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DEVELOPMENT OF A STRUCTURAL DESIGN PROCEDURE FOR RIGID AIRPORT PAVEMENTS

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PREFACE

The study described herein was sponsored by the Federal Aviation Administration as a part of Inter-Agency Agreement No. DOT-FA78WAI-377, "New Pavement Design Methodology," and by the Office, Chief of Engineers, U. S. Army, as a part of Military Construction RDTE Project No. 4A762719AT04, "Pavements, Soils, and Foundations," and Military Engineering RDTE Project No. 4A161102B52E, "Research in Military Engineering and Construction."

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Directors of the WES during the study were COL G. H. Hilt, CE, and COL J. L. Cannon, CE. Technical Director was Mr. F. R. Brown.

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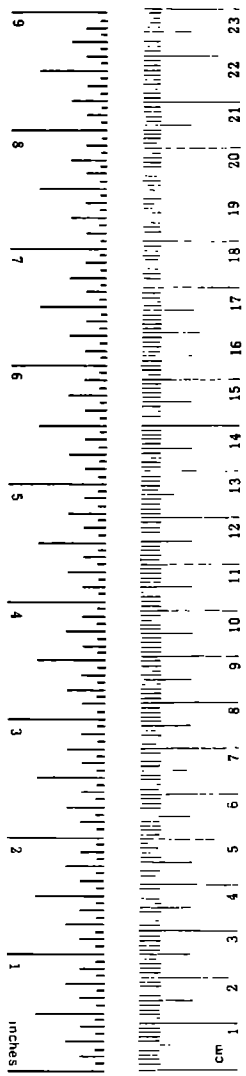
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

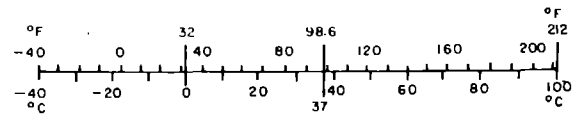
Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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INTRODUCTION

BACKGROUND

The motivation for developing a new structural design procedure for rigid airport pavements is that currently available procedures[1-5] have several weaknesses. This does not imply that the existing procedures are not valid, nor does it imply that any proposed new procedure will alleviate all of the weaknesses of the existing procedures. Rather, the proposed procedure will permit consideration of several design aspects that are ignored or approximated in available procedures.

The procedures described in References 1-5 are similar in approach with variations only in certain details. Of interest is the common approach to modeling the pavement and characterizing the supporting characteristics of the material beneath the portland cement concrete (PCC) surface layer. In all five procedures, the pavement is modeled as a two-layer system, i.e., the PCC surface layer is described as a thin elastic plate, and the underlying material is described as a dense liquid (Winkler) foundation.[6,7] Other important common features of all five procedures are: (a) the supporting characteristics of underlying materials are quantified by a single constant, referred to as the modulus of soil reaction; and (b) this constant is determined with plate load tests conducted on in situ materials.[8,9]

The validity of using a two-layer model and a plate load test for quantifying the supporting characteristics of the underlying material are questionable for pavements with relatively thin layers of bound material and for vehicle gears with large, widely spaced wheel loads. The procedure developed herein utilizes a layered elastic system for modeling the pavement and should improve the validity of the computed pavement response parameters (stress, strain, and deflection) for all layered systems and loads.

The characterization of each layer with fundamental material properties obtained from laboratory tests, as opposed to field tests on in situ material, will permit more flexibility for testing the material

at a range of field conditions. The use of a layered model and laboratory tests to determine properties of the pavement layers will permit trial of a range of types of materials arranged in various layering schemes in order to determine an optimized design. The use of laboratory tests rather than field plate load tests should permit more tests and, therefore, a better representation of the subgrade.

A final reason for developing a new design procedure for rigid pavements is the belief among some engineers involved in pavement design, evaluation, and research that there should be a universal system applicable to design and evaluation of all types of pavements, rather than separate procedures for rigid, flexible, or unsurfaced pavements. From a philosophical point of view, this goal certainly is desirable and is worth pursuing; but from a practical point of view, there are a number of obstacles that will only be overcome by advances in the state of the art of pavement technology. However, there are many more similarities between the design procedure contained herein for rigid pavements and the design procedure contained in Reference 10 for flexible airport pavement than there are between currently used procedures for rigid and flexible airport pavements. Thus, this design procedure represents a step toward achieving the goal of a universal design procedure.

PURPOSE

The purpose of this study was to develop a new, practical, and implementable procedure for the structural design of rigid airport pavements. This report documents the development of the methodology contained in the procedure and presents the procedure in a stepwise manner for implementation.

SCOPE

The study was limited to the structural design of the pavement section, i.e., the selection of required thickness of pavement layers to carry the design traffic (magnitude and number of loads) under field conditions. The key words, <u>rigid</u>, <u>practical</u>, <u>implementable</u>, and

design, are reflected in the selection of an available linear elastic response model, available procedures and specific conditions for material characterizations, and simplified alternatives for accounting for traffic and environmental conditions. Certainly more sophisticated response models (finite element) and/or more sophisticated material characterizations (nonlinear, nonelastic, or viscoelastic) are available, but these are more readily applicable to analysis rather than routine design. Available test data were used to establish performance criteria and conditions for material characterization.

No innovations are offered in the treatment of joints in the PCC surface layer, although joints are the critical location in the pavement and models are available for providing at least a rudimentary treatment. It was believed that these models had not been developed to the point where they could be used in routine design. Jointing is considered by specifying certain minimum requirements.

No new methods are offered for the design of overlays. Although the use of the basic methodology for design of new pavements appears valid for design of overlays, no acceptable procedure was found to quantify either the load deformation response of in-place pavements or interface conditions so that the structural condition of the pavement would be reflected in the required overlay thickness, or conversely the performance of the overlay.

APPROACH

The basic approach taken in the study is outlined in the following four tasks:

a. Selection of a response model.

b. Selection of material characterization procedures.

c. Development of performance criteria.

d. Assembly of the methodology into a practical, implementable, design procedure.

SELECTION OF RESPONSE MODEL

GENERAL

The layered elastic model[11] was selected for computing pavement response parameters. By changing from the presently used model (Westergaard idealization), implications are that the presently used model is inadequate and that the layered model offers significant improvements.[6,7] These implications may not be totally true and may, in fact, be completely erroneous in certain respects and for certain conditions. The problem stems from the judgment of adequacy or what is best. Such judgment is oftentimes subjective, based on opinion rather than fact, and in many cases, based on only a limited range of circumstances. Nevertheless, the selection of the layered elastic response model was based on what was believed to be sound, rational, and practical considerations.

COMPARISONS OF AVAILABLE MODELS

Crawford and Katona[12] have prepared a state-of-the-art report on the prediction of pavement response. In their discussions, they refer to three types of idealizations of pavement structures. These are the Westergaard, layered elastic, and finite element idealizations. To these primary idealizations should be added several significant mutations and combinations of the three primary idealizations.

Hudson and Matlock[13] developed a model that essentially follows the Westergaard idealization but uses a numerical technique for solving the equations of bending for the thin elastic slab representing the surface layer. The numerical technique is based on finite difference approximations of continuous functions, and the corresponding physical idealization of the elastic slab is similar to the finite element idealization. This idealization will be referred to as the discrete element idealization. A model developed by Saxena[14] combines the discrete element idealization of the elastic slab with an elastic solid idealization of the underlying material rather than a dense liquid idealization. The elastic solid idealization (Boussinesq) is a

simplified version of the layered elastic idealization (Burmister[11]) in that only one semi-infinite layer is considered.

Huang and Wang[15] developed a model that combines the finite element idealization for the thin elastic slab with the dense liquid idealization for the underlying material. A model developed by Eberhardt and Willmer[16,17] is similar to that developed by Huang and Wang, but with an additional feature, such that an intermediate layer can be considered. A procedure was developed in which the top two layers are modeled as an equivalent thin elastic plate. The finite element idealization is then used for the equivalent plate, and the dense liquid idealization for the remainder of the structure.

As stated previously, these four models are simply mutations or combinations of the three primary idealizations and are subject to similar limitations. Therefore, the following discussions are limited to the three primary idealizations.

Crawford and Katona[12] provide detailed discussions of the three primary idealizations and include discussions of various material characterization procedures that are necessary for quantification of properties of the pavement structures. For the reader interested in an in-depth comparison, Reference 12 is recommended. However, a brief comparison follows in which the primary reasons are outlined for selecting a response model based on the layered elastic idealization.

For the layered elastic idealization (Figure 1), the pavement structure is represented as a series of horizontal, uniform, elastic layers with properties defined by (a) E_i , the modulus of elasticity of the ith layer; (b) ν_i , the Poisson's ratio of the ith layer, and (c) h_i , the thickness of the ith layer. Furthermore, the layers extend horizontally to infinity in all directions, and the nth layer extends vertically to infinity. The Westergaard idealization (Figure 2) represents the PCC slab as a thin elastic plate with properties defined by E_p , ν_p , and h_p , over a dense liquid (Winkler) foundation. The liquid foundation is characterized as a bed of springs having a certain stiffness. Each individual spring represents the effect of the support

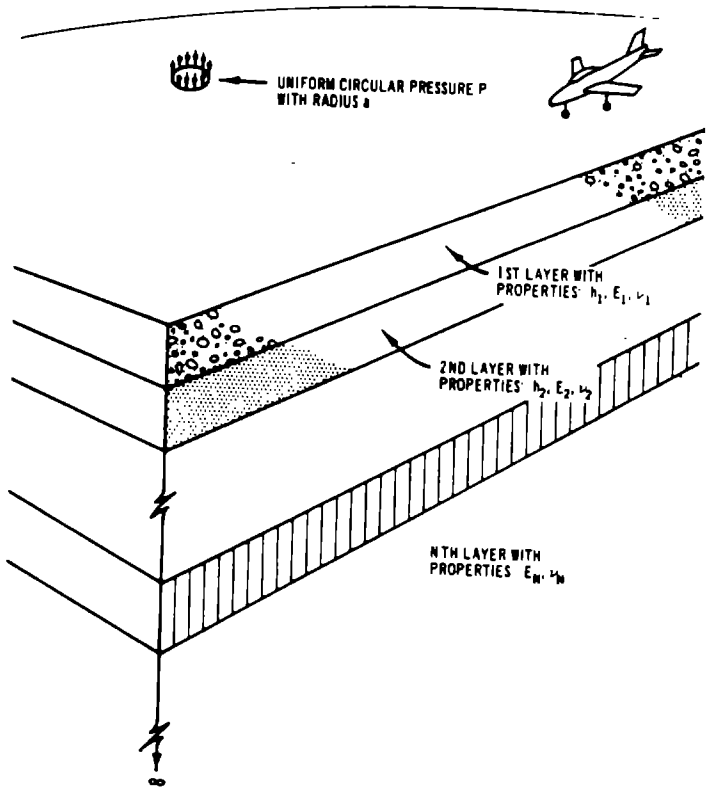

Figure 1. Elastic layered pavement idealization (from Crawford and Katona[12])

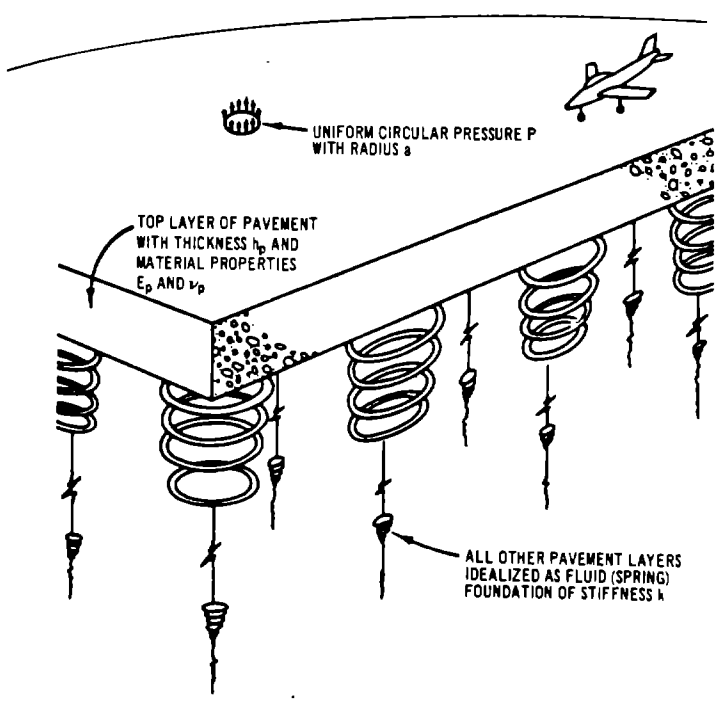

Figure 2. Westergaard pavement idealization (from Crawford and Katona[12])

provided over a unit area and is quantified by a constant k, which is the ratio of pressure on the unit area divided by the deflection. In the basic Westergaard idealization, loads were represented as uniform circular pressure distributions, but procedures developed by Pickett et al.[18] and Pickett and Ray[19] permit uniform pressure distributions with any shape to be readily handled.

The elastic layered idealization would appear to be a more realistic representation of a real pavement structure since PCC pavements are truly layered systems, although the materials may not be truly elastic. For practical loadings, however, the materials can be represented by quasi-elastic properties. The representation of the top layer as a thin elastic plate (Westergaard idealization) or as an elastic layer is really not that different when the top layer is a PCC slab, as in rigid pavements. The major difference lies in the representation of the remainder of the structure. The use of fundamental constants E and ν to represent the properties of underlying layers is theoretically sounder than a single constant k. From a practical standpoint, it is also more valid, considering that the determination of k is made with a plate test and represents the response of the material to a particular loading condition (i.e., 30-in.*-diam plate and 10-psi vertical pressure), which may be different from that actually experienced in the pavement.

Experience has shown that, for single, dual, and even dual-tandem gears with closely spaced wheels on relatively thin slabs (less than about 15 in.) laid either directly on the subgrade or on granular layers, quantification of the supporting characteristics of the underlying layer with a modulus of soil reaction produces reasonable computations of the response of the pavement. However, for larger loads transmitted to the pavement through a number of widely spaced wheels, for relatively thin, high-strength (large stiffness) base courses, and for thick PCC slabs, the validity of the idealization decreases. For the thicker slabs and

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* A table of factors for converting units of measurement is presented on page iv.

widely spaced wheels, the zone of influence (stresses in the underlying material) becomes much larger than that under a 30-in.-diam plate, although a 10-psi contact pressure may in fact be valid for both conditions. The effect of a thin, high-strength (stiffness) base course will be more pronounced on the load deformation response of a 30-in.-diam plate than on the load deformation response of a thick PCC slab. The response of the 30-in.-diam plate will be significantly reduced by the thin base, whereas the reduction in the response of the pavement will not be as significant.

Another situation in which the use of an elastic layered idealization may be more representative occurs when there exists within the subgrade different types of materials at relatively shallow depths (less than 20 ft). For instance, a stiff or a soft layer in the subgrade may not significantly affect the load deformation response of a 30-in.-diam plate, but the effect may be significant on the load deformation response of a thick slab loaded with a large load on widely spaced wheels.

Characterization of each layer with elastic constants obtained from laboratory tests, rather than one elastic constant obtained from field tests, provides the designer greater flexibility. At this point, it should be noted that it is recognized that the materials in pavements behave neither elastically nor linearly but that the assumption of linear elasticity is necessary for practical application. The state of stress under which the material is tested in the laboratory may be changed to conform to the most critical state of stress under which it may exist in the pavement. This is contrasted with the constant state of stress at which the modulus of soil reaction is selected. There is also the flexibility of being able to readily change the physical condition of the specimen (moisture, pore pressures, density, etc.), whereas this cannot be so easily accomplished on in situ material. Thus, the use of an indirect correction for saturation of the modulus of soil reaction[8] is not necessary.

The use of laboratory procedures makes it possible to test a more representative sample of the existing subgrade and a larger variety

of available base course materials. With the present design procedure, extensive plate bearing tests are the exception rather than the rule. Another factor to be considered is the repetitive nature of the loads applied to a pavement. Certainly, the use of laboratory tests will more readily permit consideration of the effects of repeated load applications than will field plate bearing tests.

In addition to the necessary and practical assumptions of linear elasticity, the layered elastic model has one major weakness when applied to rigid pavements, i.e., the inability to treat discontinuities (cracks and joints) primarily in the PCC layer. In this regard, the Westergaard idealization is better because it does permit a rudimentary treatment of the joints. With the Westergaard idealization, bending stresses in the slab, the vertical slab deflections, or the vertical reactive pressures may be computed at or near the corner or edge of a slab that is semi-infinite in the horizontal direction. Empirical adjustments to these response parameters may then be made to account for the reduction due to support provided by adjacent slabs.

The assumption of completely bonded or completely frictionless layer interfaces is not considered to be a significant weakness. A similar assumption is made by the Westergaard idealization in which the interface between the PCC slab and the underlying material is assumed to be frictionless. The interface between a PCC slab and the second layer is most likely intermediate between a completely bonded and completely frictionless condition. Between all other layers, the assumption of full bonding is probably more valid, being dependent on the type material and construction procedure. However, no data exist to adequately quantify the interface conditions, although the response model does exist that can analytically consider intermediate conditions. The computation of the various response parameters will certainly be affected by the selection of the interface condition. For computation of the design parameters, it was assumed that no bond existed at the interface beneath the PCC slab and that full bond existed at all other interfaces.

The comparison of the finite element idealization with the Westergaard and elastic layered idealizations may not be valid since it

refers basically to a computation procedure rather than to a mathematical representation of the physical structure. As has been noted previously, the finite element idealization may be employed for the upper layer with a dense liquid or elastic solid representation for the remainder of the structure. Nevertheless, consideration as a separate idealization has merit for comparisons between available techniques for computing the response of a pavement to load. As discussed herein, the finite element representation will mean that the entire structure will be broken into a number of finite elements (Figure 3).

In the finite element idealization, the continuous pavement structure is broken into a number of elements connected at nodal points. The material in each element is assigned properties that may vary from element to element. The number of elements and nodal points that may be considered is limited by computer capacity, and thus boundary conditions must be specified. The loads are applied as concentrated forces at the nodal points. With the aid of special types of elements, discontinuities, special interface conditions, reinforcing steel, and dowel bars may be introduced. Special computational techniques permit consideration of

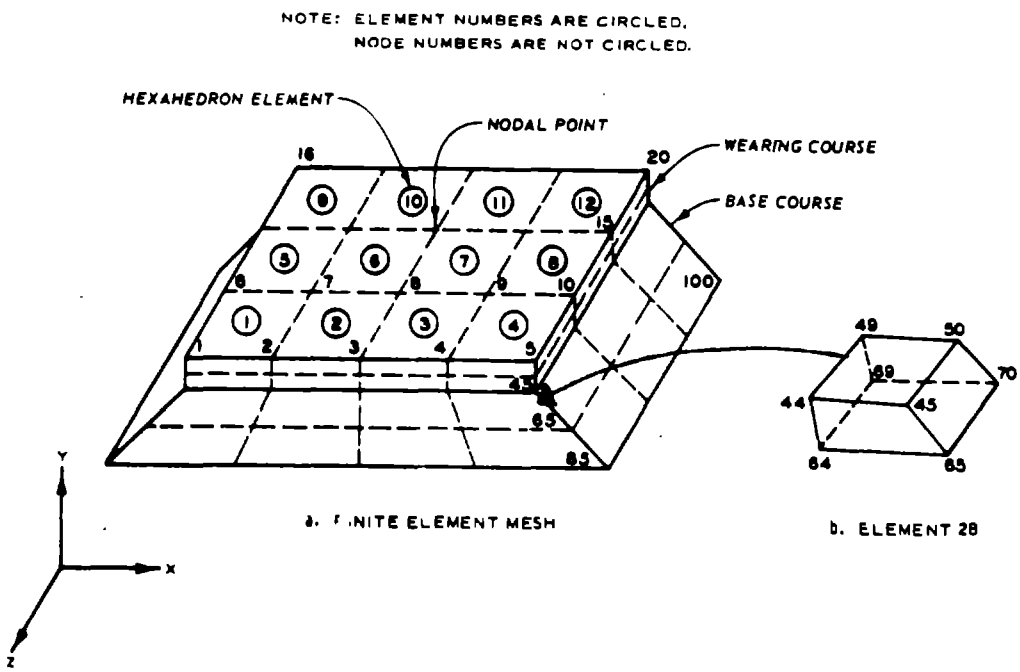

Figure 3. Pavement idealization using the finite element program (from Crawford and Katona[12])

voids and temperature and moisture gradients within the pavement structure. In addition, variable layer properties (thickness and load deformation properties) and nonlinear material response may also be treated.

However, there are limitations. For a three-dimensional idealization with only a minimum number of elements and refinements (Figure 3), the required time and cost involved in applying the procedure to pavement problems become prohibitive.

There are plane strain, axisymmetric, and prismatic solid finite element idealizations, but with all of these idealizations certain constraints are introduced. If the time, effort, and cost to apply the models are reduced to manageable levels, the applicability to a general design procedure and improvements over simpler models is likewise reduced.

After consideration of all models, the layered elastic model was selected. The model has several weaknesses, and certainly there are more sophisticated models available. However, it offers a viable alternative that can be implemented into a workable design procedure.

Consider the relationship between the interior stress computed by layer theory and 0.75 of the edge stress computed by plate theory (Figure 4). The parameter of 0.75 of the edge stress is the design parameter presently being used for design of rigid airport pavements. Although the relationship in Figure 4 is not one of equality, it is one of strong correlation; thus, if a workable design procedure could be based on edge stress, then it would be expected that a workable procedure could be developed based on the interior stress.

SELECTION OF COMPUTATIONAL PROCEDURE

The two most widely used computer codes to solve for the response of a layered elastic pavement idealization are the BISTRO[20] and the CHEVRON[21] codes. In addition, the CRANLAY[22] and the BISAR[23] codes have been used but not as extensively as the BISTRO or CHEVRON codes. From these, the BISAR code was selected to develop the performance criteria because of the author's confidence in the accuracy of the program and

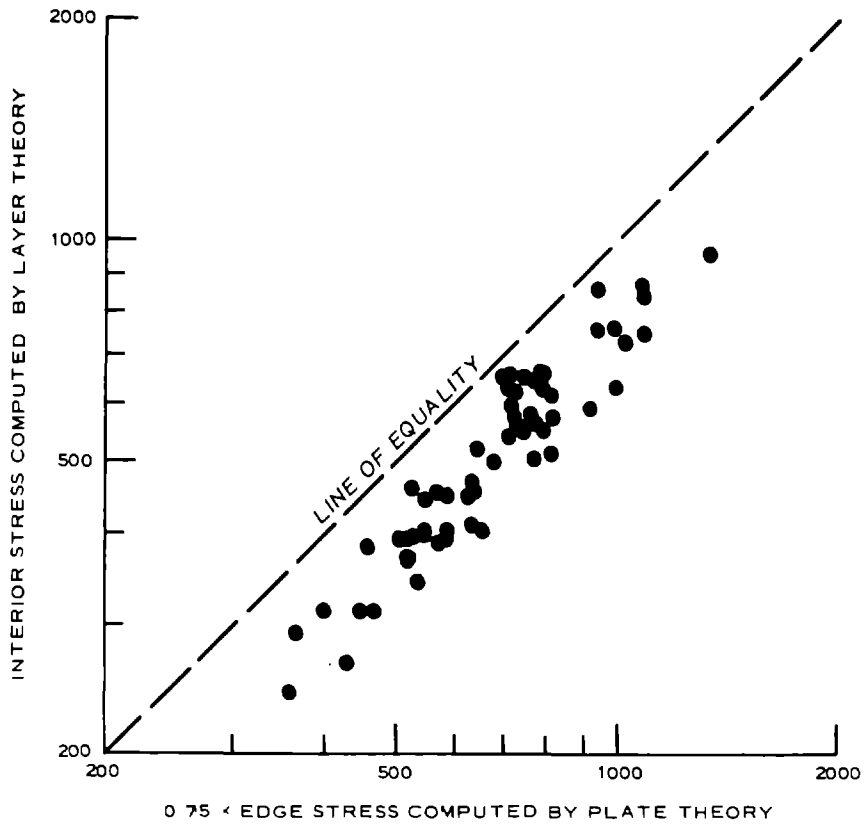

Figure 4. Comparison of stresses computed from test section data (Appendix B)

the ability of the program to consider different interface conditions. The BISAR program is an advanced version of the BISTRO program and has capabilities that, while not necessary for the use of the design procedure developed in the study, may be useful in the future for pavement analysis. The CHEVRON program does offer some advantages in terms of operation cost and may, in some design situations, be a completely adequate program. As an example, the CHEVRON code has been modified for multiple wheels and is recommended by Barker and Brabston[10] for flexible airport design. In order to maintain as much consistency between pavement types, it was the original intent to use the same code for rigid pavements.

However, for rigid pavements that have a subgrade of a low resilient modulus, the deflection basin computed with the CHEVRON

computer code becomes very distorted in the area beneath and immediately adjacent to the loaded area. This distortion is evident when compared with the deflection basins computed using the BISTRO/BISAR computer program (Figure 5). The distortion of the CHEVRON-computed basin is much greater for low subgrade moduli with the distortion decreasing as the subgrade modulus increases until for a subgrade modulus of 7000 psi the basins computed are nearly identical. The distortion is caused by inaccuracies in the numerical solution procedures for various integral equations and Bessel functions for large E_P/E_S ratios. The distortion in the curves in Figure 5 are accentuated by the location of a stiff layer (E = 1,000,000 psi) at a depth of 240 in. below the bottom of the slab (reasons for this will be discussed in the following section). When the subgrade is assumed infinite in depth, the distortion is not as severe with the CHEVRON code, i.e., there is no hump where the deflections near the load are smaller than those further removed from the load (Figure 6).

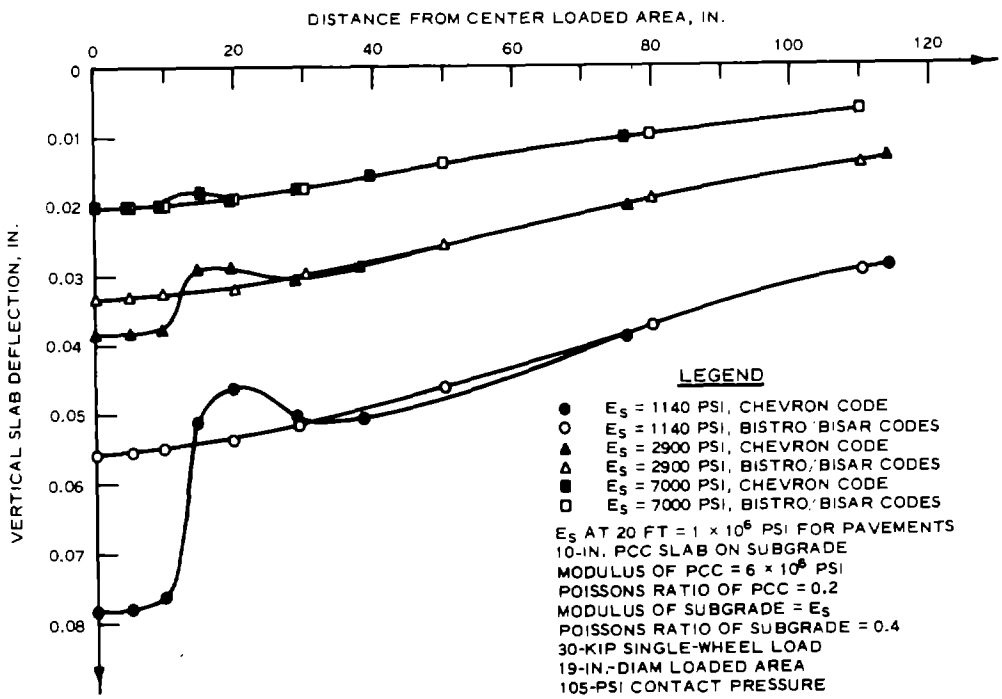

Figure 5. Deflection basins computed with the CHEVRON code compared with the deflection basins computed with the BISTRO/BISAR codes

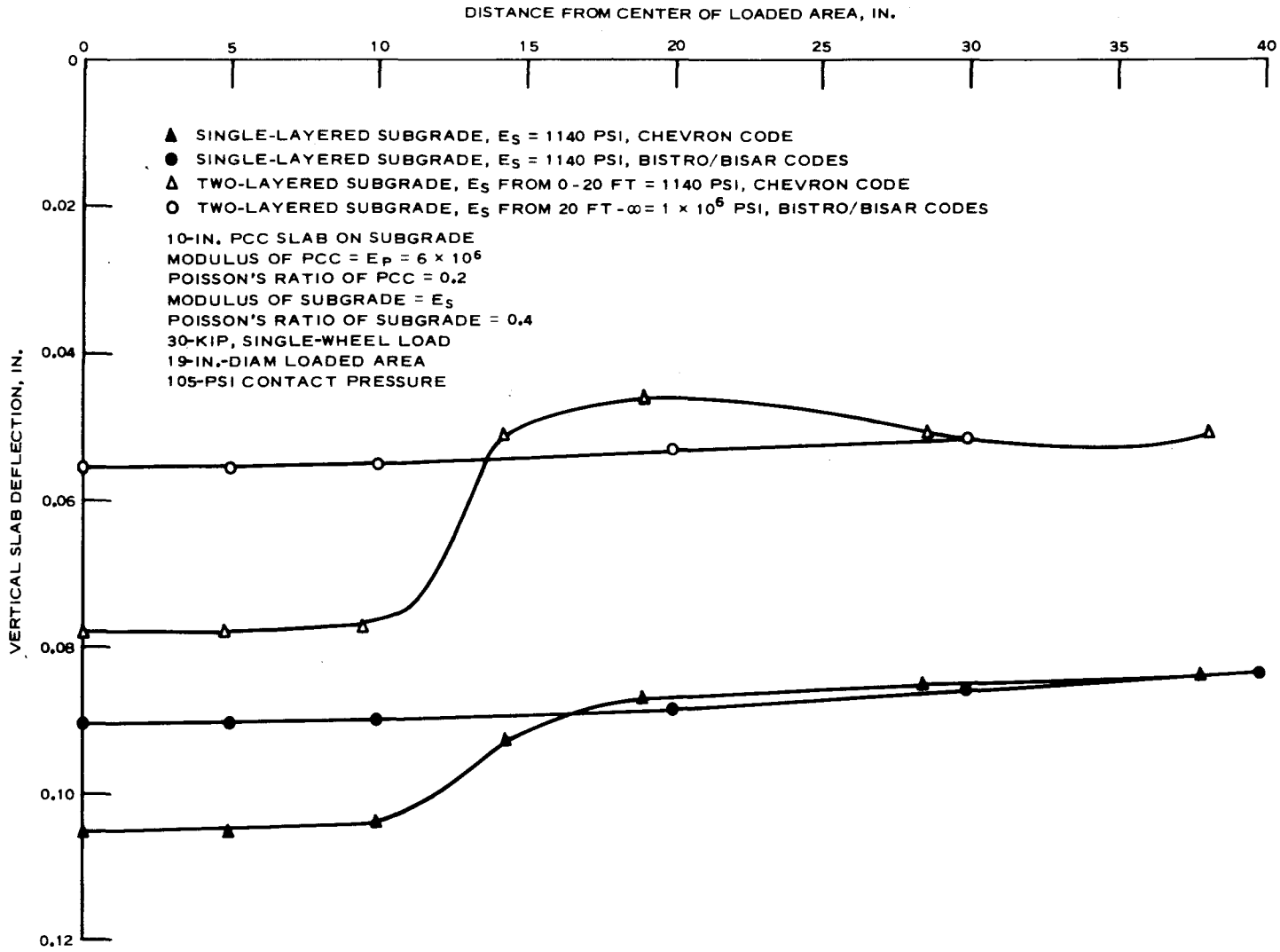

Figure 6. Illustration of effects of stiff layer on shape of deflection basin computed with CHEVRON code compared with that computed with BISTRO/BISAR codes

A much smaller effect on the stresses is apparent, as illustrated by the plot of radial tensile stress in the bottom of the PCC slab in Figure 7. However, it was believed that the additional accuracy of the BISTRO/BISAR computer codes was desirable. Because of the cost benefit and additional capabilities of the BISAR code, this program was chosen over the BISTRO code.

MODIFICATION OF MODEL FOR FINITE SUBGRADE DEPTHS

The layered elastic idealization assumes that the bottom layer extends vertically to infinity. This, along with the assumption that the layers extend horizontally to infinity, is a necessary condition for solving the integral equations and Bessel functions to obtain the pavement response. From a practical standpoint, borings are not usually made to depths sufficient to establish the location of layers below a 10-ft depth,[1,24] and the most common situation encountered is one in which the stiffness of the soil increases with depth. The results are that the model invariably overpredicts vertical deflections within the pavement structure. Strains, and therefore stresses, are also affected but to a much lesser extent.

The effect of the assumption of subgrades with infinite extent may be examined by studying the influence on two of the more important response parameters, i.e., vertical slab deflection and tensile stress in the bottom of the slab. Figure 8 illustrates the influence on vertical slab deflection. The magnitude of the influence will be affected by the subgrade modulus. As the stiffness of the underlying material(s) decreases, the effect on the vertical deflection increases. This is an obvious consequence since the vertical deflection is obtained by integrating the vertical strain function with depth from the surface of the subgrade to infinity. By introducing a relatively stiff lower layer, the effects of the integration of the strain function are masked by decreasing the magnitude of the vertical strain to very small values.

Figure 9 illustrates the influence on radial stress in the bottom of the slab. The stress is not significantly affected by the presence

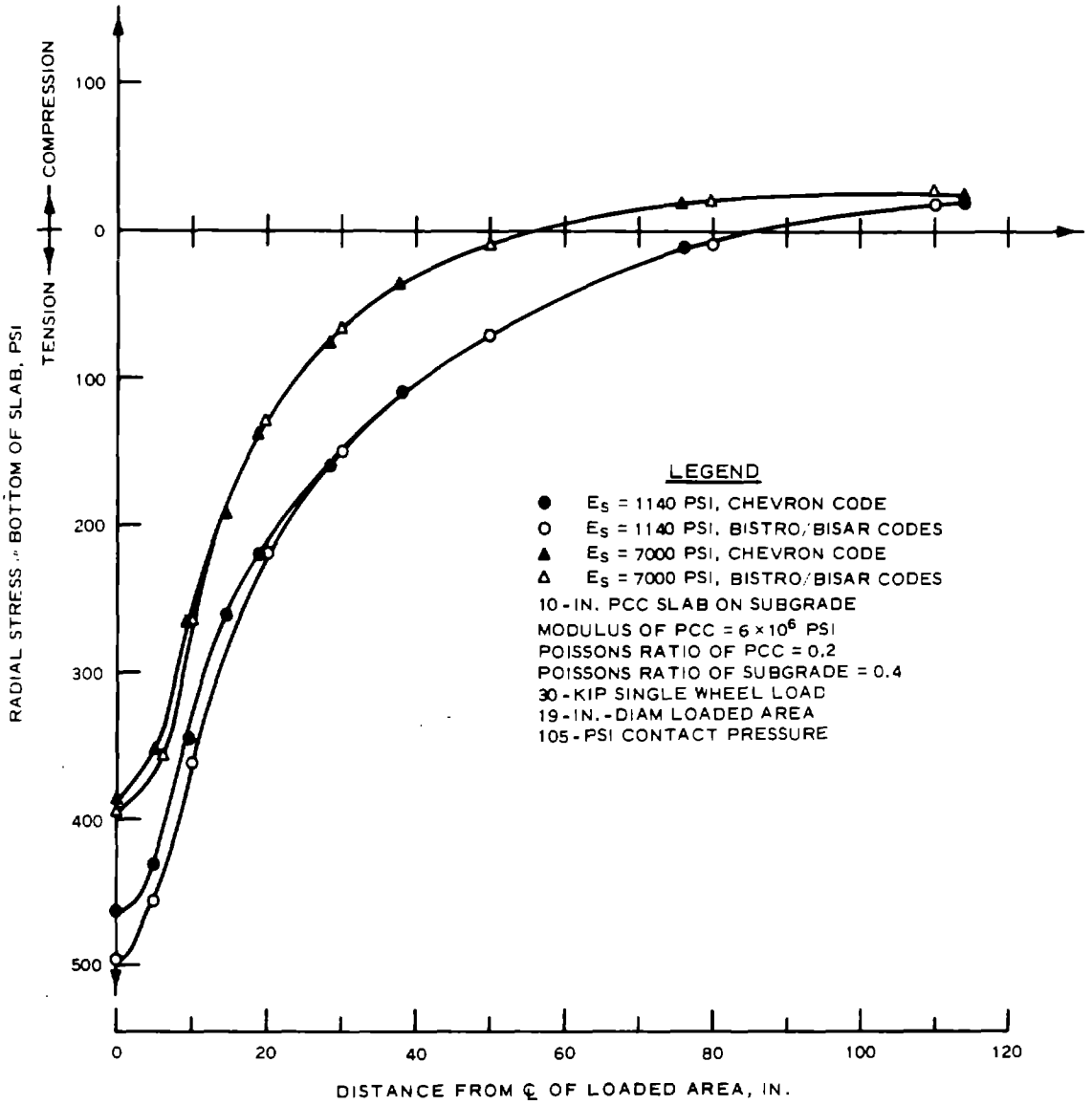

Figure 7. Comparison of tensile stress in bottom of PCC slab computed with CHEVRON and BISTRO/BISAR codes

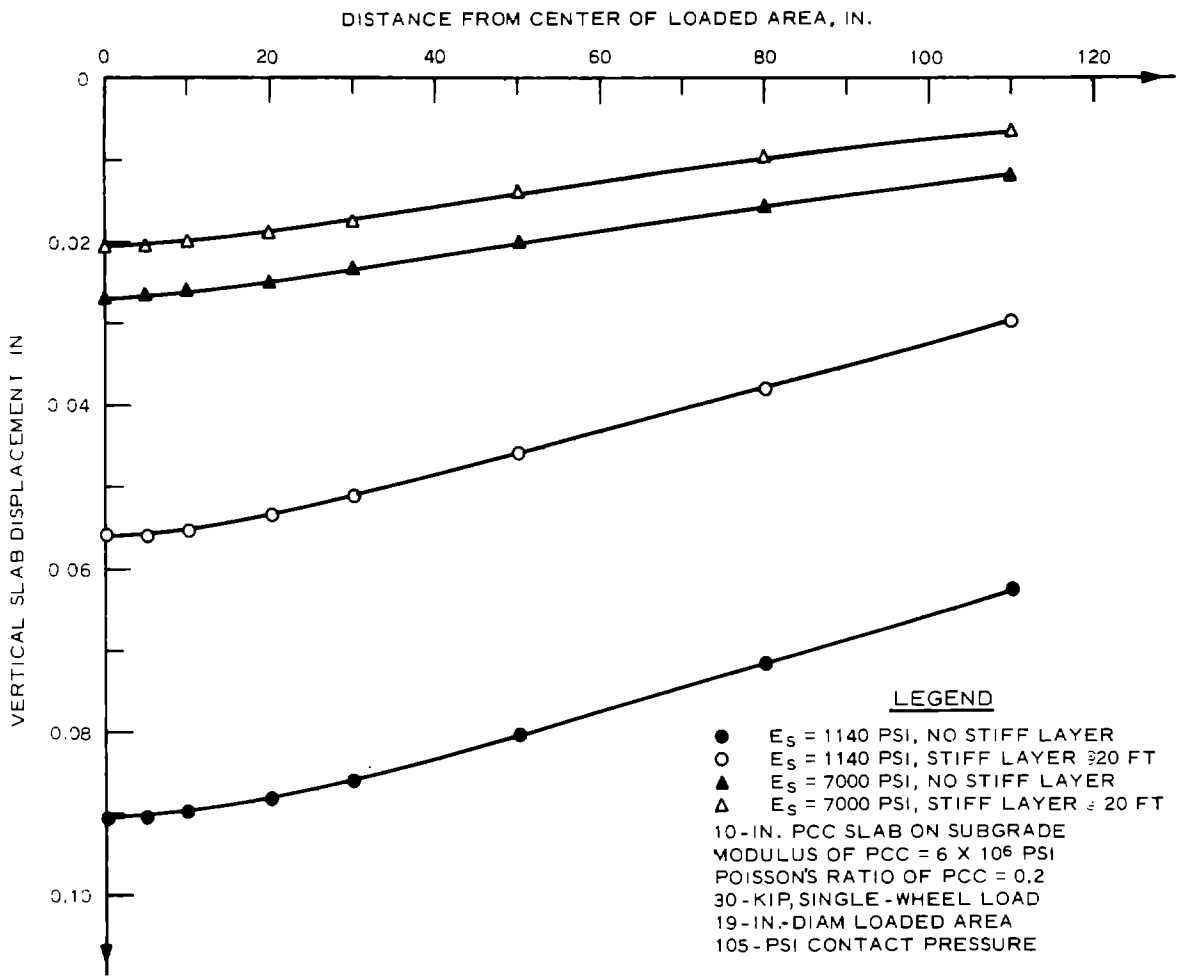

Figure 8. Effect of infinite subgrade depth of vertical slab deflection

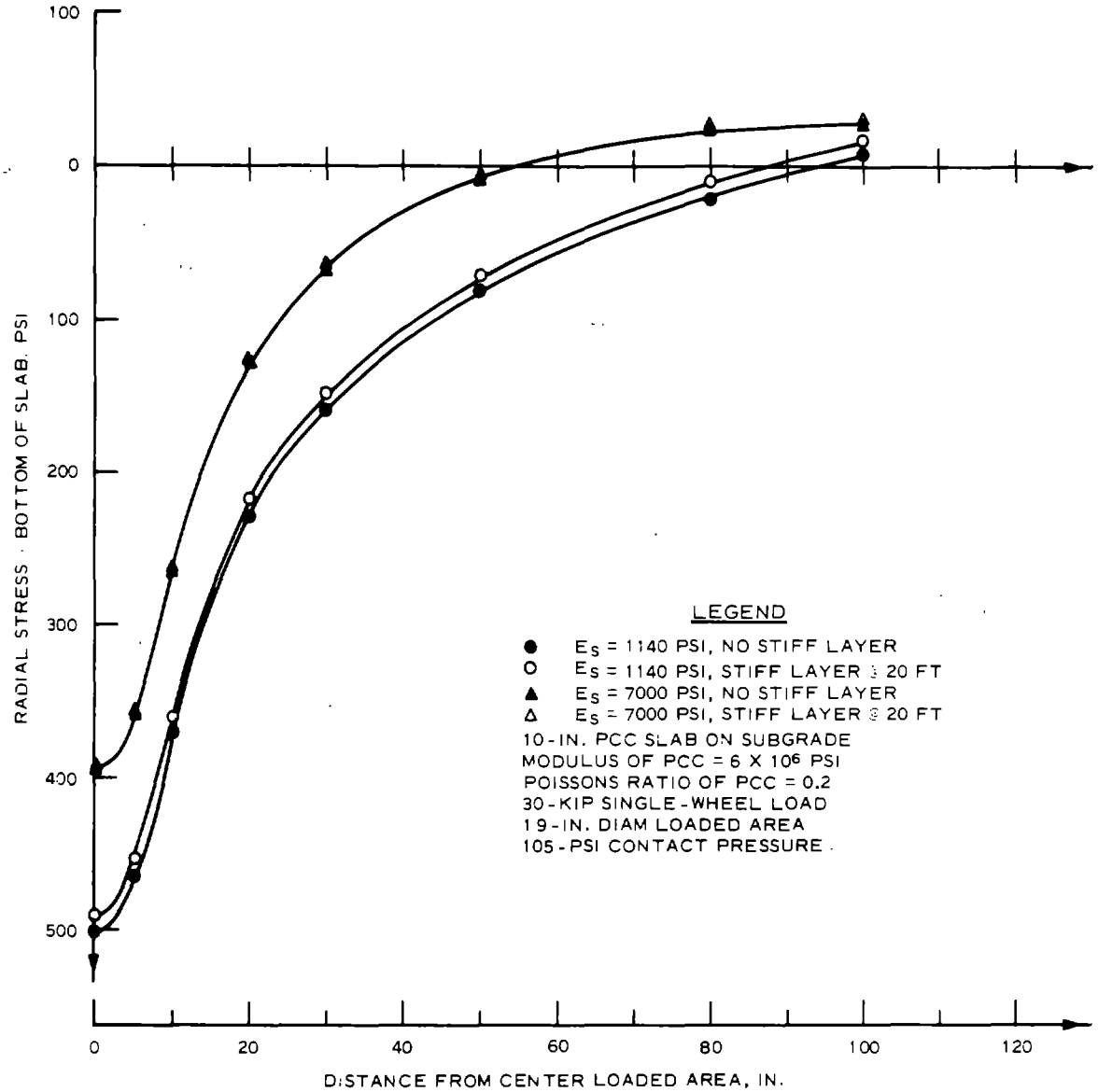

Figure 9. Effect of infinite subgrade depth on stress in bottom of slab

of the stiff layer. This is typical of not only the radial stress (Figure 9) and the tangential stress but also the vertical stress. The components of strain are affected by the assumptions of infinite layers, but since they are computed directly from the stresses and involve no integration or summation process, the effect is small. The effects of small differences are not cumulative as they are for computations of deflections. The assumption of layers of infinite horizontal extent will also affect the computed response. For rigid pavements with small thick slabs, this may be important and is one of the weaknesses of the layered elastic idealization.

The decision was made to modify the model by incorporating a layer of infinite thickness having a modulus of elasticity of 1×10^6 psi and a Poisson's ratio of 0.4 at a depth of 20 ft, unless exploration indicates the need for some other representation. This was a rather arbitrary decision but was based on observations of what was needed to improve agreement between measured and computed values. Certainly it is realized that the magnitude of the load and thickness of the PCC slab will influence the depth to which the underlying material is affected and thus should be considered in positioning the stiff layer. However, about 20 ft appeared to be a practical value for aircraft loads and a reasonable range of pavement thicknesses. The value of 1×10^6 psi for the modulus is several orders of magnitude larger than the modulus of elasticity for most subgrades and appears to work very well. This modulus value and the Poisson's ratio of 0.4 can, however, be varied considerably without affecting the results to any great extent.

