

(1) Objectively, the collision avoidance system should be designed to accommodate all airspace users.

(2) CAS equipment for use in privately-owned aircraft may be procured and maintained by some individuals to whom cost and complexity are necessarily more important than to other categories of users. Thus, it is recognized that as the cost of the airborne equipment increases, fewer aircraft will be equipped.

(3) A CAS which by its design would preclude the installation of the necessary cooperating equipment in any class of user would be significantly less desirable than a CAS which would permit all classes of users to participate.

c. Airborne Equipment Size and Weight

(1) The size and weight of airborne CAS equipment is of concern to all users, but will be an especially significant factor for smaller or lighter aircraft.

d. Number of Participating Aircraft

R/1 (1) The system must be capable of continuously assessing the potential collision threat of all aircraft in an environment in which participating aircraft will be in line of sight of other participating aircraft in constantly varying quantity as high as at least 1000 in number.

e. Independence from Ground Installations

(1) Whenever two fully equipped CAS aircraft meet, they must be protected based on the criteria established in this document. Desirably there should be no absolute need for ground installations of any type to make the airborne CAS operative. Every effort should be made to make the airborne CAS as independent of ground installations as can be justified by reasonable use of current technology.

A-2. e. (2) The system must provide for an equipped aircraft to safely re-enter and participate in a master time
R/1 oriented system after having left its influence.

f. Frequencies Utilized

R/5 (1) To provide an acceptable system capacity margin for all air space users, four frequencies are required. This will generate 2000 slots every three seconds. The center frequencies utilized are 1600 MHz, 1605 MHz, 1610 MHz, and 1615 MHz.

(2) The total allocated bandwidth for the system, to assure that 99% of all radiations are held within the bandwidth, will be from 1592.5 MHz to 1622.5 MHz.

g. Cockpit Display

(1) The airborne collision avoidance system should perform the analysis of hazards and provide an unambiguous command of the appropriate avoiding maneuver(s).

(2) The display of CAS information should not require evaluation by the crew. The system display should be an easily interpreted visual command. Aural or other commands will be utilized to keep pilot reaction time to the lowest feasible value.

h. Criteria for CAS Performance

R/3 (1) Whenever two fully CAS equipped aircraft meet, they must be protected based on the criteria established in this document.

(2) The system shall assure safe vertical separation even if only one aircraft can or does respond to the maneuver command. [See A-4 b. (1)]

R/4 (3) The CAS must provide protection to all equipped aircraft regardless of the relative bearings or relative altitudes of all aircraft using the system. This includes taking into account the radiation patterns of antennas mounted on aircraft and the resultant variations of signal.

(4) The CAS must function in normal manner during all in-flight maneuvers and in the presence of rough and smooth terrain, and man-made objects.

A-2 h. (5) All aircraft that constitute a threat to each other must exchange the required communications. For example, for two synchronized SST aircraft with a closure rate of 3,600 knots, the reliable
R/1 communication range of the equipment must be in the order of 40 NM to allow for processing time, pilot and aircraft reaction times and obtaining of the required safe separation. For two synchronized subsonic aircraft with a closure rate of 1,200 knots, the reliable communication range must be in the order of 15 miles. Communications exchange adequate to provide time synchronization is required prior to reaching the specified distances.

R/1 i. In-Flight Failure Warning - Functional Test

(1) The system should indicate to the pilot if it is not operating satisfactorily.

(2) The equipment should include a test feature to allow determination of satisfactory operation
R/1 for maintenance remote from any ground or airborne station which may be a part of the system.

(3) Refer to ARINC Report 415/ATA ANTC Report 112
R/2 for background material and basic industry policy in these areas.

A-3 <u>Techniques Employed</u>

a. Synchronized Time/Frequency Range Altitude System

(1) The most suitable technique for providing an airborne collision avoidance system at this time is one based on synchronized accurate time permitting range and range rate to be obtained from each transmission as the result of one-way propagation of a radio frequency signal.

(2) The ratio of measured range divided by range rate will be a major criterion used for threat evaluation and is designated Tau. Thus, Tau becomes the time to collision for non-maneuvering aircraft on collision courses. In view of the limitations in the accuracy of the range rate measurement and to provide adequate minimum range separation in slow closure encounters, a supplementary alarm criterion based upon range will be used.

(3) Each aircraft system will transmit pressure altitude
R/9 data based on a 29.92Hg reference. The system logic values are based on maximum aircraft altitude system errors of ± 250 Feet 3 Sigma (Gaussian) at altitudes

A-3 a. (3) of 10,000 feet or higher and ± 150 at altitudes of 10,000 feet or lower.

Comment: If it is determined that the aircraft supplied information is better or worse than these values, the threat logic values (altitude bands and warning times) must be changed accordingly.

(4) The aircraft's own indicated pressure altitude will be compared with that of all other aircraft which may constitute a threat. The CAS will establish a protected altitude band below and above its own pressure altitude (see B-3).

A-4 <u>Maneuver Commands and Advisory Displays</u>

R/9 NOTE: The present system is designed to protect against the incremental horizontal and vertical maneuvers specified in this section. Any adoption of this system intended to protect aircraft using larger values of acceleration is the responsibility of those users of the system equipping aircraft that would utilize such higher performance capability also see A-4 b (3) and Reference aa.

a. Types of Maneuver Commands

(1) Five maneuver commands will be used by the collision avoidance system; these are:

(a) Vertical Maneuver -- requires a rotation in the direction specified at an acceleration of not more than 1/4G, but not less than 1/8G up to a terminal climb or
R/1 descent rate of 2,000 feet per minute or more until the maneuver command is terminated. See A-4.b. (1) (e).

(b) Hold Altitude -- requires remaining in level flight.

(c) Level Off -- requires decreasing vertical speed to zero in
R/6 not more than ten seconds.

(d) Roll Out -- Do Not Turn requires return to linear flight in no more than four seconds; if not turning, a turn will not be
R/9 initiated.

"No Turn" means do not exceed a bank angle of 10° degrees.

A-4. a. (e) Vertical Speed Restriction --
R/9 If vertical speed exceeds the restriction, it must be reduced in not more than 10 seconds.

b. Criteria for Use of Maneuver

(1) Climb/Descend Maneuver

(a) The system shall assure safe vertical separation even if only one aircraft can or does obey the climb or descend commands. In the vast majority of all climb/descend avoiding maneuvers involving two air carriers or other completely equipped aircraft, both will maneuver cooperatively; however, since exceptional situations may be expected, system design must be predicted on safe separation should only one aircraft maneuver.

R/3 (b) The system of an aircraft will compare its own altitude (A-3(a) (3)) with that of all other cooperating aircraft, and will command a climb/descend maneuver if the altitude Tau or altitude and range threat criteria, which require a climb or descend maneuver, are satisfied.

(c) An airborne CAS detecting the requirement for a climb/descend maneuver will indicate to the pilot a command in the direction which increases the indicated altitude difference between the two aircraft. When a co-altitude climb/descend command is detected, the system logic of the aircraft system will bias the next altitude transmissions in the direction of the indicated escape. For example, assuming the decision is to climb, the display of the aircraft will show a "go up" maneuver command and the logic in the CAS will cause the succeeding altitude transmissions to correspond to an altitude higher than those which normally would have been transmitted.

R/9 (d) Intentionally omitted

R/9 (e) Although 650' is the design minumum indicated vertical separation for no alarm, the indicated vertical separation between two aircraft utilizing

A-4. b. (1) (e) the CAS and obeying the vertical maneuver commands will be 800' above 10,000' altitude.

R/9 The 800' value results from the 100' incremental digitization of altitude plus allowance for vertical entry into a threat situation at low altitude rates (e.g., 500 ft./min.) which are below the system detection threshold. At 10,000 ft. and below a value 600 ft. is used based on the same logic.

(2) Hold Altitude Command

(a) Three aircraft may be involved in co-altitude situations. In these situations one aircraft must not maneuver in either direction, although the system would in the absence of either one of the other two aircraft require it to maneuver. The system must assure adequate time for vertical separation even though one aircraft is commanded not to maneuver. (See A-4.b.(1)(a) and B-3).

(3) Roll Out Command

R/9 (a) A roll out command is required to return the aircraft to approximately linear flight (± 10° bank angle) when a potential collision situation is sensed by the system.

(b) Studies conducted by McDonnell Aircraft (reference h), and others, demonstrate that loss of warning time between two aircraft, for all angles of approach, can be sufficiently reduced by stopping the turn(s) immediately upon receipt of an appropriate alarm. Also see Reference aa.

(c) An incremental horizontal acceleration of 1G will be protected against. It is assumed that the 1G value will be the three sigma probability result of the combining of the horizontal incremental acceleration of any two aircraft flying in the ATC system. Aircraft expected to

A-4. b. (3) (c) operate with larger incremental horizontal
R/6 acceleration than 1/2G must increase its own protected volume accordingly. (NOTE: A coordinated turn of approximately 30° bank angle will produce 0.5G horizontal acceleration and a normal loading of 1.12G's.)

(d) A CAS can be built to accommodate any incremental horizontal acceleration of two aircraft by providing alarms when the appropriately increased volume of airspace is penetrated. For example, a CAS for application in a military aircraft could protect against an incremental acceleration of 2 or 3G. A change in the CAS equipment to accommodate various values of incremental acceleration is not complicated in a CAS of the type specified and might reasonably be expected to be a switch or other function for combat or acrobatic versus normal ATC Flight.

(4) Level Off Command

(a) An incremental vertical acceleration of 1/2G and a 10,000 foot-per-minute rate of climb will be protected against. The CAS system design assumes that these values will be the three sigma probability results of the combining of the vertical movement
R/6 of any two aircraft using the CAS system. Aircraft expected to operate with vertical accelerations greater than 1/4G or vertical rates greater than 5,000 feet/minute must increase its own protected volume accordingly.

(b) A level off command is utilized to command an aircraft to stop the climb or descent in
R/9 ten seconds when the CAS system detects that the protected altitude band which possesses a Tau or minimum range threat. (See B-3.). The vertical threat evaluation will be based on the assumption that the other aircraft has an equal and opposite altitude rate.

A-4 b (4) (c) An aircraft which is not changing altitude shall
R/5 not be required to discontinue turning because of the presence of another aircraft which is not co-altitude. Adequate warning time shall be provided for one of the following:

(1) Allow climbing or diving aircraft to safely discontinue their altitude maneuver before penetrating the co-altitude band of the level flying aircraft.

(2) Allow the turning aircraft to roll out if the climbing or diving aircraft penetrates its co-altitude band.

(d) Intentionally omitted.

R/9

A-4 c (1) Vertical Speed Restriction

This display uses the same Tau zone warning criteria as the level-off display, Paragraph A-4 b (4), and
R/9 is activated when the received altitude data from another aircraft indicates that it is within the above/below band but outside the level-off band of the receiving aircraft.