The acceptability of the modification may be enhanced by considering several typical examples. Figures 10-12 show influence lines of vertical deflection measured as the load traveled across the pavement and deflection basins computed with the BISTRO code with and without a stiff layer. The difference in an influence line and a deflection basin is that data for the influence lines were obtained from a gage at a fixed location in the pavement as the load moved and the deflections for the basin were computed for various locations in the pavement with the load located at a fixed position, i.e., center of loaded area located at point

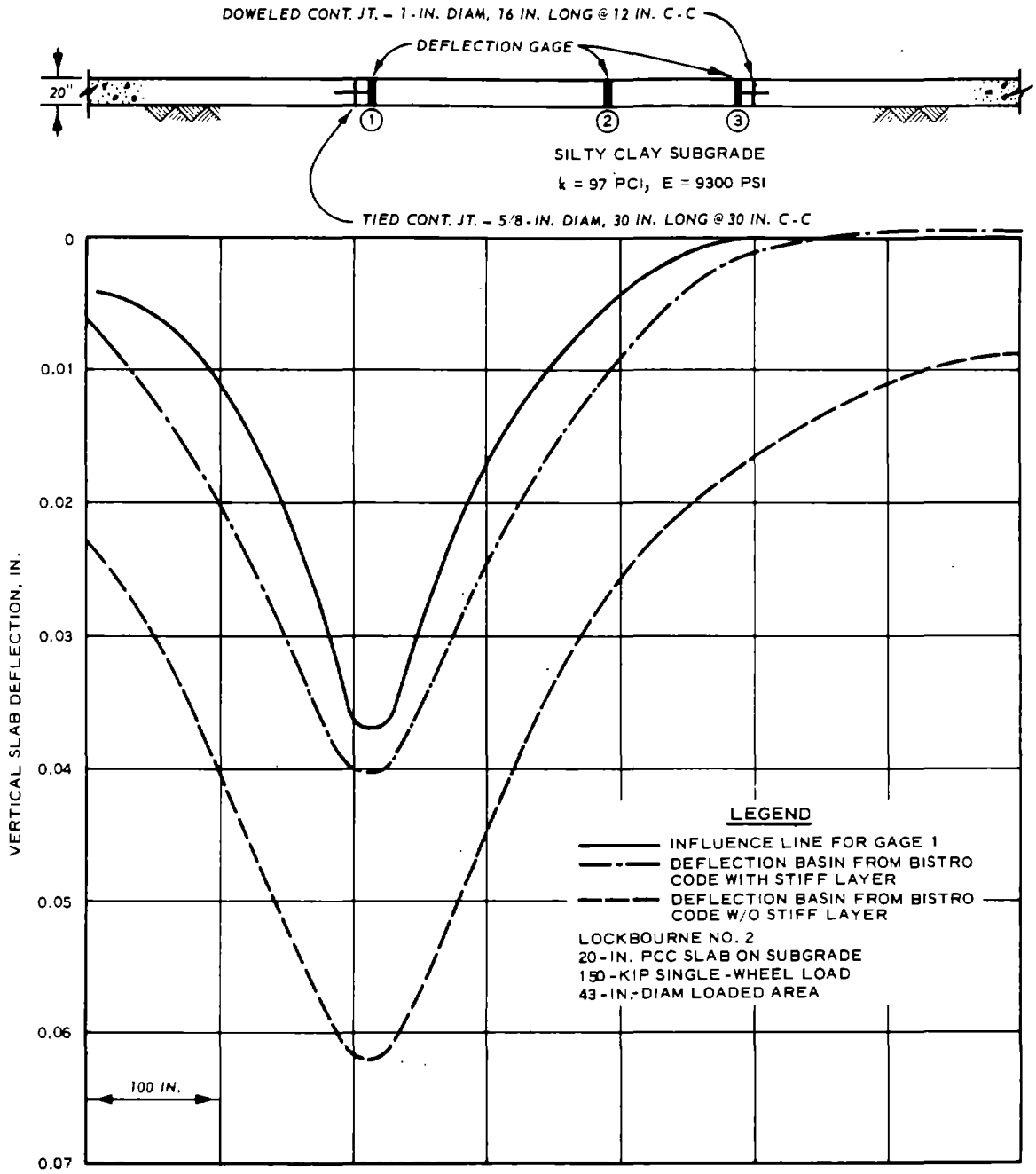

Figure 10. Comparison of measured deflection influence lines with computed deflection basins - Gage 1

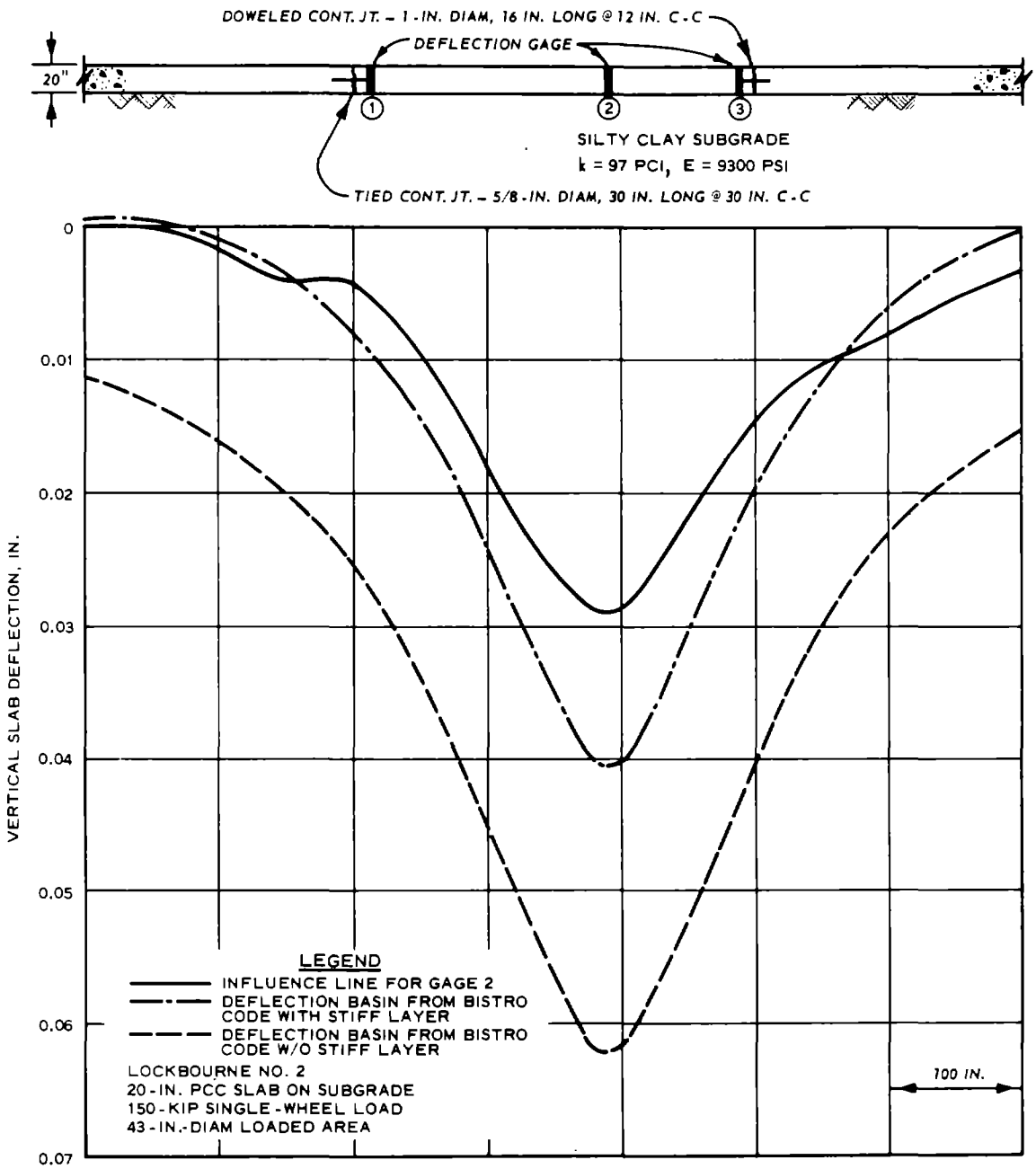

Figure 11. Comparison of measured deflection influence lines with computed deflection basins - Gage 2

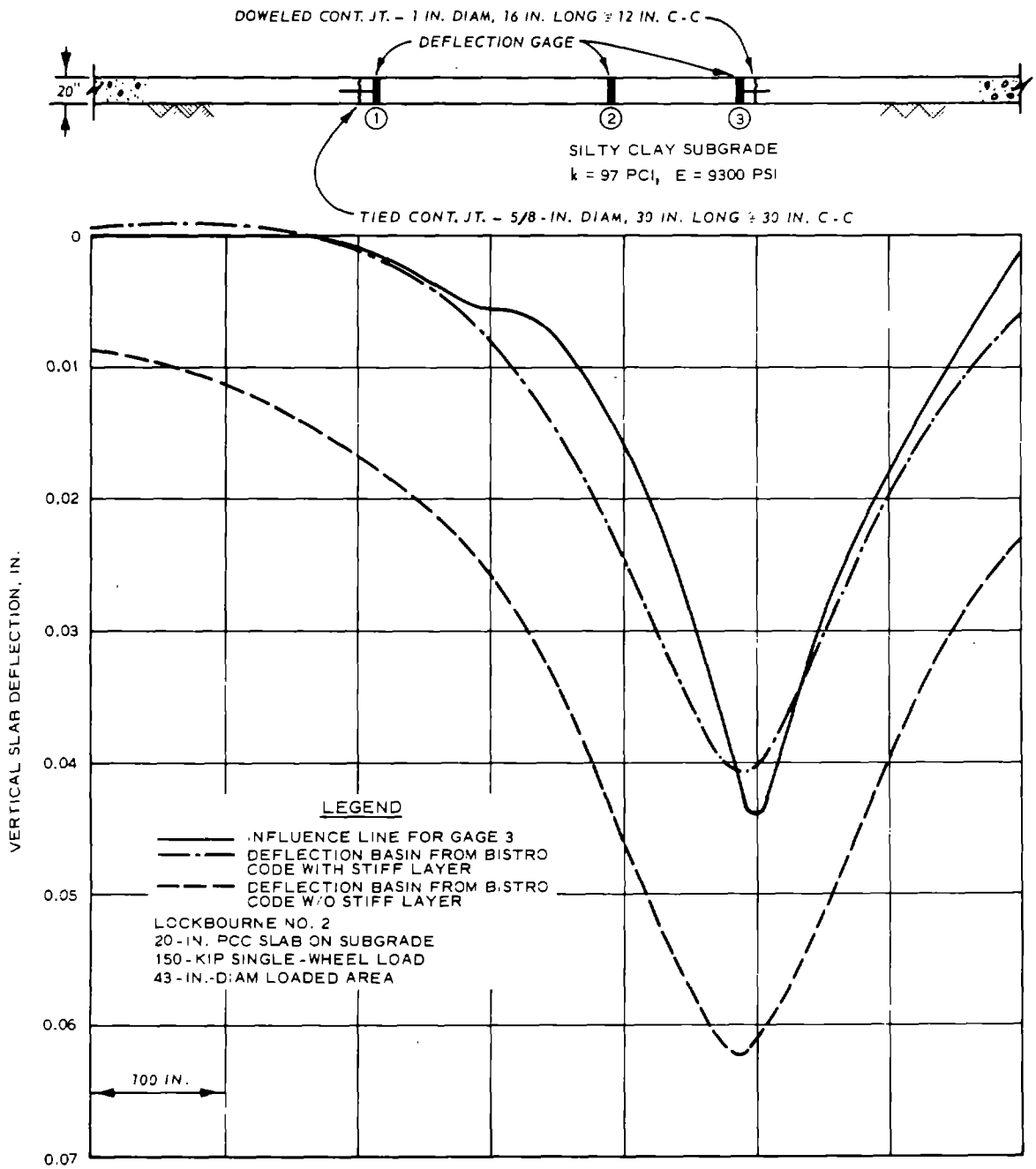

Figure 12. Comparison of measured deflection influence lines with computed deflection basins - Gage 3

of maximum response. Because of the approximations of linearity, elasticity, and time independency, the two are relatively compatible for pavements.

The results shown in Figures 10-12 are typical of the results observed for rigid pavements. The inclusion of the stiff layer improves the correlation between the measured and computed deflections and the shape of the curves. Of particular significance is the location at which the deflections become small or zero. It has been observed (even for flexible pavement structures) that the elastic layered idealization, without a stiff layer, indicates significant deflections far from the load, while measurements have shown that the deflections decrease rapidly as the distance from the load increases. This has been interpreted to mean that the zone of influence within the pavement is not as great as predicted by the elastic layered idealization in which the subgrade extends to infinity.

Additional comparisons are provided in Figures 13 and 14, which are plots of computed and measured deflection (with and without a stiff layer) for a range of pavements and loads. The pavements are from test tracks, which will be described later. The deflections are usually the largest values obtained with the load. Some of the measured values were for static loadings, and some were for slow-moving loads.

A line of equality and two linear regression lines are presented in both Figures 13 and 14. The regression analyses were accomplished with measured deflections as the independent variable and computed deflections as the dependent variable. Both regression functions are based on least square criteria, and one has the added constraint of passing through the origin.

Computed deflections without a stiff layer are shown in Figure 13. The regression function constrained through the origin is described by the equation

$$\Delta_c = 1.338\Delta_m \tag{1}$$

and the unconstrained regression function is described by the equation

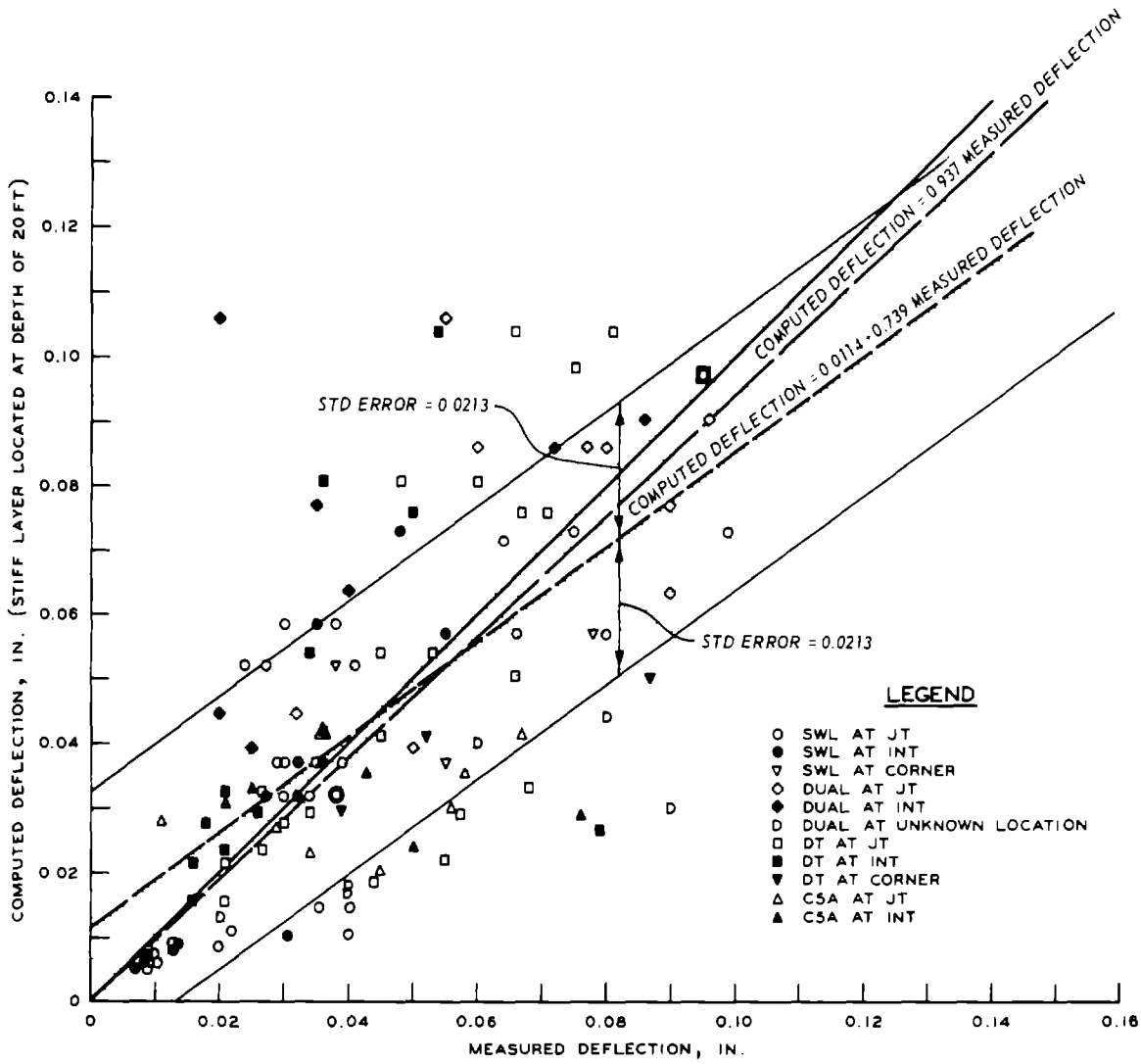

Figure 13. Comparison of measured deflections with deflections computed without a stiff layer

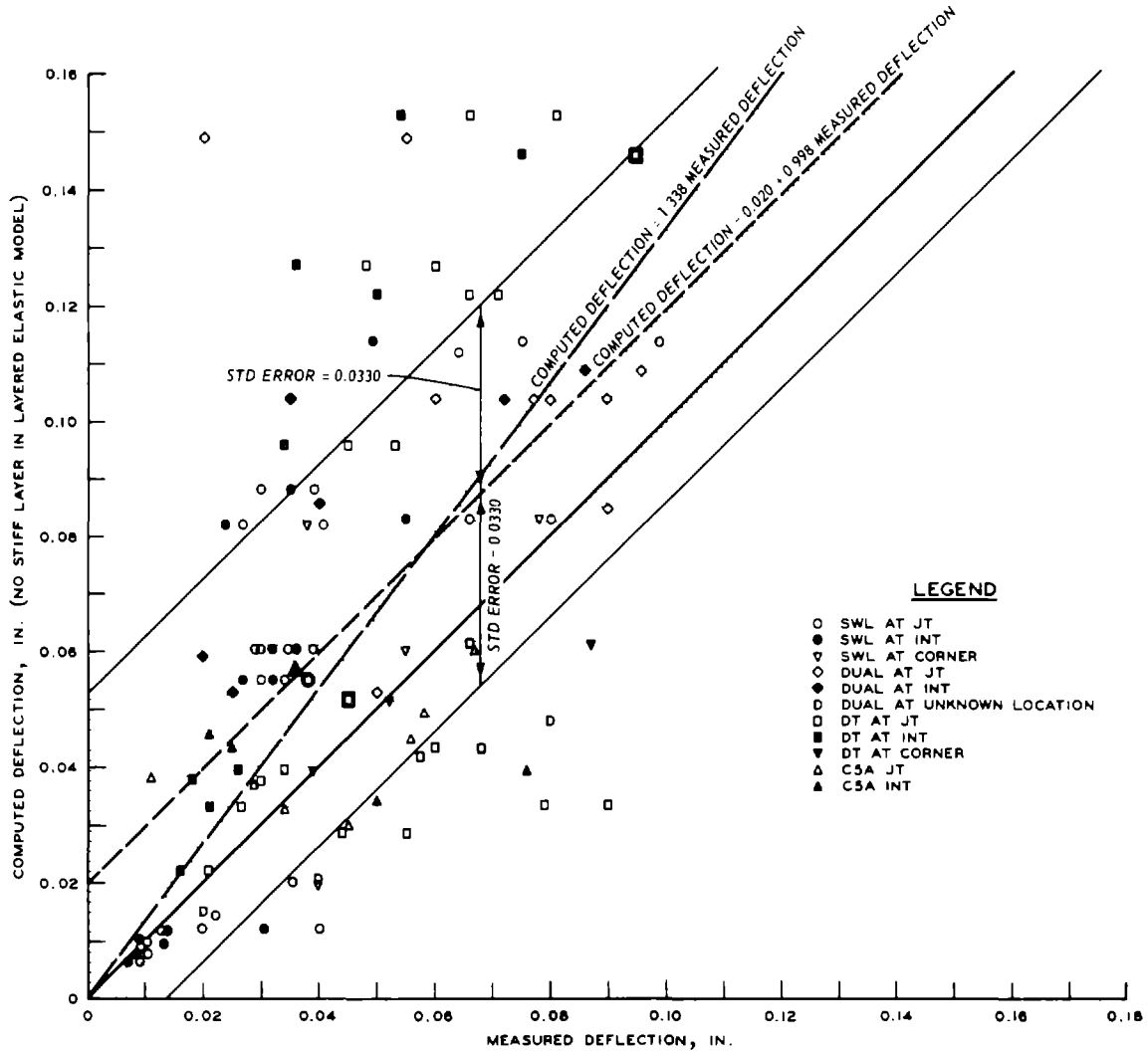

Figure 14. Comparison of measured deflections with deflections computed with a stiff layer

$$\Delta_c = 0.20 + 0.998\Delta_m \tag{2}$$

where

Δ_c = computed deflection without a stiff layer

Δ_m = measured deflection

Computed deflections with a stiff layer are shown in Figure 14. The regression function constrained through the origin is described by the equation

$$\Delta_c' = 0.937\Delta_m \tag{3}$$

and the unconstrained regression function is described by the equation

$$\Delta_c' = 0.0114 + 0.739\Delta_m \tag{4}$$

where Δ_c' = computed deflection with a stiff layer.

Examination of Figures 10-14 and the regression functions reveal the following:

a. Computation without the stiff layer overpredicts deflections.

b. The deflections computed with a stiff layer are a better approximation of the measured deflections. (Evidence of this is the position of the regression functions relative to the line of equality and the closeness to unity of the coefficients for the constrained regression functions.)

c. The variability of the computed values is reduced by inclusion of the stiff layer. (Evidence of this is the difference between the standard errors associated with the unconstrained regression analyses.)

To summarize, it was comparisons such as those just described that led to the decision to make the rather arbitrary modification to the elastic layered idealization. Although computations of stress and strain do not appear to be affected to the extent that the deflections are affected, it is believed that the modification improves the overall acceptability of the model. From a practical standpoint, it minimizes the need to explore and characterize the subgrade to large depths unless the geology of the area indicates that a soft or stiff layer is probable

between the normal exploration limits and about 20 ft or a soft layer below 20 ft.

SELECTION OF MATERIAL CHARACTERIZATION PROCEDURES

GENERAL

The amount of research that has been directed toward characterizing paving materials and subgrades with a test that is more accurate and more fundamental than a plate bearing or CBR (California Bearing Ratio) test is truly prodigious. Conversely, the formulation and application of practical, usable, widely accepted procedures for the routine design of rigid pavements is truly meager. Chou[25] has prepared a state-of-the-art report on the characterization of pavement materials including subgrades. Much of the discussion in this chapter will be based on this study.

The materials composing a rigid pavement respond neither linearly nor elastically to load, but in a complex manner. Generally, the response is nonlinear and nonelastic for a rather wide range of stress and strain conditions. The approximation of linear elasticity is more valid for such materials as PCC and bound bases (subbases) than it is for granular bases (subbases) and subgrades. PCC and bound bases (stabilized with PC or lime) tend to be more brittle and have more linear stress-strain relationships as illustrated in Figure 15. The response of many of the materials is highly dependent on the state of stress and the number of load repetitions to which the material has been subjected. Although the response of rigid pavements is not as sensitive to time and temperature* as flexible pavement, temperature and rate of loading will have an influence on the characterization of materials in which a bituminous binder is used or in other layers where freezing occurs.

The pavement designer must then approximate the complex response of materials in order to use a simple analytical procedure to obtain the response of a pavement. As it turns out, many of the necessary approximations are not nearly so drastic as might be suspected from consideration of the total response of the materials. The states of stress

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* That is, if the effects of temperature gradients within the PCC slab are ignored.

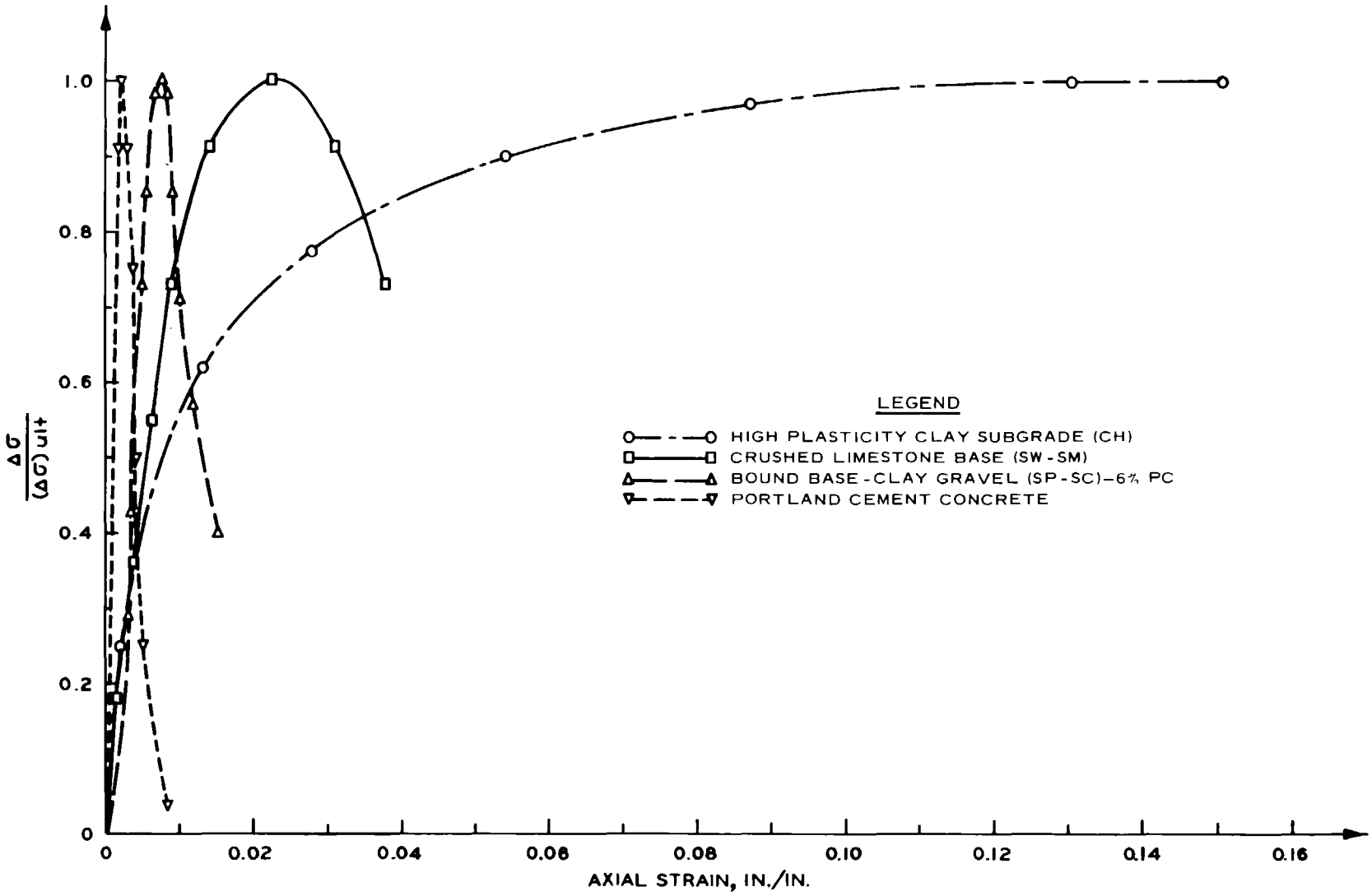

Figure 15. Comparison of typical stress-strain response of types of materials in rigid pavements

and strain within pavements are usually within the range where the assumption of linear response is reasonable. After only a limited number of load repetitions, the assumption of elastic response will usually become more reasonable, although for initial loadings the response may have been very inelastic and nonlinear. The state of stress is variable throughout a pavement structure and will depend on the loading and layer thicknesses. However, a state of stress may be selected which will provide a reasonable approximation of conditions that exist in real airport pavements.

Layers with bituminous binders are beneath the surface, where the variability in the temperature regime is not as great as it is for flexible pavements and the effects of the rate of loading are not as critical. Suitable temperatures and rates of loading can be selected that provide adequate characterization of the material containing a bituminous binder. As to the effect of freezing on the subgrade, the most critical period, in terms of magnitude of pavement response, is during periods of thaw, and these conditions can be approximated in the laboratory.

Procedures will be considered for determining the load deformation characteristics of PCC surface layers, granular bases (subbases), bound bases (subbases), and subgrades. Details of the procedures for subgrades will depend on whether the material behaves as a cohesive or cohesionless material.

At this time, some discussion of terminology regarding the four types of materials that compose rigid pavements is needed. There is general agreement as to what constitutes PCC surface layers and subgrades. Misunderstandings and the need for more precise definitions arise when considering bound or granular bases (subbases). Chou[25] discusses bound bases (subbases) under the general heading of soil stabilization, i.e., stabilizing agents, stabilization mechanism, purposes for stabilization, and resulting material properties. Wide ranges in each aspect of the stabilization are noted. Barker and Brabston[10] discuss the general area of stabilization and provide limiting values for determining when the stabilizing agents are effective in

modifying the properties of the natural material. The one fact that becomes readily apparent from these and other discussions on the subject is that there are many "gray" areas and only a few "black and white." To the myriad of existing definitions and concepts will be added several more, which are peculiar to the procedure contained herein.

Bound bases (subbases) are natural soils, prepared soils (washing, grading, crushing, etc.), or crushed stone, which has portland cement, lime (slaked or hydrated), fly ash, sodium silicate, bitumen, or a combination of the listed ingredients added. A distinction is made between bound bases in which the stabilizing agent is a bituminous binder and one in which another of the listed agents is used. Bituminous stabilized bases depend on the mechanical bond between particles provided by the bitumen binder. The other agents (primarily portland cement, lime, or lime-fly ash) depend on certain chemical reactions to provide the bond between particles and will be referred to as chemically stabilized bases. The distinguishing feature of both types of bases is that the material can be molded into a beam and can sustain flexural-type loadings.

Bituminous bases should meet requirements as set forth in References 26 and 27 for bases in which a bituminous binder is used. The references contain specifications for gradation, amount of binder, etc., which are intended to ensure that the material functions as a bound material.

Chemically stabilized materials should meet requirements set forth in References 26, 28, and 29. Among these are requirements for durability and the requirement that strength increase with age, which are intended to ensure that the materials continue to function with age and that no adverse chemical reactions occur. However, in terms of ensuring that the material functions as a bound material (sustain flexural loading), the requirement that the material attain an unconfined compressive strength of 250 psi at 28 days, as set forth by Barker and Brabston,[10] is applicable for rigid pavements. This requirement should be used in lieu of strength requirements in References 26, 28, and 29.

Those materials that have a chemical stabilizing agent added but do not meet the 250-psi compressive strength requirement should be

characterized with procedures for granular bases (subbases) or subgrades, depending on the nature of the natural material. For instance, a clay subgrade to which lime has been added but which does not meet the 250-psi strength requirement should be characterized and considered simply as part of the subgrade. The general rule is that the material should be characterized and used in the design as if no stabilizing agent had been added when the compressive strength requirement is not met.

Granular bases (subbases) are natural soils, prepared soils (washing, grading, crushing, etc.), or crushed stone, which meets gradation and durability requirements as set forth in References 1-3, 24, and 26. The characteristic of granular bases (subbases) that distinguishes them from bound bases (subbases) is that they do not possess and/or will not maintain the ability to sustain flexural loading.

The elastic constants defined for each layer will be the modulus of elasticity and Poisson's ratio. The modulus of elasticity will receive the greater attention for several practical reasons. One is that, in terms of response of the pavement, the modulus of elasticity is the dominant of the two parameters. A second is that Poisson's ratio varies only within a limited range for the different types of materials composing a pavement structure. Finally, it is difficult to accurately determine Poisson's ratio from laboratory tests.

The strength of the PCC surface layer will be defined by the flexural strength. This parameter is required in current design procedures, and the same well-established characterization procedures will be used.

EFFECTS OF LOAD REPETITIONS

To examine the effects of repetitive load applications on the response of rigid pavements, study the vertical deflection patterns that occur as a typical rigid pavement is trafficked. The total vertical slab deflection can be broken into transient (elastic) and permanent components. For rigid pavements, the elastic component remains relatively constant or decreases somewhat with traffic (Figure 16). After cracking is initiated, the elastic deflection will increase as the result of the overall decreased stiffness of the entire system.

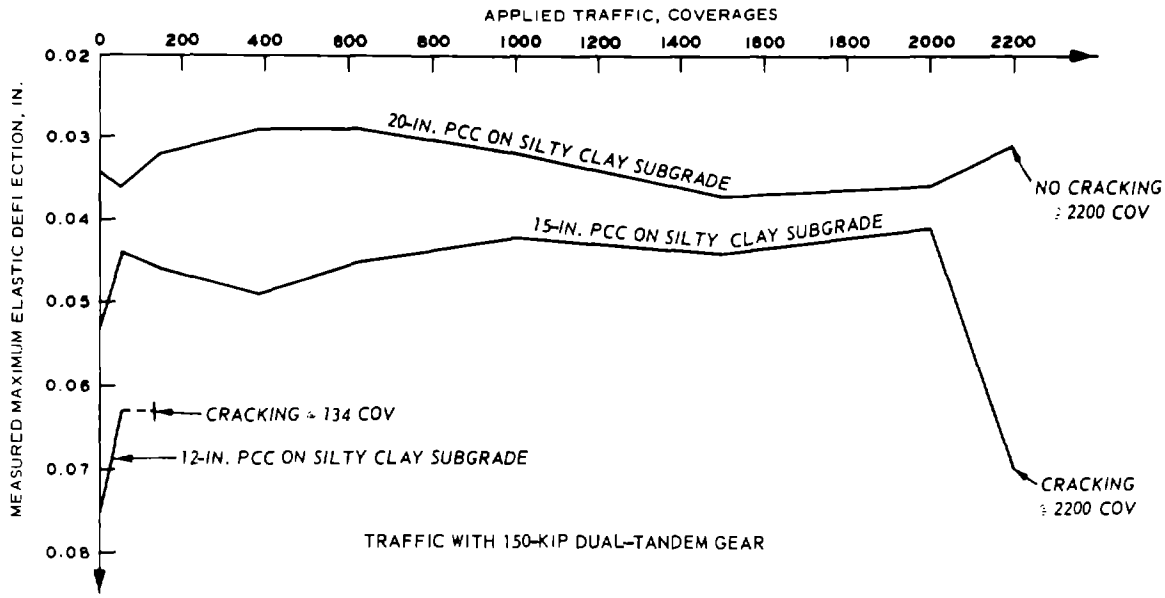

Figure 16. Elastic slab deflection versus traffic

Permanent deformation initially occurs as the slabs become seated and additional densification of loose material occurs. This deformation is rapid at first but then decreases as traffic is applied and essentially decreases to zero. Figure 17 shows that the cumulative deformation becomes relatively constant and remains so until cracking begins. In fact, Figure 17 indicates that between 600 and 2000 coverages there is a decrease in the permanent deformation, i.e., the pavement surface appears to rise. It is not known if this decrease is traffic-related or due to other causes. The only significant decrease in permanent deformation occurred between 600 and 800 coverages for the 20-in. pavement and between 600 and 1000 coverages for the 15-in. pavement. The fact that the decrease in the permanent deformation is greater for the 20-in. pavement than for the 15-in. pavement and occurred between two consecutive readings indicates that the decrease is not related to traffic; thus, it is concluded that after the initial permanent deformation, the traffic-induced permanent deformation remains relatively constant until the onset of cracking. As cracking progresses, permanent deformation again begins to increase.

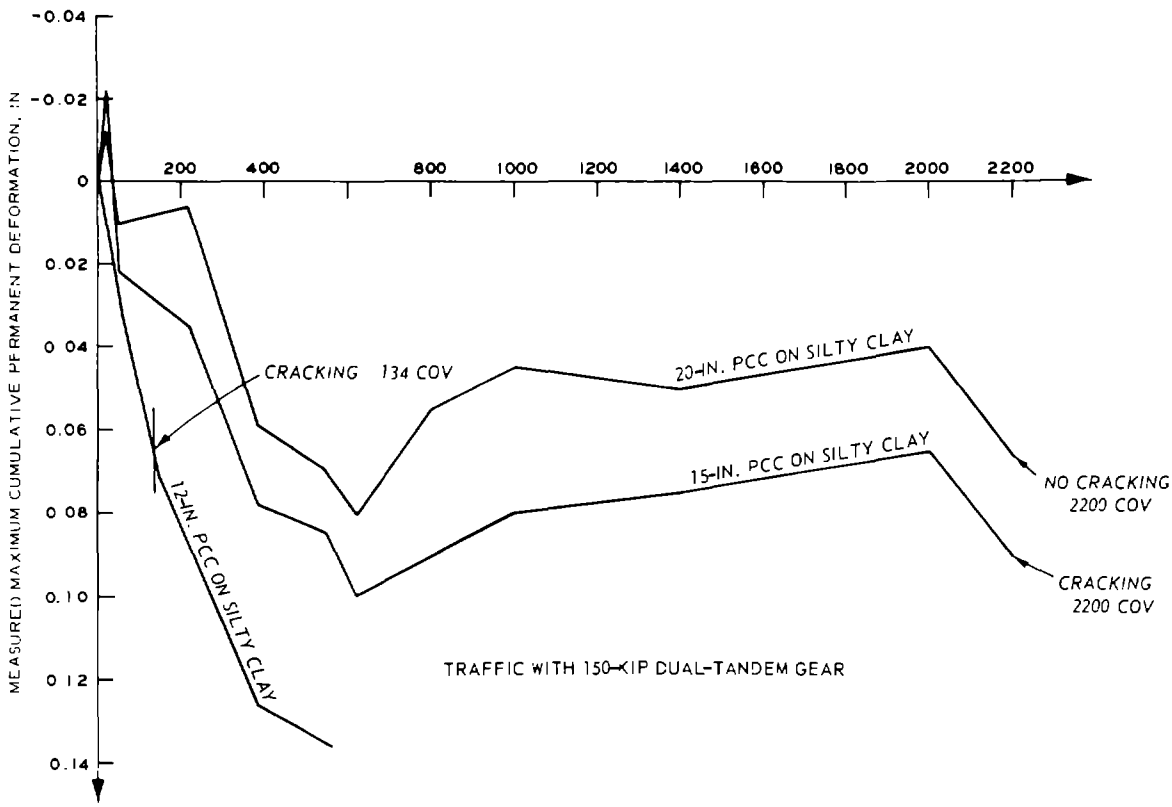

Figure 17. Cumulative permanent deformation versus traffic

The pattern of the total deflection will be similar to that for the permanent deformation since the total is the sum of the permanent and the relatively constant elastic components. The total deflection will then become practically equal to the elastic deflection after initial conditioning. For pavements designed for realistic volumes of traffic, this situation exists for a large portion of the life of the pavement.

Pavement response is a composite of the responses of the various layers. The difference in the response of the four different types of materials to static compression loading (triaxial tests) is illustrated in Figure 15. Although these results are from static tests, the same effects exist for repetitive loads. The effects of repeated loading are more pronounced for subgrades and granular materials than for PCC and bound bases. The relative influence of the assumptions of linearity

and elasticity are the same for static and resilient response. The effects of repeated loading on the PCC, granular bases (subbases), bound bases (subbases), and subgrades are discussed in detail in Appendix A.

MODULUS OF ELASTICITY AND POISSON'S RATIO

Based on the discussions presented in Appendix A, the rigid pavement system may be represented on a quasi-linear elastic system. The modulus and Poisson's ratio values are selected based upon best estimates of in-place conditions. The modulus of elasticity and Poisson's ratio of PCC are relatively insensitive to state of stress, temperature, or stress repetitions and thus can be represented with a high degree of confidence from the results of laboratory tests. The particular laboratory test considered most appropriate in representing the conditions of the PCC in a pavement system is the flexural beam test, and thus it is the flexural modulus to be used for design. This is not to eliminate the use of other testing procedures for determining a modulus, but it should be stressed that these design criteria are based upon the flexural modulus and that other modulus values should be related to the flexural modulus. There have been numerous studies relating the modulus values obtained from other test procedures, such as the uniaxial compression test, split tensile test, and resonant column test. If the designer has confidence in the correlations to the flexural modulus, then the results of these tests can be used; but the designer should also realize that the level of confidence of the design systems may be somewhat reduced by such correlations. The usage of Poisson's ratio is relatively narrow, and the effects of varying Poisson's ratio over this range on the computed response is almost negligible. Considering the difficulty in measuring Poisson's ratio, a fixed value of 0.2 for PCC will be adequate for design purposes. Thus, this has been the value of Poisson's ratio selected for development of the design criteria.

The modulus and Poisson's ratio values of base material vary over a wide range. A compensating factor is that the principal strength of rigid pavement is derived from the concrete surfacing, and thus relatively large variations in the moduli of the base can be tolerated.

For bound bases, since the properties in flexure are considered to be appropriate, the beam test as given in Appendix A should be employed. Bituminous bound bases in particular are subject to a variation in modulus and Poisson's ratio due to differences in binder type, binder content, temperature, and rate of loading. There are procedures, as discussed by Barker and Brabston[10] and Chou,[25] for estimating the modulus and Poisson's ratio of bituminous bound bases that can be used for design. Such procedures would certainly be appropriate for designs that employ bituminous bases primarily for waterproofing and not as a structural element. Bases stabilized with cement and/or lime will be similar in behavior to PCC, i.e., the modulus will depend on the strength of the material with Poisson's ratio relatively constant. As with PCC, correlations between the properties in flexure and the properties as determined by other laboratory tests have been developed. As with the bituminous bound bases, the modulus of relatively thin bound bases can be crudely estimated without seriously compromising the pavement design. Published values of the modulus of chemically stabilized materials vary over a wide range, i.e., from less than 100,000 to that approaching the modulus of lean concrete. For development of these criteria, a modulus value of 250,000 psi was used. This modulus may be considered typical of what may be expected from a stabilized base. Poisson's ratios for chemically stabilized bases vary over a wider range than the values for PCC, but 0.2 would still be an adequate estimation.

The properties of granular bases are very dependent on the state of stress, state of compaction, moisture content, and to some lesser degree on the aggregate quality. By specifying the material quality and compaction requirements and dealing with moisture conditions through assuming a nearly saturated condition or by applying some rationale for arriving at a design moisture content, the remaining variable influencing the modulus is the state of stress. Since this particular parameter does have such an important effect on the modulus of base materials, a detailed discussion is presented in Appendix A on the state of stress under rigid airport pavements. The study indicates that the first stress invariant in bases does not vary greatly and that a value of 10 psi would

be a typical value applicable for pavement design. The repetitive triaxial test is considered to be the laboratory test that can best be used to establish the characteristics of a particular granular material. Other approximate methods have been developed that do yield reasonable modulus values, but care should be exercised in the use of either the typical state of stress or the approximate methods of determining the modulus. For these procedures to be appropriate, the pavement design should be a typical design and the materials should meet the specified quality and compaction requirements.

Poisson's ratio of granular materials varies with the shear stress. In the range of stress normally encountered in rigid pavement, the value of 0.3 is an appropriate value for design. In the repetitive triaxial test, the Poisson's ratio is particularly difficult to measure and is subject to a high degree of error; thus, in most design situations, no attempt would be made to measure this material property.

Characterization of subgrade soils is normally considered much more crucial than characterization of the bases, primarily because of the relative thickness between the two and because of greater variation, which can occur in subgrade materials. In many ways, characterization of the subgrade materials is very similar to characterization of unbound granular, i.e., the material must be characterized with respect to a state of stress, moisture condition, and material density. As with the unbound base, the repetitive triaxial test is considered to be the appropriate laboratory test for characterizing the material. For the cohesionless subgrade, the characterization procedure is the same as for the unbound base, except the material quality and material densities will not meet the requirements for base materials; thus, some of the appropriation procedures will not be valid. For cohesive soils, the modulus is more a function of the deviator stress than the first stress invariant; hence, this is the stress parameter used in characterization of the cohesive subgrades. Based on the discussions in Appendix A, a 5-psi deviator stress is considered to be an adequate estimation for design of a typical airport pavement. As with the bases, since these values of stress are estimates for a typical pavement, it may be

desirable to check these estimates through computation of the stresses before the design is finalized.

As discussed in Appendix A, the moisture content at which the laboratory tests are conducted is very critical. Studies of the moisture content in airport subgrades have indicated that in most cases the subgrade material is near saturation. In selecting a moisture content at which the laboratory tests are to be conducted, the final or equilibrium water content should be considered. In some design situations, the designer may have information indicating that this final moisture content would be less than saturated. For such cases, savings may be realized by designing the pavement on the expected subgrade moisture content rather than the saturated water content.

Much effort has been applied toward developing correlations between other material parameters and resilient modulus. A study conducted by the U. S. Army Engineer Waterways Experiment Station (WES) for correlating modulus of subgrade reaction as determined by the plate bearing test is discussed in Appendix A. Such correlations can be useful in estimating the resilient modulus of the subgrade, but as with any empirical correlation care must be exercised in their use. Particular care must be taken in the case of field tests conducted to ensure that the moisture conditions at the time the tests are conducted will be representative of the final water content of the subgrade.

When the moisture content of the subgrade approaches the saturation moisture content, the Poisson's ratio of the material will approach 0.5. In the study at the WES, 0.4 has been found to be a representative value for most subgrades. For subgrades that are to be considered near saturation, it is suggested that 0.4 would be an adequate estimation of Poisson's ratio. If some lesser moisture content is considered applicable for design, then Poisson's ratio should be measured, or some lesser value of Poisson's ratio should be appropriate.

SUMMARY

Table 1 provides a summary for determining the modulus and Poisson's ratio for use in the analysis of rigid pavements. The column

Table 1

Summary Chart for Material Characterization

Material	Referenced Laboratory Test	Influencing Parameters	Expected Range for Modulus	Other Methods of Estimating Modulus by Correlations	Typical Values or Range for Poisson's Ratio
PCC	Flexural beam	Age	3×10^6 - 6×10^6	Split tensile Unconfined compression	0.2
Bituminous concrete	Flexural beam	Temperature, rate of loading	1×10^5 - 1.5×10^6	Split tensile Triaxial compression	0.3-0.5
Granular base	Triaxial compression	First stress invariant, density, moisture content	1×10^4 - 6×10^4	Empirical charts	0.3
Cohesionless subgrade	Triaxial compression	First stress invariant, density, moisture content	1×10^4 - 3×10^4	Plate bearing	0.3-0.5
Cohesive subgrade	Triaxial compression	Deviator stress, density, moisture content	4×10^3 - 3×10^4	Plate bearing	0.4

entitled "Influencing Parameters" may need additional explanation since the parameters listed are by no means all of the parameters that influence the modulus of a material. First, the table assumes that the material tested is used in the pavement and meets the material specifications. For example, for the bituminous concrete, it is assumed that the aggregate, bitumen, moisture, and density are the same as to be used and do meet the specifications. Thus, in setting up the laboratory tests many influencing parameters will be fixed but others, i.e., those parameters listed, will be variable, and particular care must be exercised in the control of these test parameters.

DEVELOPMENT OF PERFORMANCE CRITERIA

GENERAL

Historically, performance criteria for rigid pavements have been based on limiting the tensile stress in the PCC slab to levels such that failure occurs only after the pavement has sustained a number of load repetitions. Cracking or other forms of pavement distress are attributed to the repeated application of loads, and the process and criteria are referred to as fatigue and fatigue criteria. The fatigue process for rigid pavements is assumed to be similar to that for a PCC beam, and the criteria are presented in the same manner as the results of fatigue testing of concrete beams, i.e., a plot is made of the ratio of the applied stress to the strength of the PCC versus the number of stress repetitions applied.