(2) These displays command vertical speed restrictions based on the altitude difference between the threat aircraft. If vertical speed exceeds the indicated limit it should be reduced to or below the value listed. The displays also provide means to ascertain that safe separation has been obtained following the
R/9 execution of a co-altitude avoidance maneuver command. They can also be used to indicate to the pilot when it is safe to return to a preassigned altitude.

A-5 <u>Time Allowances for Maneuvers</u>

a. The maneuvers specified by A-4.a. are acceptable for routine passenger operations. Having specified such nominal avoiding maneuvers, it is essential that the pilot maneuver without delay when commanded.

b. Additional seconds allowed and added for pilot reaction time, movement of the aircraft controls, the cycling of the CAS system, and CAS error, should be kept to a minimum since all have the effect of increasing the protected volume of airspace around each aircraft.

A-5 c. The selection of the most appropriate warning times is a balance between assuring that the probability of collision is acceptably low and assuring that the interaction of the CAS system with the air traffic control system is held to an acceptable level.

d. A detailed analysis of warning times have been conducted by Collins Radio and is reported by Reference i). A computer analysis of threat logic was conducted by
R/9 McDonnell Douglas Corp. The results of this analysis (Reference aa) have permitted the selection of the most suitable warning times and protection envelopes.

R/10 e. Simulation of the CAS vertical and roll out maneuvers has been conducted by the airlines using their aircraft simulators. The results of this (and other) simulation has been reported (Reference ee).

A-6 <u>Operational Limitations on CAS Utilization</u>

a. To prevent undesirable interaction between CAS equipped aircraft during takeoff and landing some CAS functions are deleted during these operations. Limited CAS protection is provided at these times since the outer alarm zone and certain commands are eliminated. (See B-3.g.)

TECHNICAL DESCRIPTION OF SYSTEM

B-1 System Criteria, General

a. Message Format

R/9 (1) Details of the CAS message format are provided by Attachment 1. A 200-microsecond burst is employed to provide both range and range rate.

R/9 (2) Although not a threat evaluation requirement, the use of biphase modulation in the 200 microsecond burst is utilized to convey additional information. Participating system must process doppler and biphase information from these pulses.

R/9 (3) Reference z provides a discussion of the bi-phase encoding of the 200 microsecond range/doppler burst. (Attachment 1, pages 1 and 2, provides the format of the doppler burst)

b. Frequency Control

R/3 (1) The radio frequency and system time for the CAS will be based on the zero magnetic field cesium atomic resonance defined as 9, 192, 631, 770 Hz.

R/9 (2) The provision for adjustment of the CAS frequency of 700 parts in 10^{10}, previously specified, is no longer required as a result of the action taken by the International Telecommunications Union (ITU) (New Delhi resolution in February 1970 of Study Group 7 of CCIR).

R/1 (3) The absolute frequency accuracy with respect to the assigned frequencies of the airborne stations when participating in the collision avoidance environment must be within one part in 10^8.

(4) Two methods for maintaining the correct RF carrier frequencies for operation of airborne systems are specified:

R/9 (a) Use of airborne atomic standards which shall not deviate more than 2 parts in 10^{11} from the assigned frequencies.

(b) Use of stable crystal standards which can be pulled on to frequency (within one part in 10^9) during communication with synchronized stations and be accurate to within one part in 10^8 at any other time.

B-1. b. (5) Either method is expected to permit leaving the airborne equipment in service for several months of time and not require taking the oscillators out of service for adjustment.

R/9 (6) Resetting of an crystal oscillator frequency can be accomplished only when system remains in fine synchronization range of synchronized stations for 10-15 minutes. Unless an integration time of this order is used, instability can result.

(7) Power will be necessary to permit the precision atomic oscillator, or compensation of the air carrier airborne equipment, to function continuously. This continuous source of power may be used to
R/9 permit the systems with atomic clocks to keep time and retain slot counting while on the ground.

c. Slot Selection and Protection

(1) The selection of message slot shall be accomplished to afford approximately uniform use of the avail-
R/9 able slots.

(2) Intentionally omitted.
R/9

(3) The system provides 2000 time slots, but not all of
R/9 them will be available for collision avoidance messages. Slot 0000 is reserved for epoch start pulses, and one or more slots may be set aside for special purposes and test and maintenance functions. (See Attachment 1 Section 3).

(4) Aircraft arriving within air-to-air range may use the same message slot until their separation is 230 N.M. or less. To prevent interference between the transmissions of two aircraft attempting to use the same slot at too close a range, each aircraft system will stop transmitting and listen for one epoch in its own message slot on a random basis such that, on an average, co-slot occupancy is detected within forty seconds. Listening for co-slot occupancy should not occur during successive epochs nor during hazardous encounters until the encounter is past. If the message slot being used by an aircraft is found to be in use by another aircraft at too close

B-1. c. (4) a range, the listening aircraft will automatically vacate that slot and transmit in a slot previously determined at random to be vacant. When a message slot has been vacated due to co-slot occupancy, there is no need to return to that message slot after the other occupant is out of range or ceases to operate. (Back-up mode aircraft should use the same criteria verifying that its F_1 message slot is free of other F_1 transmissions) (See B-4e. [2]).

(5) Ground Stations may be assigned specific slots for message and test transmission purposes.

R/10 (a) Sixty three time slots are reserved for ground station transmission of airborne equipment tests, obstacle avoidance, and landing aid messages as follows:

F1 slots 1548 plus 32N (N=0-14)...15 slots
F2 slots 1037 plus 32N (N=0-15)...16 slots
F3 slots 526 plus 32N (N=0-15)...16 slots
F4 slots 15 plus 32N (N=0-15)...16 slots

The reservation of these message slots for ground station use imposes the requirement upon the aircraft CAS to recognize these reserved slots and not use them for normal air-to-air airplane collision avoidance.

(b) Of the above slots, sixteen slots numbered 79 plus 128N, 590 plus 128N, 1101 plus 128N, and 1612 plus 128N (N=0,1,2 and 3) are reserved for ground station transmission of test messages and shall only be used for testing CAS equipment. (Test transmission format is supplied by Att. 1, Sect. 3.7)

(c) The slots, except those which are reserved for test message transmissions (b), shall be reserved as follows:

F1 slots for basic CAS ground stations....11 slots
R/10 F2 slots for landing aid beacons.....12 slots
F3 and F4 slots for obstacle beacons.....24 slots

d. Protection from Line of Sight Interference

(1) The probability of line of sight interference from an aircraft utilizing the adjacent time slot is a very real one which must be resolved in the design of a practical collision avoidance system. A power budget analysis of a cooperating system designed to warn of a collision threat at 40 NM indicates that under some conditions of antenna look angle, transmitter power, and receiver sensitivity, the system could possibly react to signals at six hundred miles. This situation could

B-1. d. (1) result from a performance difference of about 24 db above that required between two cooperative systems for the 40 NM range -- a variation not untypical.

B-1 d. (2) The system described by this paper uses frequency shifting following the slot data period rather than increasing the slot period to include all or a portion of the line of sight transit time. After providing for the transmission and reception of the collision avoidance and synchronization data, the system shifts to an adjacent frequency.

e. Use of Time Diversity Technique

R/4 (1) The collision avoidance and synchronization data occupy 1455 microseconds of slot time. Adjacent slot interference is reduced by stepping the frequency at the end of this period of the slot. Thirty microseconds for receive and 45 microseconds for transmit are allowed to assure that all airborne systems have shifted to the next frequency in the sequence and are ready for the necessary data.

(2) The entire system begins on the lowest frequency (slot 0000), cycles upward to the highest frequency during the next three slots, a slot at a time, and then returns to the lowest frequency and repeats the cycle.

(3) The time diversity techniques used by the system requires the receiver frequency to be stepped in synchronization with the slot counts.

(4) Air carrier aircraft transmitters must be able to transmit on the four frequencies since the air carrier slot utilization requires any of the available slots to be used.

f. Multipath Protection of Data Required Each Epoch

R/9 (1) Certain data are required each epoch; these are range, range rate, and altitude.

(2) The message format has been organized to minimize the effect of multipath signals by assuring that any data necessary each epoch is provided during a period of time when multipath signals from the preceding data are no longer expected.

(3) A standard allowance is made for multipath propagation which is adequate for situations including the high altitude SST. Varying amounts of multipath protection increasing with aircraft altitude were rejected as being unnecessarily complex and would have required the making of assumptions as to the mix of aircraft with altitude, which was avoided as being undesirable.

B-1 g. Aircraft Use of Two Antennas

(1) Two aircraft antennas are required on most aircraft to provide satisfactory spherical coverage.

(2) The system power budget has been based on cable losses of 4 db to each of two antennas (B-5.a).

(3) Since upper and lower coverage antennas will be required for many aircraft, the specified antennas must be utilized during the odd and even epochs (Attachment 1). The time required to switch from one antenna to another, must not exceed 30 microseconds.

(4) During odd epochs, transmission and reception will be accomplished using the upper antennas except during own slot when transmission and reception will be accomplished using the lower antennas (Attachment 1).

(6) Diversity reception is permited. However a simple criterion such as signal level comparision is not adequate since it has been shown that multipath contamination can be accentuated. The basis for the use of diversity reception is to enhance
R/9 communications probability. Transmission must utilize only a single antenna as specified above to avoid an interference pattern being generated from the simultaneous use of two antennas.

(7) Detection of multiple slot occupancy problem must include use of lower and upper antenna switching (when utilized).

(8) If a single, spherical coverage antenna is proven feasible for some aircraft, it will be used for all communications of the CAS system.

(9) When operating in the back-up mode, the following
R/9 applies:

In the interrogating aircraft, antennas are switched every epoch for CAS message transmission. The replying aircraft always uses the upper antenna to receive and transmit. the interrogating aircraft receives replies on the antenna which was used for transmitting the interrogation [See B-4 b. (8)].

B-1. h. Limitation of CAS Transmissions by Aircraft on the Ground

(1) To eliminate the possibility of providing invalid CAS data to aircraft in flight, aircraft altitude will not be transmitted by any aircraft on the ground. This implies a strut switch or similar device. (See Attachment 1 Page 3).

(2) Ground testing of a CAS device may require the allocation of one or more "test" slots. Refer to Attachment 1, Section 3.7.

B-2. <u>System Criteria for Time Synchronization</u>

a. Introduction

(1) The system provides 2,000 time slots which occur at a rate of one each 1,500 microseconds. Thus, each time slot reoccurs once each three seconds. The three-second period is called an epoch.

(2) Each aircraft participating in the CAS system utilizes one of the time slots for transmitting its CAS data. Since the epoch is three seconds long, each participating aircraft transmits CAS data each three seconds. The transmitting frequencies of each aircraft are crystal (or atomic standard) controlled and must be within one part in 10^8, or better. (B-1 b.)