Yimprasert and McCullough[30] prepared a plot comparing performance (will not be referred to as fatigue herein) criteria from several sources. These comparisons (Figure 18) are based on both laboratory tests of beams and the results from the American Association of State Highway Officials (AASHO) road tests. The performance criteria recommended by the Portland Cement Association[5] (Figure 19) are based on laboratory tests of beams. The performance criteria developed by the Corps of Engineers are based on results of full-scale test pavements subjected to controlled accelerated simulated aircraft traffic. The evolution of these criteria is described in References 31-35. One version of these criteria is shown in Figure 20. This curve[35] is a plot of the design factor (DF) versus coverages from the equation

$$DF = R/0.75\sigma_e \tag{5}$$

where

DF = design factor

R = PCC flexural strength

σ_e = tensile stress in the bottom of the PCC slab computed with the Westergaard edge-load idealization

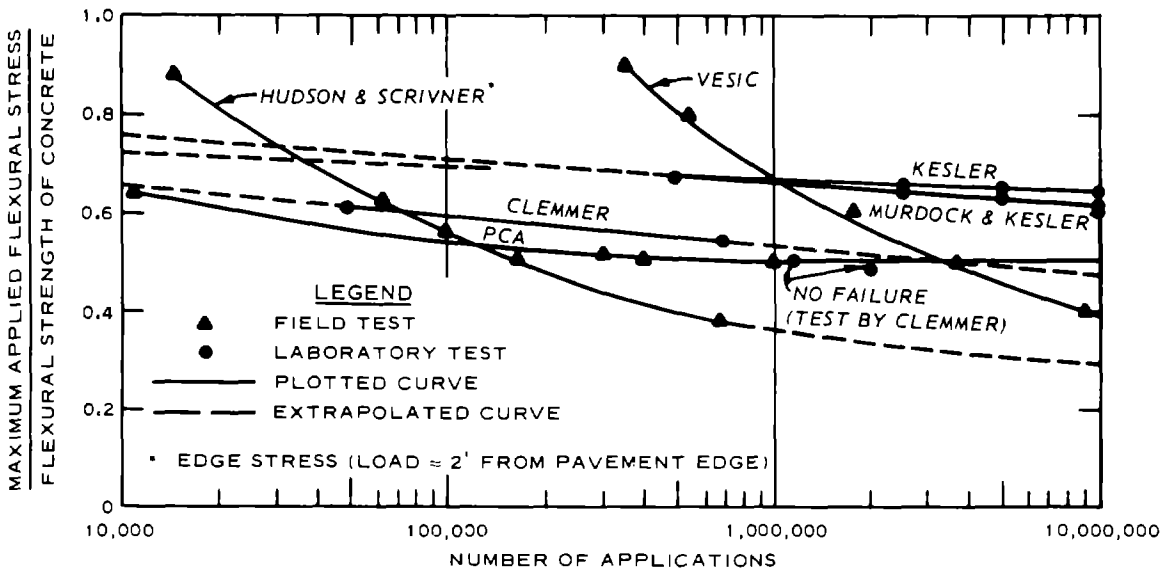

Figure 18. Comparison between field and laboratory tests on fatigue of concrete[30]

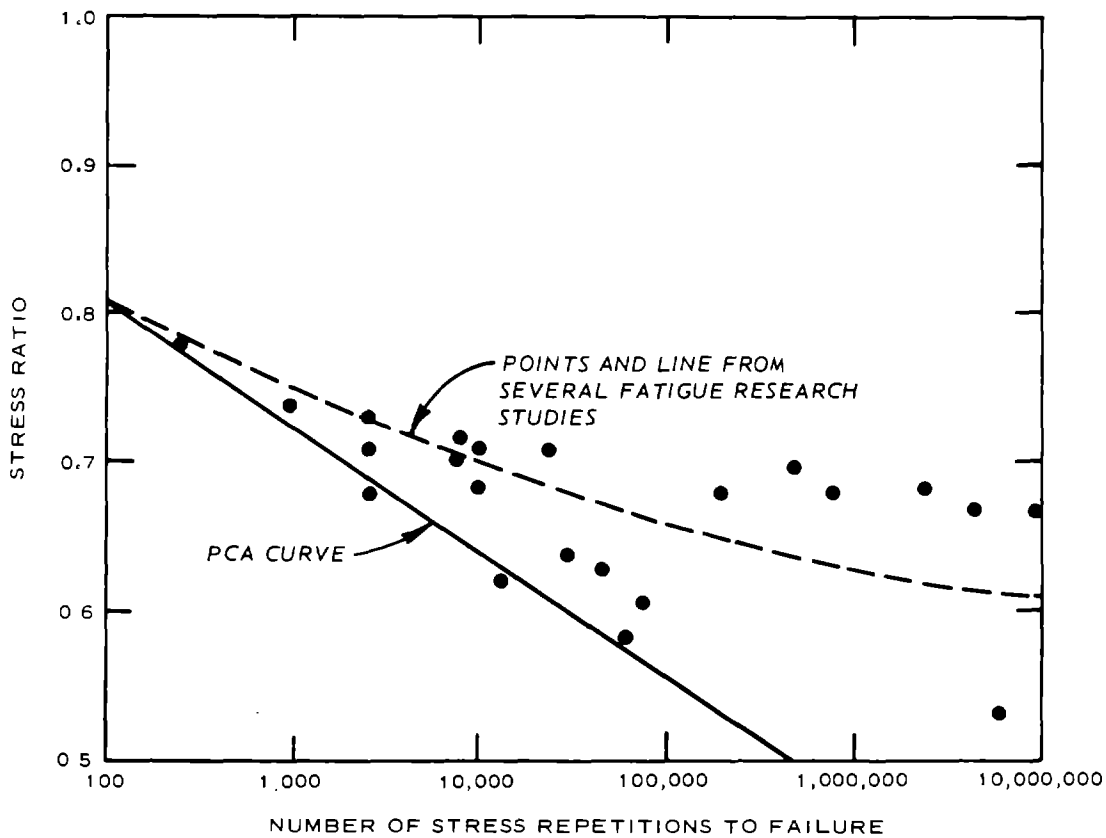

Figure 19. Fatigue curve for concrete subjected to flexural stresses[5]

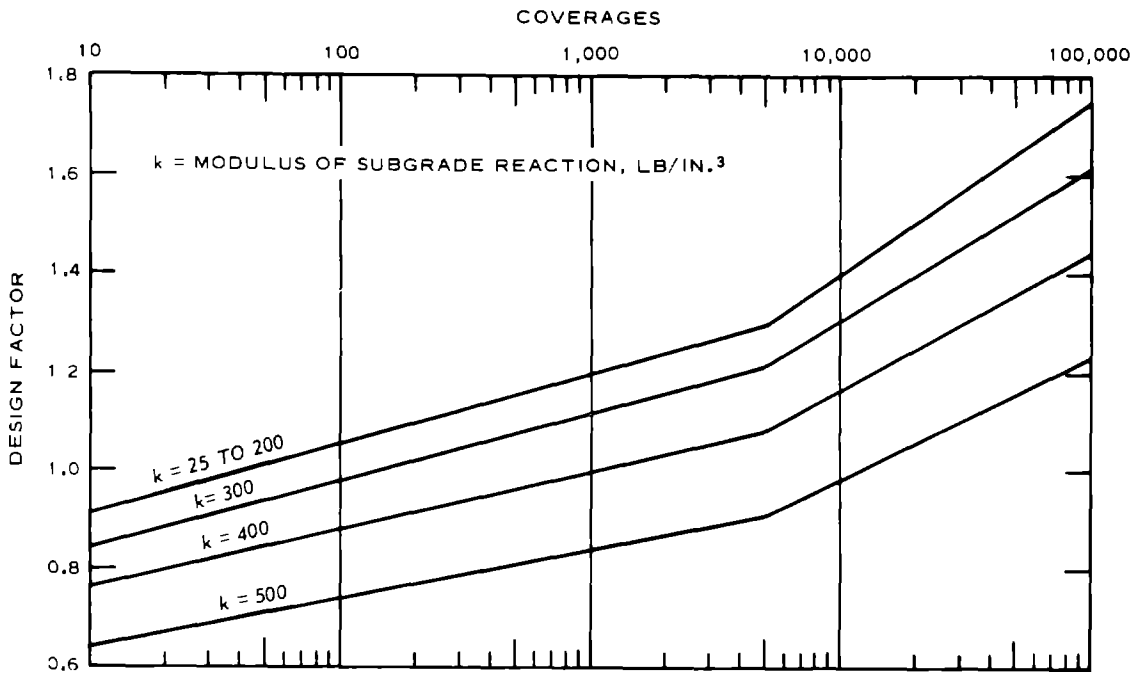

Figure 20. Current Corps of Engineers limiting stress performance criteria

The factor of 0.75 is used to account for the reduction in the edge stress resulting from the support provided by adjacent slabs. Coverages are a measure of the number of repetitions of the maximum stress occurring at a particular location in the pavement. The definition and method for converting actual aircraft operations to coverages is contained in Reference 36. The criteria developed by the Corps of Engineers are often presented in the form of a plot of percent of a standard thickness versus coverages.[37]

Upon detailed examination of the various fatigue criteria, it becomes apparent that while the basic concept and form are the same, many details are different. The data for the criteria are derived from two sources: controlled traffic tests of actual pavements, and laboratory flexural tests on concrete beams (fatigue in compression is described by compressive loading of cylinders or cubes). The data for the curves labeled Hudson and Scrivner and Vesic in Figure 18 were

developed from an analysis of data from the AASHO road tests. In Figure 20, the data for the curves resulted from an analysis of data from test pavements trafficked with simulated aircraft traffic. The data for the curves labeled Kesler, Murdock and Kesler, and Clemmer in Figure 18, and for the curves in Figure 19, were obtained from flexural loading tests of concrete beams.

There are differences in procedures for counting stress repetitions. For beam tests, the procedure is simply to count the number of applications of load, although complications may arise concerning the nature of the load. For the AASHO road tests, loads were applied with vehicles having single- and tandem-axle loads. Stress repetitions, as defined for the relationship developed by Vesic in Figure 18, were the number of load applications, with the tandem-axle load applying two-load applications with each pass. In Figure 20, the term coverages is synonymous with stress repetitions. Passes of a load are converted to coverages, which are a measure of the number of times the maximum stress will occur at a particular location in the pavement. To convert actual aircraft traffic to coverages (stress repetitions), the random lateral movement of the load across the pavement is considered. A factor, referred to as the pass-to-coverage ratio, converts the number of aircraft passes (may be referred to as operations or departures) to the number of coverages that occur at the location of maximum accumulation within the pavement.

The computation of stress is accomplished by different procedures. For laboratory beam tests, the maximum bending stress is easily calculable from simple equations of bending. For the relationship labeled Hudson and Scrivner (from AASHO road test) in Figure 18, the stresses are essentially maximum stresses measured along the pavement edge, with the loads located near the pavement edge. The word "essentially" is used, since the stresses used were not measured directly. Rather, a series of strains were measured, then converted to stresses, and an empirical relationship developed. From this relationship, the stress was computed to develop the performance relationship. The curve labeled Vesic in Figure 18 was also derived from an analysis of the AASHO road test data,

but stresses were computed using the finite element model with the Westergaard idealization of the pavement. The stresses are reported as the maximum that could have existed in the pavement. The stresses for the relationships in Figure 20 were computed as described previously.

There are differences in the definition of failure. For the relationships in Figures 18 and 19 based on flexural tests of beams, failure is defined as fracture of the beams. For the relationships based on data from the AASHO road tests, failure was defined as the pavement condition when the serviceability index equaled 2.5. The performance criteria in Figure 20 are based on what is referred to as the initial failure condition. This is defined as the condition at which 50 percent of the slabs in the traffic area have developed a crack that divides the slab into two or three pieces. The line designated $k = 25$ to 200 defines the condition at which the cracking will occur no matter what the modulus of soil reaction. However, performance of accelerated traffic tests and results of condition surveys indicated that pavements with high-strength foundations continued to satisfactorily carry loads after cracking, but that pavements with low-strength foundations developed multiple cracking and differential displacements soon after initial cracking. For this reason, additional relationships were added to relax the criterion for defining failure for pavements on high-strength foundations and thus in essence permit additional traffic after initial cracking.

For the performance criteria contained herein, the basic data will be developed from test pavements subjected to controlled accelerated simulated aircraft traffic. The term coverages[36] will serve as the measure of traffic or stress repetitions. The elastic layered idealization and in particular the BISAR computer code will compute the limiting stress. The maximum principal tensile stress occurring in the bottom of the PCC slab and the vertical PCC slab deflection were selected as the critical response parameters. The initial crack definition of failure selected stated that a pavement was considered failed when approximately 50 percent of the slabs in the traffic area had cracked.

No allowances were made for satisfactory performance after initial cracking for pavements on high-strength foundations.

FULL-SCALE ACCELERATED TEST

The pavements from which the performance criteria were developed are described in Appendix B. These pavements were constructed and tested by the Corps of Engineers from 1943 to 1973. The procedures followed in each of the test tracks were basically the same. The pavement sections were designed to answer specific questions or to solve certain problems; the pavements were constructed and the as-constructed properties measured; the type of traffic needed to answer the specific questions or to solve the specific problems was applied to the pavement; the volume of traffic and pavement condition were monitored until failure of the pavement was achieved; and the after-traffic properties of the pavement were measured.

Certain details of each test were different. The construction procedures, time of construction, geographic location, type subgrade, type joints, type load, distribution of loading, extent and type of testing, etc., varied. Although extrapolation to a general design procedure is considered justifiable, it should be recognized that the entire range of conditions that might be experienced by a pavement has not been covered. Another factor to be recognized is that although the volume of data may seem substantial, it is not sufficient to define a complete set of criteria. The relationships presented are simplifications of what are probably more extensive and complex families or systems of relationships. Because of the limited data available for each range of conditions, the relationships provided are agglomerations of a group of relationships. For example, the limiting stress criteria are defined by a single relationship. If more data had been available, it may have been possible to define a family of relationships for different loadings or possibly foundation stiffnesses.

The use of data from actual traffic tests permits a number of factors to be considered indirectly in the design. The performance is directly related to traffic, and the resulting criteria are for the

entire pavement rather than for one component or material, such as fatigue of the PCC as obtained from beam tests. Environmental effects are not considered in the computation of the response parameters for developing the criteria. Although the full-scale pavements experienced the effects of temperature and moisture changes and gradients, frictional restraint forces, etc., the range of conditions experienced was not all-inclusive.

The accelerated traffic tests were conducted over short periods of time, and the detrimental effects of exposure to the environment are not experienced. However, the beneficial effects of time are not considered either. PCC gains strength with time; the strength of cohesive subgrades does not reach the minimum strength until saturation occurs (which takes time), nor do the minimum strength conditions prevail all the time. However, the detrimental and beneficial effects of time may, to a certain extent, counterbalance each other.

The effects of joints are not considered directly although it is recognized that joints are a point of weakness in rigid pavements and the distress is usually initiated at joints. The effects of joints are handled in the following manner:

a. The test pavements all had joints and slab sizes that were similar to the standard types used in airfield pavements.

b. The traffic was applied in the most critical manner with respect to the joints.

c. The use of the criteria will result in adequate pavements provided that the standard joint types and similar slab sizes used ensure adequate vertical forces transmitted between slabs.

Each test pavement was limited in size; therefore, construction procedures and equipment were not of the size and complexity used for constructing large amounts of aircraft pavements. More manual operations were involved in the construction of the test pavements than would normally be involved in the construction of a complete runway or taxiway. This must have affected the variability and the quality of pavement. Many of the test items were only 2, 3, or 4 slabs in size. On full-scale paving jobs, the pavement at the start of a job or even at

the start of a day's operations is normally of a poorer quality than the pavement constructed after the "bugs" are worked out of the system. There are other factors, such as the difference in the consolidation of PCC with hand-held vibrators as compared with the consolidation with a slip-form paver. The question of the difference in the test pavements and actual airport pavements is unanswerable, and it must be assumed that the test pavements were representative of real pavements.

The size of the test pavements, some of which were only one slab in size, presents another problem. Considering the variability that naturally occurs, the number of additional slabs needed for a truly representative sample is not known. In assigning traffic at failure, average conditions were selected when possible.

The volume of traffic applied to the test pavements was small as compared with the volume of traffic that would be used to design pavements for today's major civil airports or the larger military facilities. The primary reason for this is the cost involved in applying large amounts of traffic. Another reason is that a number of the tests were conducted during the 1940's and 1950's when pavements were designed for much lower traffic volumes. The net result of the low applied traffic volumes is that the results from the tests will have to be extrapolated to higher traffic volumes in order to be used to design for current and projected traffic.

In summary, the use of data from full-scale accelerated traffic tests has certain disadvantages. However, when the complexity of the problem of designing pavements and alternative sources of data are considered, it apparently is the best alternative for producing criteria that are generally applicable and implementable.

ASSIGNMENT OF MATERIAL PROPERTIES TO TEST PAVEMENTS

For the test pavements described in Appendix A, the elastic properties of the various layers shown include moduli of elasticity, Poisson's ratio, and modulus of soil reaction values. For the Multiple-Wheel Heavy Gear Load,[35] Keyed Longitudinal Construction Joint,[38] and

Soil Stabilization Pavement Study[39] pavements, the values shown for all types of materials were obtained from laboratory and field tests of the various materials. However, for the remainder of the pavements (Lockbourne 1-3,[40-46] Sharonville Channelized,[34,47] and Sharonville Heavy Load[48]), the load deformation properties of the foundation material were described solely by the modulus of soil reaction obtained from static plate load tests. The modulus of elasticity for the subgrades for these pavements was obtained from the correlation between modulus of elasticity and static modulus of soil reaction (Figure 21). Values of Poisson's ratio were selected from values presented by Barker and Brabston.[10]

Data from which the correlation in Figure 21 was developed are shown in Table 2. These data were obtained by conducting static plate bearing tests[8] on in situ soils and repeated load triaxial tests (Appendix D) on "undisturbed" samples of cohesionless soils or samples prepared in the laboratory to approximate field conditions for cohesionless soils. Two points in Figure 21 were not used in establishing the correlation since the laboratory specimens did not represent the in situ conditions.

The general application of a relationship, such as shown in Figure 21, to a wide range of conditions is questionable on several accounts. However, for the purpose for which it was developed and used herein, it did appear to provide reasonable estimates for the modulus of elasticity of the subgrade in the earlier test tracks. The credibility of the correlation is enhanced by the comparisons presented in Figure 22. The three points represented three conditions in which both types of tests had been performed. Points 1 and 2 were for a high plasticity clay (CH) and point 3 was for a low plasticity clay (CL). The second curve in Figure 22 was taken from Reference 49. The proximity of the three additional data points and the additional curve to the established relationship indicates that the correlation should provide reasonable estimates of the resilient modulus of subgrade soils. It should be noted that the laboratory tests were conducted according to procedures outlined in Appendix D and that moduli for cohesive soils

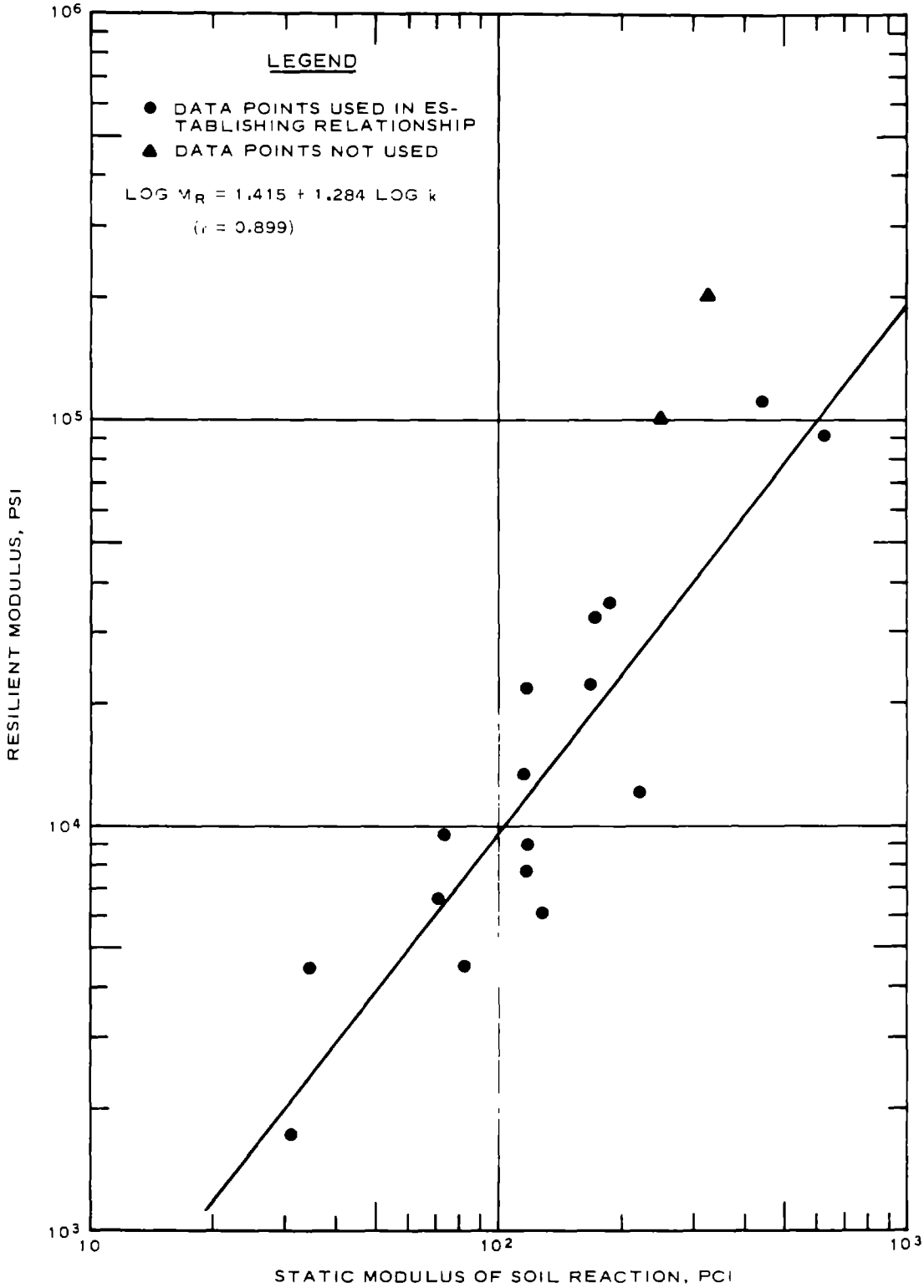

Figure 21. Correlation between resilient modulus of elasticity and static modulus of soil reaction

Table 2

Data for Correlation Between Static Modulus of Soil Reaction and Resilient Modulus

Soil Identification				Modulus of Soil Reaction	
No.	Class	LL	PI	pci	Resilient Modulus, psi
1	CL	36	12	35	4,500*
2	CH	50	32	170	22,000*
3	CH	58	33	32	1,700*
4	SM	N.P.		116	21,500**
5	CL	49	25	75	9,400*
6	ML	27	3	225	12,000**
7	SW-SM	N.P.		190	35,000**
8	SW-SM	N.P.		450	112,500**
9	SP	N.P.		630	91,000**
10	CL	43	21	175	32,500*
11	CH	73	48	85	4,500*
12	CH	73	48	118	13,200*
13	CH	73	48	120	7,625*
14	CH	73	48	120	9,000*
15	CH	73	48	130	6,213*
16	CH	73	48	72	6,500*
17	CH	53	34	325	200,000†
18	CH	64	42	250	102,500†

* Resilient modulus determined at deviator stress of 5 psi.

** Resilient modulus determined at first stress invariant of 10 psi.

† Points not used to establish correlation. Laboratory specimens were not representative of in situ material.

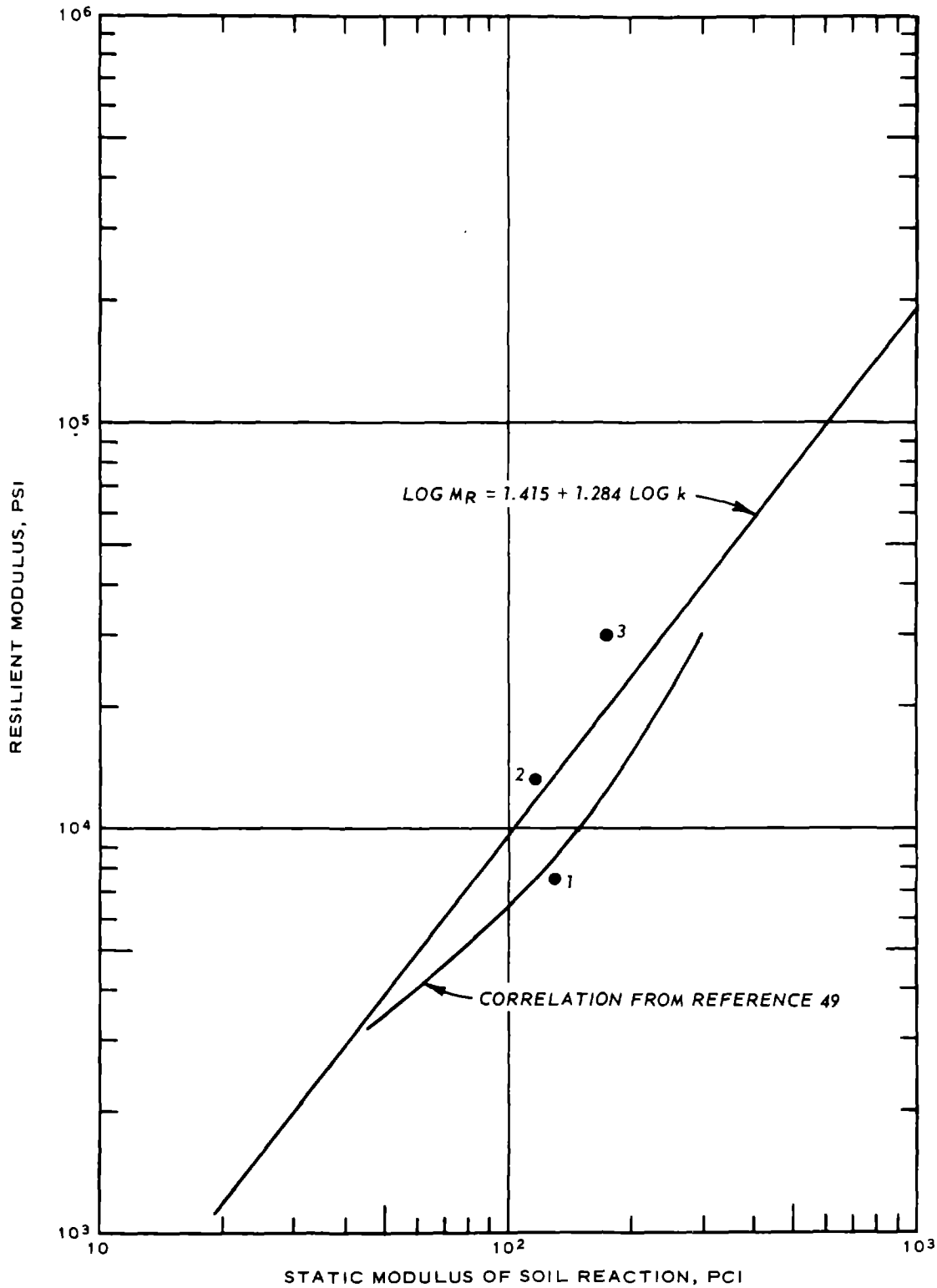

Figure 22. Comparison of correlation with measured values and correlation from Reference 49

were selected at a deviator stress of 5 psi and at a first stress invariant of 10 psi for cohesionless soils.

The moduli of elasticity for the granular base materials in the earlier test sections were selected using data from the previously described correlation (when plate bearing tests were conducted on top of the base courses), combined with information from other sources. This additional information was obtained from procedures for determining moduli of granular materials described by Barker and Brabston (Appendix G[10]) and Chou,[50] and from test results for similar materials presented by Chisolm and Townsend[51] and Hicks.[52] Values for Poisson's ratio were assigned from values recommended by Barker and Brabston.[10] The earlier test pavements did not contain bound base courses.

LIMITING STRESS CRITERIA

The limiting stress (fatigue) criteria are presented as a relationship between design factor and coverages. The basic definitions of coverages and design factor are the same as used previously for Corps of Engineers criteria. The one difference in detail is that stress as computed with elastic layered theory is substituted for the stress computed with the Westergaard idealization, and the factor to account for stress reduction due to shear between slabs is omitted. Design factor is now defined as

$$DF' = R/\sigma \tag{6}$$

where

DF' = design factor for stress computed with elastic layered theory

R = PCC flexural strength

σ = maximum principal tensile stress at bottom of PCC slab computed with elastic layered model

The limiting stress criteria are illustrated in Figure 23. The data points in this figure are listed in Table 3. The plus signs after coverages in Table 3 and the arrows on the data points in Figure 23

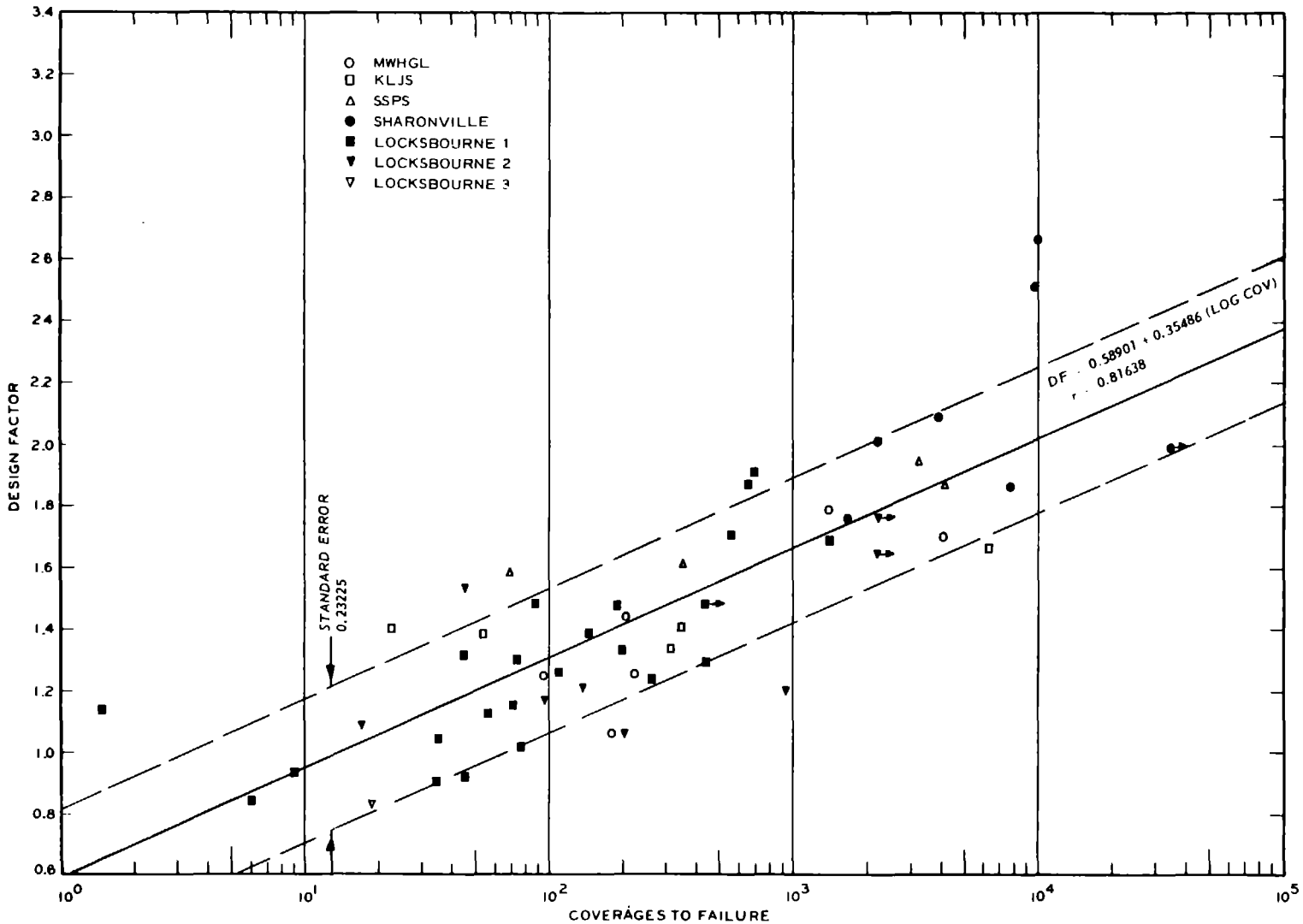

Figure 23. Performance criteria based on design factors listed in Table 3

Table 3

Data for Development of Performance Criteria

Site Location	Item Identification	Type Load	Traffic, Coverages	Design Factor	Vertical Deflection, in.
Lockbourne No. 1	A	Dual-Wheel	390+*	1.84	0.0206
	A		45	1.31	0.0356
	B		187	1.47	0.0322
	B		35	1.03	0.0558
	C		200	1.33	0.0437
	C		44	0.92	0.0757
	D		450	1.30	0.0498
	D		33	0.89	0.0863
	E		430+	1.48	0.0409
	E		77	1.02	0.0712
	F		550+*	1.87	0.0446
	F		111	1.25	0.0770
	K		72	1.29	0.0634
	K		700	1.91	0.0393
	N		150	1.39	0.0653
	N		9	0.94	0.105
	O		573	1.70	0.0575
	O		72	1.14	0.0928
	P		262	1.24	0.0894
	P		6	0.84	0.144
	Q		1,390	1.68	0.0657
	Q		57	1.12	0.106
	U		88	1.51	0.04
	U		1.5	1.16	0.0625
	A (Reconstr.)	Dual-Wheel	658	1.87	0.0435
Lockbourne No. 2	E1	Single-Wheel	97	1.15	0.0641
	E2		942	1.19	0.0584
	E3		17	1.07	0.0808
	E4		203	1.07	0.0786
	E5		43	1.53	0.0479
	E6		2,204+	1.77	0.0407
	E7	Single-Wheel	2,204+*	2.44	0.0346
	M1	Dual-Tandem	134	1.21	0.104
	M2		2,204+	1.63	0.0807
	M3	Dual-Tandem	2,204+*	2.46	0.0562
Lockbourne No. 3	1	Dual-Wheel	18	0.82	0.0903
Sharonville Channelized	57	Dual-Wheel	34,650+*	2.35	0.04
	58		34,650+	1.98	0.044
	59		7,600	1.86	0.03
	60		1,674	1.80	0.018
	61		3,867	2.13	0.017
	62	Dual-Wheel	10,082	2.72	0.013
Sharonville Heavy Load	71	Dual-Tandem	9,680+*	3.28	0.0295
	72		9,680	2.53	0.0413
	73	Dual-Tandem	2,115	2.00	0.0507
MWHGL	1	C-5A	221	1.25	0.0579
	2	C-5A	4,230	1.69	0.0502
	2	Dual-Tandem	95	1.24	0.0537
	3	Dual-Tandem	205	1.43	0.0454
	3	C-5A	1,400	1.78	0.0433
	4	C-5A	180	1.05	0.0683
KLJS	1	C-5A	54	1.39	0.0589
	2		344	1.40	0.0530
	3		22	1.40	0.0579
	4	C-5A	6,336	1.76	0.0530
	4	Dual-Tandem	320	1.43	0.0591
SSPS	3	Dual-Tandem	3,215	2.00	0.0478
	3		350	1.62	0.058
	4		4,660	2.00	0.0478
	4	Dual-Tandem	70	1.61	0.0574

Note: The plus sign (+) indicates that traffic stopped prior to item failure.
* Data points not used in analysis.

indicate that the pavements did not fail at the stated traffic level. Of the 10 data points that had not failed, six were considered invalid since these six points had not approached failure. These six points are shown in Table 3 with an asterisk preceding the coverage level.

Several curve fitting techniques were tried on the remaining 54 data points. The one finally selected was a linear relationship fitted to the data by means of least squares with the design factor as the dependent variable and the logarithm* of the coverages as the independent variable. The resulting relationship in Figure 23 is described by the equation

$$DF' = 0.58901 + 0.35486 \log(Cov) \qquad (7)$$

where log(Cov) equals the logarithm of the traffic in terms of coverages. The standard error from the regression in terms of the design factor is 0.23225. The band width of two standard deviations about the regression line is shown as the shaded area in Figure 23. The correlation coefficient from the regression analysis was 0.81638.

Several factors considered in the selection of the particular relationship in Figure 23 should be briefly noted. The selection of the design factor as the dependent variable was made for the purpose of producing a relationship that would minimize (based on least squares criteria) the variability in the required thickness, stiffness, and strength of pavement to carry a certain load. The alternative would have been to select coverages as the dependent variable and design factor as the independent variable. This solution would then minimize the variability with respect to coverages. The choice between the two relationships is subjective in nature, and only through examination of both relationships was the choice made to use the relationship having the design factor as the dependent variable for the design criteria.

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* Unless otherwise denoted, all logarithms will be to the base ten.

A single relationship, rather than multiple relationships for several ranges of conditions, was selected because of insufficient data for a range of groupings and from visual inspection of the plot, which showed no well-defined groupings of data points. Groupings according to load and foundation stiffness were tried. A linear regression was selected because its simplicity was commensurate with the nature of the data in the sense that each data point is reflective of numerous factors of a complex problem. In addition, the absence of visual trends in the data did not suggest the applicability of a more complex relationship.

The absence of data points for traffic volumes greater than 10,000 coverages should be noted. Only one point is shown for coverage levels greater than 10,000, and this pavement did not reach failure at the indicated traffic level. Use of the relationship to design for traffic volumes greater than 10,000 coverages (which will frequently be the case for current traffic volumes) will require extrapolation of the linear relationship. There is nothing to suggest that this should not be the case.

VERTICAL DEFLECTION CRITERIA

The vertical deflection criteria are presented as relationships between vertical slab deflection and traffic in terms of coverages. The vertical slab deflection data in Table 3 were computed with the elastic layered theory using the BISAR program. For each aircraft gear, the deflection was computed under each tire and under the centroid of the gear. The maximum deflection at these points is shown in Table 3.

Several types of curve-fitting techniques were tried to determine the best fit curve through the 54 data points in Table 3 that were used to develop the design factor criteria. The one finally selected was a power curve fit to the data by means of least squares with the vertical deflection as the dependent variable and the number of the coverages as the independent variable. The resulting relationship in Figure 24 is described by the equation

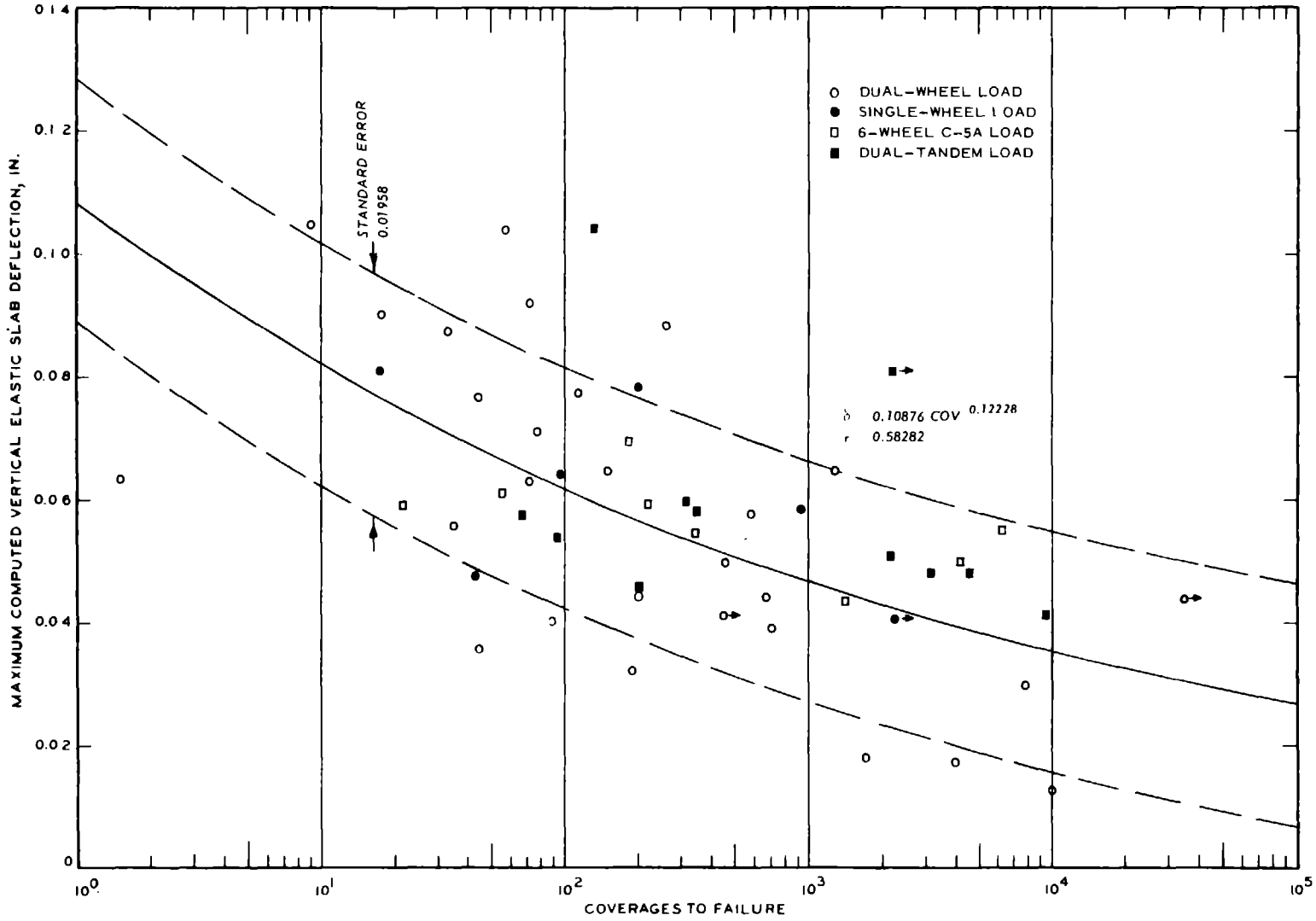

Figure 24. Performance criteria based on vertical deflections presented in Table 3

$$\delta = 0.10876(\text{Cov})^{-0.1228} \tag{8}$$

where δ represents the computed vertical elastic slab deflection. The standard error for Equation 8 is 0.01958 in. A band, two standard errors wide, is shown in Figure 24. The correlation coefficient of -0.58282 from the regression analysis signifies that deflection is a poor indicator of pavement performance.

To improve the relationship between computed deflection and pavement performance, an attempt was made to identify particular test sections that may have failed due to excessive deflection. In the plot of the test data (Figure 23), 24 test sections failed at coverage levels less than the coverage level predicted by Equation 7. If the assumption is made that the premature failure of these 24 sections may have been due to excessive deflection, a reason exists for examining these test sections alone. It still must be realized that in most of the test sections the overprediction of Equation 7 will be caused by inherent variability of the test data. Therefore, an analysis of the 24 points was conducted to see where these points were with respect to the best fit deflection curve (Figure 24). The failure coverage for 16 data points is less than the coverage predicted from the deflection-coverage relationship in Figure 24. Thus, no conclusions can be drawn from the failure of these 16 sections, but the remaining 8 test sections did indicate excessive deflection. From this analysis, it was postulated that the eight test sections failed due to excessive deflection and that deflection criteria could be obtained from these data.

A regression analysis was run on these eight points, and the curve selected as the best fit (Figure 25) is described by the equation

$$\delta = 0.098738\ (\log \text{Cov})^{-0.57910} \tag{9}$$

This curve was selected as the limiting deflection criterion. Equation 9 has a standard error of 0.00356 and a correlation coefficient of -0.98720.

As an additional analysis, the deflection data were divided into two groups: (a) 35 points in the group for single- and dual-wheel gears

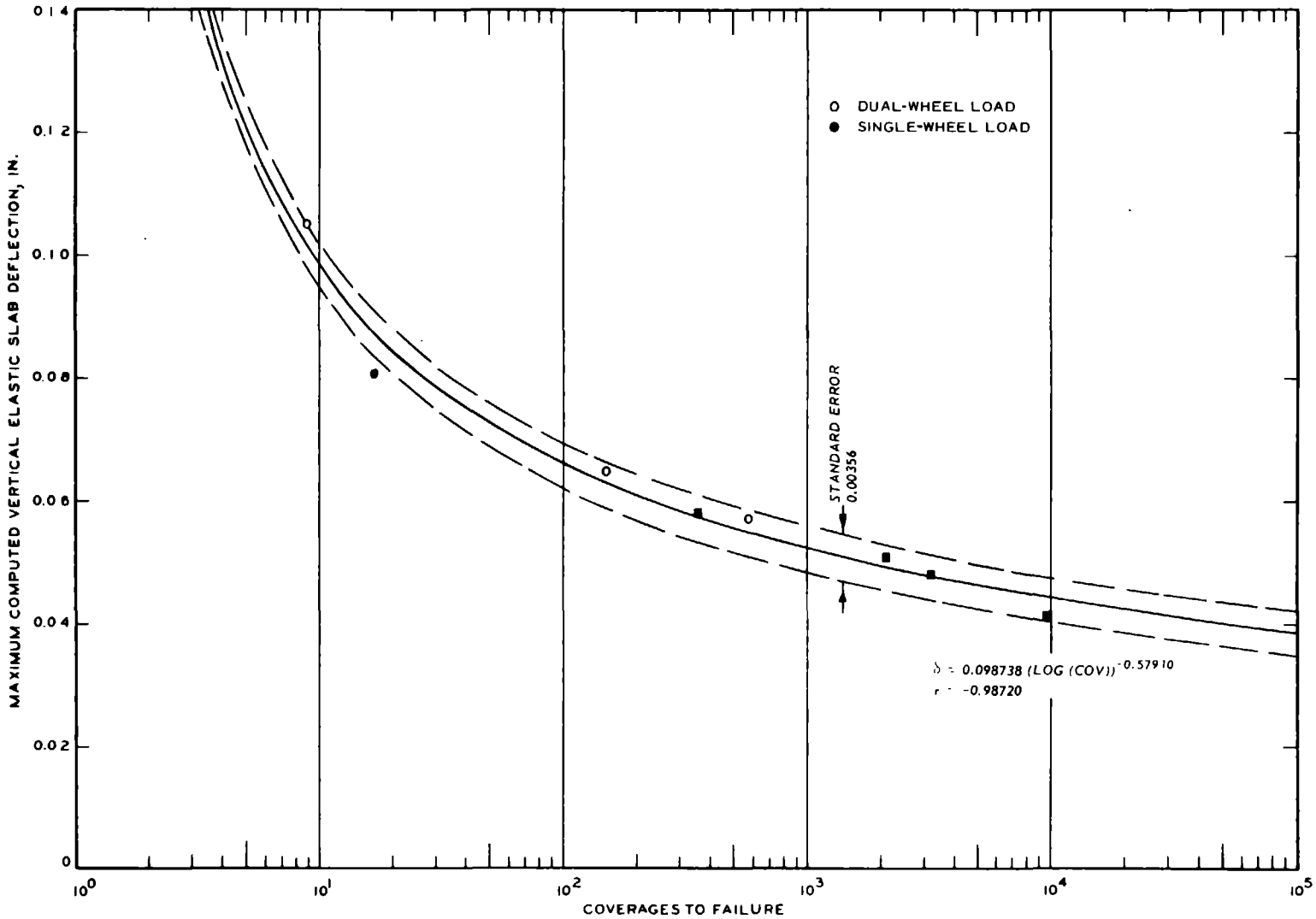

Figure 25. Performance criteria based on vertical deflections assumed to have failed because of excessive deflection

(Figure 26); and (b) 19 points in the group for dual-tandem and C-5A gears (Figure 27). The best fit curve for each group is also shown in the respective figure. The best fit equation for single- and dual-wheel gears is given by

$$\delta = 0.12657(\mathrm{Cov})^{-0.16314} \tag{10}$$

where

SE = 0.02085

r = -0.67485

The best fit equation for multiple-wheel gears is expressed as

$$\delta = 0.077005(\mathrm{Cov})^{-0.049191} \tag{11}$$

where

SE = 0.01355

r = -0.41200

The difference in the two curves in Figures 26 and 27 indicates that the vertical slab deflection varies more with the type of aircraft gear than with the performance of the pavement. Two different aircraft gears could produce about the same stress on a single pavement, but the deflection caused by the aircraft gears could vary greatly. As an illustration, consider the case of a Boeing 747 aircraft with a maximum ramp weight of 713 kips and a Boeing 727 aircraft with a maximum ramp weight of 173 kips. Both of these aircraft produce wheel loads of about 42 kips. Computations for an 18-in.-thick PCC slab ($E = 4 \times 10^6$ psi, $\nu = 0.2$) on a 12-in.-thick base ($E = 30{,}000$ psi, $\nu = 0.3$) over a subgrade with $E = 10{,}000$ psi and $\nu = 0.4$ yield maximum tensile stresses in the slab and maximum vertical slab deflections of 294 psi and 0.0435 in. for the Boeing 747 and 262 psi and 0.0238 in. for the Boeing 727. As noted, the stresses are almost the same, but the deflection produced by the Boeing 747 is almost twice that produced by the Boeing 727. The use of a single relationship for deflection criteria

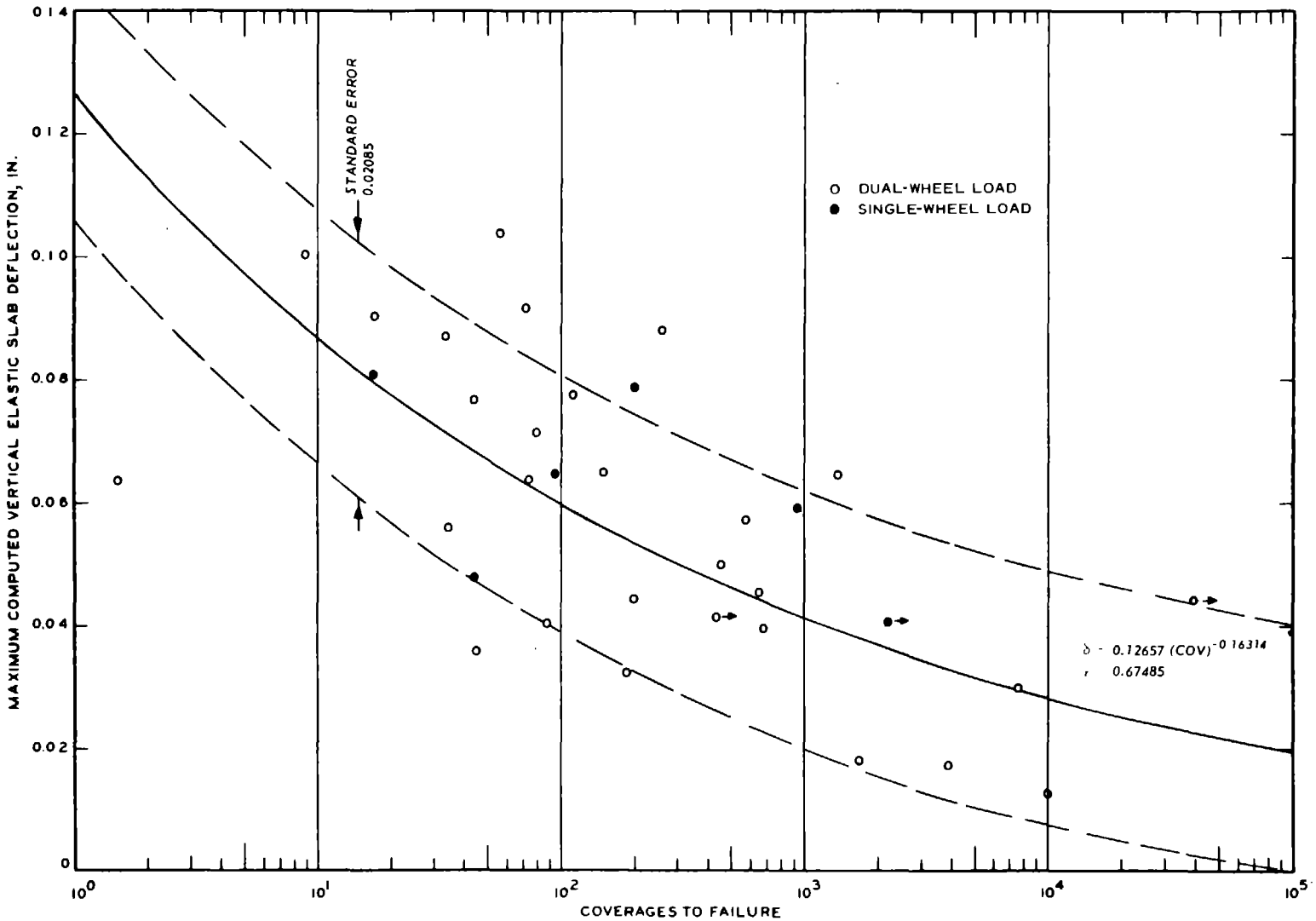

Figure 26. Performance criteria for single- and dual-wheel gears based on vertical deflections

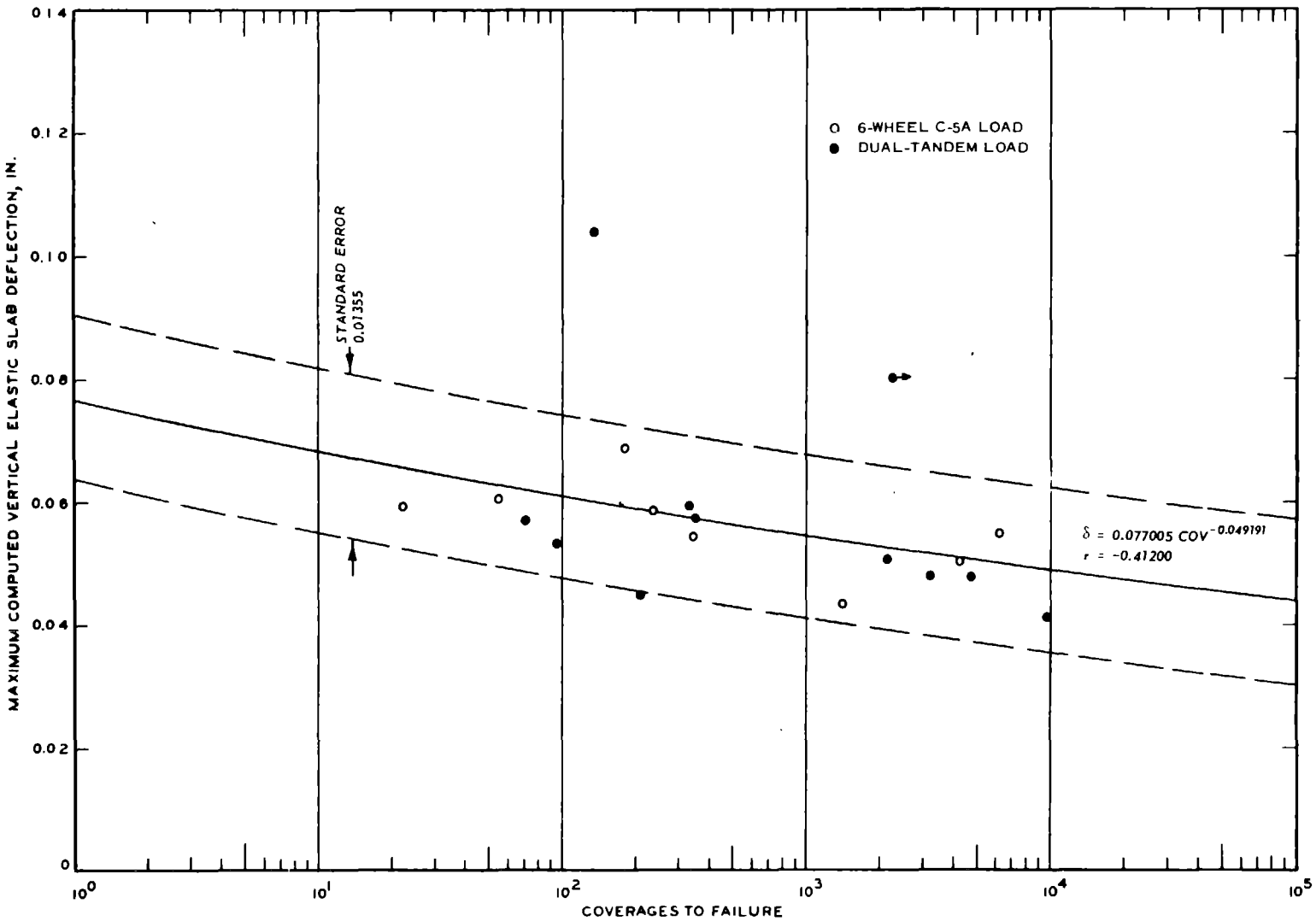

Figure 27. Performance criteria for six-wheel C-5A and dual-tandem gears based on vertical deflections

would be either too severe for the Boeing 747 or unconservative for the Boeing 727.