(3) Each aircraft will transmit at a specified time in its time slot. Since the time of transmission and the RF frequency will be known by all other aircraft in the CAS system, the range and range rate of all aircraft will be mutually available. The ratio of measured range divided by range rate will be a mojor criterion used for threat evaluation and is normally designated "Tau". Thus, Tau becomes the time to collision for non-maneuvering aircraft on collision courses.

(4) Normally, aircraft time will be set by listening to and being synchronized by ground stations. A network of ground stations will be provided for this purpose. A means has been provided to extned this ground station master time through aircraft-to-aircraft relay. This extension of ground station time establishes time of descending accuracy - called a time hierarchy. Fully equipped aircraft will provide an estimate of the expected accuracy of

B-2. a. (4) their time by including their time hierarchy number in their transmission. Aircraft out of range of a ground station will request master time from other aircraft of better time hierarchy status. This request will be accomplished by discrete address to the desired donor using the donor's message slot number in the address. This process is called "fine synchronization". Refer to Attachment 1 and Ref. M for details.

(5) Two major categories of airborne equipment have been designed and have been identified as:

(a) Air carrier or other complete equipment
(b) Limited Equipment (See B-6).

(6) All air carrier aircraft transmission will include the following elements:

(a) Two hundred microsecond range, range rate burst.
R/9 (b) A 25.6 (16.4 for BUM) microsecond pulse, providing pressure altitude based on a reference of 29.92" Hg.
(c) The 200 microsecond RF burst is biphase modulated to provide for airborne station propagation of master time and aircraft identity:
R/9

i) Position in time hierarchy;
ii) An optional airframe identification;
iii) Slot address for fine synchronization request;
iv) Parity checks

b. Ground Station Propagation of Master Time

(1) Ground station will transmit a unique epoch start triplet in slot 0000 of every other epoch. These ground station epochs are called "odd"
R/3 epochs for identification. The time at which the leading edge of the first of the epoch start pulses is transmitted is referred to a T_o.

(2) The timing of the epoch start pulses of each ground
R/4 station will be kept within plus or minus one half microsecond (three sigma value) of a designated CAS worldwide time.

(3) During the odd epochs, the ground station supplies fine synchronization replies to all aircraft (cont.)

B-2. b. (3) within fine synchronization range timed from receipt of the leading edge of the aircraft
R/9 200 microsecond range/doppler burst. A fine synchronization request shall be validated by the ground station. (Attachment 1 Section 3).

(4) If two ground stations are separated by a distance less than twice the fine synchronization range, both ground station replies may be heard by some aircraft, which may bring about the requirement for various ground stations to alternately utilize the odd epochs.

R/4 (5) Ground stations normally will not reply to either of two or more aircraft occupying the the same message slot when more than one range doppler pulse is received prior to the time of transmission of the resync signal.

(6) Ground stations will transmit their epoch starts using the frequency 1600 MHz and will supply fine synchronization replies on each of the four CAS frequencies.

(7) A ground station will supply fine synchronization during even epochs to any aircraft which addresses it.

c. Airborne Station Propagation of Master Time

(1) Air carrier stations in hierarchies 1 through 59 will transmit unique epoch start triplets in slot 0000 of the even epochs using the frequency of 1600 MHz and reply to fine synchronization requests on each of the four CAS frequencies.

(2) An air carrier station, after coarse synchronization by receipt of the epoch start pulse, will request fine synchronization by transmitting the slot number of the station from which fine synchronization is desired and will indicate own synchronization hierarchy as #63.

(3) Some suitable method of arbitrarily advancing own station time is permissible to extend the range of start up fine synchronization, but is not intended for normal fine synchronization, since such fine synchronization range extension could jeopardize CAS performance after an airborne station is operating in the system.

R/8 (4) During any even epoch, the synchronized air carrier stations will supply fine synchronization to other air carrier aircraft of lower hierarchy rank (higher hierarchy number) which transmit an addressed request which matches the donor's (cont.)

B-2. c. (4) message slot number. During the air (even) epochs, each synchronized air carrier station will supply, on a random duty cycle, fine synchronization to all aircraft which (a) transmit hierarchy 63 and a 2047 sync request, (b) do not transmit biphase data. The average duty rate shall be 1/n where n is equal to the number of synchronized air carrier aircraft within communication range (counting own aircraft). Thus, for example, when there are two other synchronized aircraft within range the
R/9 duty rate is 0.33. A duty cycle of greater than 0.5 is not desired.

R/4 (5) Air carrier stations will not reply to either of two (or more) aircraft occupying the same slot when more than one range doppler pulse is received prior to transmission of the resync signal.

(6) Ground stations will be the source of master time and are designated zero (0) in the time hierarchy.

(7) An airborne air carrier station receiving fine synchronization from a ground station will carry a designation of #1 in the time hierarchy. (See details of message format, Attachment 1, for information on binary data transmitted for this purpose.) The time accuracy of the #1 status aircraft station must be within plus or minus .25 microseconds (three sigma value) of the time of the ground station which synchronized it at the time the fine synchronization occurred.

(8) Airborne air carrier stations receiving fine synchronization from other air carrier stations having a #1 status will carry a designation of #2 in the time hierarchy. The time accuracy of a #2 status station must be within plus or minus .25 microseconds (three sigma value) of the #1 station that synchronized it at the time synchronization occurred.

(9) Sixty steps are provided in the time hierarchy. Any air carrier aircraft arriving in fine synchronization range of other air carrier aircraft with better time hierarchy status will request a fine synchronization from the aircraft in fine synchronization range which (cont.)

B-2. c. (9) carries the best (lowest numeric) designation
R/4 in the time hierarchy.

(10) As a result of the averaging process (some aircraft time fast--others slow), the time accuracy of the Nth step in the time hierarchy will be $.25\sqrt{N}$. Therefore, the 36th step, for example, is statistically expected to have an accuracy of 1.5 microseconds (three sigma value) exclusive of initial ground station tolerance.

(11) Aircraft-to-aircraft relay (chaining) of master time is acceptable to time hierarchy level #60.

R/10 (12) Upon receipt of fine synchronization, an aircraft will commence timing to determine how long it may remain in the time hierarchy just achieved (ground station or donor aircraft hierarchy, plus one). In the event the next fine synchronization does not arrive before the system would develop an expected .05 microsecond of time error (based upon the manufacturer's specification for clock stability), the system will demote itself one step in the time hierarchy and continue to demote itself at the rate of one hierarchy per .05 microsecond until 40 is reached. At any time in the process if fine synchronization from an eligible donor (i.e., one whose hierarchy rank is better) is achieved, the system will designate itself the donor's hierarchy number plus one.

(13) Aircraft-to-aircraft relay (chaining) may continue above hierarchy #40 through #60;
R/3 however, hierarchy status above #40 may be continued for only five epochs in the absence of a fine synchronization from an aircraft of better hierarchy status.

(14) An aircraft system which demotes itself beyond
R/9 hierarchy #40 will stop transmitting; unless it is in back-up-mode, or in a threat condition, or receiving coarse synchronization (see Attachment 1).

(15) An aircraft will not discontinue normal CAS
R/9 synchronized mode operation during a threat encounter.

B-2. d. System Time Validation

(1) Aircraft may receive fine synchronization from either ground or aircraft systems, however, aircraft stations within fine synchronization range of ground stations will always utilize the ground station fine synchronization, when
R/8 provided in odd epochs, or request fine synchronization from ground stations in even epochs where bi-phase modulation is provided.

(2) A three-pulse code is provided to permit detailed validation of the fine synchronization signal before its utilization. Several fine synchronization cycles should be utilized by air carrier systems before own station system time is adjusted; for example, an average of three or more fine synchronization replies all falling within a few tenths of a microsecond of each other.

R/3 (3) Aircraft operating in the range of other aircraft all of the same hierarchy, and not in range of an eligible donor of lower hierarchy, may periodically check (but not reset) own station system time by utilizing fine synchronization signals. If own station time consistently falls outside the system time tolerance the system should demote itself to the next higher numbered time hierarchy.

(4) It is the intent that the interpretation of the foregoing synchronization procedural rules establish a time which is traceable to the CAS master time standard, and any interpretation of the foregoing rules which would establish a local time (i.e., one which is not traceable to the CAS time standard) is expressly forbidden.

B-3 Threat Evaluation, Separation Standards and Output Logic
R/10

The altitude threat status and the tau-range threat status of the other aircraft shall be evaluated, then coupled to produce the appropriate logic output. Air-to-air threat evaluation will not be performed in message slots reserved for ground station or beacon transmissions (B-1. c. [5]).

a. Altitude Threat Evaluation

The CAS equipment shall compare the received altitude data relative to "own altitude data". On the basis of this evaluation, the equipment shall classify the altitude threat status in-bands as shown in Figure 1.

(1) Co-altitude bands are based on the difference between the indicated altitudes of the two aircraft. These zones shall include all values from zero to ± 800.

(2) Above-below bands each extend an additional 2500 feet beyond the outer edge of the co-altitude bands and are subdivided as shown in Figure 1.

(3) Predicted co-altitude band shall be extended from the outer edge of the co-altitude band in the direction of altitude rate when the rate is
R/9 greater than 500 feet/minute. The height of this band shall equal to "own" altitude rate times 30 seconds.

(4) No altitude threat exists when the other aircraft is outside the bands listed in B-3.a. (1) through B-3. a. (3).

(5) A Back Up Mode (BUM) identified altitude pulse shall not be used for threat evaluation while
R/9 operating in the synchronized mode.

b. Range and Range Rate Threat Evaluation

The CAS equipment shall evaluate the measured range and range rate data and classify the threat as shown in Figure 2. The design values of the tau threat zones are;

B-3. b.

Tau Zone	R_M (N.M.)	R_O (N.M.)	1/Slope* (Sec.)
1	0.5	0.25	25
2	-	1.8	40

R_M = Minimum Range R_O = Offset Range

* R.H. Edge of Threat Zone as shown in Figure 2.