Additional data will be presented in a following section, which will strongly indicate that the deflection is also dependent on the subgrade strength. The test data were insufficient to define the relationship between the deflections and the subgrade strength; therefore, at this point, the deflection criteria are not considered to be a good design parameter.

It should be noted that the deflection criteria are independent (at least directly) of the PCC strength. Because the modulus of rupture is related to the modulus of elasticity, and the modulus of elasticity is used in computing the deflections, the strength criteria are indirectly related to the PCC strength but in a very complicated manner. An attempt was made to utilize the PCC strength in a more direct manner. The attempt involved developing from the test section data a relationship between computed deflection and the design factor (Figure 28). The relationship is best described by the equation

$$\delta = 0.082794DF^{-1.2209} \tag{12}$$

As with the previously developed deflection criteria, the usefulness of the criteria for design is considered limited. These criteria may also be used by selecting a limiting computed deflection, but for these criteria the limiting deflection is based on the design factor, which contains the PCC strength. The relationship may find greater use in pavement evaluation since it does provide a means of relating deflection to design factor criteria, which are the principal criteria for rigid pavement design. Again, a warning must be issued that the criteria are somewhat dependent on gear configuration and care must be exercised in the use of the criteria.

CRITERIA FOR OVERLAY AND REINFORCED SLAB DESIGN

The criteria developed thus far have been for nonreinforced PCC slabs placed on bound or granular base courses or directly on the

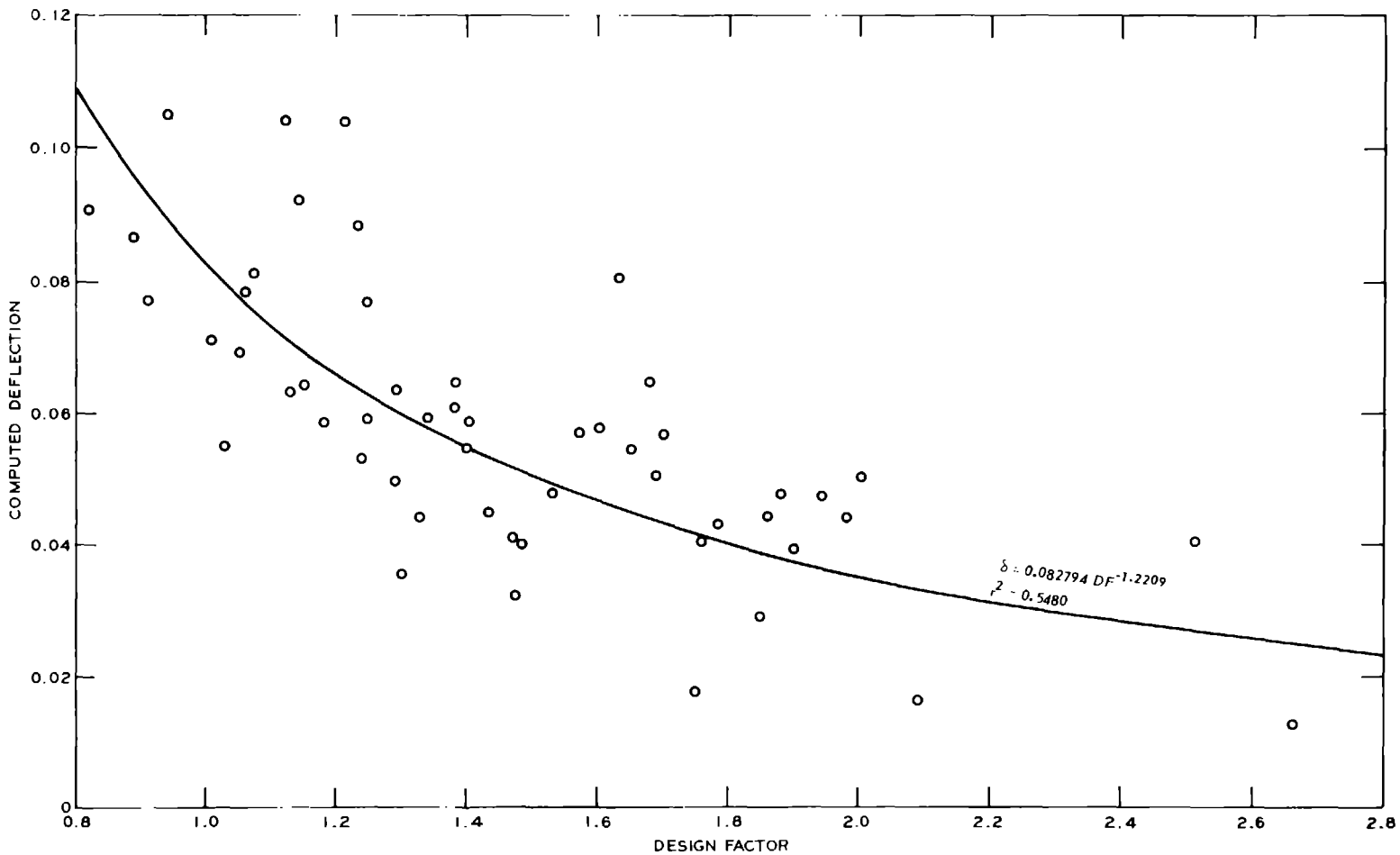

Figure 28. Relationship of computed deflection versus design factor as developed from test section data

subgrade. Criteria for unreinforced rigid overlays and reinforced pavements and overlays will be considered in this section.

The criteria recommended for rigid overlay design and for reinforced pavement and overlay design are the same as are currently employed for military and civil facilities.[1-3] The criteria were developed from data from full-scale accelerated test pavements, just as the criteria previously described. Many of the test tracks listed in Appendix A contained overlays and reinforced pavements as well as the unreinforced pavements described. No reanalysis was performed on data from the overlays and reinforced pavements because it was apparent that no significant improvements could be made.

As the criteria were developed and applied, it was noted that they would be applicable for the proposed system. The reason for this is that the basic parameter used in the development of both criteria is the thickness of unreinforced pavement. The overlay criteria were developed and then applied in terms of the thickness of overlay needed to provide the same performance of an equivalent reinforced pavement thickness.[33,53] The criteria for reinforced pavement and overlay design were developed and then applied in terms of the steel increasing the effective slab thickness as a percentage of the required unreinforced pavement thickness.[33,34,53,54] Provided the proposed criteria result in adequate thicknesses of unreinforced pavement, the use of these thicknesses with existing procedures should result in adequate unreinforced overlay, reinforced pavement, and reinforced overlay thicknesses.

OVERLAY CRITERIA

Three equations are provided for determining the thickness of rigid overlays of rigid pavements. Equations are provided for the three conditions of bond achieved between the overlay and the base pavement. When a deliberate and concentrated effort is made to achieve bond, overlay thickness is given by

$$h_o = h_d - h \tag{13}$$

where

h_o = thickness of overlay

h_d = thickness of PCC slab placed directly on foundation

h = thickness of existing PCC slabs

This equation is based on the assumption that the bond is sufficient for two slabs to act as one, thus the direct one-for-one relationship in thickness. Certainly, the equation should be applicable no matter what the basis for determining the thickness of the slabs directly on the foundation. Therefore, it is deemed adequate for use with the proposed criteria.

Bonded overlays are recommended when the existing pavement is in sound structural condition, i.e., no cracking. This permits the direct substitution of thickness. The required equivalent thicknesses of PCC slabs directly on the foundation are selected based on the flexural strength of the overlay. The use of Equation 13 is predicated on the assumption that the flexural strength of the overlay is approximately equal to the flexural strength of the base pavement. Should the flexural strength of the overlay be 100 psi or more greater than the flexural strength of the base pavement, the flexural strength of the base pavement should be used to compute the equivalent slab on foundation thickness.

When no deliberate effort is made to achieve bond and a condition of partial bond exists between the overlay and the existing pavement, overlay thickness is given by

$$h_o = \sqrt[1.4]{h_d^{1.4} - Ch^{1.4}} \tag{14}$$

where C equals a coefficient that depends on the structural condition of the existing pavement. When a deliberate effort is made to ensure that there is no bond between the overlay and the existing pavement, overlay thickness is given by

$$h_o = \sqrt{h_d^2 - Ch^2} \tag{15}$$

As based on a visual inspection of the existing pavements, the numerical value of C is established as follows:

C = 1.00 when the slabs are in good condition, with little or no structural cracking.

C = 0.75 when the slabs show initial cracking caused by loading, but little or no multiple cracking.

C = 0.50 when a large number of slabs show multiple cracking, but the majority of slabs are intact or contain only single cracks.

C = 0.35 when the majority of slabs show multiple cracking.

Equations 14 and 15 were developed empirically from the results of full-scale test tracks and adjusted where necessary based on actual performance data. The performance of various overlay items was compared with varying thicknesses of slabs on similar foundations under the same load and traffic. The overlay thickness is related to an equivalent thickness of slab on foundations. Therefore, if it is assumed that the equations establish a valid relationship between overlay and equivalent slab on foundation thickness, then appropriate overlay thicknesses can be determined provided the criteria for selecting the equivalent slab on foundation is adequate. Because of their wide usage, it was assumed that the equations provide adequate overlay design; thus, they are recommended for use with the proposed criteria for overlay design.

Although the presently used overlay design equations are recommended, a design procedure with elastic layered theory as the basic response model provides the framework within which the overlay design may be directly incorporated. This will, however, require development of capabilities in several areas, such as the ability to quantitatively define the load-deformation characteristics of cracked pavements (which will be reflective of support provided and the resulting performance of the overlay), and the ability to quantitatively define the degree of bond developed between the overlay and the existing pavement.

A system, such as that currently existing, might be employed for characterizing cracked pavements, i.e., when C = 1.00, the modulus of elasticity of the PCC layer is as measured for the intact material; when C = 0.5, the modulus of the PCC layer might be 0.4 times the measured

modulus of the intact material, etc. Further, the same type system might be used to describe the degree of bond achieved between the overlay and the base pavement. The effect of such a system would be the same as the different exponents in Equations 14 and 15. The BISAR code, as recommended for use with the criteria developed herein, does have the capability of considering variable bond between layers, but the problem is in defining the degree of bond.

The bonded case could be handled with the proposed system, provided it is only applied when the existing pavement is structurally sound. However, the only improvement over Equation 13 that could be effected would be to consider differences in the moduli of the PCC in the overlay and the existing pavement. The effect of different PCC moduli are not considered significant. When the moduli are the same, Equation 13 should give the same results as direct application of the design procedure since the assumption of full continuity between the two PCC layers, which is made in the equation and can be made in the response model (BISAR code).

The currently used procedure for design of rigid overlays of flexible pavements is to design the overlay thickness as a slab on foundation where the load deformation characteristics of the existing flexible pavement are defined by a modulus of soil reaction measured with a plate bearing test. It is recommended that the same procedure be employed with the proposed procedure, i.e., the overlay designed as a slab on foundation. Material characterization procedures discussed previously with test procedures outlined in Appendixes C-E should be followed.

When the asphalt concrete (or other type bituminous material) surface layers of the existing flexible pavement are badly fractured and, in the estimation of the designer, will not behave as a bound material, the designer may characterize with either of the following:

a. The material may be assigned the same properties as granular base or subbase material in the pavement.

b. The material may be assumed to be granular, and a modulus may be estimated using the procedure described by Barker and Brabston in Appendix G.[10]

If the bituminous surface course is less than 3 in. thick, it may be assigned the same modulus as the base or subbase course regardless of its structural condition.

Occasionally, the use of a plate bearing test to characterize the load deformation characteristics of flexible pavements provides questionable results. The characterization of the materials composing a flexible pavement with the prescribed procedures and the use of these properties in an elastic layered model should improve the design of rigid overlays of flexible pavements.

CRITERIA FOR REINFORCED PAVEMENTS AND OVERLAYS

The basic criterion for the design of reinforced pavements and overlays is shown in Figure 29. This relationship was developed empirically from data from full-scale accelerated traffic tests. The test tracks designated Lockbourne No. 1,[40-42] Lockbourne No. 2,[43,44] and Sharonville Channelized Traffic[34,47] contained reinforced test sections of varying thicknesses and percentages of reinforcement.

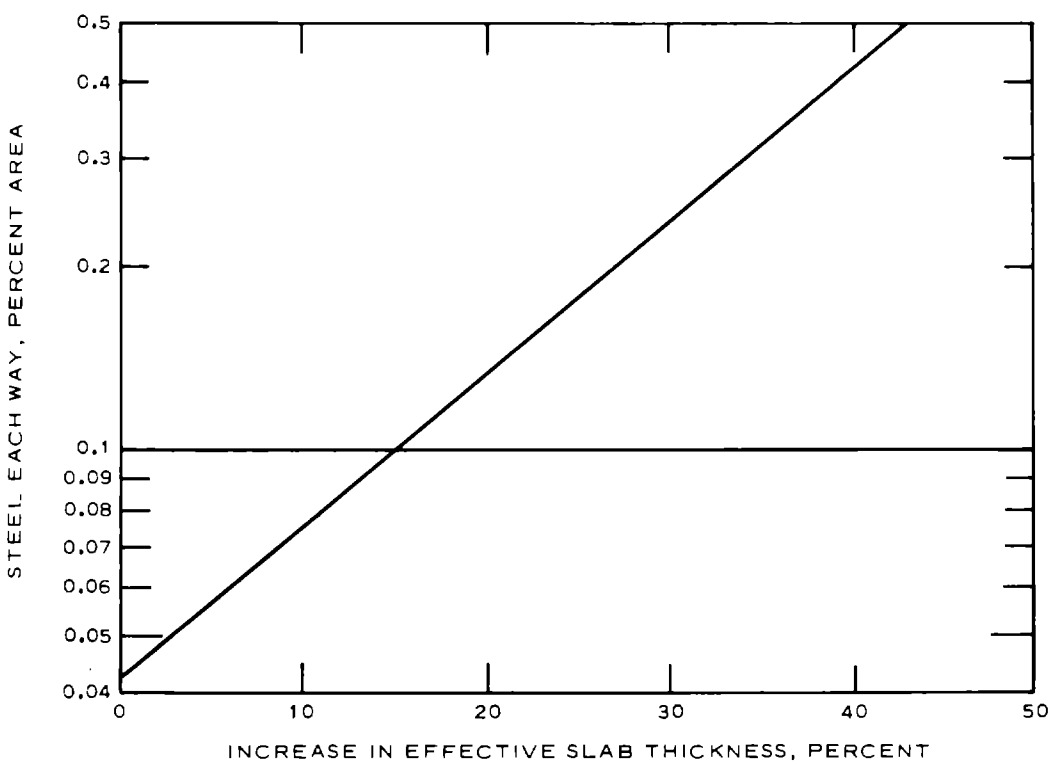

Figure 29. Effect of steel reinforcement on rigid pavements[53]

The relationship in Figure 29 was developed by comparing the performance of plain and reinforced pavements and relating the improvements in performance to the amount of steel. The basis for the comparison was the thickness of unreinforced pavement. Therefore, assuming that the proposed procedure will result in adequate thicknesses of unreinforced pavement, application of the criteria illustrated in Figure 29 should result in adequate thicknesses of reinforced pavements and overlays. Limitations on application of the criteria, as presently employed, should continue to be used.

ASSEMBLY OF PROCEDURE FOR DESIGN

GENERAL

Herein, procedures for application of the response model, material characterization tests, and performance criteria to the design of rigid airport pavements will be set forth. Much of the information provided will be general in nature since the problems faced by the engineer will vary widely for each design situation. Although a general procedure is provided by which rigid pavements can be designed, the emphasis has been placed on presenting the development and background in lieu of presenting a cookbook-type design procedure.

MATERIAL SAMPLING

Only general guidance can be provided for material sampling. The amount accomplished should depend on how much the designer feels he needs to adequately define the properties of the materials so that the resulting pavement will have the desired reliability. The variability of the subgrade, borrow and aggregate sources, number of material sources considered, the type of facility being designed, etc., should determine the extent of the material sampling program.

SUBGRADE AND BORROW

Guidance for soils investigations for currently used procedures are contained in References 1-3 and 24. Provisions for exploratory surveys and preliminary investigations, as contained in these references, are applicable to the procedures outlined herein. The depth and spacing of borings contained in these references for cut, fill, and borrow areas are applicable. Samples should be obtained from the borings or from pits for classification and development of compaction data.

The extent of undisturbed sampling will depend on the results from the preliminary soil survey. At least four samples of each distinct type of subgrade soil should be obtained for resilient modulus testing. Bag samples of each distinct type of borrow material proposed for use as

fill should be taken to provide for preparation of four samples of each distinct type material for resilient modulus testing.

BASE (SUBBASE) MATERIAL

Samples from all sources of aggregate that are to be considered in the design process should be obtained in quantities sufficient to perform classification tests, durability tests, and at least four specimens for resilient modulus testing. If a binder is to be used to produce bound base material, the samples from available sources should be obtained and used in the preparation of specimens for resilient modulus testing. Procedures as currently employed to ensure that binders (cement, lime, fly ash, or bituminous materials) meet required specifications should be employed.

PORTLAND CEMENT CONCRETE

Samples from available sources of aggregate, portland cement, and additives should be obtained and tested to ensure compliance with material specifications. In addition, samples of aggregate from available sources and representative samples of portland cement and additives should be obtained for mixture proportioning studies to determine ranges of flexural strength and modulus.

EXISTING PAVEMENTS
TO BE OVERLAID

The basic requirements for a preliminary survey of pavements to be overlaid is the same as for a new pavement, i.e., to define the materials within the pavement. If as-built plans and specifications are available, the preliminary investigation may be omitted. However, if this information is not available a series of borings will be necessary to determine the layered system comprising the existing pavement. Undisturbed samples, where possible, or disturbed samples for cohesionless materials should be obtained for all materials in quantities sufficient to provide four specimens of each distinct material type for resilient modulus testing.

For rigid pavements, samples should be obtained for the subgrade and base course layers. Condition coefficients should be established, as outlined in References 1-3, to define the structural condition of the PCC surfacing. Beams of the existing PCC should be obtained for flexural testing, or if this is not possible, cores should be obtained for split tensile testing. Procedures for obtaining and testing drilled cores and sawed beams are outlined in ASTM Standard Method C 42-68[55] (CRD-C 27-69[56]).

For flexible pavements, samples should be obtained for the subgrade, subbase course, and base course layers. For bituminous surfacings, layers 3 in. thick or less need not be sampled, and the layer may be included with the base layer when assigning material properties. Likewise, when the bituminous surface layers are cracked to the point where they would not function as a continuous layer of bound material, then the same material properties may be assigned as for the base layer. No specific guidance can be provided to determine the amount of cracking that will destroy the integrity of the bituminous surfacing. It will be a matter of the opinion of the designer.

MATERIAL CHARACTERIZATION

The standard tests for classifying materials, establishing compaction requirements, and determining if material specifications are met will be the same as those presently employed.[1-3,24] Procedures, different from those currently used, will be needed to characterize the load deformation properties of each material. These properties will be determined from laboratory tests rather than from field tests.

Variations will be obtained in material properties. For design, it is recommended that the average value of modulus of elasticity be used. The 80 percentile value is recommended for use when determining the design value of flexural strength. Selection of Poisson's ratio will not be so precise. All available data should be studied, including recommended typical values, and a representative value selected.

Procedures are available for estimating modulus values as well as values of Poisson's ratio. The reliability desired in the resulting

design should be considered before relying on a procedure that will result only in an approximation of the material properties. Thickness design is not overly sensitive to Poisson's ratio and approximations are acceptable, although it is recommended that tests be conducted and used in conjunction with typical values for selecting a representative value.

SUBGRADE

Test procedures for subgrade soils are outlined in Appendix E. Cohesionless soils will be insensitive to moisture content, but very sensitive to density. Samples should be prepared as close as possible to field densities. Moduli values should be computed at first stress invariants of 10 psi, and average values computed. Computations of Poisson's ratio should be made and compared with typical values for selecting representative values for design.

Cohesive soils are sensitive to moisture content and density and should be tested at the most critical conditions. Normally, this will be in a condition of complete saturation. However, for arid locations where experience indicates that saturated conditions are never attained, moisture contents less than saturation may be used. Undisturbed samples should be used where applicable and where possible. For fill material, specimens should be compacted to simulate field conditions (density and moisture) as close as possible and saturated from the as-compacted condition. Moduli values should be computed at a deviator stress of 5 psi, and the design value selected as the average value from test results.

When the potential for frost action exists and the pavement is to be designed based on zero or partial frost protection, the subgrade should be tested according to procedures outlined in Appendix B of Reference 10. Moduli values should be selected at a deviator stress of 5 psi, and the average value selected for design. When the subgrade is to be protected against frost penetration (even though climatic and soil conditions are conducive to frost action), procedures outlined in Appendix E should be used.

Computations of Poisson's ratio should be made and compared with the typical values to select a representative design value.

Procedures for characterization of subgrades beneath existing pavements for overlay design are basically the same as those described above, exceptions being that the materials should be tested at their in situ moisture and density conditions. Exceptions to this would be conditions where frost penetration may produce higher water contents during thaw periods or special conditions (sample taken near edge of paved area or in areas where the surface was not sealed) where the moisture conditions are not those normally prevailing in the subgrade.

BASE (SUBBASE) MATERIAL

Separate procedures are provided for granular and bound base materials. Granular materials should be tested in compression according to procedures outlined in Appendix D. The response of granular bases, while insensitive to moisture, is dependent on the density of the material. Therefore, specimens should be prepared as close as possible to field densities. Moduli values should be selected at first stress invariants of 10 psi, and average values selected for design. Computations of Poisson's ratio should be made and compared with typical values for selecting a representative design value.

Bound base materials should be tested in flexure according to procedures outlined in Appendix C. Separate procedures are provided for chemically stabilized and bituminous stabilized bases. Chemically stabilized materials are sensitive to curing time and conditions and should be moist-cured for 28 days prior to testing. Bituminous stabilized materials are sensitive to temperature, and the design modulus should be selected for average temperature conditions in the base. The average temperature may be selected from available temperature prediction models. The values selected for design should be the average of all values obtained. Poisson's ratio should be selected from the following recommended values:

Material	Poisson's Ratio
Bituminous Stabilized	0.5 for E < 500,000 psi 0.3 for E > 500,000 psi
Chemically Stabilized	0.2

PORTLAND CEMENT CONCRETE

Mixture proportioning studies should be conducted to establish a practical range of flexural strength that may be obtained with available materials. Flexural strength tests are conducted according to ASTM Standard Method of Test C 78-75[57] (CRD-C 16-66[56]). During the conduct of flexural tests, deflections should be measured, and moduli of elasticity computed according to procedures outlined in the Corps of Engineers procedure CRD-C 21-58.[56]

The design strengths specified should depend on the method used in the specifications. If mixture proportions are contained in the job specifications, the design strength should be the strength obtainable 80 percent of the time with the specified mixture proportions. If flexural strength is specified in job specifications, the design strength should be the strength reasonably obtainable 80 percent of the time with locally available materials that meet other material requirements. The 80 percentile strength should then be used for construction quality control.

Normally the modulus of elasticity of PCC will not vary over a very wide range for aggregates from a particular locality. For the mixtures meeting strength requirements, a representative value of modulus of elasticity should be selected. Should a range of over 1,000,000 psi exist between the moduli for proposed mixtures, limiting moduli from the ends of the range should be used in the design procedure. Should this result in differences in the required thickness, each thickness design would then have to be tied to particular mixtures. However, situations that require different thicknesses based on the modulus of elasticity of the concrete will, in all likelihood, be rare.

Strength of PCC in existing pavements may be obtained by testing beams sawed from the pavement according to ASTM Standard Method of Test

C 78-75[57] (CRD-C 16-66[56]) or from tensile strength obtained from split tensile tests according to ASTM Standard Method of Test C 496-71[58] (CRD-C 77-22[56]) on cores from the pavement. The split tensile strength should be converted to flexural strength with a correlation recommended by Hammitt[59] or other correlations as available.

LAYERED SYSTEM DESIGN

Once the properties of available materials have been determined the optimum (economic and structural considerations) layered system may be selected by a trial-and-error process. PCC slab thickness and base course type and thickness will be the primary parameters that may be varied. The results will be an array of acceptable designs, which are structurally acceptable but must be evaluated economically to determine the optimum design.

The array of acceptable layered systems will be bounded by practical limitations. The minimum thicknesses of bases and PCC slabs are based on structural requirements, construction constraints, thickness limitations based on soil and environmental conditions (swelling soils, frost action, etc.), and limitations of locally available materials. Awareness and adherence to these practical limitations will reduce the effort required by the designer.

The procedures for selecting layered thicknesses, as outlined herein, will be based solely on structural considerations. However, special requirements, as currently specified in References 1-3 and 24, should control when thicker or higher quality layers are indicated.

LIMITING STRESS CRITERIA

The limiting stress criteria (Figure 23) are used to select a PCC slab thickness for a given set of foundation conditions. Base course thickness is controlled by the stress criteria, but the control is indirect. Base course thickness will normally be held constant, and the thickness of the PCC slab varied until an acceptable structure is determined. The process can then be repeated with a different base thickness. The entire process may be repeated for all available materials. In areas where frost problems exist, the base course thickness

will probably be controlled by requirements for protection of the subgrade against frost penetration.

The use of the limiting stress criteria can best be demonstrated by a set of example problems. Three foundation conditions are illustrated in the examples. One case will be with the PCC slab directly on the subgrade, a second is with the PCC slab on a 12-in. base course layer with properties similar to a granular material, and a third case is with the PCC slab on an 8-in. base course layer with properties similar to a bound material. Two conditions of traffic are illustrated in each of the examples, one similar to that which might be encountered at a civil aviation facility and a second similar to that which might be encountered at a military installation.

Four procedures for handling traffic or loads are illustrated. For the examples for civil facilities, the procedures recommended in Reference 1 are followed. To determine a pavement thickness, all traffic is equated to equivalent DC-8-61 traffic with the relationship

$$\log R_1 = \left(\log R_2\right)\left(\frac{W_2}{W_1}\right)^{1/2} \tag{16}$$

where

R_1 = equivalent DC-8-61 departures

R_2 = adjusted departures of aircraft in question

W_1 = wheel load for equivalent DC-8-61

W_2 = wheel load for aircraft in question

A second thickness of the pavement is determined only for the wide-body jet aircraft in the traffic mixture. When Equation 16 is used, wide-body jet aircraft are substituted for equivalent DC-8-61 traffic on a one-to-one basis. The number of departures for aircraft with single- or dual-tire gears should be adjusted using the following conversion factors:

To Convert	To	Multiply R_2 by
Single wheel	Dual tandem	0.50
Dual wheel	Dual tandem	0.60

For the examples for military installations, the pavements are designed for each aircraft, and the required thickness is selected for the critical aircraft. For both civil and military designs, the cumulative damage concept is applied in a rather crude fashion, and the pavement designed for the entire mixture of traffic.

For the four methods of handling traffic, the proposed criteria will be an approximation. The test pavements were loaded with only one type load, and the loads were applied across the pavements in specific patterns. In many of the earlier tests, the loads were distributed uniformly across the pavements; in the later tests, the loads were applied in a distribution simulating a normal distribution. Interwoven into these criteria is also the concept of "coverages." The concept of coverages as applied herein is discussed in Reference 36. It is a difficult concept to comprehend, and impossible to extend or use with other criteria, such as fatigue data developed from flexural tests of concrete beams. However, conceptually it is analogous to the more general terminology of stress or load repetitions even though the two are not interchangeable. The traffic at failure (in terms of coverages) assigned each of the test pavements was for one type load and is implicitly tied to the manner in which the traffic was applied.

From the standpoint of applied load, the procedure of equating all traffic to equivalent DC-8 traffic and the use of this one load with the criteria is conceptually correct since the test pavements were loaded with only one type load. The assumption necessary is that Equation 16 realistically accounts for the differences in load configuration and distribution of load on the pavement. Certainly the equation provides only an approximation, but as will be seen in the examples, it is an acceptable approximation.

The procedure of designing for only the wide-body jet aircraft and comparing this design with the design for the equivalent DC-8 design is similar to the procedure employed for military design, i.e., the most critical design (thickest slab) is selected. Conceptually the use of this procedure with the proposed criteria is correct from the standpoint of load and load application. However, the assumption necessary is that the pavement life or thickness required is not affected by any loading

except the critical loading, i.e., the loading requiring the thickest pavement. This assumption may not be totally valid, but as will be seen in the examples, it also provides an acceptable approximation.

The use of any of the procedures described has the advantage of simplifying the process of treating a mixture of traffic. This simplification appears to be in order when considering the impreciseness of the performance relationship, the precision of material characterization procedures, and the accuracy of estimates of traffic (both numbers and magnitude of loads).

The use of cumulative damage concepts to account for the effects of mixed traffic is a more complex procedure than those previously described. Conceptually it is a more satisfying procedure since theoretically the effects of each load, the differences in the distribution of each load across the pavement, and their cumulative effects on pavement life (or thickness required) can be considered. However, several factors should be noted to caution the user when using this procedure.

As noted previously, the data for development of the limiting stress criteria (Figure 23) were from pavements where only one load was used, and this load was distributed across the pavement in a particular manner. The effects of different loads are not accounted for in the criteria, and one is forced to make the assumption that each load has a detrimental effect on the pavement and that Miner's hypothesis, in some form, can be used to accumulate the damage from each aircraft. The feeling that most people have is that each aircraft operation does have some detrimental effect on the pavement; however, no definitive data exist to show this (to the author's knowledge). Conversely, there is a certain amount of evidence available from laboratory tests to indicate that the fatigue strength or life of concrete is not adversely affected by loads less than some fraction of a failure load and that they may even have a beneficial effect.[60,61] Certainly a pavement is a more complex system than a beam or cylinder, and in addition to fatigue of the concrete, such things as foundation support are affected by load repetitions. However, because of the lack of definitive data to prove

or disprove its validity, the use of Miner's hypothesis must be considered only as a rough approximation.

When cumulative damage concepts are applied, one is faced with the task of choosing the complexity of the analysis to be performed. The limiting stress criteria (Figure 23) are based on the maximum stress existing in the pavement, i.e., design factor is defined as the ratio of flexural strength of concrete to the maximum induced stress. Stresses less than the maximum do exist in the pavement, and one must decide whether to include their effects in the analysis. Similarly, the location of the maximum stress will vary as the position of the gear varies. In addition, the peak of the load distribution for each aircraft will vary. This can best be illustrated by considering Figure 30. This figure illustrates, conceptually, two distributions of loads across a pavement and the distribution of stress within the pavement for loads located at the center of the distribution, i.e., the location where they are most likely to occur. The criteria, as developed, are based only on the maximum stress that occurs, and through the concept of coverages, the critical location within the pavement, i.e., the location (one tire width wide) where the maximum number of maximum stress repetitions will occur. When an accumulated damage approach is used, one must decide whether or not to try to account for the effects of stresses other than the maximum. This is represented by the unshaded area in Figure 30. The next decision that must be made is whether or not to consider the differences in the locations of the center line of the distributions of the various aircraft (aircrafts A and B in Figure 30). With the criteria, as developed, inclusion of either effect will be only an approximation.

In the examples, only the maximum stress is considered, and the location of the center line of the distributions of each aircraft is considered to be coincident, i.e., the pass-to-coverage ratios developed in Reference 36 were used to convert aircraft departures or operations to coverages. This is a rather crude application of cumulative damage concepts but is as complex as can be justified considering the approximations involved.

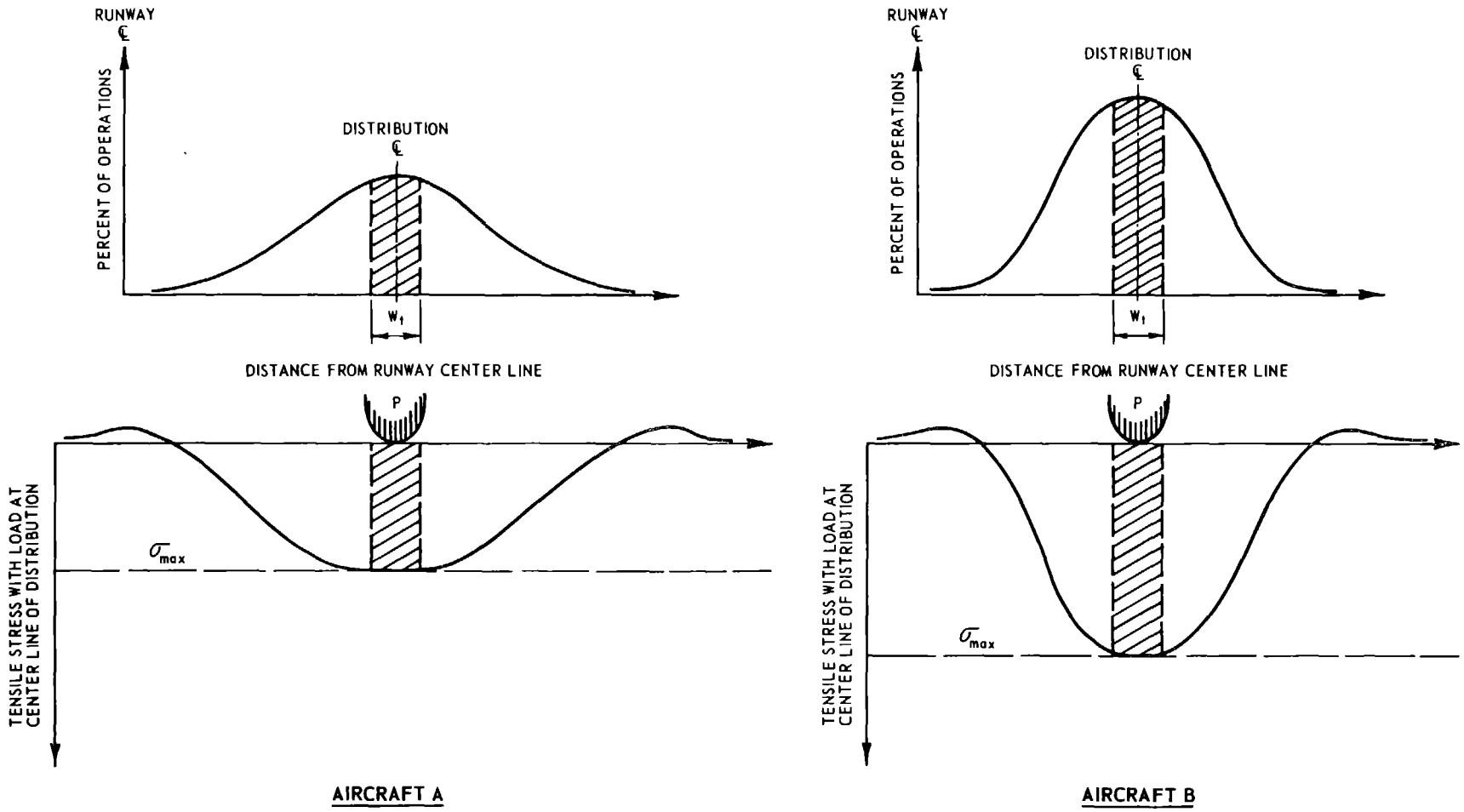

Figure 30. Distribution of aircraft load and stress within pavement

Any of the four procedures for handling traffic can be used. When the approximations involved and the effort required are considered, the simpler approaches have certain advantages. This is especially true, since a trial-and-error procedure will be required to select a PCC slab thickness for each foundation condition tried and to select the optimum combination of foundation condition and PCC slab thickness. The trial-and-error process will be illustrated later with example problems for three foundation conditions for the four methods for handling traffic.

LIMITING DEFLECTION CRITERIA

Although the deflection is not recommended for design, examples are worked including these criteria. The design examples provide some information as to the comparison of thickness, as determined by both the stress criteria and deflection criteria, and justify not recommending the criteria for design.

The limiting deflection criteria (Figures 24 and 25) are used to check designs obtained based on the limiting stress criteria. If the deflection of the pavement is less than the permissible deflection, then the design is acceptable. Otherwise, the slab thickness must be increased, or the base course thickness or quality modified.

The four methods of handling traffic are basically the same. The loading requiring the largest thickness is the critical loading. For design of civil facilities, when all traffic is related to equivalent DC-8-61 traffic, the limiting deflections are for the equivalent DC-8-61 traffic and are computed for the equivalent DC-8-61 load. When the design is for the wide-body jet traffic, the limiting deflections computed by layered elastic theory are for the traffic for the particular wide-body aircraft used in the design. When the accumulative damage traffic procedure is used, the limiting and computed deflections and the volume of traffic are determined for each individual aircraft and the largest required thickness is selected. The same procedure is followed for design of military facilities, i.e., limiting and computed deflections and traffic volume are computed for

each individual aircraft load and the largest thickness is selected as the design thickness.

Examples of all of these procedures will be illustrated in the following section. In the examples, it will be noted that deflection criteria control in all cases where the loads are applied through multiple-wheel gears, but the stress criteria control for the single-wheel gears.

EXAMPLE PROBLEMS

Design examples follow for both civil and military facilities. The traffic for the civil facility is tabulated in Table 4 and for the military facility in Table 5.

The subgrade soil for the examples is assumed to have a resilient modulus of 10,000 psi and a Poisson's ratio of 0.4. For one foundation condition, the PCC slab is placed directly on the subgrade. The second foundation condition is one in which a 12-in. base course having a resilient modulus of 30,000 psi and a Poisson's ratio of 0.3 is placed between the PCC slab and the subgrade. The properties of the base course are similar to what might be contained with a granular material. The third foundation condition is one in which an 8-in. base course having a resilient modulus of 300,000 psi and a Poisson's ratio of 0.2 is placed between the slab and the subgrade. The properties of this base course are similar to what might be obtained with a bound material. PCC slab thicknesses are selected for these three foundation conditions based on the limiting stress criteria and checked, for purposes of illustration, against the limiting deflection criteria. Assuming that the materials used represent realistic ranges of available materials, then additional designs for different foundation conditions would have to be made for economic comparisons. However, the examples serve to illustrate the procedures required to select acceptable pavements based on structural considerations. The PCC is assumed to have a modulus of 4×10^6 psi, a Poisson's ratio of 0.2, and a design flexural strength of 700 psi.

The step-by-step procedures followed for several of the loading and foundation conditions are outlined in detail. These will be typical

Table 4

Traffic for Example of Design for Civil Facility

Aircraft	Traffic Volume Annual Departures	Gross Aircraft Weight, kips	Contact Area, in.^{2}
DC-9	7000	115	165
B-727	7000	173	237
DC-8	2000	358	209
B-747	1000	713	245

Note: Designs assume 95 percent of gross aircraft weight on main gears.

Table 5

Traffic for Example of Design for Military Facility

Aircraft	Traffic Volume Annual Departures	Gross Aircraft Weight, kips	Contact Area, in.^{2}
B-52	1000	480	267
KC-135	2000	300	267
C-141	2000	320	208
F-111	4000	110	241

Notes: 1. Designs assume 90 percent of gross aircraft weight on main gears.
2. Load for B-52 includes 15 percent overload factor.

of the procedures followed for other conditions. The results for all the loading and foundation conditions will then be tabulated and compared.

Design for Civil Facility--Granular Base. For a 20-year design for a civil facility, a 12-in. granular base over the previously described subgrade and all traffic equated to equivalent DC-8-61 traffic, the first step is to convert the annual departures in Table 4 to total departures by multiplying the annual departures by 20. The next step is to apply Equation 16 to convert the total departures to equivalent DC-8-61 departures. This results in the following:

Aircraft	Total Departures	Equivalent DC-8-61 Departures
DC-9	140,000	8,800
B-727	140,000	69,400
DC-8	40,000	40,000
B-747	20,000	20,000
		138,200

The next step is to convert the total equivalent DC-8 departures to coverages by dividing by the pass-to-coverage ratio of 3.35.[36] This results in traffic as expressed in coverages of 41,300. Applying Equations 7 and 9, respectively, results in a required design factor of 2.23 (stress criteria) and a limiting deflection of 0.0407 in.

The next step is to run the BISAR computer program for several trial slab thicknesses. The design factor is computed, and the results may be tabulated in the following manner:

PCC Slab Thickness, in.	Design Factor	Slab Deflection, in.
14	1.52	0.0594
16	1.80	0.0516
18	2.10	0.0453
20	2.44	0.0400
22	2.79	0.0357

These results may be plotted as illustrated in Figure 31, and a required slab thickness of 18.8 in. is selected. A check of deflection criteria reveals that a PCC slab thickness of 19.7 in. would be required to meet the deflection criteria. This completes the structural design for this one load and foundation condition.

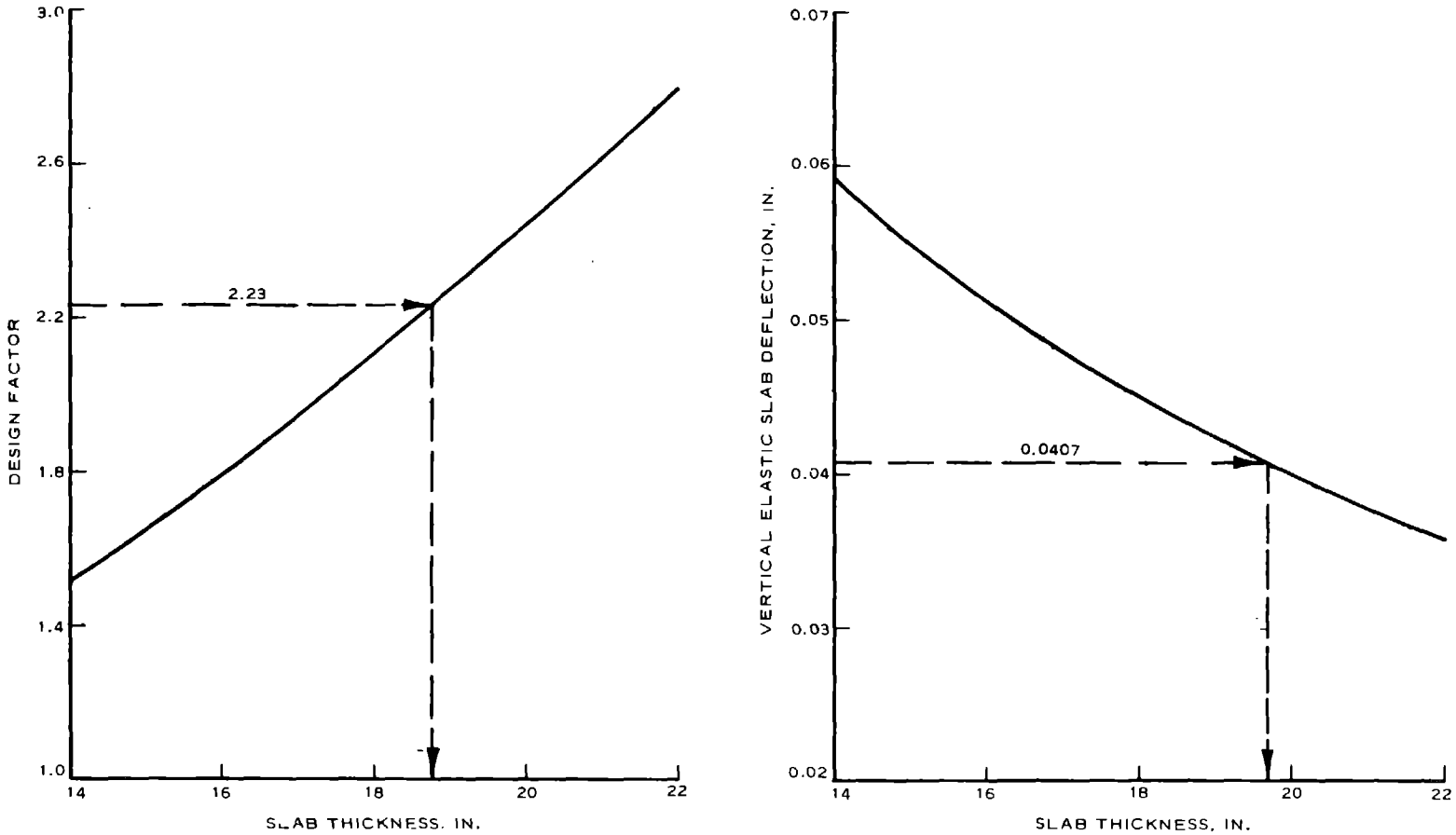

Figure 31. Selection of required slab thickness - civil facility - equivalent DC-8 traffic

The design for the wide-body jet aircraft in the traffic mix is similar. The 20,000 total departures are converted to 5,750 coverages with a pass-to-coverage ratio of 3.48.[36] Applying Equations 7 and 9, respectively, results in a required design factor of 1.92 and a limiting deflection of 0.0459 in. The BISAR computer code was run for several trial slab thicknesses, and design factors computed with the maximum stress. The results are tabulated as follows:

PCC Slab Thickness, in.	Design Factor	Slab Deflection, in.
12	1.45	0.0650
14	1.73	0.0565
16	2.04	0.0494
18	2.38	0.0435
20	2.75	0.0387

These results may be plotted as illustrated in Figure 32, and a required slab thickness of 15.2 in. is selected. A check of deflection criteria reveals that a 17.1-in.-thick PCC slab is required for satisfying the deflection criteria. Thus, the equivalent DC-8-61 traffic is critical, and the design would be based on the slab thickness requirement of 18.8 in. for the DC-8-61.

Design for Military Facility--Bound Base. This example illustrates the procedure for selecting PCC slab thickness for the traffic tabulated in Table 5 and a foundation composed of the 8-in.-thick bound base course ($E = 300{,}000$ psi and $\nu = 0.2$) over the previously described subgrade ($E = 10{,}000$ psi and $\nu = 0.4$). For a 20-year design for a military facility, the first step is to convert the annual aircraft passes in Table 5 to total passes by multiplying by 20. The next step is to convert the total passes for each aircraft into coverages by the appropriate pass-to-coverage ratios from Reference 36. Equation 7 is then applied to determine the design factor, and Equation 9 is applied to determine limiting deflection. The results of these steps on the traffic in Table 5 are tabulated as follows:

Aircraft	Total Passes	Total Coverages	Design Factor	Design Deflection, in.
B-52	20,000	13,100	2.05	0.0435
KC-135	40,000	12,800	2.05	0.0436
C-141	40,000	11,700	2.03	0.0438
F-111	80,000	14,200	2.06	0.0433

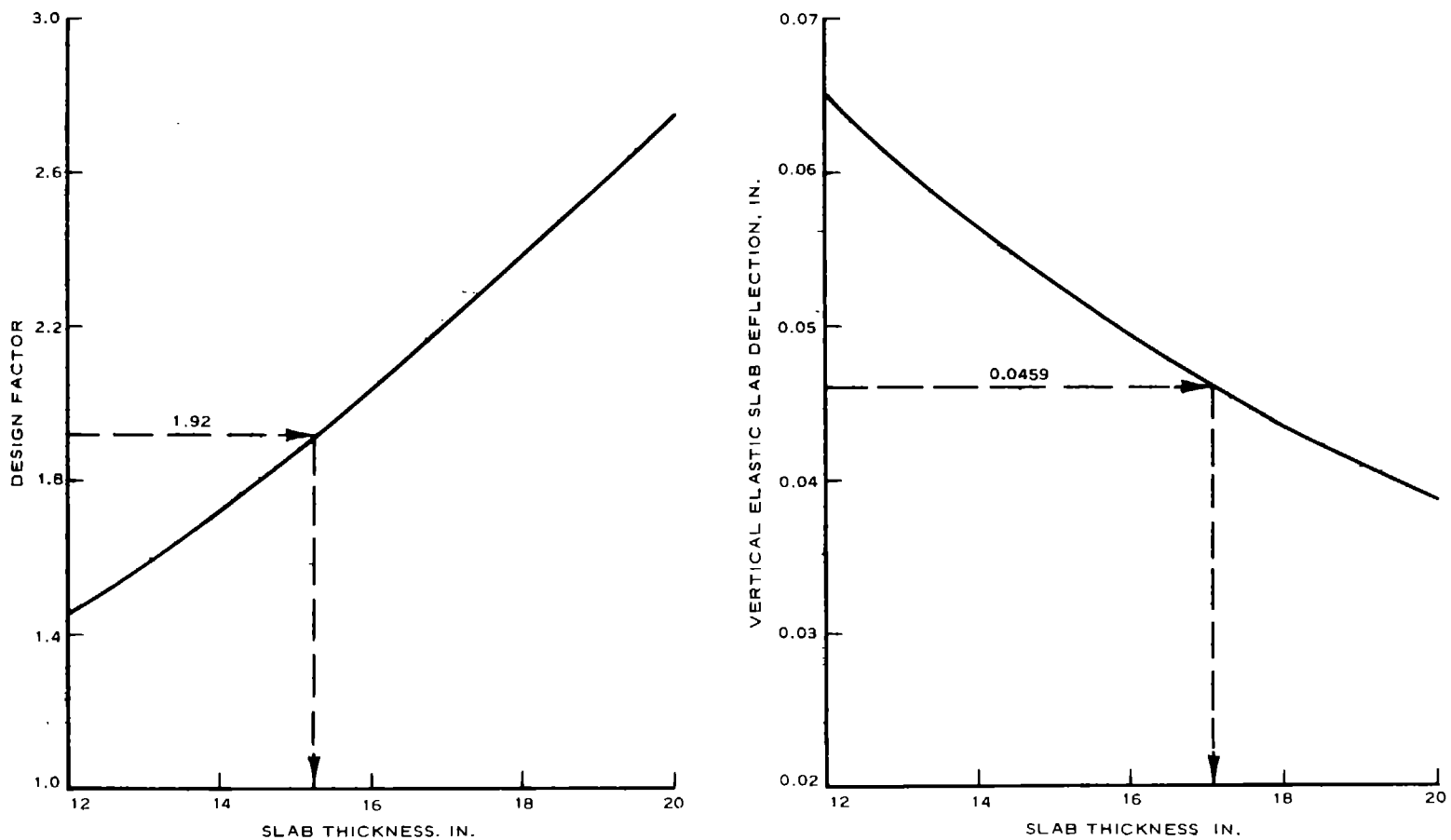

Figure 32. Selection of required slab thickness - civil facility - wide-body jet traffic

The next step is to run the BISAR computer code for several trial slab thicknesses to generate for each aircraft an array of data similar to the following for the B-52 aircraft:

PCC Slab Thickness, in.	Design Factor	Slab Deflection, in.
20	1.56	0.0576
22	1.79	0.0520
24	2.02	0.0472
26	2.27	0.0431

These results are plotted for each aircraft (B-52 aircraft in Figure 33), and a slab thickness is selected based on the limiting stress criteria. Again, for illustrative purposes the thickness is checked against the deflection criteria (Figure 33). The results for all four aircraft are tabulated as follows:

Aircraft	Required Slab Thickness (Stress Criteria), in.	Required Slab Thickness (Deflection Criteria), in.
B-52	24.3	25.8
KC-135	12.6	13.8
C-141	14.7	15.5
F-111	13.3	<10

The design thickness of 24.3 for the PCC slab would then be selected based on the B-52 traffic and the stress criteria.

Design for Civil Facility--Mixed Traffic--Bound Base. This example will illustrate how the criteria can be used with cumulative damage concepts to design the PCC slab thickness to account for the effects of a mixture of traffic. The foundation is composed of the 8-in.-thick bound base layer over the subgrade, and the PCC has the properties previously enumerated.

The first step is to convert the traffic in Table 4 to total departures for a 20-year life and then to convert the total coverages by application of appropriate pass-to-coverage ratios from Reference 36. The next step is to apply Equations 8 and 9 as appropriate to determine the limiting deflections for each aircraft at the applied traffic level. The total coverages and limiting deflection for each aircraft in the mix are tabulated as follows:

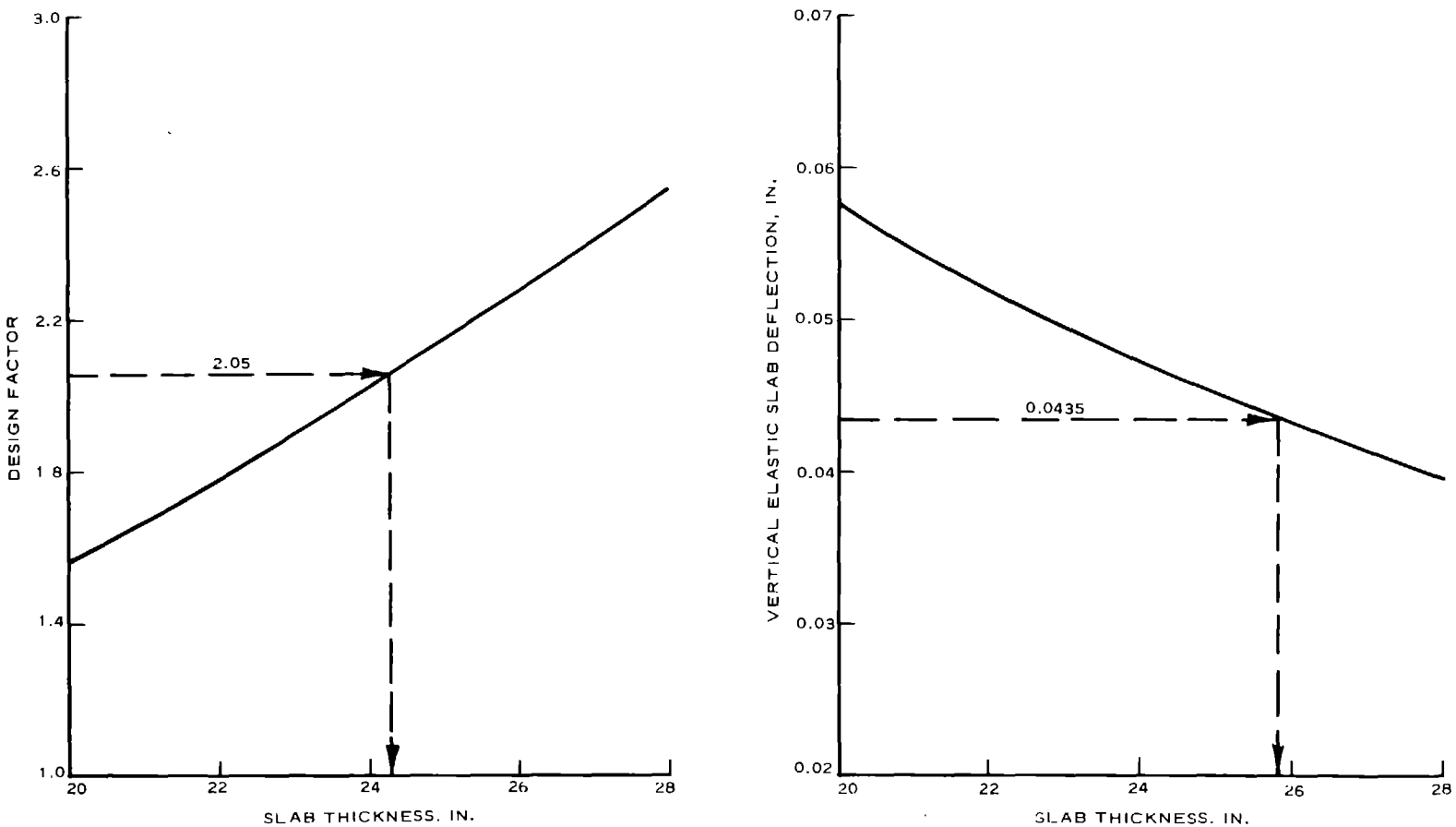

Figure 33. Selection of required slab thickness - military facility - B-52 aircraft

Aircraft	Applied Coverages	Limiting Deflection, in.
DC-9	38,000	0.0409
B-727	42,900	0.0406
DC-8	11,900	0.0438
B-747	5,750	0.0459

The BISAR computer code is run for a number of trial slab thicknesses for each aircraft. Design factors are then computed, and maximum deflections are selected from the results of the computer runs. With the computed design factor, the allowable coverage level for each aircraft for the particular thickness in question may be computed. Finally, the ratio of the applied to allowable coverage level for each aircraft, plus the sum, is computed. The results of this procedure are contained in Table 6. Figure 34 presents a plot of the summation of the ratios of applied to allowable coverages versus thickness. Application of Miner's hypothesis, i.e., failure will occur when the ratio of applied to allowable coverages reaches one, yields a required thickness of 16.8 in. A comparison of limiting deflections for each aircraft listed above with the computed deflections in Table 6 reveals that the computed deflection is less than the limiting deflection for a thickness of 17 in. for all aircraft except the DC-8. For the DC-8, the limiting value is 0.0438 as compared with the computed value of 0.0461. For the 18-in.-thick slab, the computed deflection for the DC-8 is 0.0433. By plotting the deflection versus slab thickness, it is determined that a slab thickness of 17.8 in. would be required to satisfy the deflection criteria. Again, the results indicate that for the twin-tandem gear the deflection criteria would dictate the design and help to confirm the conclusion that design on the basis of deflection would be overly conservative.

By comparing the ratios of applied to allowable coverages in Table 6, it can be noted that the DC-8 aircraft dominates the design. Although the B-747 has about the same gear load, the flotation of the gear is more efficient; therefore, the damaging effect of the B-747 traffic is negligible compared with the effect of the DC-8 traffic.

Summary of Design Examples. The array of designs obtained for the civil facility traffic is shown in Table 7 and for the military facility traffic in Table 8. Presented also in the tables is the slab

Table 6

Mixed Traffic Analysis--Civil Facility--Bound Base

Aircraft	Maximum Deflection, in.	Design Factor	Allowable Traffic, Coverages	Applied Coverages/ Allowable Coverages
		Slab Thickness = 16 in.		
DC-9	0.0178	3.26	3.36×10^7	0.00
B-727	0.0261	2.39	119,000	0.36
DC-8	0.0491	1.96	7,300	1.63
B-747	0.0471	2.22	39,500	0.15
				2.14
		Slab Thickness = 17 in.		
DC-9	0.0165	3.57	2.51×10^8	0.00
B-727	0.0243	2.62	529,000	0.08
DC-8	0.0461	2.11	19,300	0.62
B-747	0.0441	2.38	111,000	0.05
				0.75
		Slab Thickness = 18 in.		
DC-9	0.0154	3.91	--	0.00
B-727	0.0228	2.86	2.51×10^6	0.02
DC-8	0.0433	2.27	54,600	0.22
B-747	0.0416	2.56	358,000	0.02
				0.26
		Slab Thickness = 19 in.		
DC-9	0.0145	4.27	--	0.000
B-727	0.0213	3.11	1.27×10^7	0.003
DC-8	0.0407	2.44	164,000	0.073
B-747	0.0394	2.75	1,230,000	0.005
				0.081

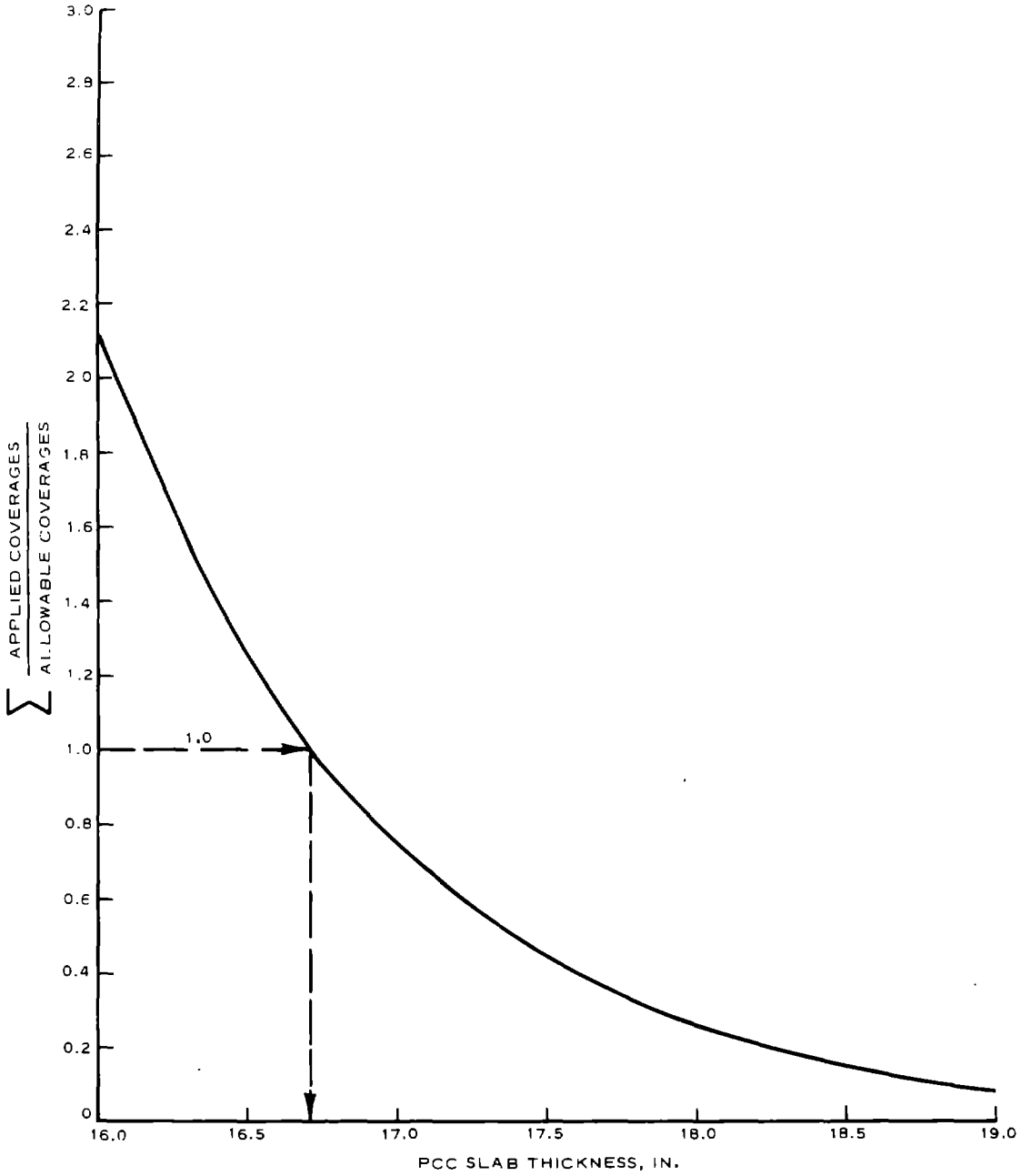

Figure 34. Summation of the ratios of applied to allowable coverages versus slab thickness

Table 7

Slab Thickness for Civil Facility

Pavement Type	Required Slab Thickness, in.		
	Stress Criteria	Deflection Criteria	Present Design Criteria
Equivalent DC-8 Design			
No base	19.2	20.2	18.1
12-in. granular base	18.8	19.7	15.7
8-in. bound base	17.8	19.1	--
Wide-Body (B-747) Design			
No base	15.7	17.6	13.8
12-in. granular base	15.2	17.1	11.8
8-in. bound base	14.2	16.4	--
Accumulative Damage-Mixed Traffic Design			
No base	18.2	19.0	
12-in. granular base	17.8	18.5	
8-in. bound base	16.7	17.8	

Table 8

Slab Thickness for Military Facility

Pavement Type	Required Slab Thickness, in.		
	Stress Criteria	Deflection Criteria	Present Design Criteria
B-52 Aircraft			
No base	25.6	27.0	23.2
12-in. granular base	25.3	26.5	20.8
8-in. bound base	24.3	25.8	--
KC-135 Aircraft			
No base	14.2	15.2	13.0
12-in. granular base	13.7	14.6	10.9
8-in. bound base	12.6	13.8	--
C-141 Aircraft			
No base	16.3	16.8	14.7
12-in. granular base	15.8	16.2	12.7
8-in. bound base	14.7	15.5	--
F-111 Aircraft			
No base	14.2	<10	14.1
12-in. granular base	13.9	<10	13.4
8-in. bound base	13.3	<10	--
Accumulative Damage-Mixed Traffic Design			
No base	25.6	27.0	

thickness as determined by present Corps of Engineers* design criteria. The comparison between the different slab thicknesses should be noted. Although the design examples provide some comparison between the different criteria, these examples cover only a limited design condition. A more comprehensive comparison is provided in the following section.

COMPARISON OF CRITERIA

In working through the design examples, it was obvious (Tables 7 and 8) that the slab thicknesses obtained by applying the stress criteria (Equation 7) or the deflection criteria (Equation 9) were different from the thicknesses obtained from the present Corps of Engineers design procedure. Although the difference due to the dependence of the criteria on the gear was expected in the case of deflection criteria, for the stress criteria the magnitude of the difference was unexpected and felt to be unacceptable. To understand the nature of the differences in the criteria, additional analyses were conducted.

First, the required slab thicknesses for different design situations were computed using each of the design criteria. Figures 35 and 36 present the comparisons of the required slab thicknesses over a range of subgrade strengths for the DC-8 and C-141 aircraft. From these figures, it is seen that the slab thickness determined by the deflection criteria is highly sensitive to changes in subgrade strength. This would strongly indicate these deflection criteria to be dependent on the subgrade strength. It would appear that for weak subgrades, the deflection criteria would be overly conservative. From these studies, the conclusion was reached that the surface deflection would not be a good design parameter. The stress criteria also appeared, at least for the higher levels of traffic, to give slab thicknesses that were significantly greater than those determined from the present design criteria.

For additional comparison between the stress criteria and present design criteria, coverage predictions (Table 9) were made

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* These criteria are also the same as FAA design criteria contained in FAA Advisory Circular AC 150/5320-6C. Thereafter, the criteria will be referred to as the present design criteria.

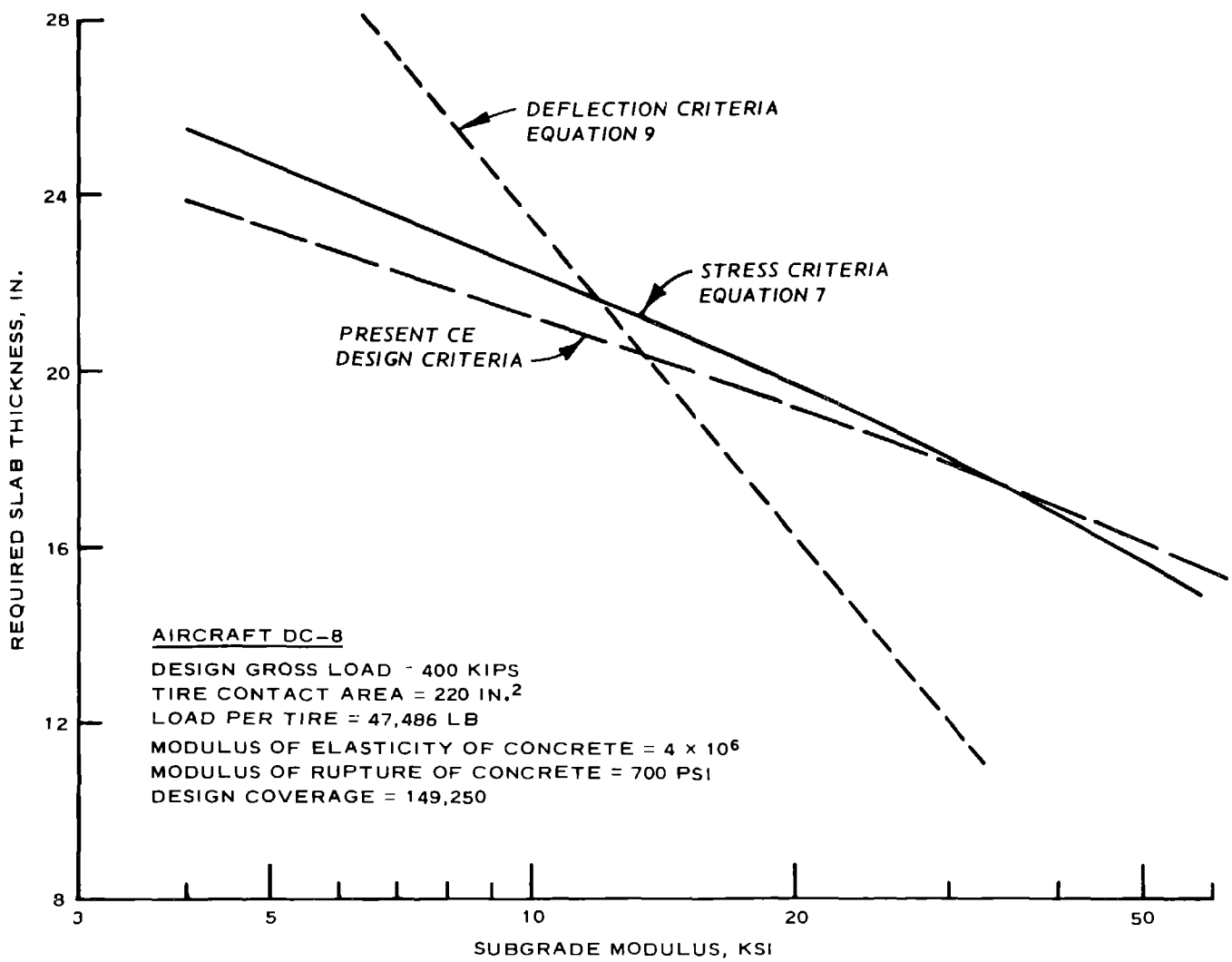


Figure 35. Comparison of deflection and stress criteria with present design criteria - DC-8 aircraft

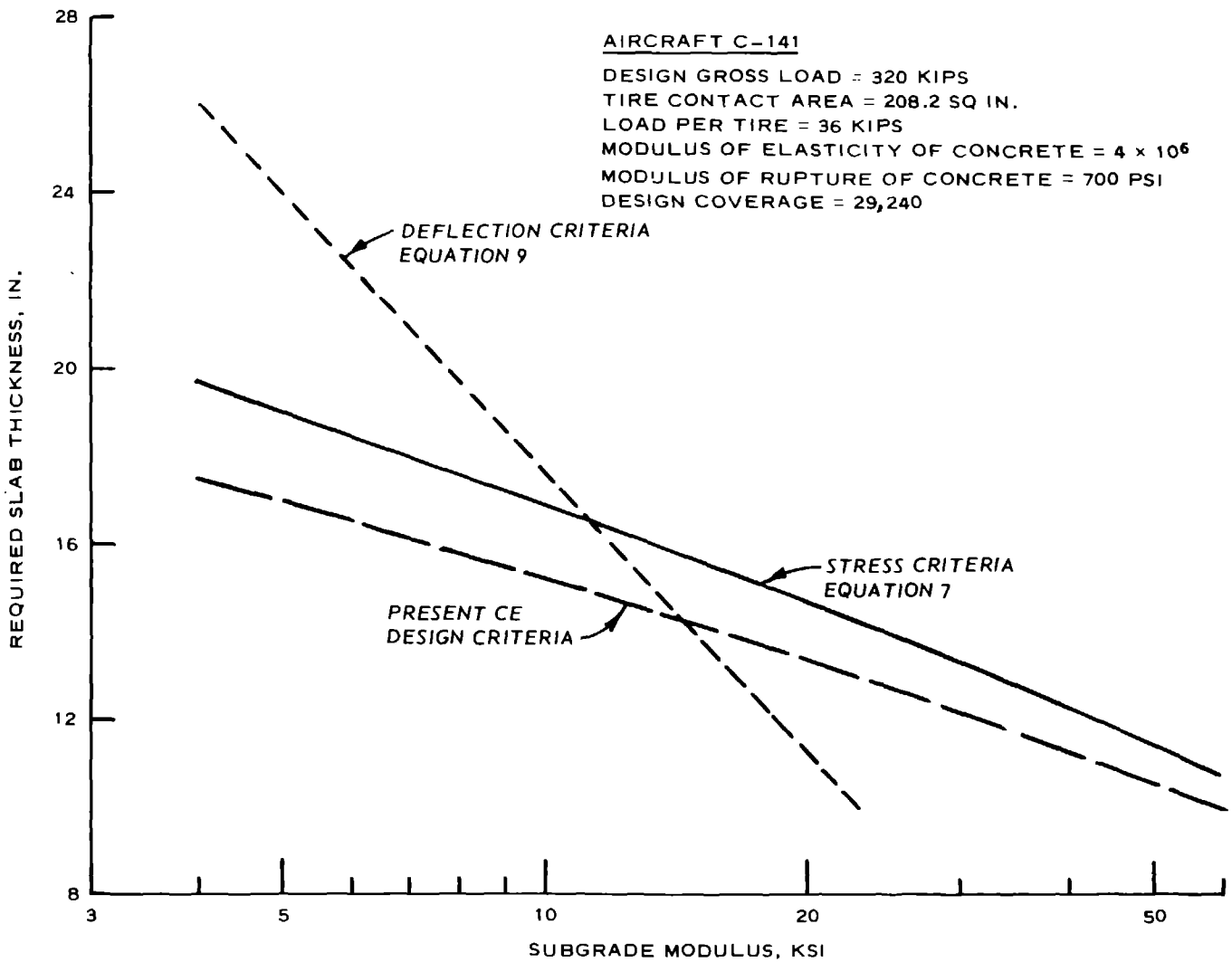


Figure 36. Comparison of deflection and stress criteria with present design criteria - C-141 aircraft

Table 9

Data for Comparison of Stress and Present Criteria

Test Series*	Test**	Thickness (in.)	K (pci)	R (psi)	0.75 σ_e (psi)	H (percent)	σ_{LM} (psi)	Coverages Actual	Coverages Present	Coverages Equation 7
Lockbourne No. 1	A-1	5.72	150	740	627	92.7	405	390.0+	461.000	7,888
	A-2	5.72	150	780	921	71.3	599	45.0	0.400	96
	B-1	5.50	75	740	776	79.4	504	187.0	6.000	286
	B-2	5.50	75	780	1145	60.6	759	35.0	0.010	17
	C-1	5.50	70	740	789	78.6	555	200.0	5.000	116
	C-2	5.50	70	780	1165	59.8	853	14.0	0.010	8
	D-1	5.50	75	740	776	79.4	772	450.0	6.000	90
	D-2	5.50	75	780	1145	60.6	877	33.0	0.010	7
	E-1	5.75	104	740	680	87.5	505	430.0+	55.000	286
	E-2	5.75	104	780	998	65.9	771	77.0	0.100	15
	F-1	7.75	52	740	525	106.0	396	550.0+	10,068.000	3,734
	F-2	7.75	52	780	804	80.6	625	111.0	9.000	70
	K-3	9.44	90	735	730	81.4	570	72.0	12.000	90
	K-2	9.44	90	780	547	107.3	410	700.0	14,606.000	4,527
	N-2	8.00	75	780	709	88.1	564	150.0	103.000	161
	N-3	8.06	75	735	936	66.7	785	9.0	0.100	10
	O-2	9.46	75	780	569	104.2	458	573.0	9,265.000	1,253
	O-3	9.46	75	735	764	78.9	647	72.0	5.000	34
	P-2	7.58	95	780	721	86.9	632	265.0	69.000	61
	P-3	7.58	95	735	937	66.0	583	6.0	0.100	5
	Q-2	9.44	109	780	524	110.7	465	1,390.0	24,064.000	1,102
	Q-3	9.44	109	735	694	84.4	659	57.0	31.000	30
	U-2	5.83	207	780	818	77.4	527	88.0	3.000	305
	U-3	5.83	207	735	995	60.0	551	1.5	0.100	32
	A-REC	9.81	107	725	451	118.3	390	656.0	73,462.000	3,502
Lockbourne No. 2	E-1	15.00	90	725	796	78.5	629	97.0	5.000	37
	E-2	15.00	150	680	721	79.8	574	942.0	7.000	44
	E-3	15.00	99	710	782	78.2	663	17.0	4.000	22
	E-4	15.00	164	680	707	80.7	642	203.0	9.000	21
	E-5	18.75	91	695	573	95.5	451	43.0	1,149.000	421
	E-6	20.26	97	700	505	104.4	397	2,204.0+	9,541.000	1,543
	E-7	24.00	88	760	397	129.2	312	2,204.0+	364,000.000	145,115
	M-1	12.00	55	725	719	81.7	600	124.0	13.000	54
	M-2	15.00	55	725	543	102.1	446	2,204.0+	6,806.000	800
	M-3	20.00	55	725	364	136.1	295	2,204.0+	1,003,000.000	165,000
	Lockbourne No. 3	6.00	62	800	1339	57.3	976	18.0	0.004	4
Sharonville Channelized	57	20.00	27	740	447	115.6	315	34,650.0+	57,233.000	81,420
	58	18.00	30	740	519	105.8	373	34,650.0+	11,788.000	7,567
	59	16.00	47	730	585	97.4	394	7,600.0	2.135.000	3,234
	60	12.00	335	730	654	89.8	416	1,674.0	179.000	1,728
	61	14.00	300	730	533	103.1	349	3,867.0	7,853.000	15,335
	62	16.00	360	730	428	120.9	271	10,082.0	107,620.000	595,949
Sharonville Heavy	71	32.00	100	800	359	145.4	249	9,680.0+	3,900,000.000	20,368,141
	72	28.00	70	800	455	120.9	319	9,680.0	107,620.000	827,455
	73	24.00	70	800	585	103.6	401	2,115.0	8,483.000	8,604
MWHGL	1-C5	10.00	60	725	820	73.6	580	221.0	0.910	70
	2-C5	12.00	70	800	632	97.8	473	4,230.0	2,433.000	1,176
	3-C5	14.00	74	700	510	104.1	394	1,400.0	9,130.000	2,095
	4-C5	8.00	74	775	1014	64.3	735	180.0	0.040	19
	2-DT	12.00	70	700	779	75.5	566	95.0	1.700	65
	3-DT	14.00	74	660	637	85.2	461	205.0	40.000	221
KLJS	1-C5	8.00	250	905	747	94.7	656	54.0	885.000	160
	2-C5	11.00	100	730	641	89.9	522	344.0	185.000	182
	3-C5	10.00	80	810	763	81.8	580	22.0	35.000	182
	4-C5	10.00	235	850	576	112.1	522	6,336.0	29,557.000	909
	4-DT	10.00	235	860	709	94.3	643	320.0	752.000	124
SSFS	3-200	15.00	120	900	621	108.9	463	3,215.0	18,474.000	5,853
	3-240	15.00	120	900	745	91.0	564	350.0	794.000	659
	4-200	15.00	150	870	588	110.5	463	4,660.0	23,368.000	3,982
	4-240	15.00	150	870	706	96.0	555	70.0	1,353.000	544

* For description of test series, refer to Appendix B.

** Column headings:

K = the modulus of subgrade reaction
R = the modulus of rupture for the concrete slab
σ_e = the edge stress as computed by plate theory
H = the percent standard thickness computed for development of design criteria
σ_{LM} = the interior stress as computed by layer theory
Actual = actual coverage to failure or if a + is used would mean section did not fail and the coverage given is when traffic was terminated
Present = the predicted coverage to failure as determined from present design criteria
Equation 7 = the predicted coverage to failure as determined from the design criteria (Equation 9) developed in this study

for the test sections using both criteria. Figure 37 shows the comparison of the two criteria. These data are the evidence that the two criteria are different even though much of the same test data were used in the development of both criteria.

The difference is particularly noticeable at the lower coverages (below 200) and the higher coverages (from 1,000 to 30,000). Figure 38 presents a comparison of both criteria with the actual coverages to failure. The comparison of Figure 38 clearly shows that the stress criteria as developed in this study better predicts the actual performance, at least for the lower and higher coverages, than the present design criteria.

To better understand why the present design criteria are failing to predict performance in certain coverage ranges, the test data were reanalyzed in the same manner, i.e., plate theory was used to compute edge stresses, as was done to develop the original Corps of Engineers design criteria. The data for the analysis are given in Table A1. Also, the present design criteria were used to develop interior stress criteria of the form of Equation 7. This was accomplished by designing, based on present design criteria, a number of pavements covering a range of design conditions. The design factors for these hypothetical sections were computed in the same manner as the design factors for the test sections. The result was interior stress criteria, which best fit the present design criteria.

Figure 39 shows the comparison between the present design criteria and the data developed from the test sections. It should be repeated that these data shown were computed using <u>edge stresses</u> in the <u>same way</u> as was used to develop the present design criteria. The specific reasons for the disparity between the criteria and the test section data are not known. One contributing factor may have been that a considerable amount of the data shown is from test sections constructed and tested after the development of the present design criteria. The lack of data would not completely explain the disparity, but no other explanations can be offered. Figure 40 provides a comparison of stress criteria (Equation 7) with the present design criteria. From this

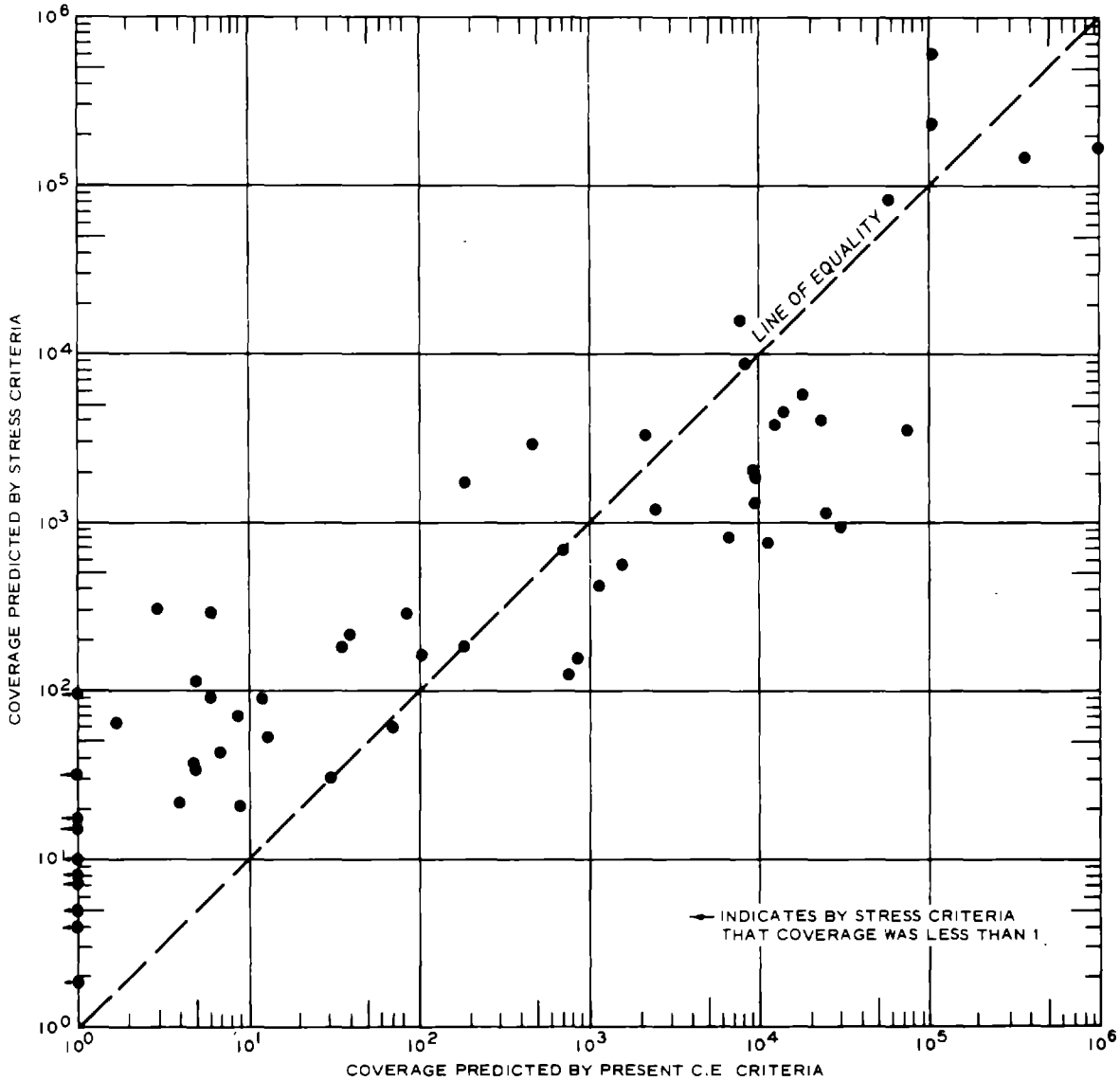


Figure 37. Comparison of coverage predicted by present design criteria with coverage predicted by stress criteria

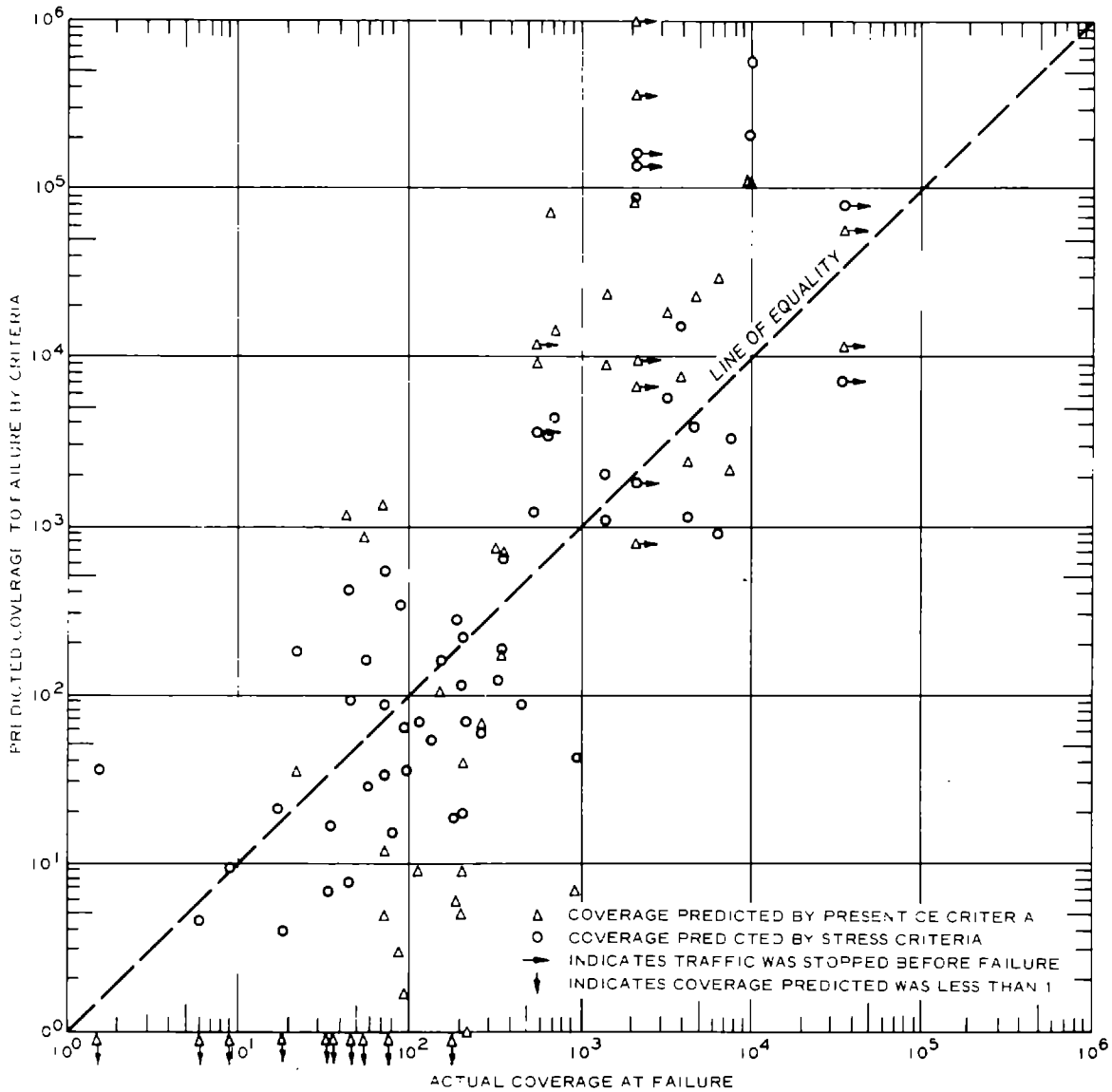


Figure 38. Comparison of actual coverage to failure with predicted coverage to failure

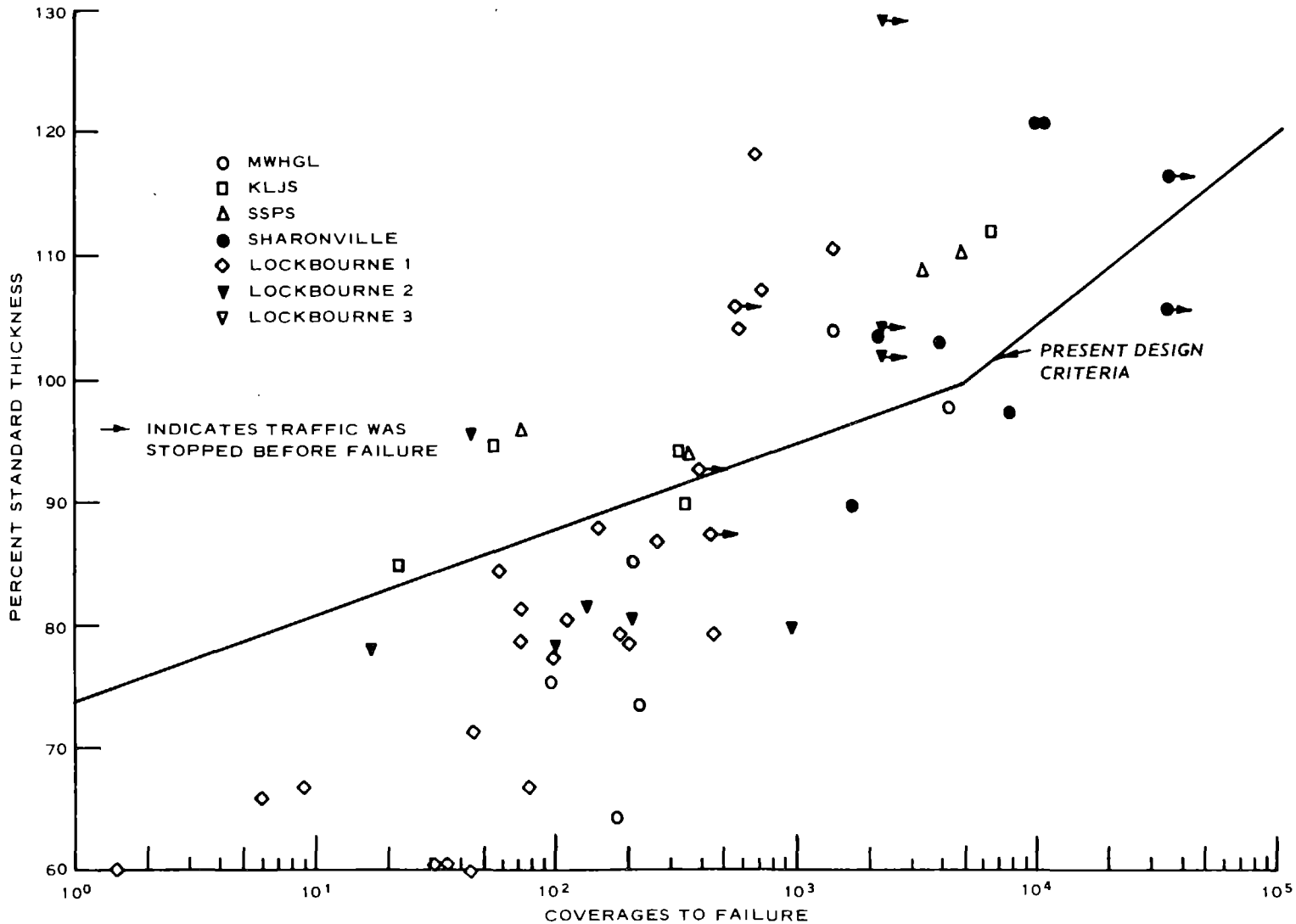


Figure 39. Comparison of present design criteria with test section data

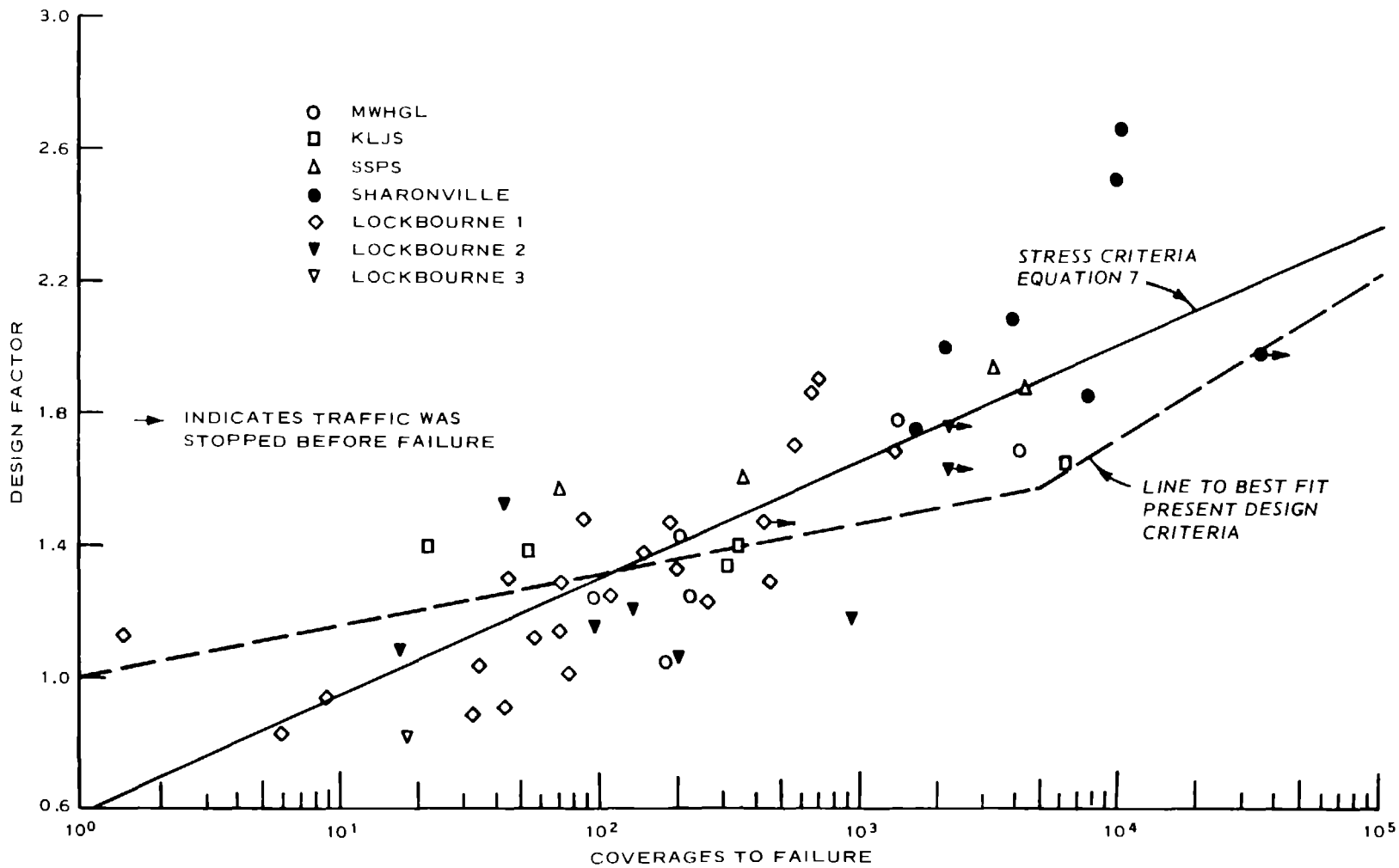


Figure 40. Comparison of stress criteria (Equation 7) with present design criteria

comparison, it is clear that for low coverages the new stress criteria will require pavements of lesser thickness and for high coverages will require pavements of greater thicknesses.