This is the slope of the line, not the linear time to collision

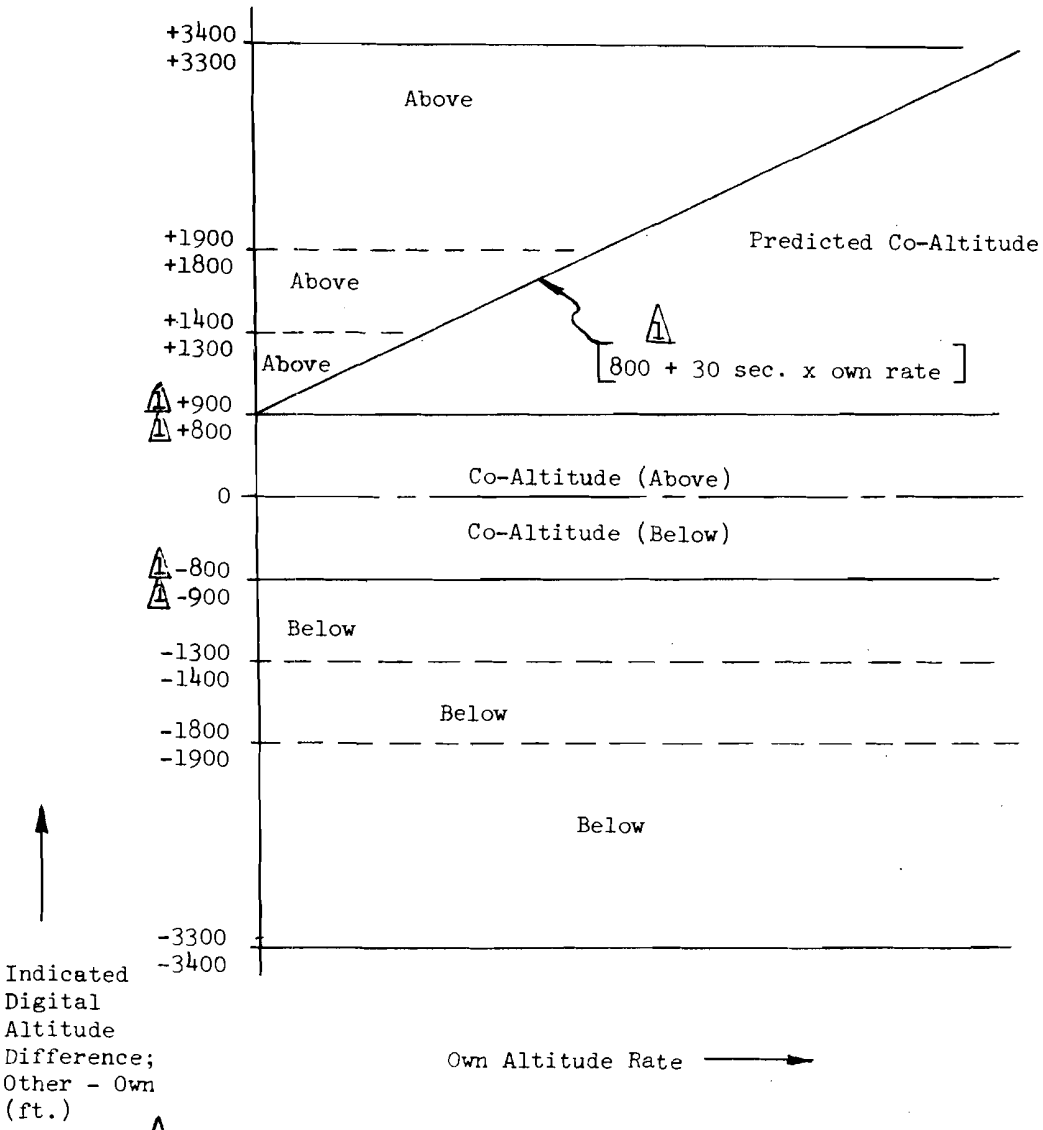
A minimum range warning zone is added as an extension of the tau threat zones as shown in Figure 2 to insure safe operation at low closing rates, provide protection under turn maneuver conditions, and accommodate signal noise in the range rate measurements.

There is no threat when the range and range rate point is to the right of the tau zone 2 as shown in Figure 2.

c. (Incorporated into B-3.b.)

d. This section has been omitted.

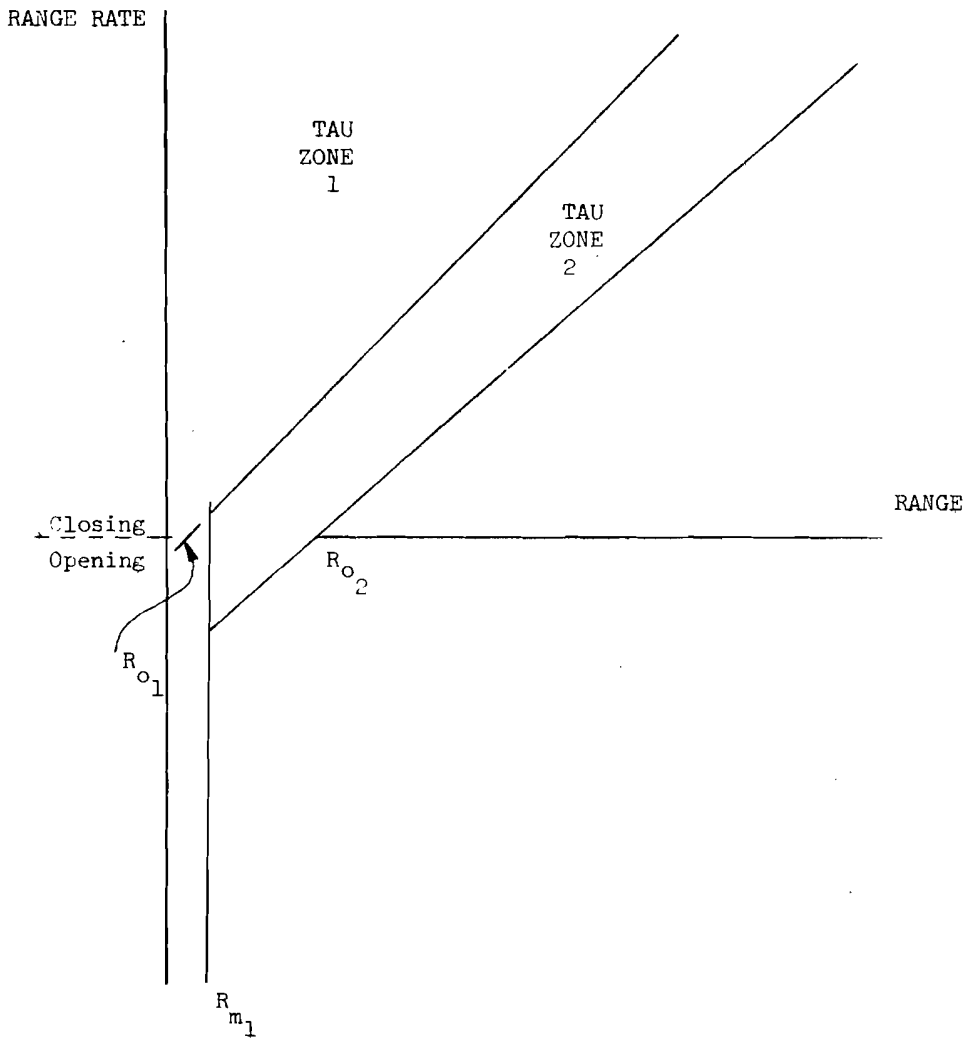
NOTE: Predicted co-altitude is extended only in direction of own altitude rate. This figure only illustrates own A/C ascending.



These values reduced by 200 feet at an altitude of 10,000 or less.

THREAT EVALUATION
ALTITUDE AND ALTITUDE RATE

Figure 1



THREAT EVALUATION
RANGE AND RANGE RATE

Figure 2

B-3. e. Output Logic

Using the threat evaluation criteria described in B-3. a. and b., the equipment shall provide command outputs in accordance with the logic shown in Figures 3, 4a and 4b for the respective cases of two and three aircraft encounters.

(1) Output Logic Feedback

To assure complementary maneuvers in each aircraft, in a threat encounter, the transmitted altitude signal is biased when certain output commands are formulated, and 400 feet or less of altitude separation is indicated. The bias is 200 feet in the direction of the following commands:

Command	Bias
Climb - do not turn	+200
Dive - do not turn	-200
R/9 Do not turn - Don't descend	+200*
Do not turn - Don't climb	-200*

At all other times, unbiased altitude data shall be transmitted. "Own" unbiased altitude data is always used on board for input into the threat evaluation logic.

* This bias must not be used unless accompanied by a display to the pilot showing the direction of the imminent maneuver.

f. Low Altitude Threat Logic

With barometric altimetry errors the system logic values are based on a minimum aircraft altitude system error of ± 150 feet 3 sigma (Gaussian) at lower altitudes, the following reductions in threat logic values are recommended. The switchover from high altitude logic to low altitude logic will be automatically accomplished at an indicated altitude of 10,000 feet.

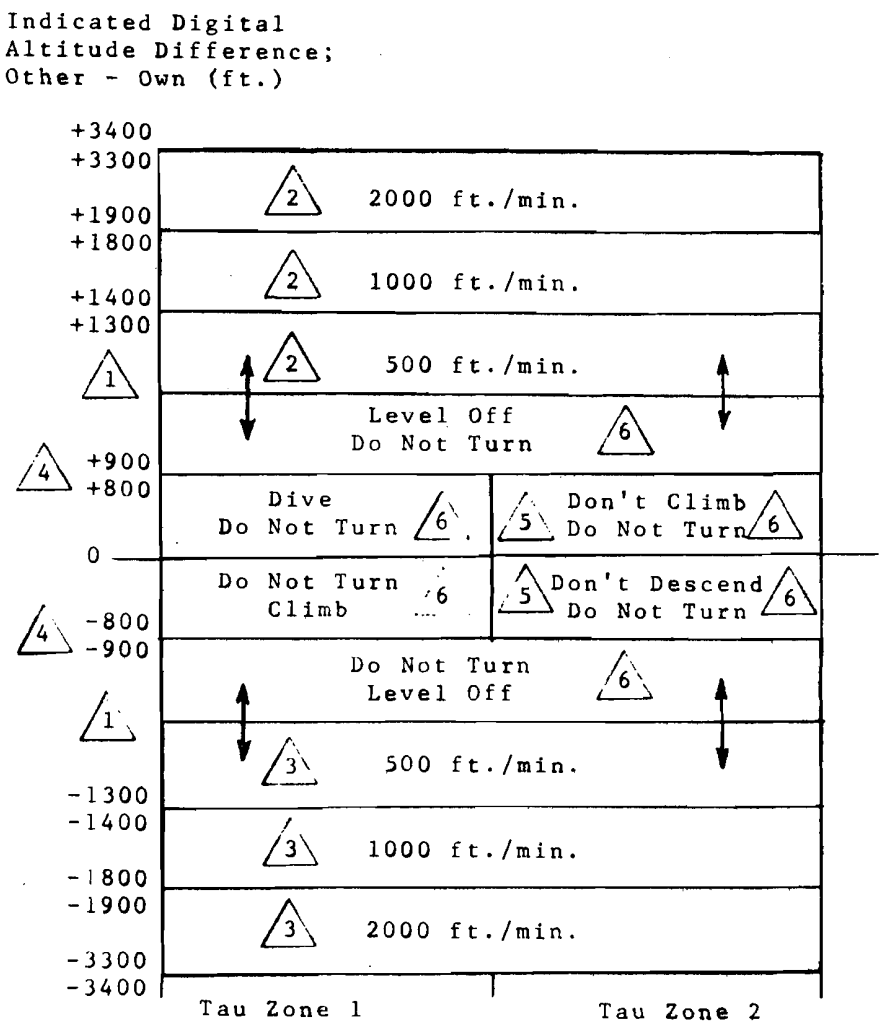
(1) Altitude Threat Evaluation

(a) Same as paragraph B-3.a except co-altitude difference are reduced by 200 ft. and;

$|\text{Predicted co-altitude}| = 600 + 30 \text{ sec.} \times |\text{own altitude rate}|$

(b) Range and Range Rate Threat Evaluation

Same as paragraph B-3.b and Figure 2.



NOTES:

1. Predicted co-altitude $\dot{h}$ = 800 + 30 sec. x own $\dot{h}$
2. Do not exceed climbing $\dot{h}$ value shown.
3. Do Not exceed descending $\dot{h}$ value shown.
4. These values are decreased 200 feet when at 10,000 feet altitude or below.
5. Cockpit display of a vertical spped limit within the range of 0-400 feet per minute is acceptable.
6. Restrict bank angle to ± 10° or less (Paragraphs A-4. a. 1 and A-4. b. 3 (a)).

HIGH ALTITUDE
THREAT LOGIC OUTPUT
TWO AIRCRAFT ENCOUNTER

Figure 3

Δh≤2300ft Tau 2	Dive Do Not Turn LVS 0fpm Up 2000fpm Down	Do Not Turn LVS 200fpm Up 2000fpm Down	Level Off Do Not Turn LVS 500fpm Up 2000fpm Down	LVS 500fpm Up 2000fpm Down	Level Off Do Not Turn LVS 1000fpm Up 2000fpm Down	LVS 1000fpm Up 2000fpm Down	Level Off Do Not Turn LVS 2000fpm Up 2000fpm Down	LVS 2000fpm Up 2000fpm Down
Predicted Coaltitude Δh≤2300ft Tau 2	Level Off Do Not Turn LVS 500fpm Up 2000fpm Down	Level Off Do Not Turn LVS 200fpm Up 2000fpm Down	N/A	Level Off Do Not Turn LVS 500fpm Up 2000fpm Down	N/A	Level Off Do Not Turn LVS 1000fpm Up 2000fpm Down	N/A	Level Off Do Not Turn LVS 2000fpm Up 2000fpm Down
Δh≤1800ft Tau 2	Dive Do Not Turn LVS 0fpm Up 1000fpm Down	Do Not Turn LVS 200fpm Up 1000fpm Down	Level Off Do Not Turn LVS 500fpm Up 1000fpm Down	LVS 500fpm Up 1000fpm Down	Level Off Do Not Turn LVS 1000fpm Up 1000fpm Down	LVS 1000fpm Up 1000fpm Down	Level Off Do Not Turn LVS 2000fpm Up 1000fpm Down	LVS 2000fpm Up 1000fpm Down
Predicted Coaltitude Δh≤1800ft Tau 2	Level Off Do Not Turn LVS 500fpm Up 1000fpm Down	Level Off Do Not Turn LVS 200fpm Up 1000fpm Down	N/A	Level Off Do Not Turn LVS 500fpm Up 1000fpm Down	N/A	Level Off Do Not Turn LVS 1000fpm Up 1000fpm Down	N/A	Level Off Do Not Turn LVS 2000fpm Up 1000fpm Down
Δh≤1300ft Tau 2	Dive Do Not Turn LVS 0fpm Up 500fpm Down	Do Not Turn LVS 200fpm Up 500fpm Down	Level Off Do Not Turn LVS 500fpm Up 500fpm Down	LVS 500fpm Up 500fpm Down	Level Off Do Not Turn LVS 1000fpm Up 500fpm Down	LVS 1000fpm Up 500fpm Down	Level Off Do Not Turn LVS 2000fpm Up 500fpm Down	LVS 2000fpm Up 500fpm Down
Predicted Coaltitude Δh≤1300ft Tau 2	Level Off Do Not Turn LVS 500fpm Up 500fpm Down	Level Off Do Not Turn LVS 200fpm Up 500fpm Down	N/A	Level Off Do Not Turn LVS 500fpm Up 500fpm Down	N/A	Level Off Do Not Turn LVS 1000fpm Up 500fpm Down	N/A	Level Off Do Not Turn LVS 2000fpm Up 500fpm Down
Coaltitude Tau 2	Dive Do Not Turn LVS 0fpm Up 200fpm Down	Do Not Turn LVS ⚠ 200fpm Up 200fpm Down	Level Off Do Not Turn LVS 500fpm Up 200fpm Down	Do Not Turn LVS 500fpm Up 200fpm Down	Level Off Do Not Turn LVS 1000fpm Up 200fpm Down	Do Not Turn LVS 1000fpm Up 200fpm Down	Level Off Do Not Turn LVS 2000fpm Up 200fpm Down	Do Not Turn LVS 2000fpm Up 200fpm Down
Coaltitude Tau 1	Hold Altitude Do Not Turn LVS 0fpm Up 0fpm Down	Climb Do Not Turn LVS 200fpm Up 0fpm Down	Level Off Do Not Turn LVS 500fpm Up 500fpm Down	Climb Do Not Turn LVS 500fpm Up 0fpm Down	Level Off Do Not Turn LVS 1000fpm Up 500fpm Down	Climb Do Not Turn LVS 1000fpm Up 0fpm Down	Level Off Do Not Turn LVS 2000fpm Up 500fpm Down	Climb Do Not Turn LVS 2000fpm Up 0fpm Down
Intruder below / Intruder above	Coaltitude Tau 1	Coaltitude Tau 2	Predicted Coaltitude Δh≤2300ft Tau 2	Δh≤1800ft Tau 2	Predicted Coaltitude Δh≤1800ft Tau 2	Δh≤1300ft Tau 2	Predicted Coaltitude Δh≤1300ft Tau 2	Δh≤2300ft Tau 2

THREAT STATUS

THREAT LOSS ONLY

THREAT / NON THREAT INCONSISTENT

NOTE: LVS = "Limit Vertical Speed" to values listed

⚠ See Note 5, Figure 3 for all 200 FPM values on this page.

Figure 4a

	Coaltitude Tau 1	Coaltitude Tau 2	Predicted Coaltitude Δh≤1800ft Tau 2	Δh≤1800ft Tau 2	Predicted Coaltitude Δh≤1800ft Tau 2	Δh≤1800ft Tau 2	Predicted Coaltitude Δh≤3300ft Tau 2	Δh≤3300ft Tau 2
Δh≤3300ft Tau 2	Dive Do Not Turn LVS 0fpm Up	Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	LVS 500fpm Up	Level Off Do Not Turn LVS 1000fpm Up	LVS 1000fpm Up	Level Off Do Not Turn LVS 2000fpm Up	LVS 2000fpm Up
Predicted Coaltitude Δh≤3300ft Tau 2	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	N/A	Level Off Do Not Turn LVS 1000fpm Up	N/A	Level Off Do Not Turn LVS 2000fpm Up	Level Off Do Not Turn LVS 2000fpm Up
Δh≤1800ft Tau 2	Dive Do Not Turn LVS 0fpm Up	Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	LVS 500fpm Up	Level Off Do Not Turn LVS 1000fpm Up	LVS 1000fpm Up	N/A	LVS 1000fpm Up
Predicted Coaltitude Δh≤1800ft Tau 2	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	N/A	Level Off Do Not Turn LVS 1000fpm Up	Level Off Do Not Turn LVS 1000fpm Up	Level Off Do Not Turn LVS 1000fpm Up	Level Off Do Not Turn LVS 1000fpm Up
Δh≤1300ft Tau 2	Dive Do Not Turn LVS 0fpm Up	Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	LVS 500fpm Up	N/A	LVS 500fpm Up	N/A	LVS 500fpm Up
Predicted Coaltitude Δh≤1300ft Tau 2	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 500fpm Up	Level Off Do Not Turn LVS 500fpm Up
Coaltitude Tau 2	Dive Do Not Turn LVS 0fpm Up	Do Not Turn LVS ⚠ 200fpm Up	Level Off Do Not Turn LVS 200fpm Up	Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 200fpm Up	Do Not Turn LVS 200fpm Up	Level Off Do Not Turn LVS 200fpm Up	Do Not Turn LVS 200fpm Up
Coaltitude Tau 1	Dive Do Not Turn LVS 0fpm Up	Dive Do Not Turn LVS 0fpm Up	Level Off Do Not Turn LVS 500fpm Up	Dive Do Not Turn LVS 0fpm Up	Level Off Do Not Turn LVS 500fpm Up	Dive Do Not Turn LVS 0fpm Up	Level Off Do Not Turn LVS 500fpm Up	Dive Do Not Turn LVS 0fpm Up

Intruder above

Intruder above

INTRAL STATUS

INTRAT LOGIC OUTPUT

THRIL ATT [illegible]

Figure 4b

NOTE: LVS = "Limit Vertical Speed" to values listed.

⚠ See Note [illegible] Figure 3 for all 200 FPM values on this page.

B-3. (c) Output and Mode Control

All the logic functions of paragraphs B-3. c. thru B-3. e. remain unchange.

g. CAS Threat Logic, Automatic Operation Controls

It is necessary to eliminate the command to descend and not turn during the final phase of approach to an airport so that pilots need not be trained to ignore this command. Since the CAS commands are complementary in all equipped aircraft the vertical avoiding maneuver can be made by the higher aircraft when the command to descend is not displayed in the lower aircraft. A requirement also exists to inhibit unwanted CAS command being generated by
R/9 aircraft immediately after takeoff. Dive and do not turn commands must therefore be inhibited for a short period of time. The oleo strut switch can be used to sense takeoff and thus the field altitude can be captured and used or (alternatively) time after takeoff can be fed to the CAS system. The requirements of Figure 5a must be incorporated into the CAS and will use the aircraft oleo strut and landing gear (or flap position) as airframe inputs. Figure 5b shows a method of complying with these requirements.