The conclusion from the comparisons of the two criteria is that a basic difference does exist between the two criteria. The test section data as analyzed in this study definitely support the new criteria, and acceptance of the new criteria is justified.

JOINT DESIGN

The subject of joints has been sorely neglected in this report. This was done deliberately even though joints in rigid pavements are necessary and are critical to the performance of the pavement. Joints were neglected primarily because the elastic layered computational model does not have the ability to simulate the discontinuities in the layers, and no new data or analyses were developed during the study that would improve joint design. As a result, the current joint types and joint requirements as contained in References 1-3 should be used.

The criteria developed will indirectly reflect the influence of joints since all the test pavements contained joints that were similar to those currently used. The traffic, in many of the tests, was applied in the critical location with respect to the joints. Therefore, the criteria should be adequate, provided currently specified joint systems are used.

Joints in PCC pavements are critical. In order to improve joint design (and therefore improve the design of the entire pavement system), a computational model is needed that will permit the computation of the response of a layered system in which the layers contain discontinuities. A model such as this would permit a reanalysis of the results from the test pavements and the development of criteria that are based on the most critical stress and deflection in the pavement. Designs could then be accomplished wherein the design of the joints would be an integral part of the procedure, just as selection of base and PCC thickness. This would then permit consideration of the effectiveness of various types of joints, i.e., doweled, keyed, keyed and tied, thickened edge, etc.

TRANSLATION FROM DESIGN TO CONSTRUCTION

The production of plans and specifications for constructing pavements with the design procedure contained herein will require some modifications to presently used procedures. For subgrade soils and borrow material, no changes will be needed. The materials to be used may be identified, and with specifications of thickness of compacted layer, density, and moisture, there is a reasonable degree of certainty that the material constructed will be as intended by the designer.

For bases (subbases) and PCC, changes will be required. For bases, there are several alternatives. One alternative is to completely specify the material as to source, gradation, density, moisture, type and amount of additive, and layer thickness. A second alternative is to specify alternate types (granular, chemically stabilized, or bituminous stabilized) along with the required thicknesses and moduli of elasticity of each type. A third alternative is to specify a relationship between modulus of elasticity and layer thickness. This will permit the contractor to select the type material to use. Certain ranges of acceptable moduli and thicknesses would have to be established, and different quality control measures would be required for the third alternative.

For PCC, the alternatives are similar to those for bases, i.e., a complete description of the material to be used, including source of aggregates, mix design, etc., may be specified, or the desired properties (flexural strength and modulus of elasticity) may be specified, allowing the contractor to select necessary materials and ingredient proportions.

The selection of the procedure to use for translation from design to construction will depend on how and when the economic analysis is to be made. The use of a layered model permits a number of structurally acceptable layered systems to be generated. If the designer performs the economic analysis, then definite narrow specifications should be provided to ensure that the pavement that is built conforms to that which the designer intended. When end-product specifications (such as

the third alternative for bases) are used, it is implicitly assumed that the contractors will consider all possible alternatives in preparation of their bids and thus perform the economic analysis. In this case, the specifications would be very broad.

The user agency should select the type procedure that best fits within its management system. Any of the alternatives discussed above should prove satisfactory.

A final note on translation from design to construction should be made regarding rounding off of thicknesses. Thicknesses of base (subbase) courses should be considered in increments of 1 in. When fixed side forms are used, the thickness of PCC should be rounded to the nearest full inch. For fractions of an inch equal to or less than 0.25 in., the thickness is rounded down; for fractions of an inch greater than 0.25 in., the thickness should be rounded up. When slipform pavers are to be used, the thickness of PCC should be rounded to the nearest 0.5 in. with the quarter points (0.25 and 0.75 in.) as the limits for rounding up or down.

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are offered regarding the design procedure developed herein:

a. There exists a basic difference in the design criteria developed in this study and the present design criteria. The data from test sections support the acceptance of the stress criteria as developed herein.

b. The design criteria as developed in this report should be implemented for the design of aircraft pavements. The Shell BISAR computer code should be used for computation of stresses and deflections. Materials should be characterized with procedures recommended in this report.

c. Data points are needed at traffic volumes greater than 10,000 coverages to verify extrapolation of the limiting stress and limiting deflection performance relationships, or to provide means for modification of the relationships for higher traffic volumes.

d. Efforts should be continued toward the development of a more generalized computational code, which will permit direct inclusion of the effects of discontinuities in the layers and variable interface conditions between layers.

e. Efforts should be made to develop procedures for quantifying the load deformation characteristics of deteriorated rigid pavements and interface conditions between PCC layers. Achievement of these goals will permit design of rigid overlays of rigid pavements with same methodology used for new rigid pavements.

APPENDIX A: DEVELOPMENT OF MATERIAL CHARACTERIZATION PROCEDURES

CONSIDERATION OF REPEATED LOADING

PORTLAND CEMENT CONCRETE

Chou[25] devotes a chapter to the characterization of PCC. One of the many factors noted by Chou, which affects the modulus of elasticity of concrete, is the repeated application of stresses. The consensus from this review and two additional, rather extensive, reviews of the fatigue of PCC[60,61] is that the stiffness as measured with any of a variety of procedures (flexural, compression, etc.) and as computed by several different methods (secant, tangent, etc.) is decreased by the application of load repetitions. However, as far as is known, it has not been shown that traffic causes a significant reduction in the modulus of in-place PCC. The extensive evaluation program conducted on military airfields has not shown this to be a major factor.[62] This presumes, of course, that the concrete is intact. Exposure to freezing-thawing and deicing salts, aggregate reactivity, sulfate attack, etc., will affect the modulus of elasticity, but this is usually manifest in visible deterioration other than structural cracking.

The complexity of the relationship between modulus of elasticity and repeated loads and the apparently small magnitude of change caused by traffic has led to the omission of the effects of repeated load on PCC modulus of elasticity. There may be some decrease in modulus due to repeated loads or exposure, but conversely there should be some increase because of the effects of long-term hydration. The net result is that the computation of the modulus of elasticity from the stress-strain relationship obtained from the initial loading of a PCC specimen is considered adequate for characterizing the material for the life of a pavement.

Poisson's ratio for PCC normally receives very little attention. This may be unjustified, but as pointed out by Chou,[25] the range of statically determined Poisson's ratio is only from about 0.11 to 0.21,

and the average of dynamically determined values was about 0.24. Added factors are the difficulty of measurement and the relatively small influence that varying Poisson's ratio within a reasonable range has on the computed response. Several studies referenced by Chou show that the value of Poisson's ratio increases with load repetitions, but that this occurs primarily for high stress levels. No evidence was found to indicate that load repetitions should seriously be considered in describing Poisson's ratio for PCC.

The effects of repeated loads on the strength of PCC is a well-established and extensively researched phenomenon.[60,61] It is universally accepted that the magnitude of stress that can be sustained by PCC before cracking is a function of the number of repetitions of the stress and that the magnitude of this stress decreases as the number of stress repetitions increases. The number of stress repetitions of a given magnitude that a material can sustain is dependent on numerous factors, i.e., age, mix proportions, type aggregate, rate of loading, range of loading, etc. The most important, however, is the static strength of the material. Fatigue data are normally presented in the form of a plot of the ratio of the static strength to the applied stress versus the number of repetitions. This would appear to be the characterization needed for PCC and is the approach taken indirectly by several design agencies.[1,4,5] Safety factors are applied to keep stress levels within tolerable limits, and the material is characterized by the static strength.

The approach taken by other agencies[2,3] is similar but the effects of load repetitions on the entire pavement system are considered, and the fatigue relationships used are for the entire pavement system. The number of load repetitions the pavement can sustain is related to the static strength of the PCC and the stress within the pavement. The result is that the effects of load repetitions are handled indirectly and a fatigue relationship for the concrete to be used is not required for each design situation. Rather, the performance of the pavement system is related to the static strength and is the one parameter needed to characterize the material. This is also the

approach taken for the procedure developed herein.

BOUND BASES (SUBBASES)

When considering bound bases, chemically stabilized materials (portland cement, lime, fly ash, etc.) and bituminous-stabilized materials need to be discussed separately, even though the conclusions regarding inclusion of effects of repeated loading are the same for both types of bound bases. Due to the viscous and temperature-dependent behavior of the bituminous binder, bituminous-stabilized materials are affected by temperature and rate of loading to a much greater extent than any other component in a pavement structure.

A great deal of the work done on the characterization of bituminous mixtures has been directed toward determining the rather complex response of the material and the effects of temperature and rate of loading. Also, much of the work has been performed for the purpose of characterizing the material for flexible pavements.

Chou[25] has a detailed review of characterization procedures for bituminous mixtures. The various types of available tests are discussed including repeated load flexural tests. The effects of rate of loading and temperature are noted as dominating factors. Therefore, the inclusion of the effects of repeated loading, while important, does not account for other important factors. Complete characterization would require that a different rate of loading be used for various features (runways, taxiways, and aprons) and that a range of temperatures be used for defining the modulus of elasticity and Poisson's ratio. From a practical point of view, the ranges of rates of loading and temperatures encountered are limited, and the inclusion of the effects of repeated flexural loads at approximate temperatures and rates of loading adequately characterizes bituminous bases (subbases) for rigid pavement design.

Chemically stabilized bound bases (subbases) are not as dependent on the rate of loading and the temperature as bituminous bases. They do have an effect, but this effect would be minor in comparison with other factors. Chou[25] cites numerous references of studies made of the load

deformation properties of chemically stabilized material. Emphasis is placed on the modulus of elasticity, and the effects of repeated loads are noted. The effects on compressive, tensile, and flexural loadings are noted and are essentially the same, i.e., the modulus increases with the number of loadings. The magnitude and nature of the increase would be dependent on the type loading, magnitude of applied stress, curing time, etc. Modulus values computed from compressive tests generally appear to be more sensitive to load applications than do modulus values computed from flexural tests.

Static and resilient modulus values for several chemically stabilized bound materials are compared in Table A1. The results shown are averages from tests of several samples. The materials were field-mixed, but the samples were compacted and cured in the laboratory. The larger ratios are for the more flexible materials. Indications are that the consideration of the effects of repeated loading on the modulus of elasticity of chemically stabilized bases is a justifiable requirement.

GRANULAR BASES (SUBBASES)

Granular materials are extremely difficult to characterize. For bituminous mixtures, the rate of loading and the temperature are the dominating factors affecting the properties of the material. For granular materials, the state of stress, particularly the confining stress, is the dominating factor in determining load-deformation properties. Repeated loading also affects the modulus of granular materials. In summarizing the results of numerous studies, Chou[25] states: "The consensus from these studies has been that the response of granular materials to repeated loading is different from their response to static loading." The general pattern noted was that repeated loadings increased the stiffness provided shear failure was not progressing. This implies that the modulus of elasticity is increasing.

The effect on Poisson's ratio of repeated loading may be different from the effect of repeated loading on modulus of elasticity. The nature of any change in Poisson's ratio that may occur with repeated

Table A1

Comparison of Static and Resilient Moduli of Chemically Stabilized Bound Bases

Material	Loading	Static Modulus, psi	Resilient Modulus, psi	Ratio M_R/E_{static}
Low plasticity clay (CL) with 12 percent portland cement	Flexure	200,000	273,000	1.37
Low plasticity clay (CL) with 12 percent portland cement	Compression	79,000	105,000	1.33
Low plasticity clay (CL) with 2 percent portland cement, 3 percent lime, and 10 percent fly ash	Compression	30,000	80,000	2.67
Low plasticity clay (CL) with 10 percent portland cement	Compression	80,000	110,000	1.38
Sandy clay (SC) with 5 percent portland cement	Compression	37,000	125,000	3.38

applications will depend on the initial density of the material. If the relative density is low, then densification may occur and there would be an apparent decrease in Poisson's ratio (possibly negative values) as loads are applied. However, Poisson's ratio would reach a relatively constant value where it would remain unless shear failure began to occur. Then, there would be an apparent increase in Poisson's ratio as the material underwent an increase in volume during shear failure. To summarize, the use of repeated loadings to characterize granular materials is a well-established, generally accepted procedure.

SUBGRADES

The thickness of the PCC surface layer and the properties of the subgrade are the two most important parameters in determining the response of rigid pavements to loads. As noted previously, subgrades are generally the components of rigid pavements where the assumptions of linearity and elasticity are least valid. The subgrade is also the pavement component that is most affected by repeated load applications.

Subgrades may be divided into the general classes of cohesive and cohesionless soils. The majority of soils possess properties of both, but in a saturated condition, where it is generally appropriate to characterize subgrade soils, most natural subgrade soils behave primarily as a cohesive material. Repeated loading affects both cohesive and cohesionless soils. Cohesionless sands, gravels, or sand-gravel combinations will respond much like granular bases or subbases. Cohesive soils are more sensitive to repeated loadings. The resilient modulus of cohesive subgrades generally increases with load repetitions provided the level of stress is lower than that required to initiate shear failure. However, the number of stress repetitions required before a stable condition is reached may be greater than for bound bases, granular bases, or cohesionless subgrades.

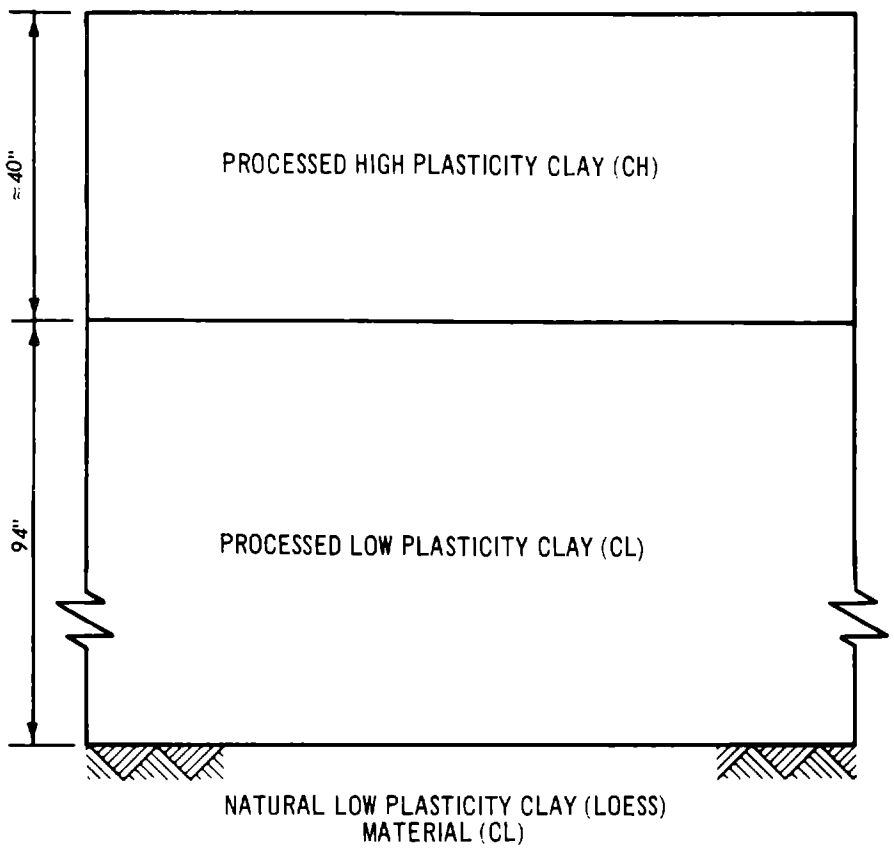
The effects of repeated loadings on the response of cohesive subgrades may be examined by studying the response of several test pavements. These pavements consisted of PCC slabs directly on a prepared clay (CH) subgrade as part of the Multiple-Wheel Heavy Gear Load (MWHGL)

Pavement Tests.[35] The pavements consisted of 8-, 10-, 12-, and 14-in.-thick PCC slabs directly on the prepared clay subgrade. The prepared clay subgrade (Figure A1) was composed of two processed materials (a clay of high plasticity and one of low plasticity) and a natural material of low plasticity. The materials were placed with a moisture content and a density that result in a degree of saturation near 100 percent. This condition remained relatively constant as the pavements were constructed and tested.

Table A2 presents the results from static plate bearing tests on the subgrade. As noted, the values of modulus of soil reaction from the tests conducted after traffic are larger than those from tests conducted prior to slab construction. This is due partly to the compaction effect of traffic and partly to the thixotropic effect as the clay ages. The same type patterns are apparent from the results of plate bearing tests conducted for later test pavements constructed on the same subgrade (Table A2).

In Table A3, additional evidence of the effects of repeated loading and the modulus of soil reaction values computed from both static and cyclic plate bearing tests are illustrated. Since both tests were run after completion of traffic, such large differences were unexpected. However, removal of the slab, causing a certain amount of disturbance and allowing relaxation of stresses within the soil, may account for at least part of the difference between the static and cyclic response.

The difference in the response to moving wheel loads and a cyclic load applied at one location on the pavement may account for part of the difference, i.e., the conditioning or stiffening effects of moving wheel loads distributed across a pavement are different from the effects of a cyclic load applied at one location. Ledbetter[63] investigated the response of pavements to moving loads and showed that the response is rather complex (especially for flexible pavements). Cyclic load tests conducted in test pavements at the WES have shown that the response of the pavement to the first load in a series of loadings is different from the response to subsequent loads in that



CONSTRUCTED - 1968 - MULTIPLE WHEEL HEAVY GEAR LOAD TESTS

DEPTH OF PROCESSED HIGH PLASTICITY CLAY (CH) WILL VARY WITH THICKNESS OF PAVEMENT ABOVE. TOTAL DISTANCE FROM TOP OF PAVEMENT TO NATURAL UNPROCESSED MATERIAL IS 144 IN.

FOR THE KEYED LONGITUDINAL JOINT STUDY AND THE SOIL STABILIZATION PAVEMENT STUDY THE TOP 6 TO 12 IN. OF THE CH MATERIAL WAS REPROCESSED AND MATERIAL CUT OR ADDED AS REQUIRED TO MEET GRADE.

Figure A1. Clay subgrade for WES rigid test pavements

Table A2

Results from Plate Bearing and Field Density and Moisture Content Tests

Item	Modulus of Soil Reaction, pci		Dry Density, pcf		Moisture Content percent	
	Before Traffic	After Traffic	Before Traffic	After Traffic	Before Traffic	After Traffic
Multiple-Wheel Heavy Gear Load Study (Reference 35)						
1	62	154	86	84	32	34
2	70	94	84	83	33	34
3	74	87	86	83	32	35
4	74	125	86	87	32	32
Average	70	115	85	84	32	34
Keyed Longitudinal Joint Study (Reference 38)						
1	70	--	85	--	31	--
2	110	100	92	81	28	36
4	47	--	86	--	30	--
5	40	--	81	--	37	--
Average	67		86		32	
Soil Stabilization Pavement Study (Reference 39)						
1	47	180 & 200	86	89	32	30
2	85	118	89	--	34	--
3	84	164	87	--	32	33
4	40	68	86	86	33	33
5	--	120 & 143	--	87	--	32
Average	64	142	87	88	33	32
Overall Average	67	129	86	85	32	33

Table A3

Results from Cyclic Plate Bearing Tests - Multiple-Wheel Heavy Gear Load Study

Item	Cyclic Modulus of Soil* Reaction, pci	Static Modulus of Soil** Reaction, pci	Dry Density, pcf	Moisture Content percent
1	370	169	84	33
2	270	111	85	33
3	300	115	84	33
4	330	128	88	32
Average	318	131	85	33

* Modulus of soil reaction computed with 10-psi plate pressure after 10 cycles at 5-psi and 10 cycles at 10-psi pressure.

** These values are different from those shown in Table A2, since the values shown in Table A2 are the average of several tests and the values shown here are for one test conducted at the same location as the cyclic tests.

series even though the pavement may have sustained considerable traffic prior to the tests. This indicates that the conditioning effect of traffic is different from a cyclic load at one point. There may also be a time factor involved wherein a certain amount of relaxation occurs when there is a rest period between load applications. Certainly the phenomena of pavement response and material characterization are not thoroughly understood.

Much of the discussion presented thus far appears to be tearing down evidence accumulated to justify the use of repeated load tests for characterizing paving materials. However, this is not the case because the response of a pavement to vehicle loading (Figure 14 in main text) relates well with the response computed with material properties obtained from repeated load tests. The validity of the use of repeated load testing for cohesive subgrades can be illustrated by comparing vertical slab deflections and horizontal bending strains in the PCC slabs in the MWHGL pavement tests with corresponding values computed using moduli of elasticity values computed from various type tests. Poisson's ratio of the clay subgrade was assumed as 0.4, and as discussed previously, a stiff layer ($E = 1 \times 10^6$ psi) was located at a depth of 20 ft in the elastic layered simulation.

A composite modulus of elasticity for the subgrade was obtained from the static and the cyclic modulus of soil reaction values. An average value was used for all four pavements. The test results shown in Tables A2 and A3 indicate some variability within the test section, but this was probably due to test variability and natural variability within the entire test section rather than a real difference between individual sections. The composite modulus of elasticity was computed with the formula

$$E = 19.8k \tag{A1}$$

where

E = modulus of elasticity, psi

k = modulus of soil reaction computed by dividing plate pressure by plate deflection, pci

Equation A1[64] is derived from Boussinesq's theory for Poisson's ratio of 0.4 and rigid 30-in.-diam plate. Application of Equation A1 yields moduli of 6300 psi for the cyclic tests and 2300 psi for the static tests (after-traffic plate bearing tests).

Static triaxial and unconfined compression tests conducted on undisturbed samples from the subgrade yielded moduli of 1850 psi for the high-plasticity clay (CH) and 1600 psi for the low plasticity clay (CL). Repeated load triaxial tests on companion samples yielded the plots of resilient modulus versus deviator stress shown in Figures A2 and A3. From a composite or average relationship designated by the heavy dashed line in Figures A2 and A3, moduli of 7,500 psi and 13,500 psi were selected for the high plasticity and low plasticity clays, respectively. These were selected at a deviator stress of 5 psi. In a triaxial test, the deviator stress is defined as the difference between the applied axial stress and the confining stress. These tests were run on material sampled during the Soil Stabilization Pavement Study (SSPS),[39] which was conducted several years after the MWHGL tests. However, it is noted that the condition of the material was similar for both test tracks. Table A2 shows the results of plate bearing tests conducted before and after traffic for both the MWHGL and SSPS tests, as well as for the Keyed Longitudinal Joint Study (KLJS),[38] which was conducted between the MWHGL and the SSPS. The average moduli of soil reaction, dry density, and moisture contents are comparable.

In the plots of measured versus computed slab deflections (Figures A4-A7), the computed values were obtained with the subgrade characterized with the different moduli. In addition to the data points, a line of equality and a least-square-regression relationship constrained through the origin are shown. These comparisons indicate that the use of repeated load test characterizations results in underprediction of the deflection and that the use of static load test characterizations results in overprediction of the deflection. The computed values with repeated load characterizations relate more closely with the measured values (Figures A4 and A6). The correlation with the results from the cyclic plate tests produced the relationship

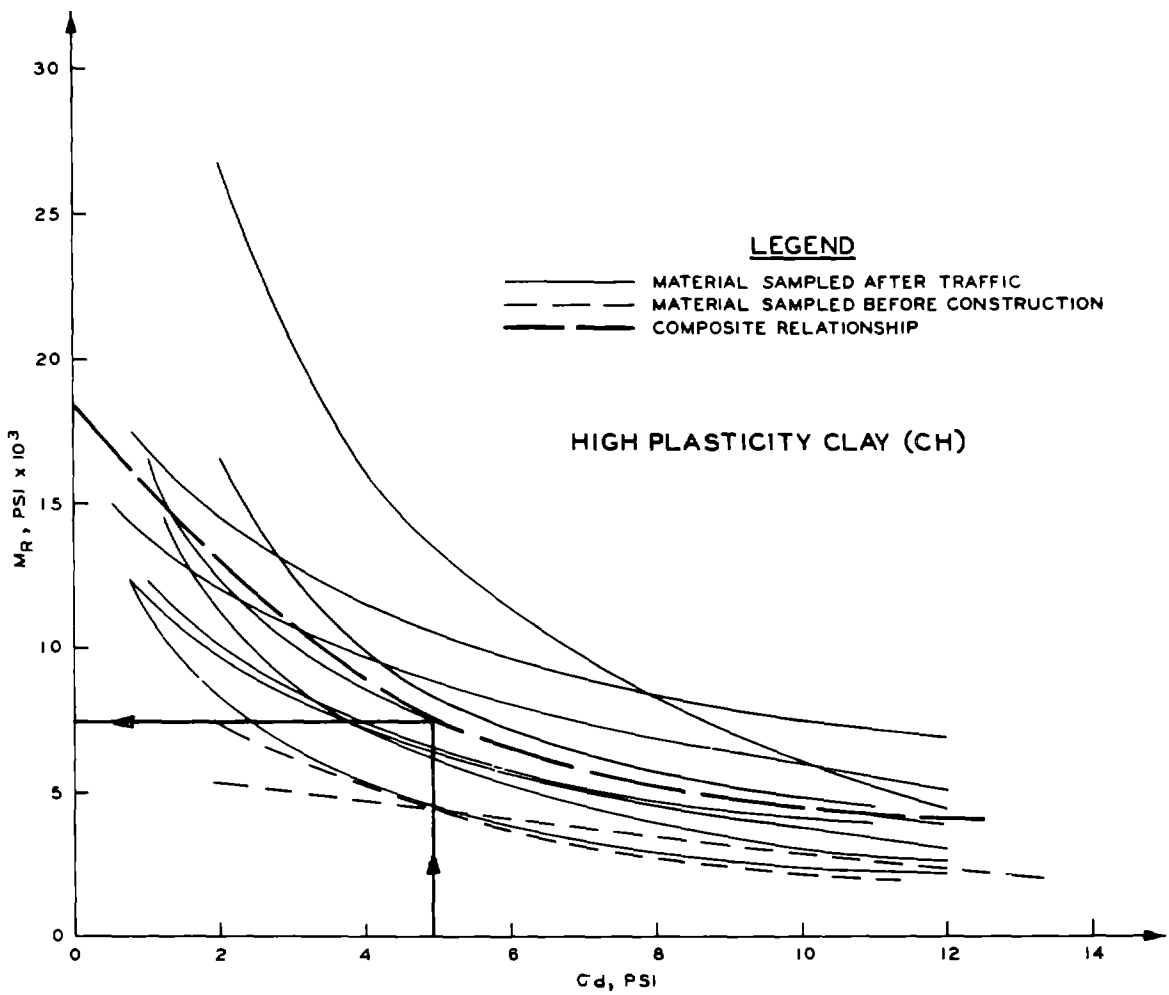


Figure A2. Resilient modulus versus deviator stress for high plasticity clay

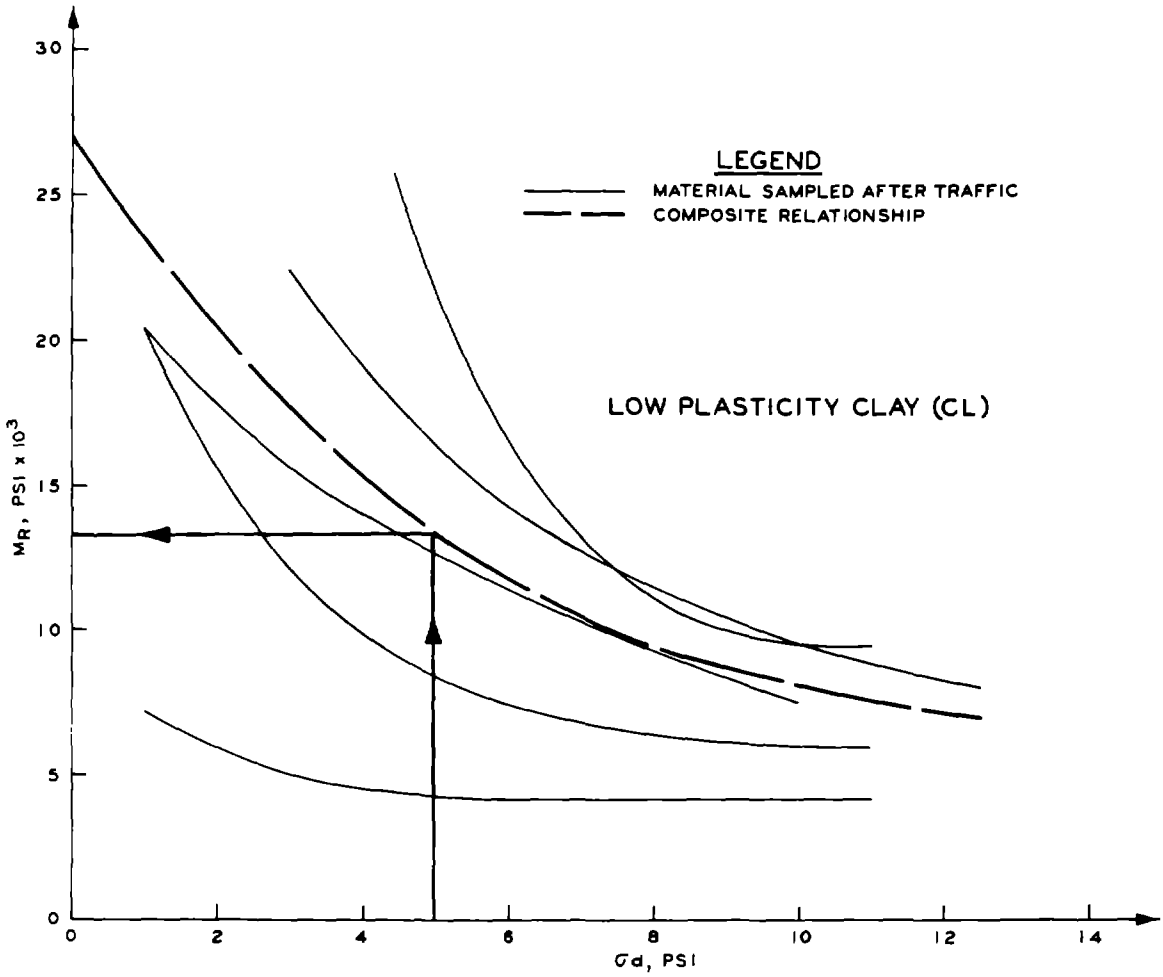


Figure A3. Resilient modulus versus deviator stress for low plasticity clay

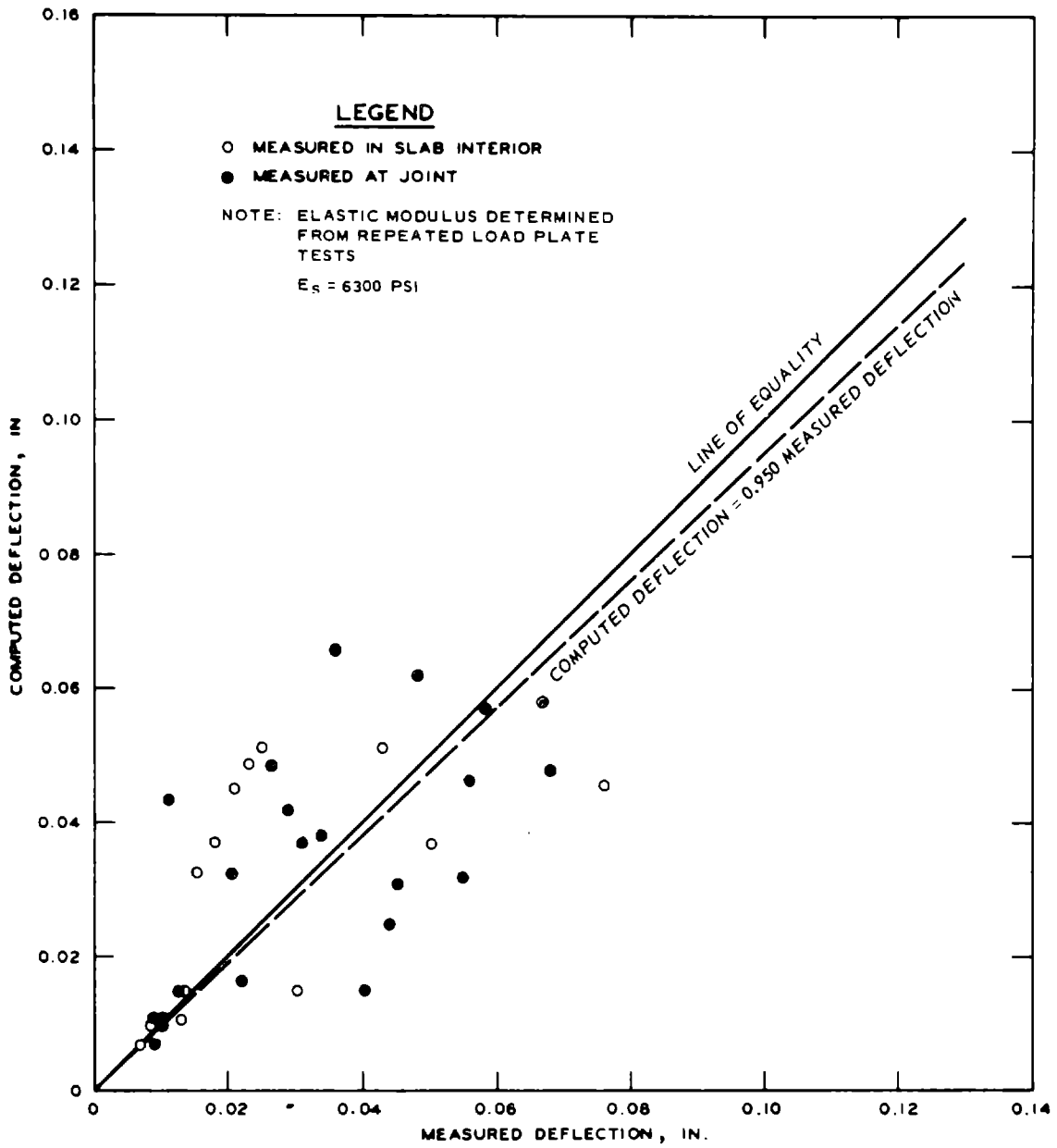


Figure A4. Comparison of measured slab deflection and deflections computed with the subgrade characterized with cyclic plate bearing tests

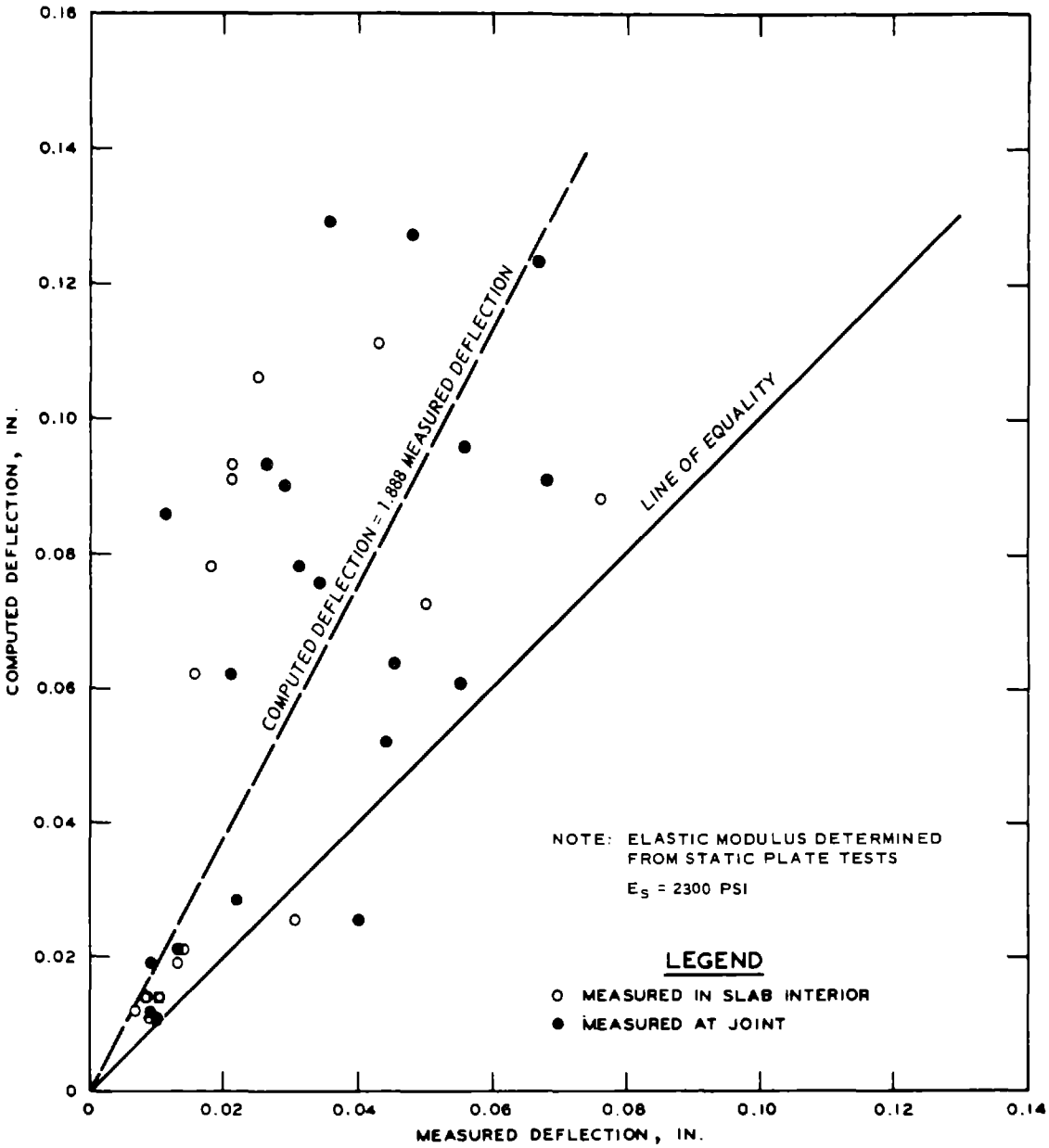


Figure A5. Comparison of measured slab deflection and deflections computed with the subgrade characterized with static plate bearing tests

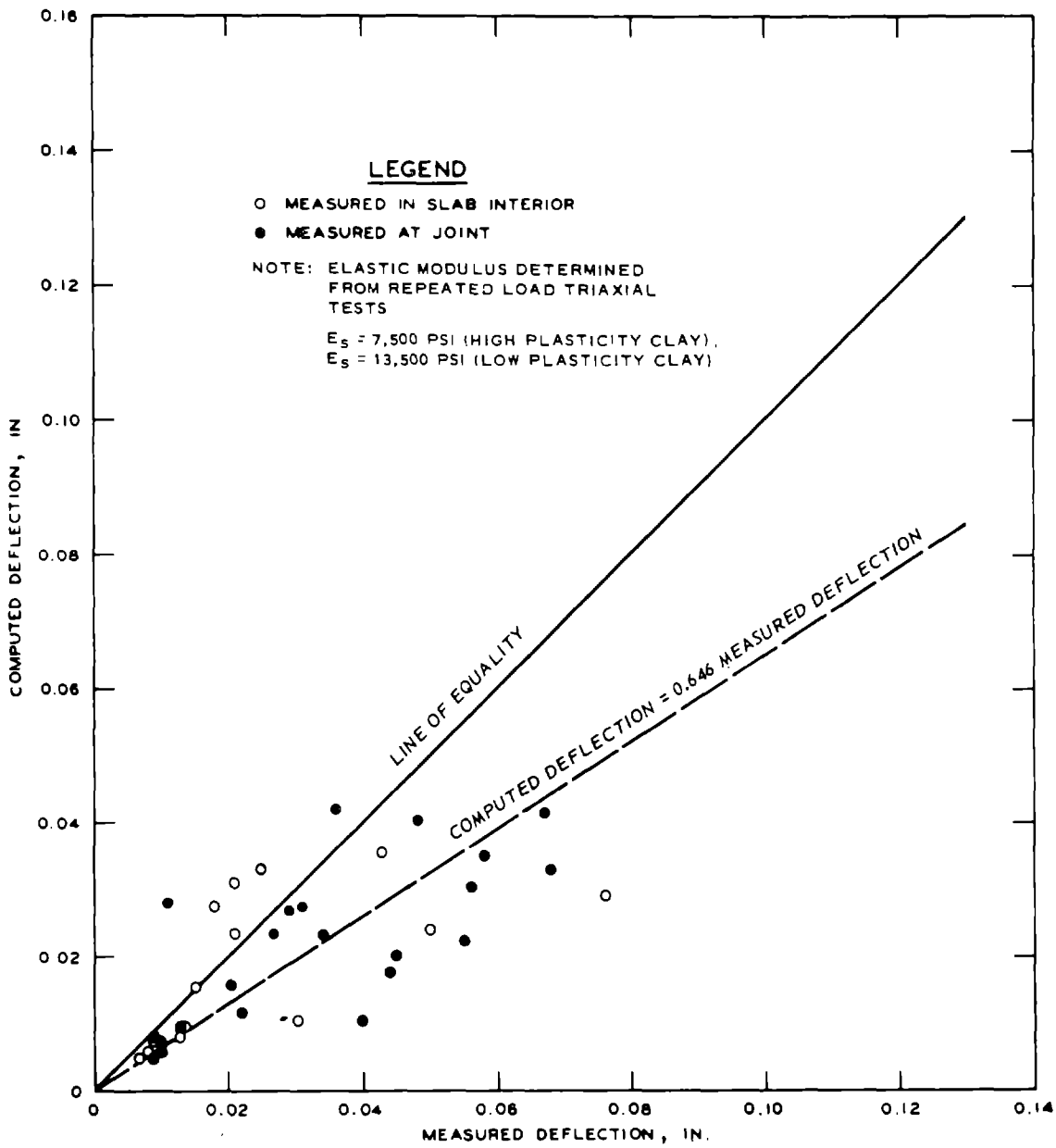


Figure A6. Comparison of measured slab deflection and deflections computed with the subgrade characterized with repeated load triaxial tests

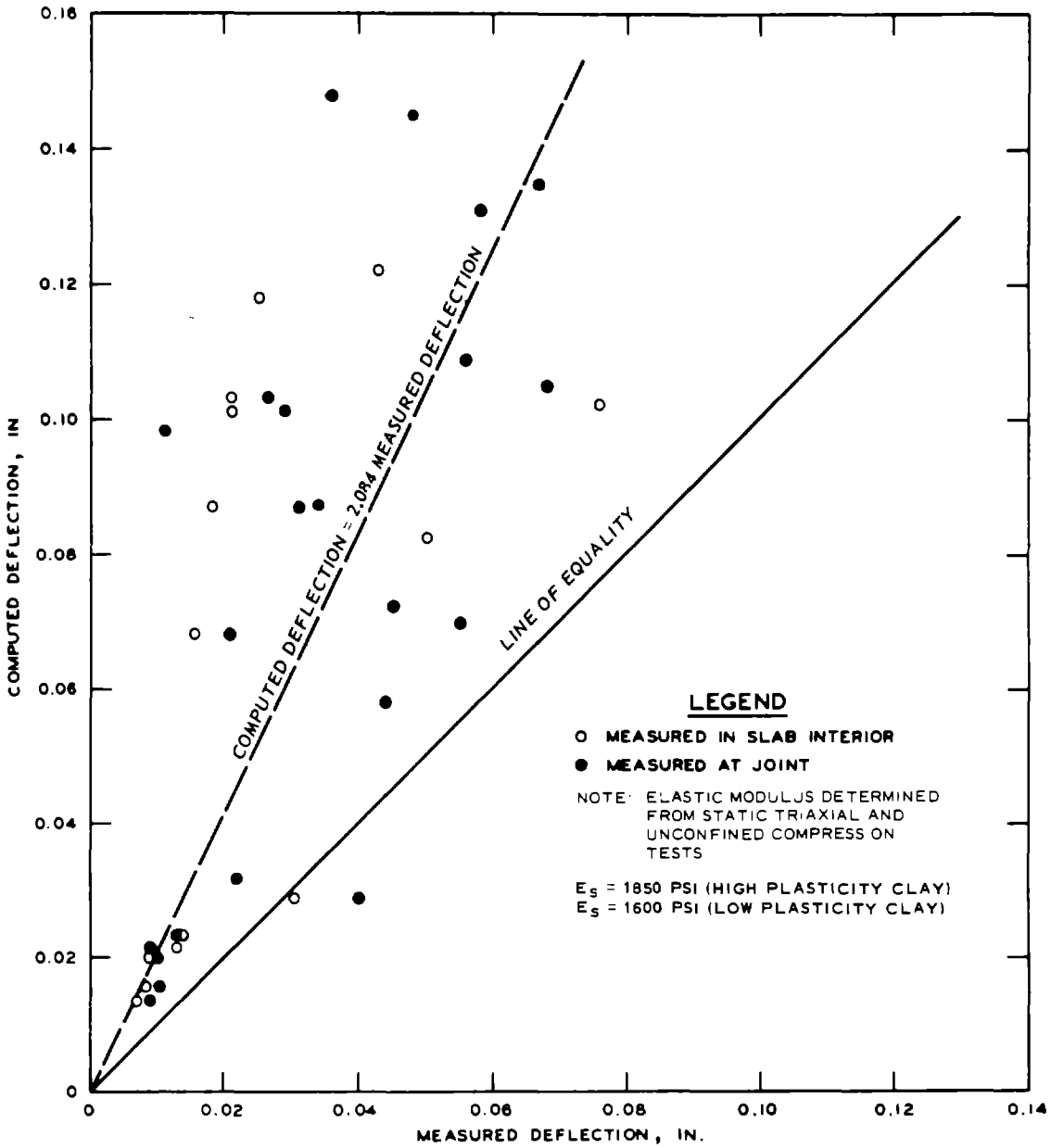


Figure A7. Comparison of measured slab deflection and deflections computed with the subgrade characterized with static triaxial and unconfined compression tests

closest to the line of equality. The coefficient from the linear regression was 0.950 for this case.