CAS THREA. LOGIC

AUTOMATIC OPERATION CONTROLS

A/C OPERATION	INPUTS	TRANSMIT		THREAT LOGIC		
		RANGE	ALTITUDE	Synchronized Mode		Back-Up Mode
				Process Receptions	Logic [1]	Participation
On ground	Oleo Strut switch [2]	yes	no	no	---	Inhibit
Takeoff						
0 thru 12 sec. [3] [5] 0 thru 400 ft. alt.	"	yes	no	no	---	Inhibit
12 thru 42 sec. [3] [5] 400 thru 1500 ft. alt.	"	yes	yes	yes	Inhibit; "Dive" and "Do not turn commands" and Tau 2	Inhibit
Climb out, cruise & let down		yes	yes	yes	Full	Normal
Landing	Landing gear [4] down and locked	yes	yes	yes	Inhibit; "Dive" & "Do not turn commands" & Tau 2	Inhibit

[1] Altitude band values depend on A/C altitude, see Paragraph B-3f.

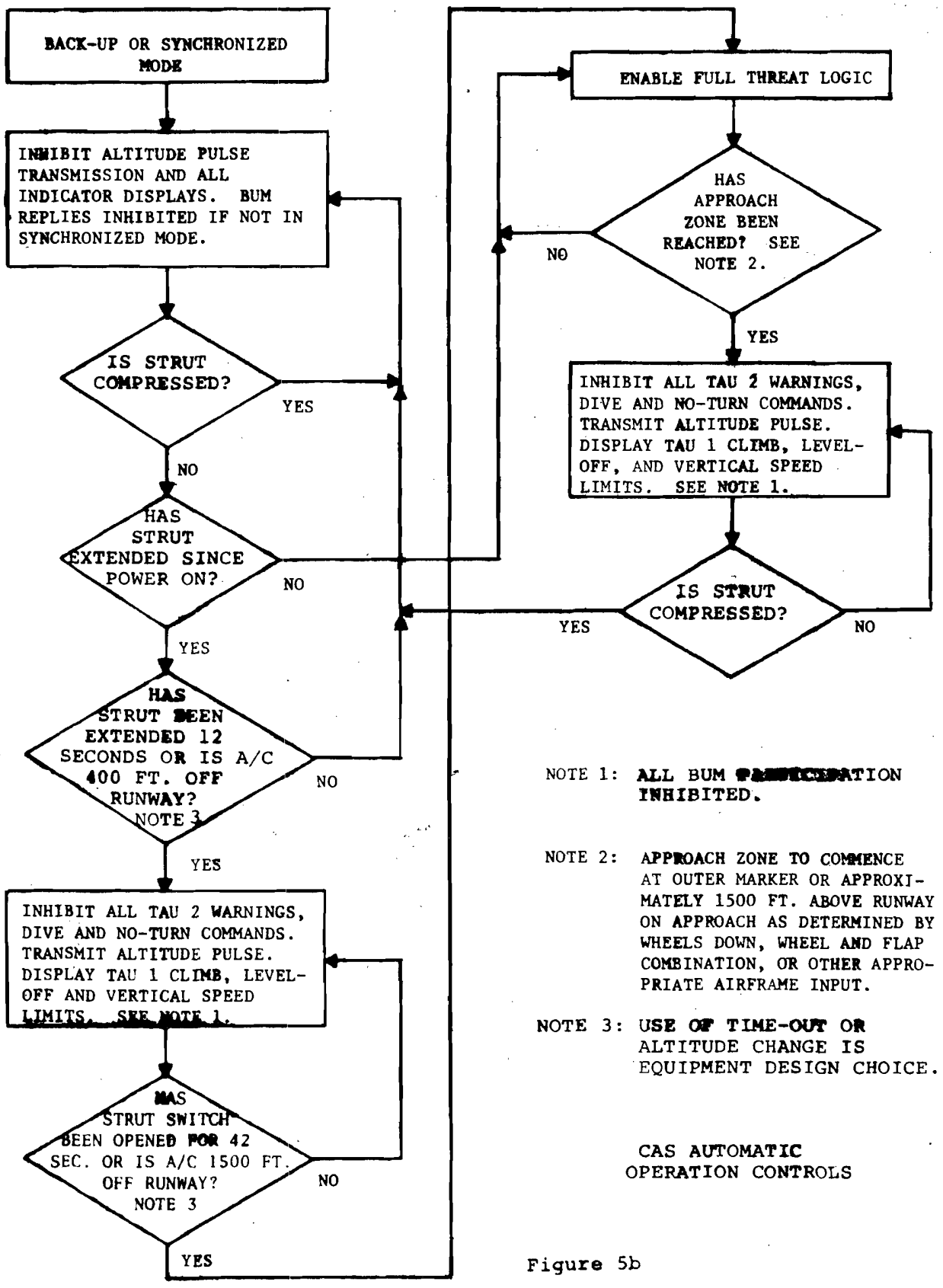
[2] Also referred to as "Air/Ground" circuit.

[3] Relative to time of takeoff or field altitude. Time-out or altitude change recognition is done with CAS

[4] On some airplanes, where the landing gear is ocassionally used as a speed brake, this input may need to be combined with "landing flaps down".

[5] Use of time-out or altitude change is equipment design choice.

Figure 5a



NOTE 1: ALL BUM [illegible]ATION INHIBITED.

NOTE 2: APPROACH ZONE TO COMMENCE AT OUTER MARKER OR APPROXIMATELY 1500 FT. ABOVE RUNWAY ON APPROACH AS DETERMINED BY WHEELS DOWN, WHEEL AND FLAP COMBINATION, OR OTHER APPROPRIATE AIRFRAME INPUT.

NOTE 3: USE OF TIME-OUT OR ALTITUDE CHANGE IS EQUIPMENT DESIGN CHOICE.

CAS AUTOMATIC OPERATION CONTROLS

Figure 5b

B-4. Back Up Mode

a. General

A back up mode is utilized when the airborne clock errors exceed system requirements. It is expected that this will occur only in situations involving small numbers of aircraft within line-of-sight range; typically, fewer than 30 to 50 aircraft. The back up mode uses the same general techniques as the fully synchronized mode for evaluating closing rate and altitude, and an interrogate/respond technique for sensing range. Because of system signalling and logic limitations, the back up mode is limited (in certain physical combinations of aircraft encounters) to encounters involving only two aircraft.

b. Back Up Mode (BUM) Requirements

(1) Operating Frequency

All back up mode communication uses only the lowest of the four R-F frequencies (F_1), 1600 MHz, to assure that all aircraft are able to communicate (since there is no single system time and thus cycling from frequency to frequency is not feasible) and to permit hearing epoch start pulses as soon as they are within range. Limited equipment capable of transmitting on one frequency only and not having BUM capability, shall not transmit on 1600 MHz.

(2) Message Slots

Message slots are 6000 microseconds in duration (four normal 1500 microsecond slots).

(3) Epoch Start/Sync Reply Transmission

During back up mode operation, epoch start pulses are not transmitted, and synchronization replies are not made.

(4) Interrogation Transmission

The back up mode interrogation consists of the normal 200-microsecond range/doppler and shorter altitude pulse transmission as shown in Attachment 1. If a bi-phase message is transmitted, the hierarchy portion must indicate hierarchy numbers which do not identify the transmitting aircraft with having synchronization status.

B-4. b. (a) Back Up Mode Slot Selection

The time of the interrogation transmission must be selected so that the leading edge is separated by at least 6000 Microseconds from the leading edge of any other CAS (synchronized or back up mode) message received on F_1. The interrogation transmission normally occurs once per epoch.

(5) Interrogation Reception and Evaluation

An aircraft, upon receiving the interrogation of another (intruding) aircraft, determines whether the intruder is a potential threat. The intruder is judged to be a potential threat if range rate is algebraiclally greater than $\dot{R}_a$*<u>and</u> is off altitude by less than g_2*. If the intruder is not a potential threat, no action is necessary.

(a) Reply Transmission

If the interrogator is a potential threat, a reply consisting of a warning pulse and a maneuver command pulse is transmitted using the antenna on which the message was received, except that warning and maneuver command replies are not transmitted by aircraft on the ground. The timing for the reply is based on T_w*, a fixed length of time after reception of interrogator's doppler pulse leading edge. See figure 6.

1) Warning Pulse

The warning pulse of the reply is transmitted earlier than T_w by the amount $(2/c)(\dot{R}\tau + R_o)$ or $2R_m/c$ microseconds, whichever is larger. Here $\dot{R}$ <u>1</u>/ is the range rate, τ is the back up mode tau value and c is the propagation velocity. The R_o constant is included to provide protection for turning acceleration cases. The $2R_m/c$ constant represents a range alarm of R_m nautical miles.

* Terms are defined elsewhere in section B-4.
<u>1</u>/ Closing range rate is taken as positive.

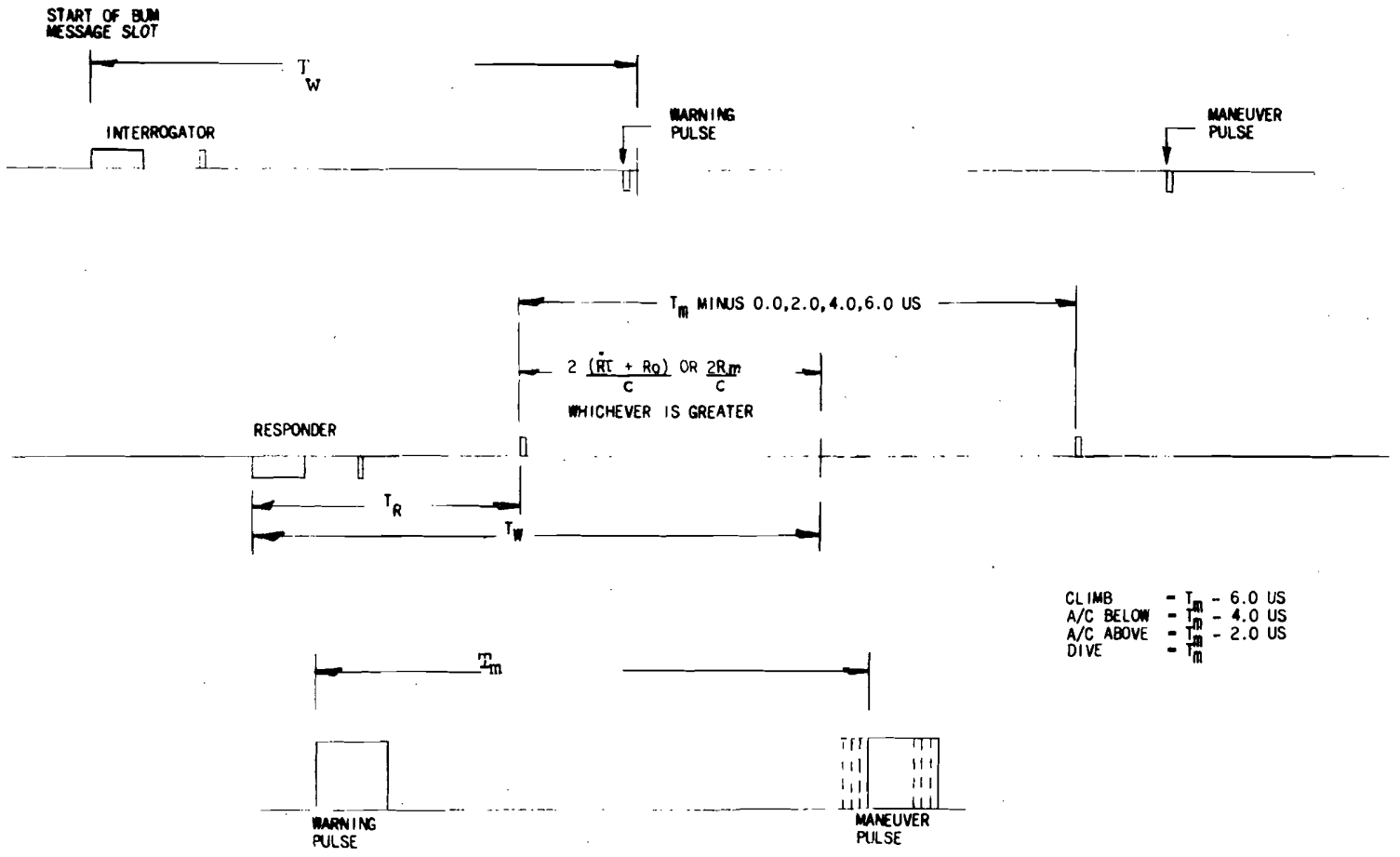


FIGURE 6- BACK-UP MODE

R/10

B-4. b. (2) Maneuver Command Pulse

The maneuver command pulse of the reply is spaced T_m - 6.0, T_m - 4.0, T_m - 2.0 or T_m microseconds after the warning pulse (leading edge to leading edge) to indicate the commands CLIMB, AIRCRAFT BELOW, AIRCRAFT ABOVE, and DIVE. A CLIMB command is sent when the interrogator is within g_1 and above the responder in altitude. Likewise, a DIVE command is sent when the interrogator is within g_1 and below the responder in altitude. When altitude separation is between g_1 and g_2, an AIRCRAFT BELOW command is sent if the interrogator is above the responder and an AIRCRAFT ABOVE command is sent if the interrogator is below the responder.

(6) Reply Reception by Interrogator

The interrogating aircraft determines whether the warning pulse of the reply arrives at or prior to T_w. A signal falling in this area is received from an aircraft that could be within τ seconds of collision or is within R_m nautical miles. The interrogator must act on the first of any replies received at or prior to T_w. Whenever an aircraft is following a back up mode command of CLIMB or DIVE, it biases its own internal altitude by 200 feet in the direction of the maneuver for the purpose of computing relative altitude from the interrogations of other aircraft; however, it does not bias transmitted altitude.

(7) Altitude Rate Protection

The back up mode includes alternative methods of providing altitude protection in the event an AIRCRAFT ABOVE (BELOW) warning/command is present:

R/9 (a) Own altitude rate must be limited to 500 feet per minute in the direction of the intruder and a "Rollout-Do not turn" command must be

B-4. b. (7) (a) applied, until own altitude rate is reduced to 500 FPM or less in the closing direction, or

(b) If closing rates in excess of 500 feet per minute are to be allowed, a second interrogation transmission must be made each epoch with the transmitted altitude biased in the direction of altitude maneuver by an amount
R/9 equal to own altitude rate times projection time (Zt). If a reply to this second interrogation shows AIRCRAFT BELOW (ABOVE), or CLIMB, or DIVE, a LEVEL-OFF and ROLLOUT or equivalent command must be displayed.

(8) Antenna Switching

Back up mode operations uses both antennas
R/9 normally required to provide satisfactory coverage on most aircraft. See paragraphs B-1. g. (1), B-1. g. (8), and B-1. g. (9).

(a) Transmission

Antennas are switched for transmitting as follows:

1) Interrogation Transmission

Back up mode interrogations are transmitted using upper and lower antennas, alternating each epoch.

2) Reply Transmission

Back up mode warning and maneuver command replies are always transmitted on the upper antenna.

(b) Reception

If diversity reception is not employed, antennas are switched for receiving as follows:

B-4. b. (8) (b) 1) Reception of Replies

For reception of replies to an interrogation, the interrogator uses the antenna which was used for interrogation transmission.

2) Reception of Interrogations

For reception of interrogations from other aircraft, the upper antenna shall be used.

(9) Detection of Co-Slot Occupancy

Each aircraft using back up mode must randomly inhibit interrogation transmission and listen during its own slot so that on the average co-slot occupancy is detected within 40 seconds. Co-slot occupancy is defined as any range/doppler pulse received on F_1 occuring in own 6000 microsecond back up mode slot. Listening for co-slot occupancy shall not occur in consecutive
R/9 epochs nor for the duration of hazardous encounters. While listening in own slot, transmission in an alternate, unused slot, is permitted.

B-4. c. Back Up Mode Constants
R/9

Back Up Mode Constants are:

τ= 40 seconds

R_o= 1.8 nautical miles

R_m= .5 nautical miles

R_a= -117 knots

g_1= 800 feet

g_2= 3300 feet

t = 30 seconds

T_w= 2951 microseconds

T_m= 1500 microseconds

Back Up Mode Warning Pulse and Maneuver Command Pulse Width = 8 microseconds.

B-4. d. Synchronized Mode Join-Up From Back Up Mode

R/9 The process of synchronized mode join-up is initiated by the reception of the synchronized mode epoch start triad. Aircraft operating in the back up mode, upon receipt of an epoch start triad, will attempt synchronized mode join-up using a sequence similar to Figure 7 which:

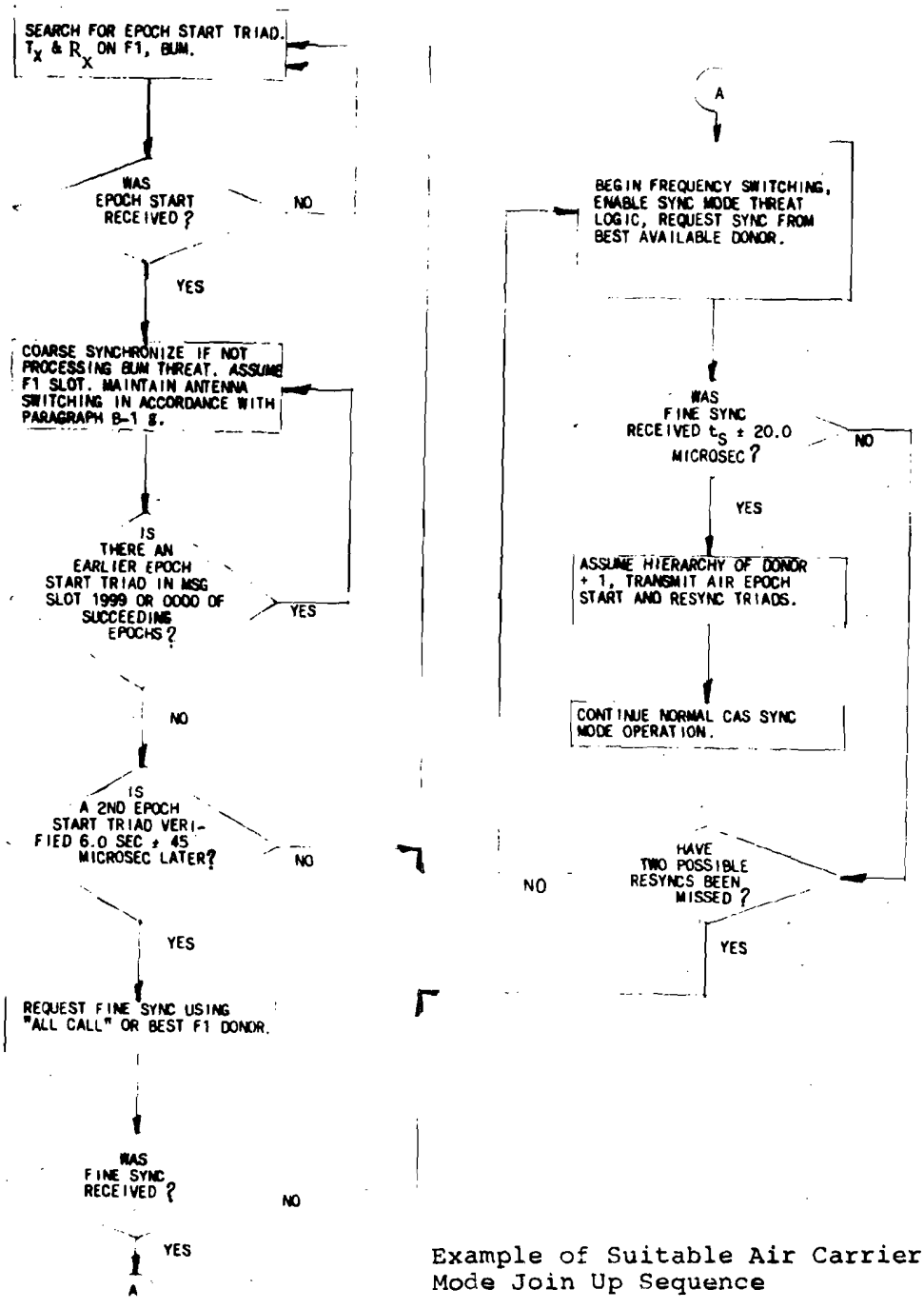
(1) Inhibits coarse synchronization for the duration of a back up mode CLIMB or DIVE maneuver command.

(2) Provides use of epoch start triads from closest synchronized system.

(3) Provides verification of two or more epoch start triads prior to utilization of fine synchronization replies. Verfified epoch start triads must occur 6.0 seconds $\pm$ 45.0 microseconds apart.

(4) Inhibits frequency switching and continues back up mode operation in an F1 time slot until a fine synchronization reply is received.

(5) Adjusts antenna switching sequence in accordance with paragraph (B-1. g.), and transmit synchronized message format.



Example of Suitable Air Carrier Sync Mode Join Up Sequence

Figure 7

B-5 Power Budget

a. Air Carrier System

(1) A power budget for an air carrier CAS system is provided by Attachment 1, Section 2.

R/3 (2) To establish uniform power and sensitivity requirements that will provide adequate CAS hazard and synchronization ranges, a CAS hazard range of 40 NM (to accommodate an SST Mach 6 closure rate) and a fade margin of 10db was utilized for the computation.

(3) The 40 NM hazard range will provide reasonable synchronization performance and some fade margin at the 97 mile nominal maximum synchronization range. It is assumed that fine synchronization will be repeated as often as required so that the fade margin may be excluded on a valid basis.

R/3 (4) Uniformity requirements make it necessary to establish maximum receiver noise temperature and minimum transmitter power at the equipment input/output terminals. Cable losses specified will be the maximum permitted in CAS installations without offsetting improvements in transmitter or receiver performance and/or remote location.