In the plots of measured versus computed slab bending strains (Figures A8-A11), the comparisons indicate that the use of repeated load test characterizations results in regression relationships closer to the line of equality than the use of static load tests. The use of static load tests results in overprediction of the strains. Once again, the use of the characterization with a repeated plate bearing test resulted in the correlation closest to the line of equality. The coefficient from the linear regression was 1.023 for this case.

The position of the regression lines relative to the lines of equality is interpreted to mean that the repeated load tests yield more representative characterizations than the static tests. In addition, it appears that the cyclic plate load test is the most accurate for the particular cases compared. Although accuracy of prediction of pavement response was a major factor, other factors entered into the final selection of the repeated load triaxial tests for characterizing subgrade soils.

CONSIDERATION OF STATE OF STRESS

EFFECTS OF STATE OF STRESS

The load deformation and strength characteristics of paving materials and subgrade soils are dependent on the state of stress at which they exist. For this discussion, consider that the materials are linearly elastic and that the magnitude of the modulus of elasticity and Poisson's ratio will depend on the state of stress.

Certain materials are more sensitive to the state of stress at which they exist in the pavements than are others. The modulus of granular materials has been related to confining pressure and overall state of stress. Two such relationships are illustrated in Figures A12 and A13 for two granular base course materials. In terms of strength, the effect of confinement may be explained by the influence of the angle of internal friction, as defined in the Mohr-Coulomb failure theory. The fact that granular materials have a relatively large angle

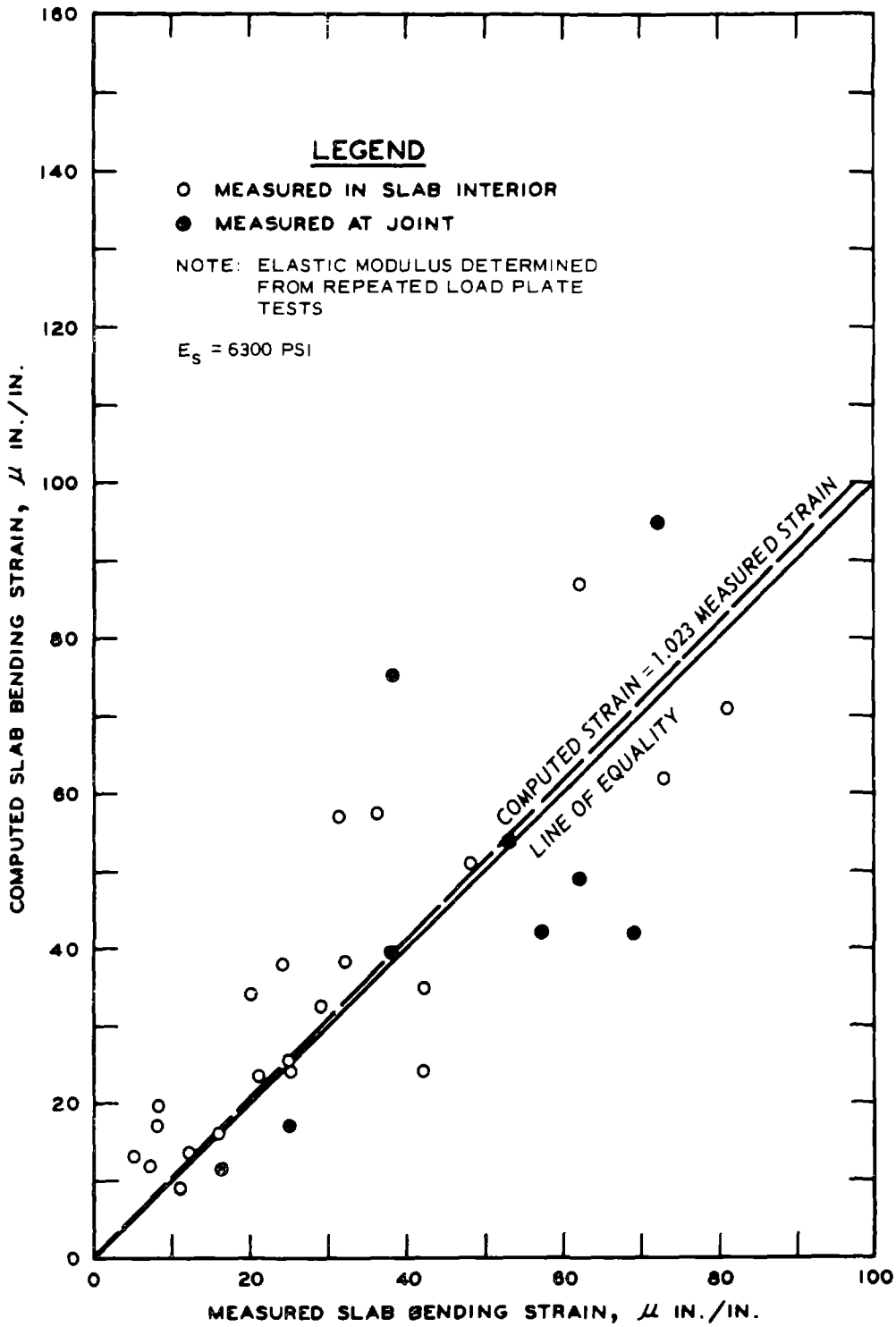


Figure A8. Comparison of measured slab bending strain and strains computed with the subgrade characterized with cyclic plate bearing tests

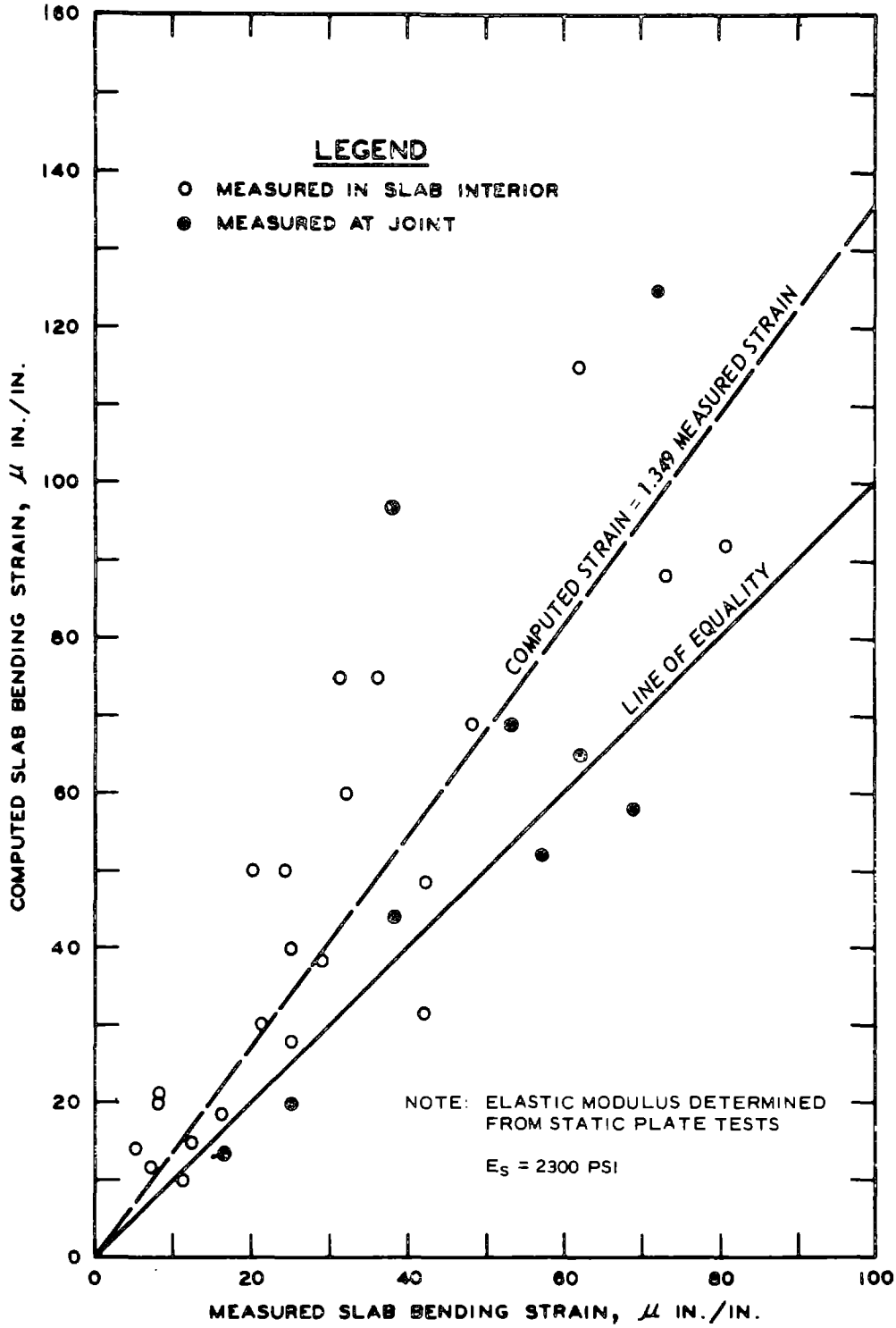


Figure A9. Comparison of measured slab bending strain and strains computed with the subgrade characterized with static plate bearing tests

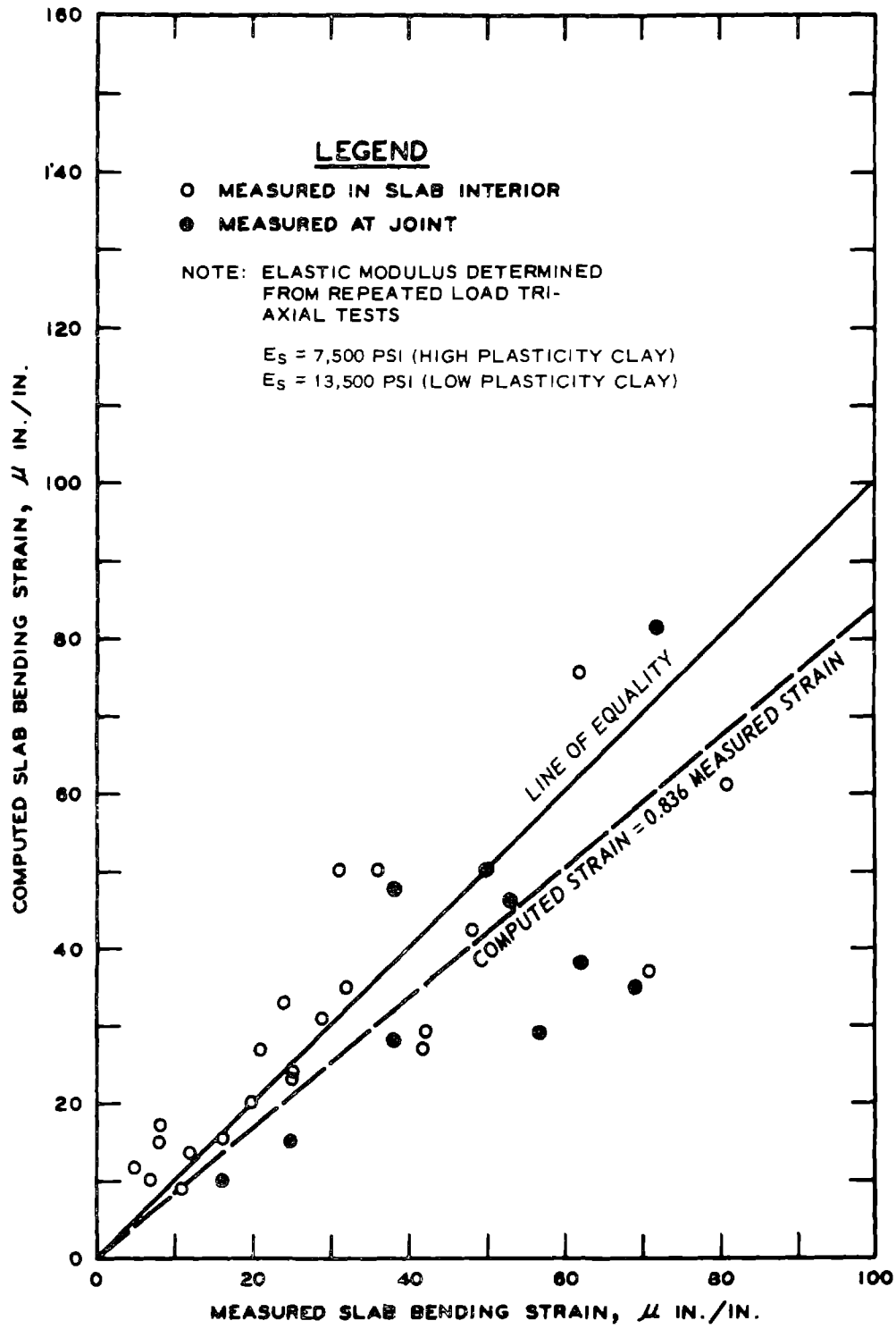


Figure A10. Comparisons of measured slab bending strains with strains computed with the subgrade characterized with repeated load triaxial tests

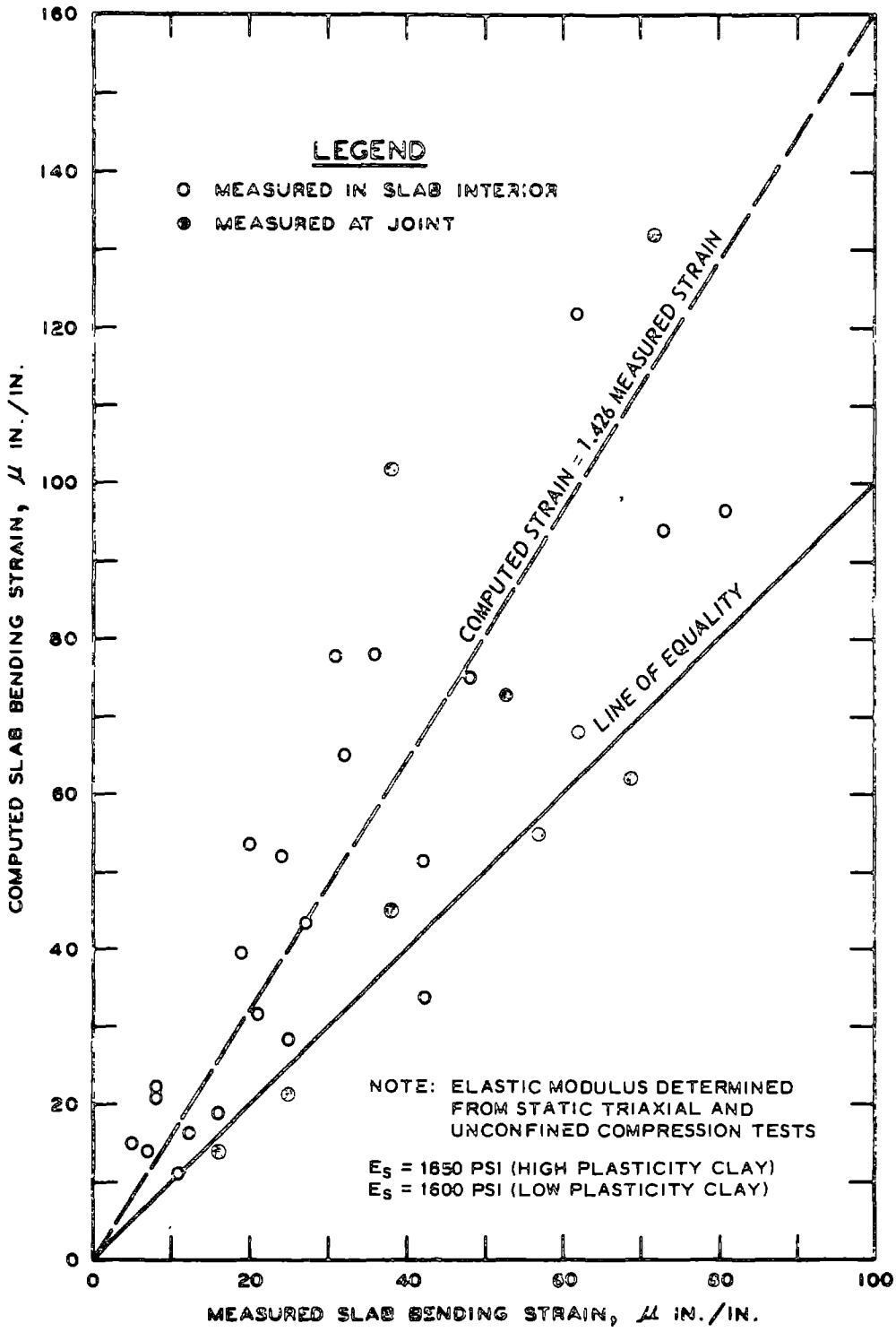


Figure A11. Comparison of measured slab bending strains with strains computed with the subgrade characterized with static triaxial tests

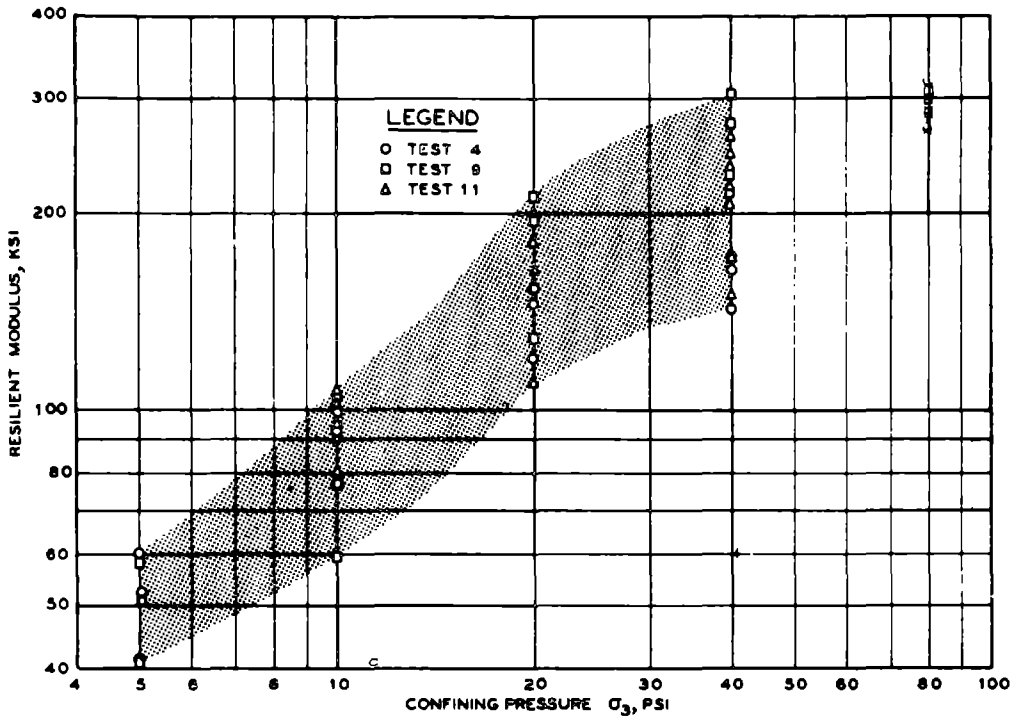


Figure A12. Resilient modulus data for gravelly sand as function of confining pressure[51]

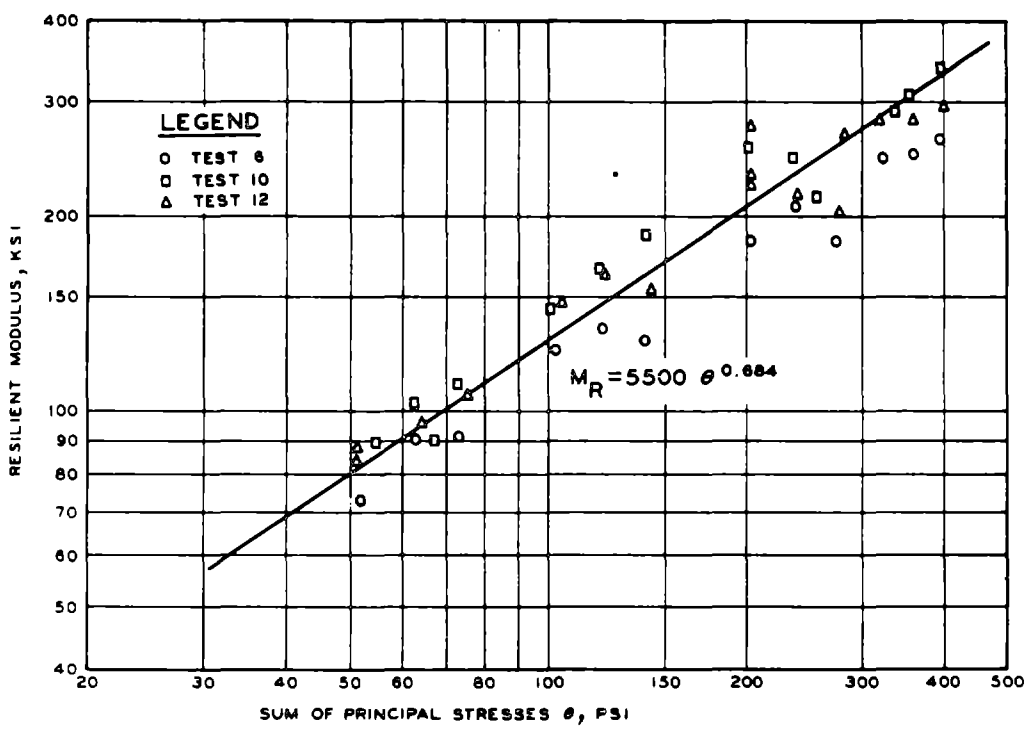


Figure A13. Resilient modulus data for dense crushed limestone[51]

of internal friction also accounts for the large influence of confining stress on the stiffness of granular materials.

Cohesive soils, in a saturated condition, are insensitive to confining pressure. This may be explained by the low angle of internal friction. However, cohesive soils are highly sensitive to the magnitude of the deviator stress applied (Figures A2 and A3).

PCC is sensitive to the state of stress, but the magnitude of the stresses at which the effects become significant is much larger than exists in pavements. Except directly beneath a tire, no large vertical or confining stress exists in PCC. The stress in the slabs is due primarily to bending, and a flexural test is considered the most appropriate for characterizing PCC.

The effects that the state of stress has on the properties of a bound base (subbase) are similar in certain aspects to PCC and in other aspects to the natural material without the stabilizing agent added. The relationship that exists between the strength and load deformation characteristics will depend on the type of natural material (cohesive or granular) and the degree of stabilization attained (defined as the development of bond between particles that results in the ability of the material to sustain flexural loading). The stresses in a bound base (subbase) are different from those in a PCC slab, which are essentially simple bending, and different from those in a granular base, which cannot sustain tensile stresses of any appreciable magnitude. The predominate stress mode is bending, but a vertical compressive stress component is also present. However, with the minimum strength requirement employed to ensure that the layer behaves as a layer of bound material, characterization with flexural tests is considered most appropriate.

Selection of States of Stress. Now that the influence of the state of stress on the properties of paving materials and subgrade soils has been established, a practical usable procedure for accounting for this effect is needed. As so often happens, a number of approximations of actual conditions are necessary.

The state of stress in a pavement layer or subgrade varies with

depth and with horizontal location. The exact distribution of stresses within a pavement structure will depend on the composition of the structure and the loading. Figures A14-A19 show examples of the variability that may be expected. Pavement structures are infinite in variety, and loading conditions may vary widely; however, Figures A14-A19 represent real pavement structures and loads. The general trends illustrated in these examples are representative of pavements for aircraft operation.

Interpretation of Figures A14-A19 and subsequent figures requires a definition of terms and the establishment of a sign convention for stresses. The sign convention used herein will be that compressive stresses are positive and tensile stresses are negative. The following terminology and relationships are established:

σ_1 = major principal stress
= principal stress with the largest numerical value

σ_3 = minor principal stress
= principal stress with the smallest numerical value

σ_2 = intermediate principal stress
= principal stress with a numerical value between the major and minor principal stress

$\sigma_d = \sigma_1 - \sigma_3$
= deviator stress

$\theta = \sigma_1 + \sigma_2 + \sigma_3$
= first stress invariant

The definition of deviator stress is consistent with the definition previously given for conditions in a triaxial test.

The stresses plotted in Figures A14-A19 were computed with the elastic layered model. The material properties used were obtained from a rather extensive testing program. However, the material characterization would not reflect precisely the influence of the state of stress. Only two constants are used for each material, but the stresses vary throughout each material. The validity of the computed stresses then is questionable and should be considered only as an approximation. One quickly gets into a "vicious cycle," and the futility of precise material characterization becomes apparent. To be precise, the material

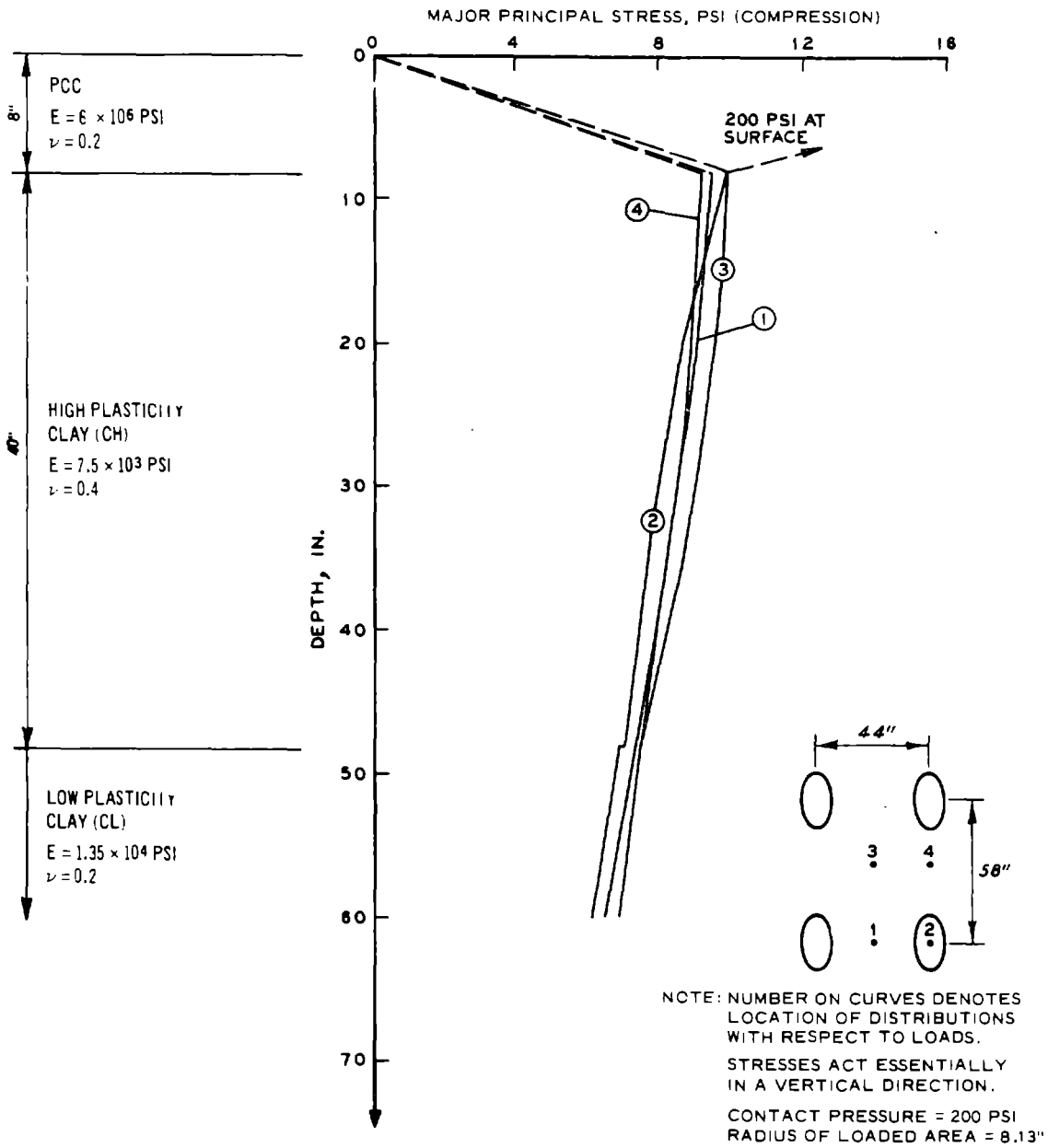


Figure A14. Distribution of major principal stress in a rigid pavement with no base loaded with a dual-tandem gear

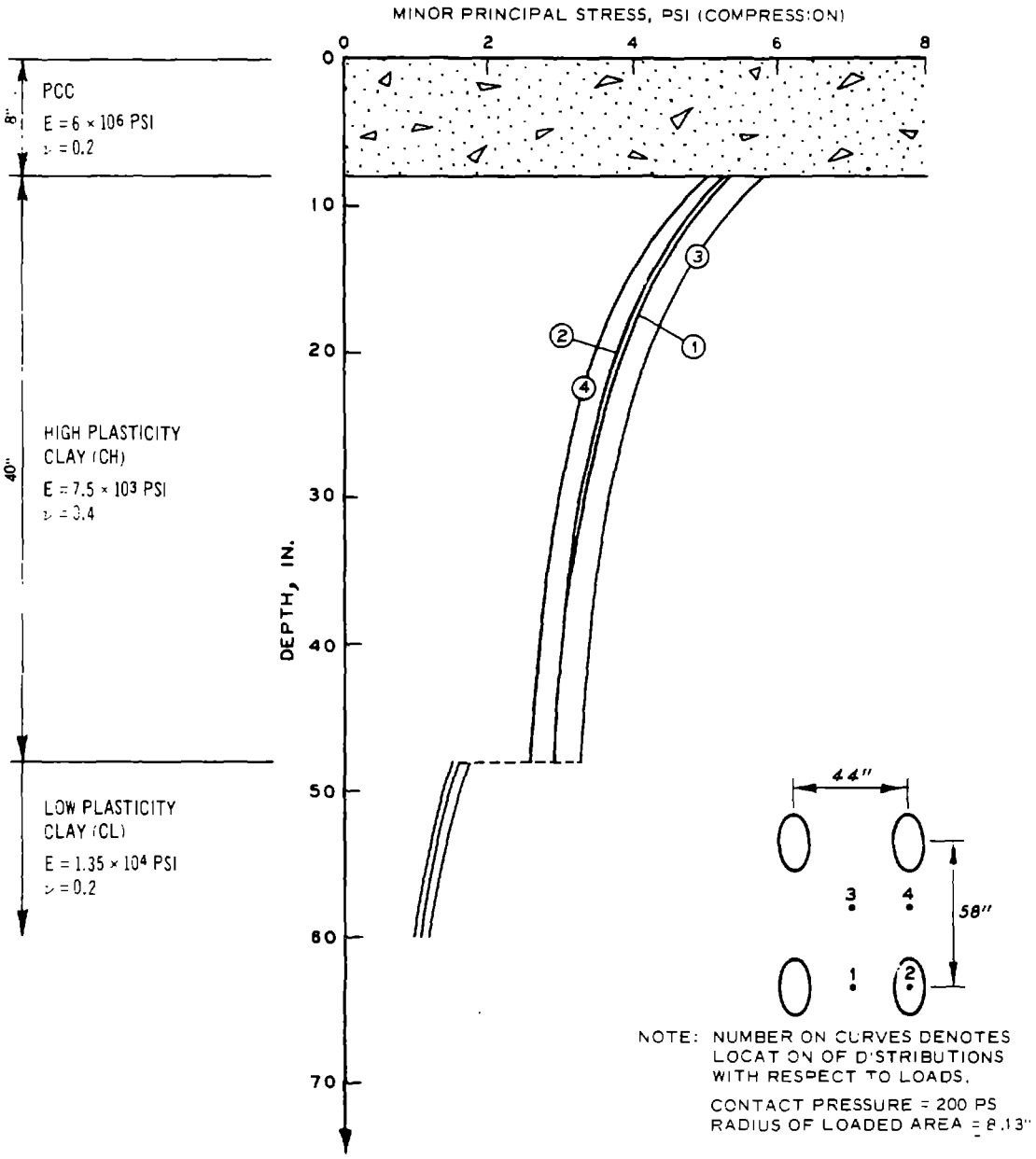


Figure A15. Distribution of minor principal stress in a rigid pavement with no base loaded with a dual-tandem gear

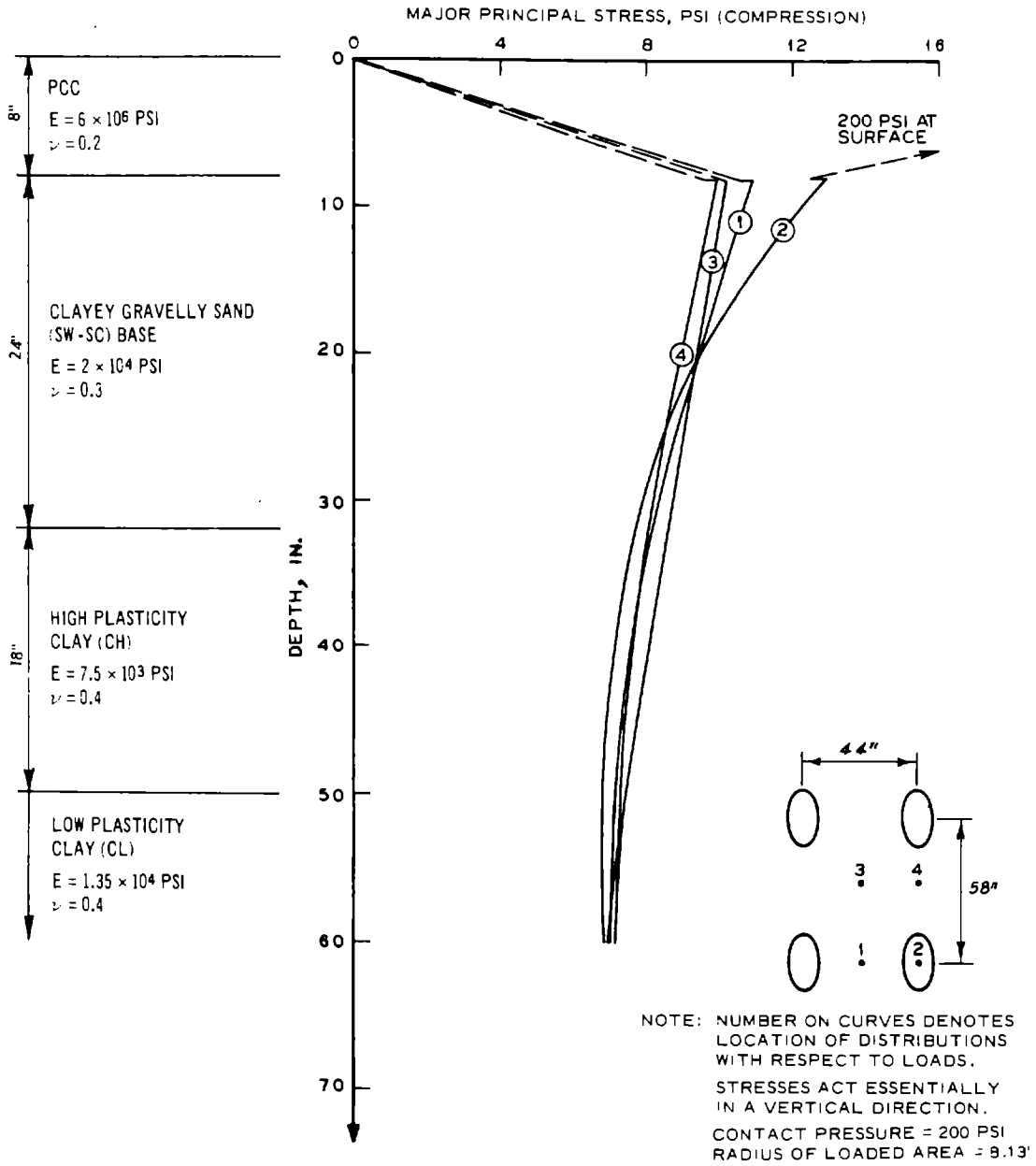


Figure A16. Distribution of major principal stress in a rigid pavement with a granular base loaded with a dual-tandem gear

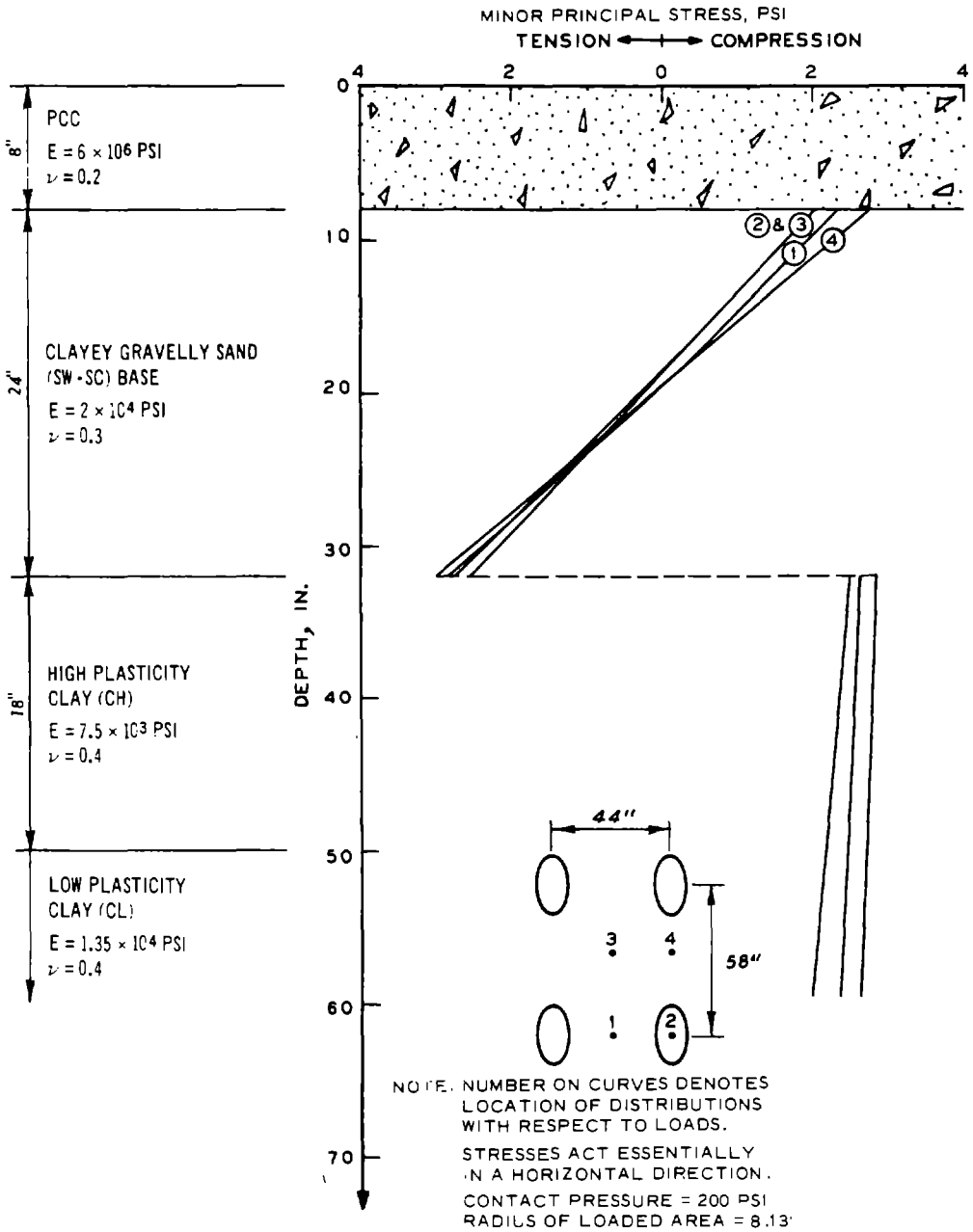


Figure A17. Distribution of minor principal stress in a rigid pavement with a granular base loaded with a dual-tandem gear

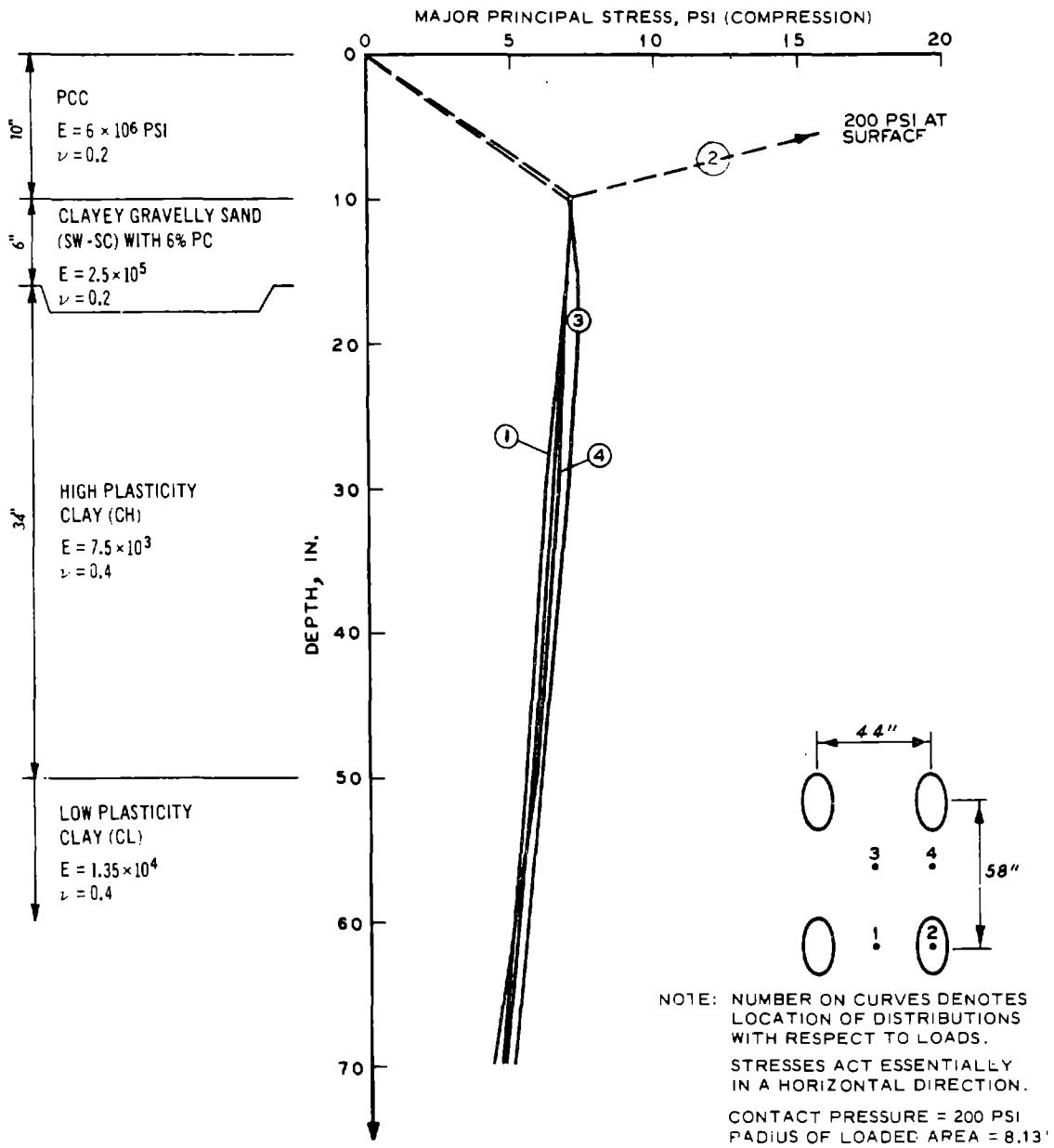


Figure A18. Distribution of major principal stress in a rigid pavement with a bound base loaded with a dual-tandem loading

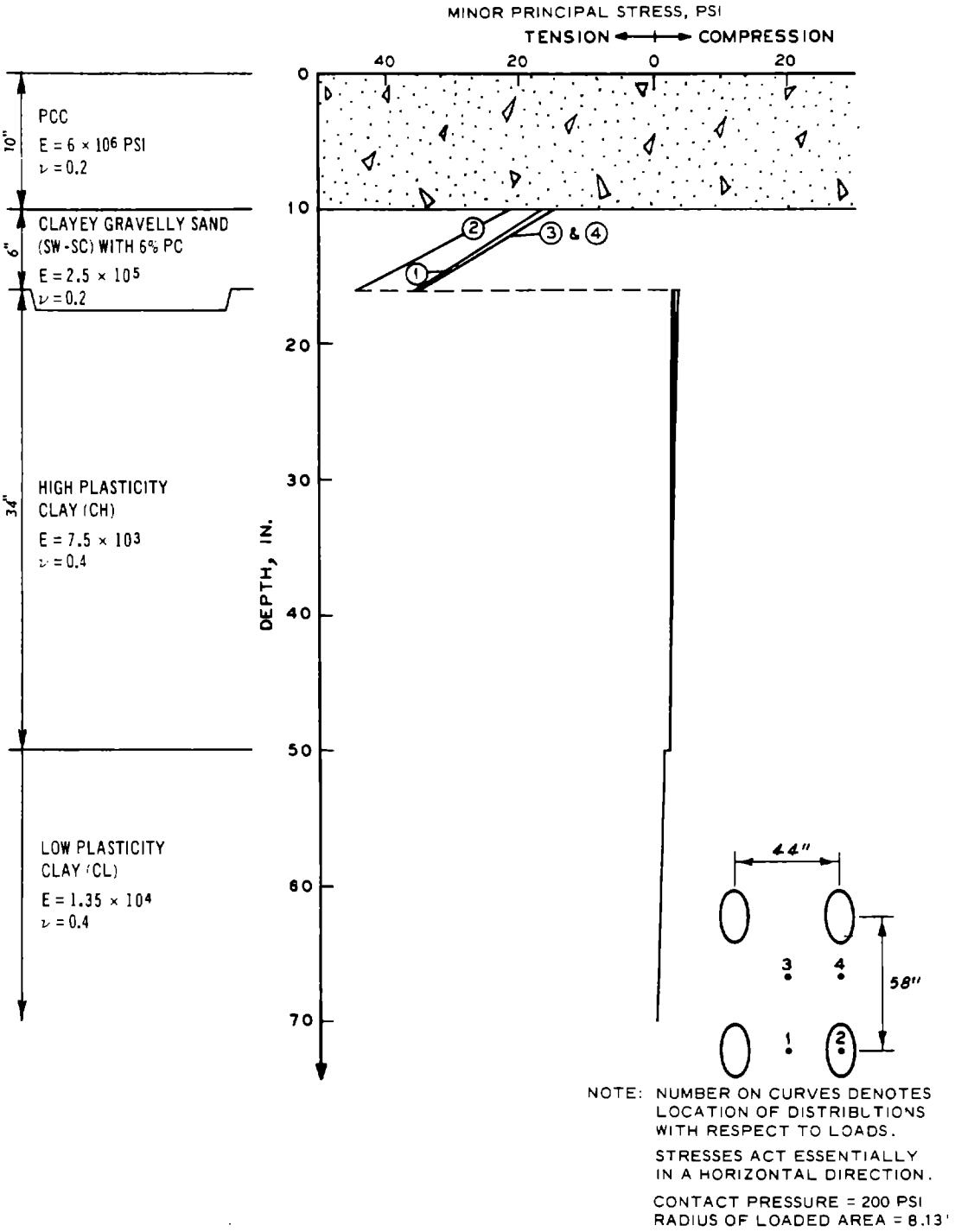


Figure A19. Distribution of minor principal stress for a rigid pavement with a bound base loaded with a dual-tandem gear

properties would have to be a continuous function of location within the pavement structure.