(5) The two db antenna gain reflects the performance of expected typical forward looking antennas, recognizing that the collision geometry requires less range performance in overtaking situations where antenna gain is not optimum.

B-6. Limited Equipments

R/3 This technical description provides for the possibility of the design of limited equipments which omit as many of the complexities as possible, but which retain limited capability which may be acceptable to some categories of users.

a. Limited Equipment with Full Hazard Logic

Limited equipment in this category is that which

B-6. a. operates compatibly within the communication format specified by this technical description and specifically includes the capability of cooperatively commanding avoidance maneuvers when required. On this basis, examples of system requirements which can be relaxed in this equipment category include the following:

(1) Generation and demodulation of bi-phase modulation not required. Note: Doppler processing of received bi-phase modulated signal is required.

(2) Use of synchronization hierarchy is not required. Time synchronization is received from ground stations or airborne stations, without addressing. (See para. e).

(3) High power transmissions are not required.

R/9 (4) Coherent transmission on one of the four specified frequencies other than F_1 is required with the frequency selected to be determined at random at the time of manufacture. Paritcipation in the back-up mode will also require an F_1 capability.

R/9 (5) All equipments which are (1) synchronized, (2) flying and (3) capable of transmitting on F_1 are required to transmit air epoch start, T_o' pulses.

(6) Section 4 and 5 of Attachment 1 provide the technical details including power budgets for two classes of equipment falling in this category.

b. Further Reductions in Limited Equipment

(1) Additional reductions in the complexity of the limited equipment could be made through modification or deletion of threat computation capability or the back-up mode or other functions. Depending on the specific operational situation and the extent of reduction the possibility of

B-6. b. (1) a reduced protection in encounters with fully
R/9 equipped system and/or between similarly equipped aircraft must be considered. Any radiated signals must be coherent and must comply with the signals in space defined by Attachment 1. Section 6 of Attachment 1 discribes a beacon type of equipment which only transmits synchronized range and altitude pulses. It does not evaluate the threat environment.

(2) Attachment 1, Section 4 provides technical characteristics of a limited equipment capable of compatible operation at jet altitudes and speeds. This system is refered to as the Level 1 limited system. The power budget has been scaled to provide for operation of 600KT
R/9 aircraft in the same airspace as the 1800KT SST; (2400KT maximum range rate). It does not provide for the use of synchronization hierarchy and biphase modulation. Transmission on a single frequency, and provision for Back Up Mode are optional. Since this system must operate compatibly with the full system in the same air space, certain performance criteria must be maintained intact, although wherever possible the technical requirements have been reduced.

(3) Attachment 1, Section 5 provides technical characteristics for a further reduction in equipment complexity for aircraft whose performance is less than 200 knots air speed and less than 10,000 ft. in altitude. This system is referred to as the Level 2 limited system. The power
R/9 budget has been scaled to permit operation with full and Level 1 limited systems which are constrained to 250KTS velocity below 10,000 (450KTS maximum range rate). As with the limited equipment, relief is granted wherever possible so long as this relief does not affect systems compatibility or create situations where inadequate protection exists. The characteristics provide only for single frequency transmission and do not include a Back Up Mode option, in the interest of total system complexity. As with the "Limited Collision Avoidance System:, Level 1, performance criteria has been maintained where necessary for compatible operation with full systems in the same airspace.

B-6. b. (4) Attachment 1, Section 6 provides technical characteristics for a Collision Avoidance Beacon Level 1. This system, described for the below 200 knot, 10,000 ft. environment, is a fully compatible but reduced function system that does not provide maneuvering information in the aircraft containing it, but permits aircraft with the "full", "limited", "Level 1 or 2", collision avoidance systems to maneuver to avoid the aircraft containing the "beacon" system requests and maintains synchronization and broadcasts in its slot a normal minimum range altitude message. It must receive for synchronization only and does not process received Collision Avoidance messages from other aircraft. Two aircraft equipped with the "beacon" system are not protected one from another. As with the "limited" systems, performance has been reduced wherever possible without jeopardizing compatible operation in the same airspace.

R/9

c. Other Limited Equipment

(1) It is recognized that information of possible value for pilot Proximity Warning Indication (PWI) can be obtained by airborne equipment having significantly less capability then that described. For example, one user has indicated that an indication of the "mere presence" of other aircraft in the vicinity would be of value in many instances. A crystal video receiver would provide this function and could be built inexpensively. Relative altitude information could be added to the simple crystal video receiver if some users indicated a desire to have that information. A rough indication of the range of aircraft being heard could be provided.

(2) The value of such "receive only" devices or more complete versions which would include transmit capability, has not been determined. The purpose of this paragraph is to point out the latent capability of the system and invite the application of ingenuity to use the system capability in an ever increasing number of aircraft. Transmissions on the CAS frequencies in an incompatible manner could degrade or destroy the system described by this document and, therefore, must not be permitted. Before proceeding with significant effort to develop concepts or equipment not described by this document, it would be essential to assure that such equipemnt would be fully compatible with the system(s) set forth herein.

B-6. d. Limited Equipment Capabilities

(1) The buyer-seller market is believed to be the best forum for determining the capability which will be contained in the Limited Equipment to satisfy the requirements of various categories of users. It is feasible to have doppler demodulation, Tau, minimum range threat computation, and other useful information (e.g. identity decoding) in the Limited Equipment as required by various categories of users.

e. Limited Equipment Fine Synchronization

R/7 (1) A limited equipment requests fine synchronization with each transmission as a result of including no bi-phase modulation in its 200 microsecond range/doppler burst.

R/6 (2) Ground stations reply (during odd epochs) to each transmission by providing fine synchronization on the same frequency as the request.

(3) Each air carrier station answers the portion of all limited equipment heard during even epochs as determined by the inverse proportion of the number of air carrier aircraft within resynchronization range (97 NM) including counting own station. (References w and x)

(4) If, during odd epochs, a limited equipment can obtain a valid fine synchronization reply in its slot, fine synchronizations from air carrier aircraft (if any) during even epochs are not to be utilized.

(5) Limited equipment will not provide fine synchronization.

(6) The validation of system time through use of sequential fine synchronizations (See B-2 (d)) is not required of limited equipment since they do not propagate master time except as provided by B-6. a. (5).

a) FUNDAMENTAL PHYSICS OF THE AIRCRAFT COLLISION PROBLEM by Dr. J.S. Morrell, Bendix Radio Division, Bendix Aviation Corporation; Technical Memorandum 465-1016-39, June 1, 1956.

b) FUNDAMENTALS OF COLLISION AVOIDANCE by Collins Radio Co., Cedar Rapids, Iowa, May 1958; includes engineering reports CER 700 through 705.

c) EXPERIMENTAL EVALUATION OF COOPERATIVE COLLISION AVOIDANCE SYSTEM - Flight Test Data for Phase II Tests - by Bendix Corporation, Bendix Radio Division, Baltimore, Maryland; Contract No. FAA/BRD-353, January 26, 1962.

d) AIRBORNE COLLISION AVOIDANCE SYSTEM by Bendix Corporation, Bendix Radio Division, Baltimore, Maryland; July 1962.

e) DESIGN STUDY REPORT - EXPERIMENTAL EVALUATION OF PWI/CAS INTERROGATOR-TRANSPONDER TECHNIQUES by Sperry Gyroscope Company, Division of Sperry Rand Corporation, Great Neck, New York; Sperry Report No. CA-4283-0157-1, Contract FAA/ ARDS-444; December 1962.

f) COMPUTER/SIMULATION STUDY OF AIR-DERIVED SEPARATION ASSURANCE SYSTEMS IN MULTIPLE AIRCRAFT ENVIRONMENT by Collins Radio Company, Cedar Rapids, Iowa; Contract No. FA-WA-4598, Project No. 247-001-01C; First Interim Report - October 1964. Second Interim Report March, 1966, Final Report October, 1969.

g) NEW ANTI-COLLISION SYSTEM STIRS INTEREST by Philip J. Klass April 25, 1966 issue of Aviation Week & Space Technology.

h) EFFECTS OF TURNING MANEUVERS ON COLLISION THREAT EVALUATION by F.D. Watson and R.E. Perkinson, McDonnell-Douglas; Report No. E 759, dated August 1, 1966.

i) ANALYSIS OF WARNING TIMES FOR COLLISION AVOIDANCE SYSTEMS by John M. Holt and Ronald M. Anderson, Collins Radio Co; October 1967.

* j) BI-PHASE-BARKER CODED DATA TRANSMISSION by K.E. Toerper, Conductron, Missouri; June 28, 1967.

k) COLLISION AVOIDANCE by Paul G. Thomas, Space Aeronautics, June 1967.

l) FINAL REPORT Contract No. FA-WA-4598, Project No. 247-001-01C. (AD697-942, Government Clearing House).

m) COLLISION AVOIDANCE SYSTEM SYNCHRONIZATION by Richard L. Jaycox of Collins Radion Company; IEEE Transactions Vol. AES-4, March 1968.

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o) A SURVEY OF ATOMIC FREQUENCY STANDARDS by A.O. McCoubrey, Reprinted from the PROCEEDING OF THE IEEE Vo. 54, No. 2, February, 1966, Pages 116-135.

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q) AIRBORNE COLLISION AVOIDANCE SYSTEM DEVELOPMENT - Introduction by Frank C. White, Air Transport Association, IEEE Transactions Vol. AES-4, March 1968.

r) HISTORY OF TIME-FREQUENCY TECHNOLOGY by M.R.Bates, H.K. Fletcher, L. Michnik and J.W. Prast; IEEE Transactions Vol. AES-4, March 1968.

s) MASTER TIMING OF CAS GROUND STATIONS by C.O. Thornburg, IEEE Transactions Vo. AES-4, March 1968.

t) CAS MESSAGE FORMAT by R.E. Perkinson; March 1968. IEEE Transactions Vol. AES-4, March 1968.

u) SLOT LENGTH AND LINE-OF-SIGHT PROTECTION by H.A. Steinberg; IEEE Transactions Vol. AES-4, March 1968.

v) ELEMENTS OF THE ATA COLLISION AVOIDANCE SYSTEM by W.G. Shear; IEEE Transactions, Vol. AES-4, March 1968.

w) LIMITATION OF FINE SYNCHRONIZATION TRIAD REPLIES BY DONOR STATUS THROUGH USE OF THE 1/n CRITERION, by Martin Bates, Sierra Research, May 1968, Paper TR-0676.

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* dd) BENDIX, AVIONICS DIVISION, Technical Note 361-1103-WS-1 CAS Ground Station Test Messages, Revised March 1, 1971.

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* Copies may be obtained from the Air Transport Association.