For the following discussion, assume that the stresses are reasonable approximations of the stresses that would actually exist in a real pavement loaded with a real aircraft. The following general observations can be made:

a. The stresses within a rigid pavement structure vary with depth, and the nature of the variation is dependent on the composition of the structure.

b. Stresses in the vicinity of the loads are fairly constant.

c. Stresses within a pavement structure vary with horizontal location as well as with depth.

The implications of these general observations on the selection of the state of stress at which to characterize paving materials and subgrade soils will be discussed in detail in the following paragraphs.

Figures A14-A19 show stress distributions with depth for three rigid pavements (PCC slab directly on a clay subgrade, PCC slab on a granular base, and PCC slab on a bound base) loaded with a dual-tandem aircraft gear. The stresses are the major principal stress (essentially the vertical component of stress) and the minor principal stress (essentially one of the horizontal components of stress). The minor principal stress is the stress that will result in the maximum difference when subtracted from the major principal stress. Signs of the stress are considered when computing the deviator stress and first stress invariant. As an example, if the principal stresses at a point were -12.0, -9.82, and 5.55 psi, the major principal stress would be 5.55 psi and the minor principal stress of -12.0 psi would be used to compute a deviator stress of 17.5 psi. However, if the principal stresses were 2.66, 2.79, and 5.58 psi, the major principal stress would be 5.58 psi and the minor principal stress of 2.66 psi would be used to compute a deviator stress of 2.92 psi. The general trends illustrated in Figures A14-A19 are that the major principal (vertical) stress decreases at a rather slow rate with depth and that the variation in the minor principal (horizontal) stress will depend on the composition of the pavement structure.

Figures A14 and A15 show distributions for a pavement composed of a PCC slab directly on a clay subgrade. The stress parameter of primary interest for characterizing cohesive soils is the deviator stress. Combining the distributions in Figures A14 and A15 results in the distribution of deviator stress in Figure A20. Since the modulus of cohesive materials varies with deviator stress, the modulus of the clay subgrade should also vary with depth. However, from a practical point of view, the variability is not really that great and the change is rather gradual. This implies that characterization at a given deviator stress will be applicable for appreciable depths (i.e., 5 psi for at least 5 ft for the example under consideration). The same type patterns were observed in the subgrade beneath the granular and bound bases.

Figures A16 and A17 show distributions for PCC slabs on a 24-in.-thick granular base over the clay subgrade. Of interest in Figure A16 is the fact that the nature of the distribution and magnitude of the stresses are about the same as in Figure A14 for the PCC slabs directly on the clay subgrade. However, the minor principal stresses in the granular base (Figure A17) are quite different from the stresses in the subgrade (Figure A15). There is some small compressive (confining) stress at the top of the granular base layer, but at the bottom, the bending action has resulted in tensile stresses. Although the tensile stresses are small, granular bases are normally considered incapable of sustaining tensile stresses. Part of this effect may be caused by the assumption of complete continuity at the interface between the PCC slab and the granular base. If some slip were permitted at the interface (which is probably what actually occurs), the compressive stress at the top of the layer would be larger and the tensile stress at the bottom of the layer would be smaller. This is illustrated in Figure A21, which shows distributions of horizontal stresses for the case where the PCC layer is free to slip horizontally relative to the granular base layer and the case where there is complete continuity at this interface. These horizontal stresses are principal stresses since they are located along a line through the center of a circular loaded area where the shear stresses are zero. For comparative purposes, these may be considered equivalent to the minor principal stresses previously discussed.

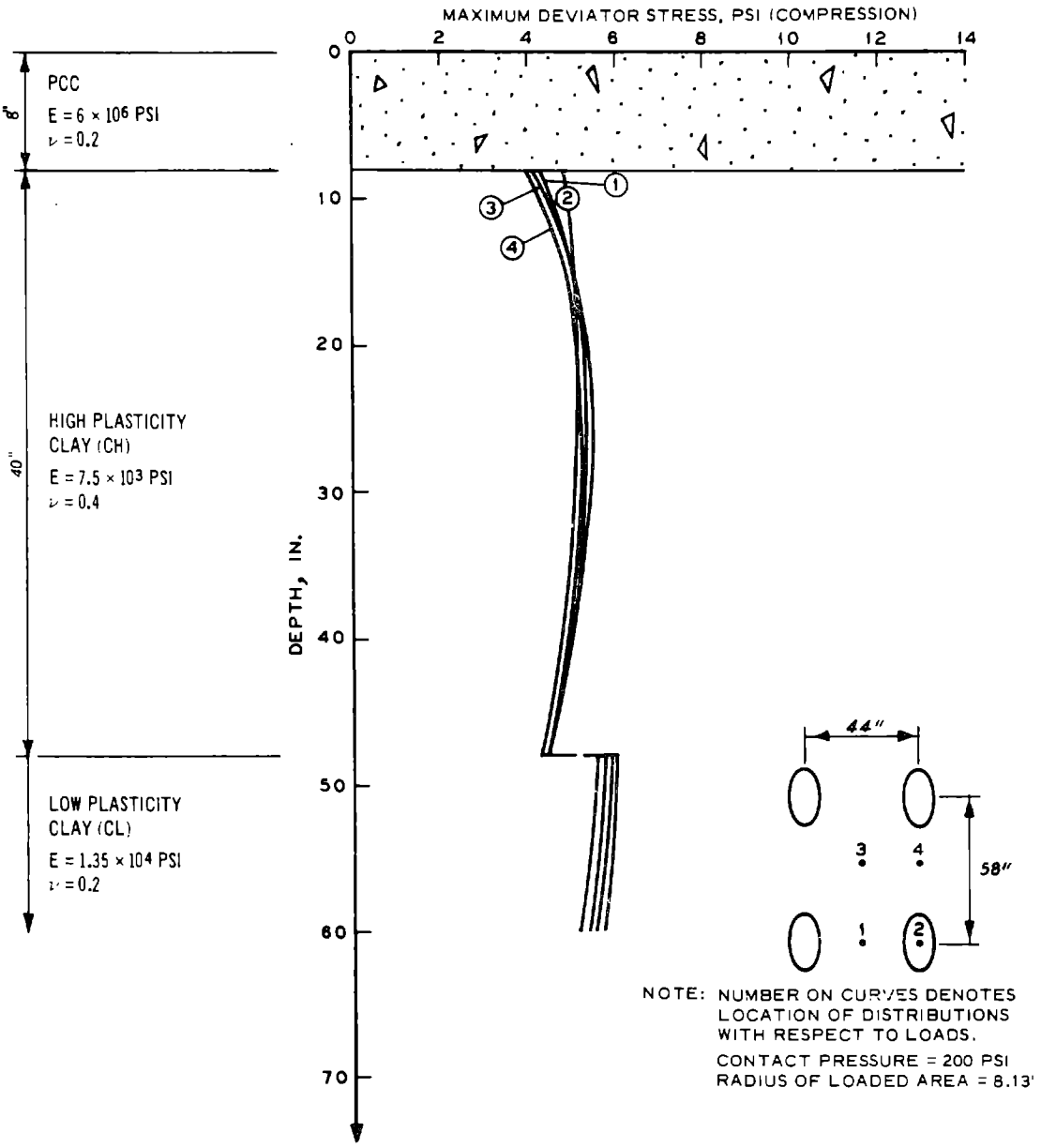


Figure A20. Distribution of maximum deviator stress in a rigid pavement with no base loaded with a dual-tandem loading

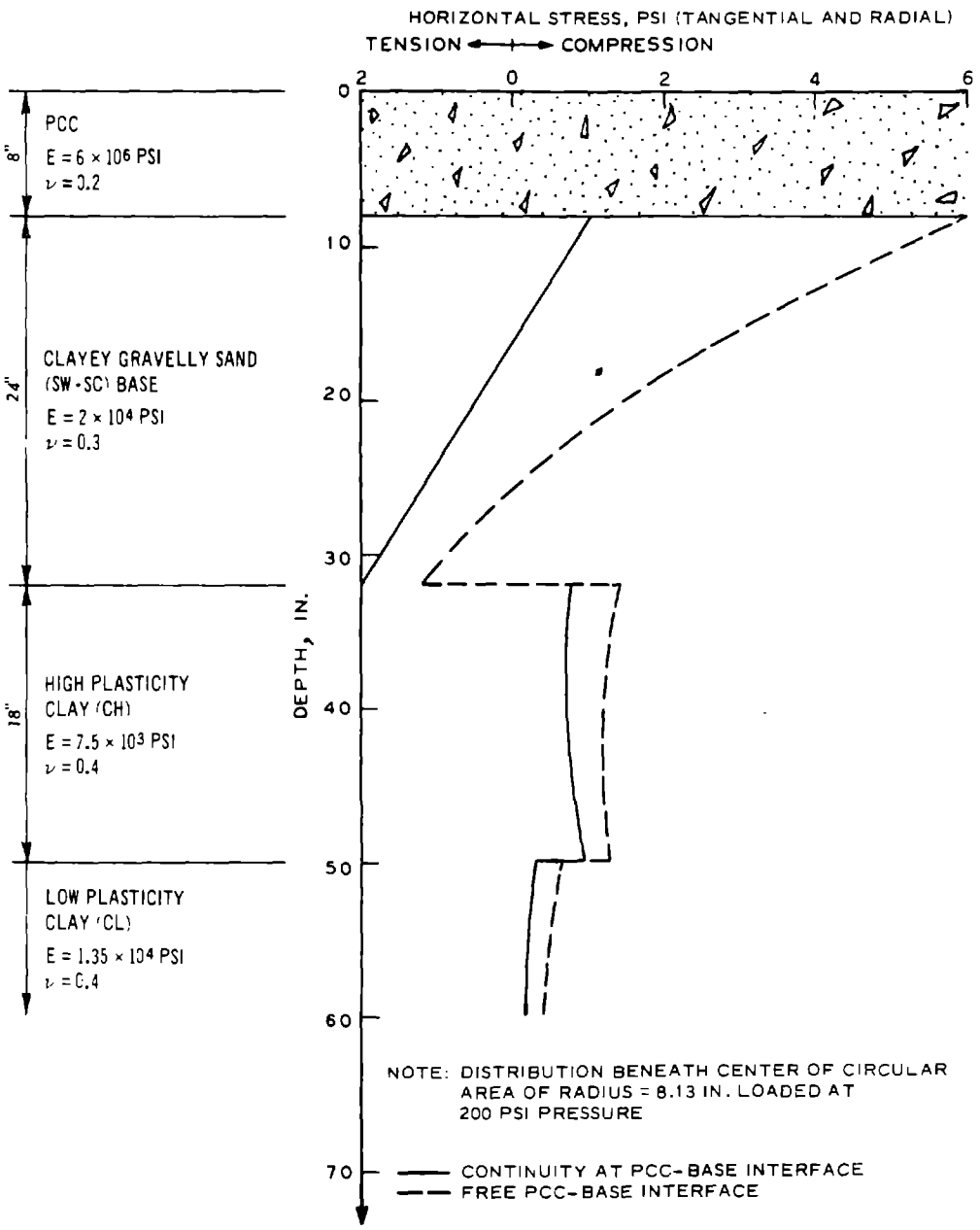


Figure A21. Horizontal stress distribution illustrating effect of interface condition between PCC slab and granular base

However, for the case of a free interface condition, the horizontal (compressive) stress in the top of the base layer may be greater than the vertical (compressive) stress and, thus, will be the major principal stress.

The distributions were generated with the CRANLAY computer code described in Reference 22. Continuity is maintained in the vertical direction at all interfaces at all times and in the horizontal direction at all interfaces other than the one case previously described. Since granular materials are highly dependent on the state of stress, particularly the confinement, the important fact is that there is a variation with depth for both interface conditions.

One of the primary stress parameters affecting the properties of granular materials is the first stress invariant. The general trend is that the resilient modulus of granular materials increases as the first stress invariant increases. In Figure A22, the first stress invariant, and thus the resilient modulus, decreases with depth through the granular base course.

Figures A18 and A19 show stress distributions for a 10-in. PCC slab on a 6-in. bound base over a clay subgrade. In these figures, note that the distribution of major principal stress is about the same as that in Figures A14 and A16 and that the minor principal stress is tensile throughout the entire depth of the bound base.

The magnitude of the stresses shown was influenced by the assumption of full continuity at the PCC slab-bound base interface and the ratio of the moduli of the PCC slab and the bound base material. Figure A23 illustrates the effect of interface condition in the horizontal stress. The effect of horizontal slip at the interface between the PCC slab and the bound base layer is to reduce the tensile stresses in the bound base layer. When horizontal slip is permitted at this interface, the two layers respond as separate slabs. The stresses in the top of the bound base layer are compressive and may be larger than the vertical component. When full continuity is assumed at the interface, the PCC and bound base layers tend to function as a composite beam and the stresses in the top of the bound base layer are tensile.

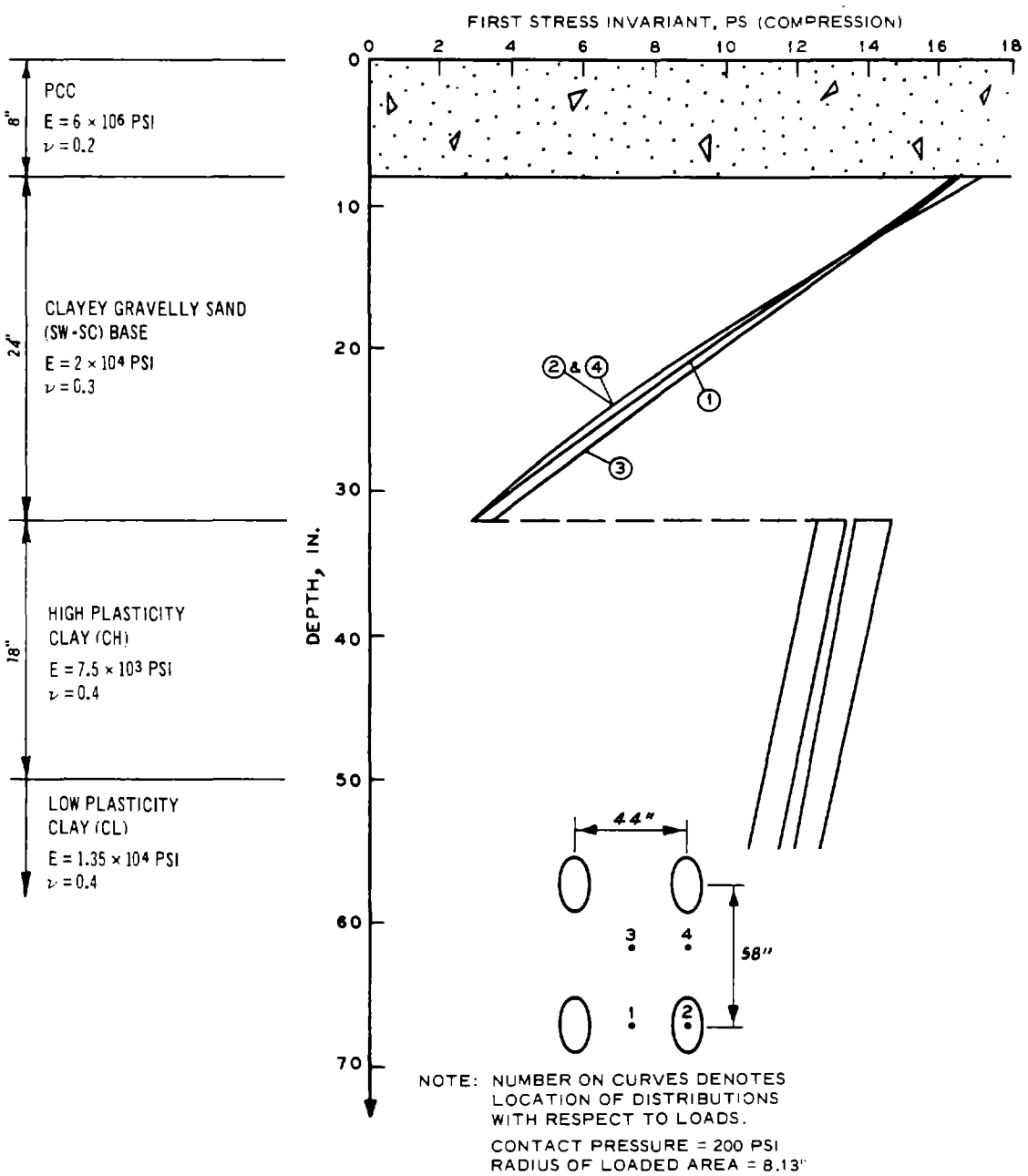


Figure A22. Distribution of the first stress invariant in a rigid pavement with a granular base loaded with a dual-tandem gear

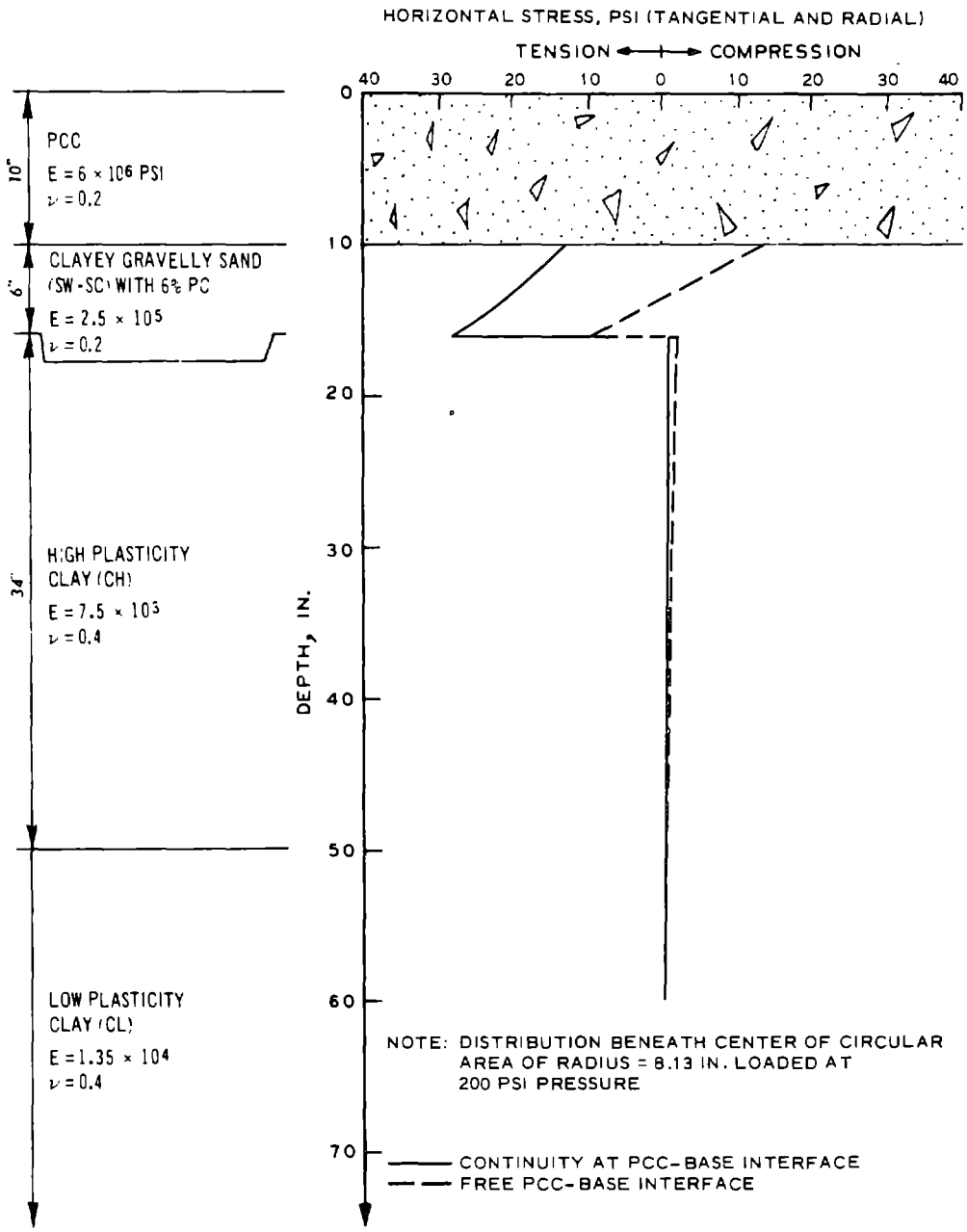


Figure A23. Horizontal stress distribution illustrating effect of interface condition between PCC slab and bound base

Figure A24 shows the effect of the modulus ratio between the PCC slab and the base material. The smaller the ratio the more pronounced the bending. For full continuity, this means larger tensile stresses in the top of the base course.

From the standpoint of material characterization, the important fact is that the deformation pattern in the bound base course is basically the same for free or fully continuous interface conditions and for a range of modulus ratios. This basic pattern of deformation, bending, is illustrated in Figure A25, which shows the variation of the first stress invariant. Implications are that a flexural test is appropriate for testing bound base material.

The effects of gear configuration on stresses within a rigid pavement may be studied by comparing Figures A26-A29 with Figures A16 and A17. These six figures are for the pavement containing the granular base, but Figures A26 and A27 are for a dual-wheel load and Figures A28 and A29 are for one of the four C-5A gears, which contain six wheels. The dual-wheel load is the same as two of the tires of the dual-tandem. The basic patterns and even the actual magnitudes of the stresses are similar. The implication of these comparisons is that for practical purposes, the gear configuration is not a major factor in determining the distribution of stress with depth in a rigid pavement.

The second general observation made was that the stresses in the vicinity of the loads on any horizontal plane are fairly constant. This is illustrated by the proximity of the curves, even for the C-5A gear (Figures A28 and A29) that is composed of six widely spaced wheels. The relatively large stiffness of the PCC slab is primarily responsible for this. Compared with a flexible pavement, the load is distributed uniformly to the underlying material over a large area. The implication of this is that selection of the state of stress at which to test material for rigid pavement design is not as critical as one might determine from a study of the overall sensitivity of the material properties to the state of stress.

The third general observation is related to the second. Although

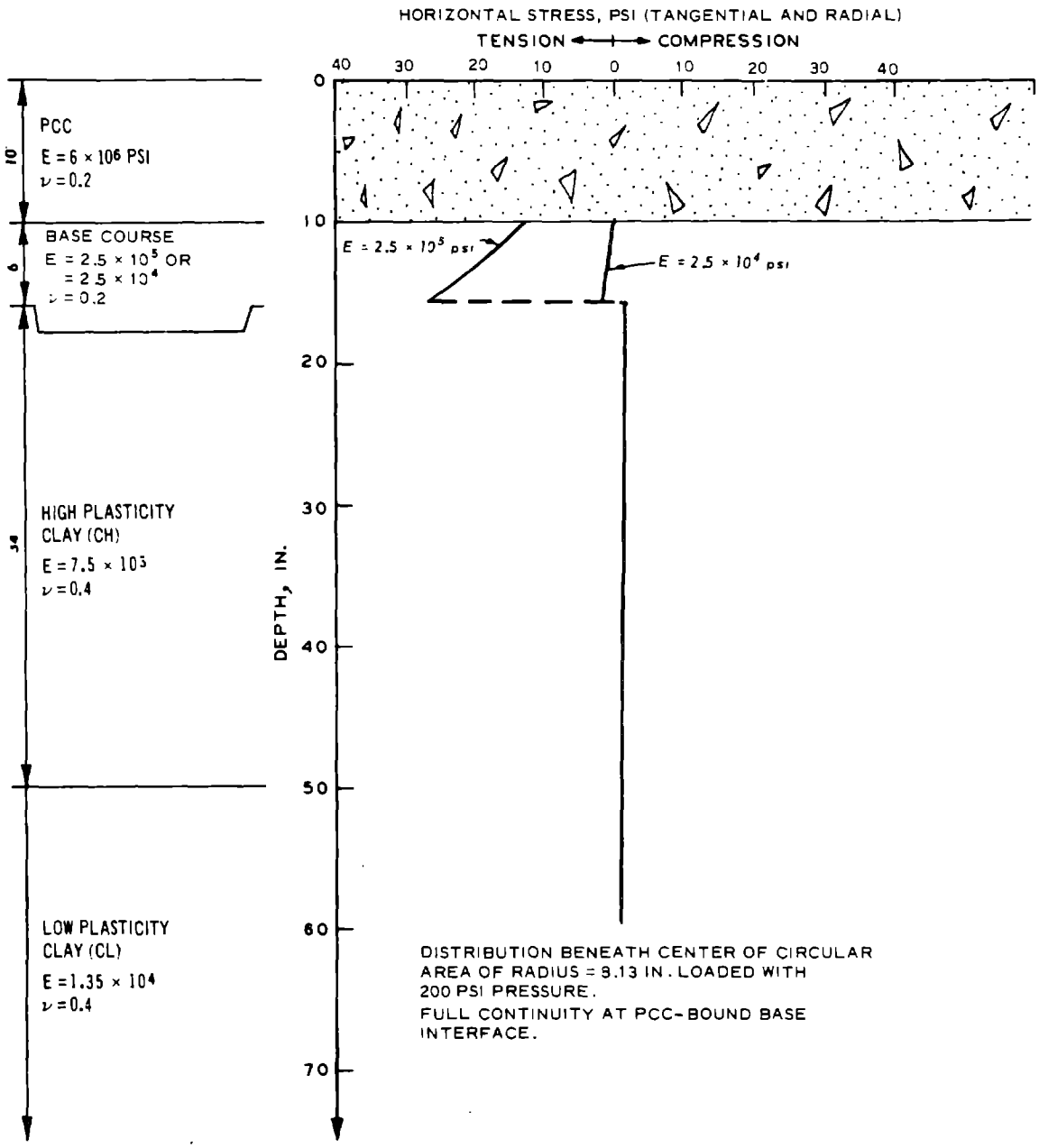


Figure A24. Horizontal stress distribution illustrating effect of ratio of moduli of PCC slab and base layer

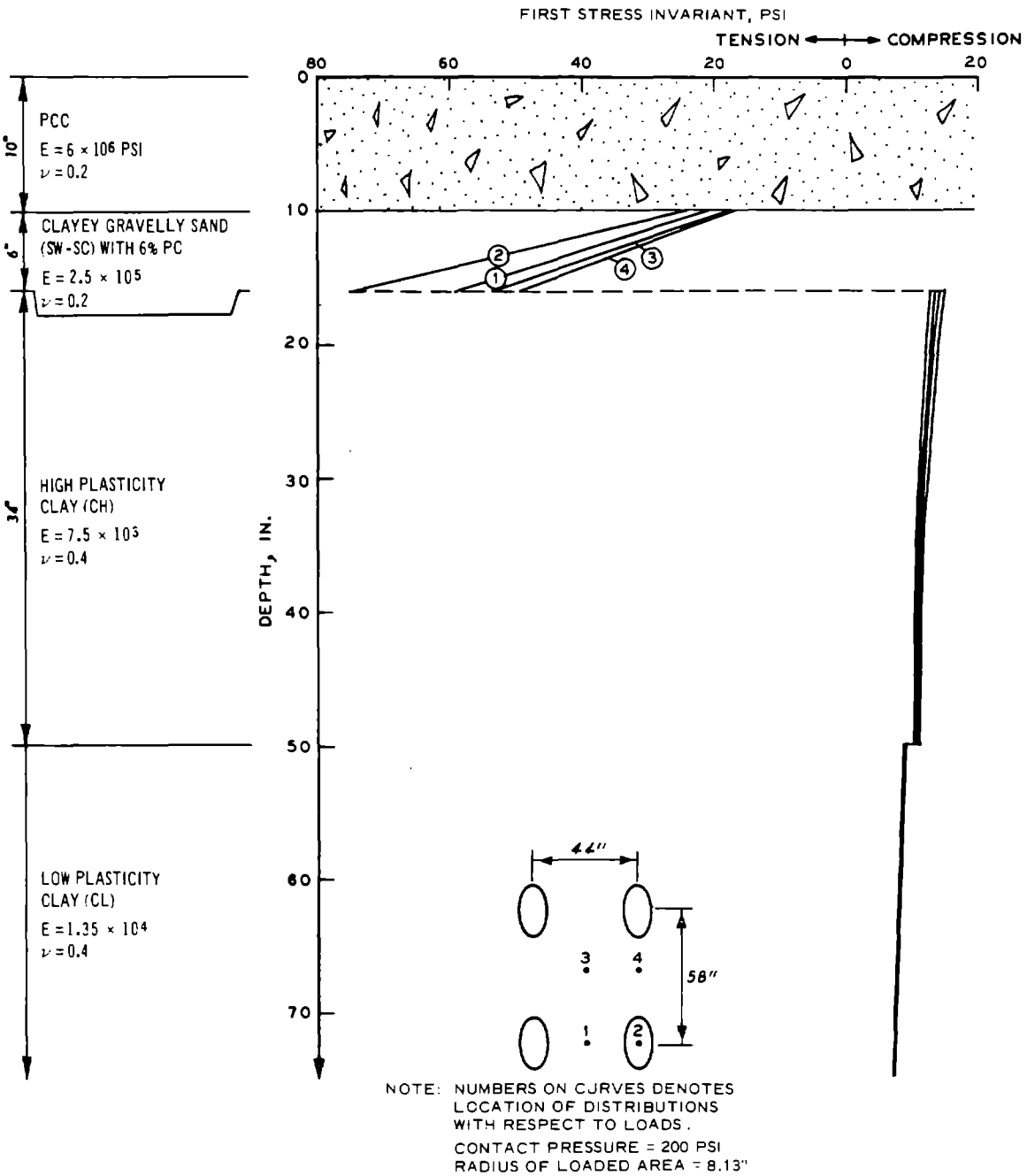


Figure A25. Distribution of the first stress invariant for a rigid pavement with a bound base loaded with a dual-tandem gear

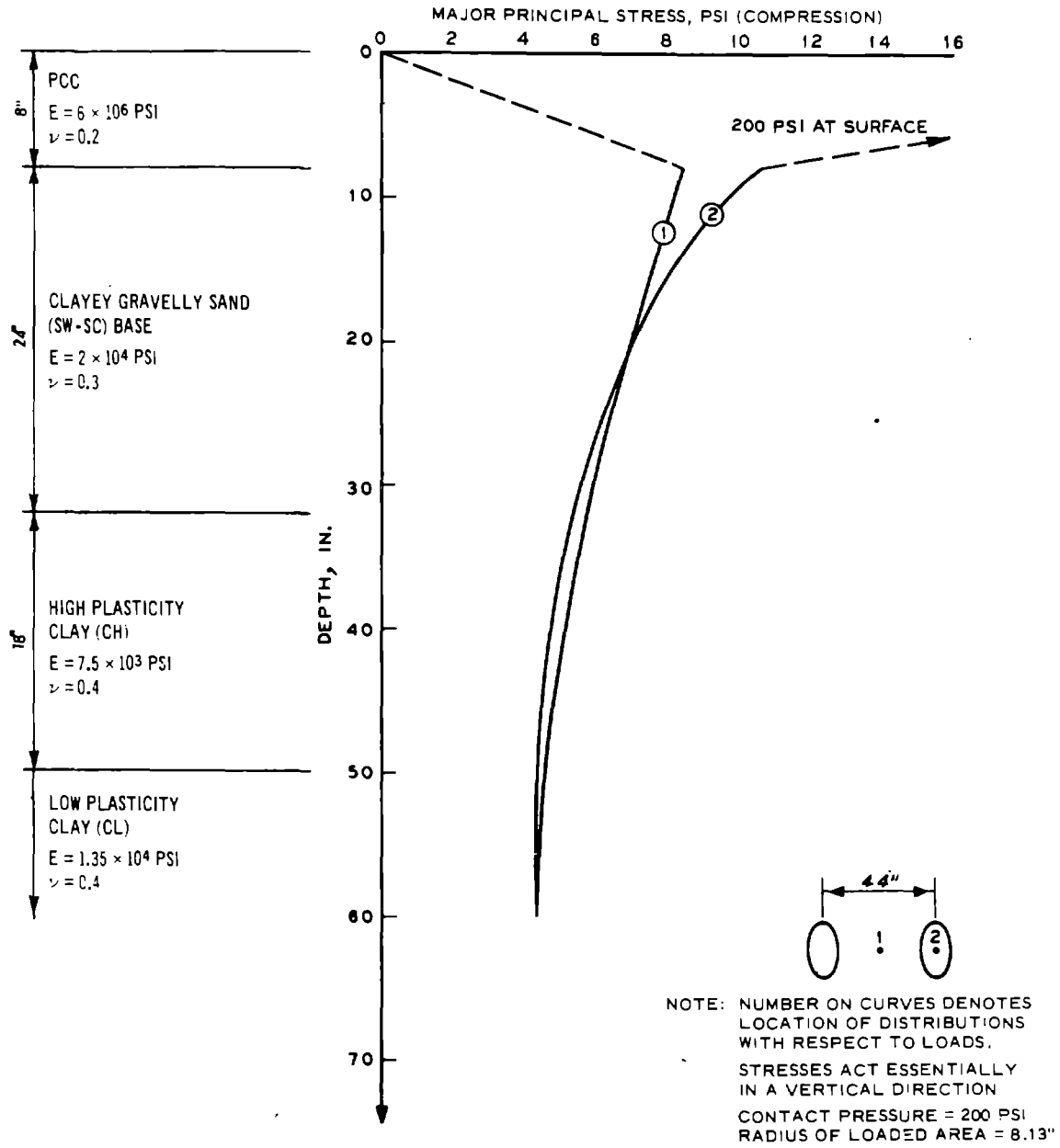


Figure A26. Distribution of major principal stress in a rigid pavement with a granular base loaded with a dual-wheel gear

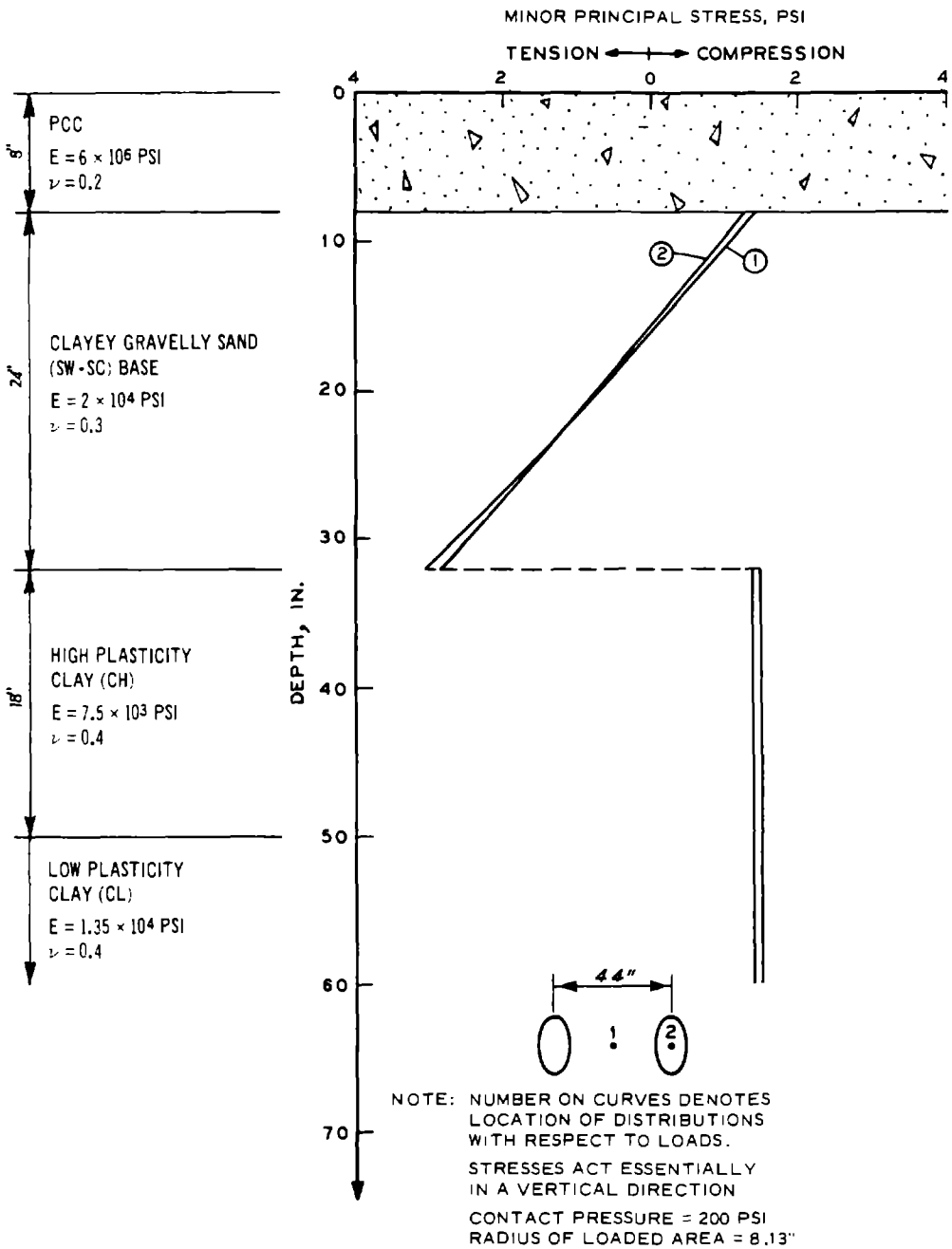


Figure A27. Distribution of minor principal stress in a rigid pavement with a granular layer loaded with a dual-wheel gear

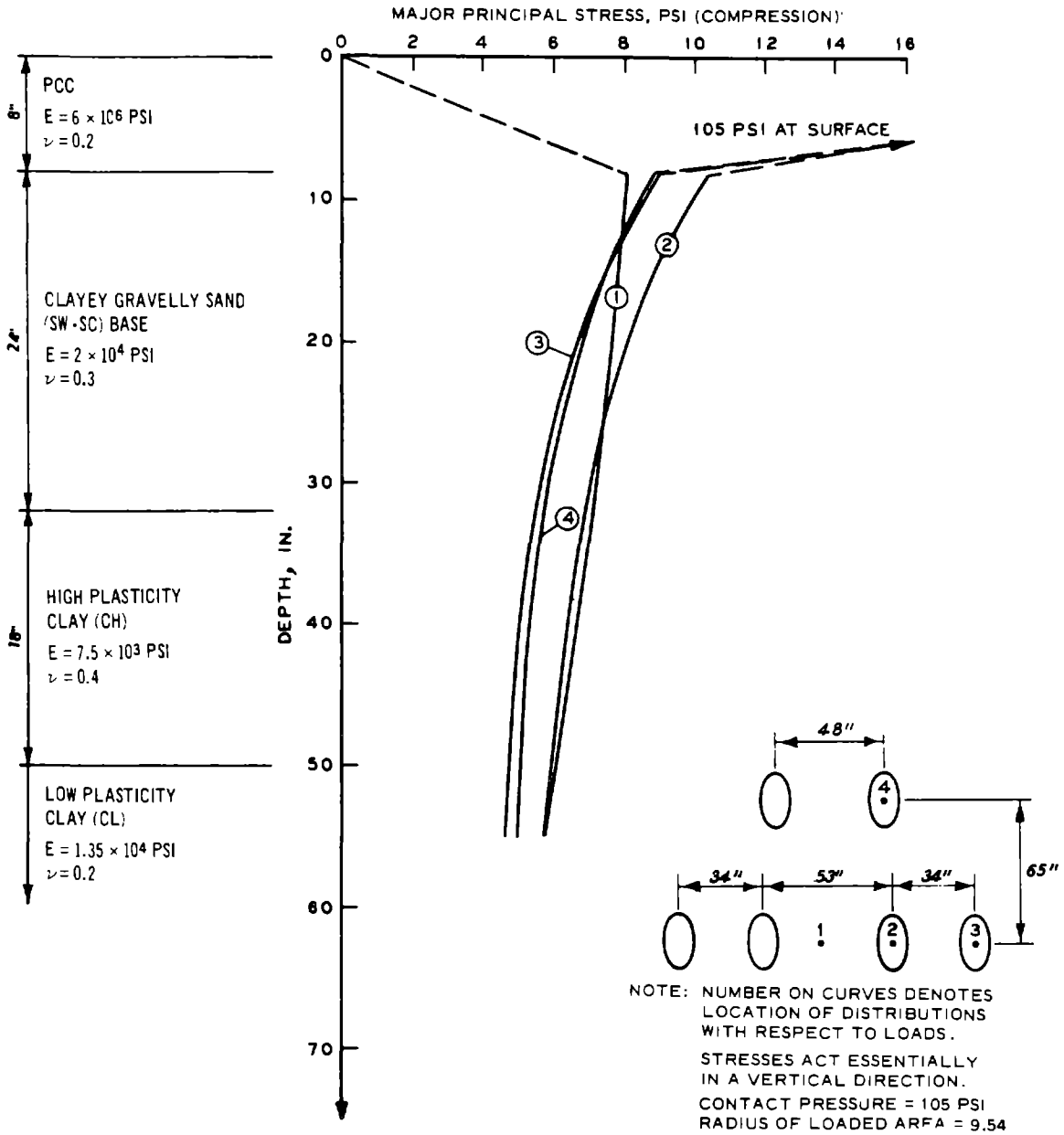


Figure A28. Distribution of major principal stress in a rigid pavement with a granular base loaded with a C-5A gear

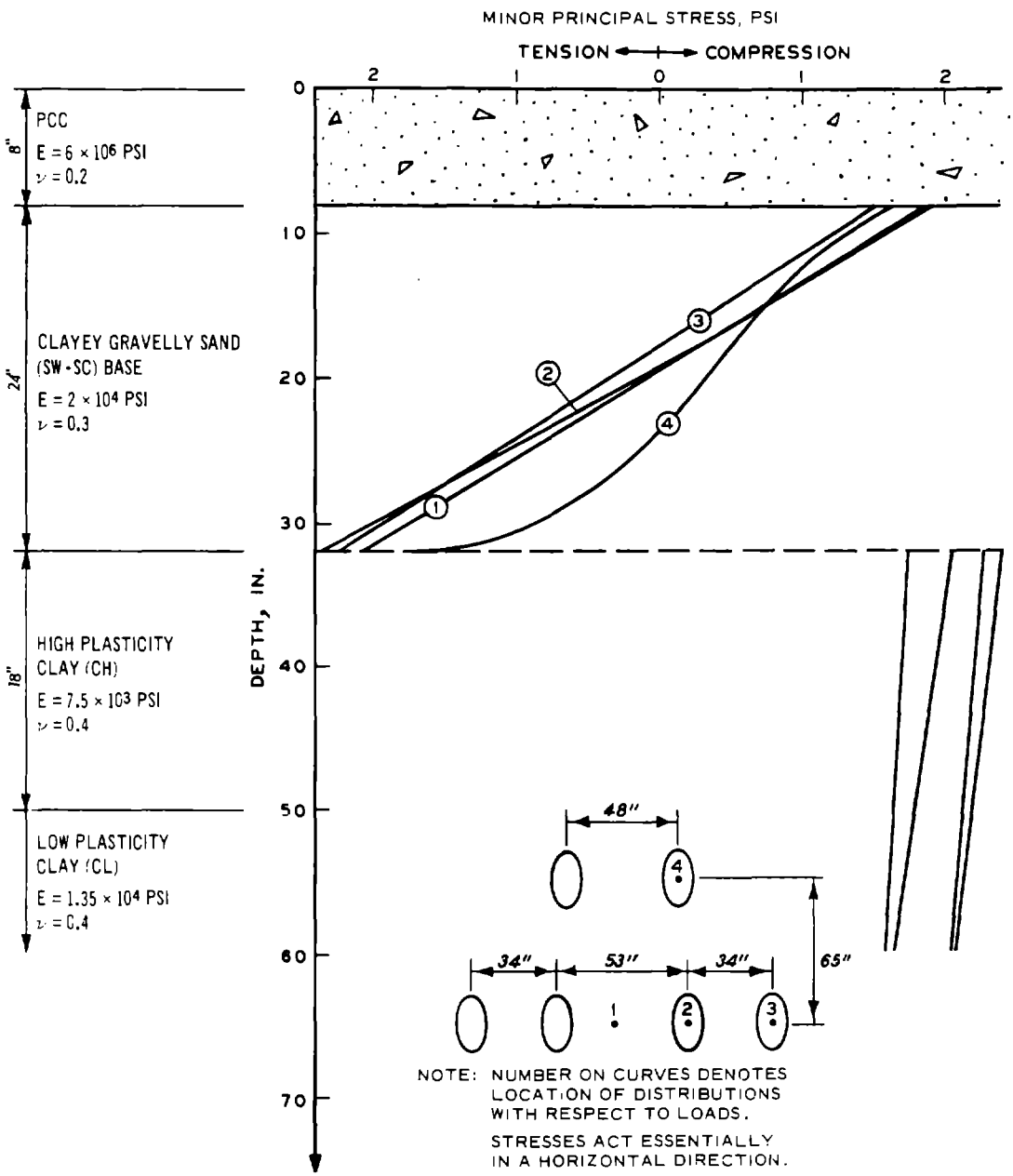


Figure A29. Distribution of minor principal stress in a rigid pavement with a granular base loaded with a C-5A gear

the stresses in a pavement at a particular depth may be relatively constant in the vicinity of the loads, they will decrease as the distance from the load increases. Examples of this are shown as plots of major and minor principal stress versus horizontal location for the pavement with the granular base loaded with the C-5A gear in Figures A30 and A31. The rate at which the stresses decrease will be largely dependent on the thickness of the PCC slab, but implications are that the properties of a material that are sensitive to the state of stress will vary as a moving load passes a given point on the pavement.

The effects of the assumptions of full continuity at layer interfaces and the modulus ratio of PCC and base course materials on the stresses within the pavement have been mentioned previously. The major principal stress (vertical stress) is not affected to any great extent by a base course. The shape and the magnitude of the stress distributions in Figures A14, A16, and A18 are similar.

Figure A32 illustrates the effect of the interface condition between the PCC slab and the adjacent material. In this particular illustration, the base course has properties of a bound material, and the stresses in the upper part of the base course are different for the fixed and free conditions. Within the subgrade, the stresses are similar. This is the general pattern for all types of pavements. Due to the magnitude of the stresses relative to the compressive strength and deformation characteristics of paving materials, the difference is not considered significant. The minor principal stress (horizontal stress) is significantly affected by the modulus ratio between the PCC slab and the adjacent material, as shown by comparing the stresses beneath the PCC slab in Figures A15, A17, and A19. Also, Figures A21 and A23 illustrate the effects of the interface condition on the minor principal stress (horizontal stress).

The ratio of the thicknesses of the PCC slab and the base course will also affect the stresses in the base course. In general, the smaller the thickness ratio, the more pronounced the bending. With full continuity between the PCC slabs and the second layer, the second layer will function in conjunction with the PCC slab as a composite beam

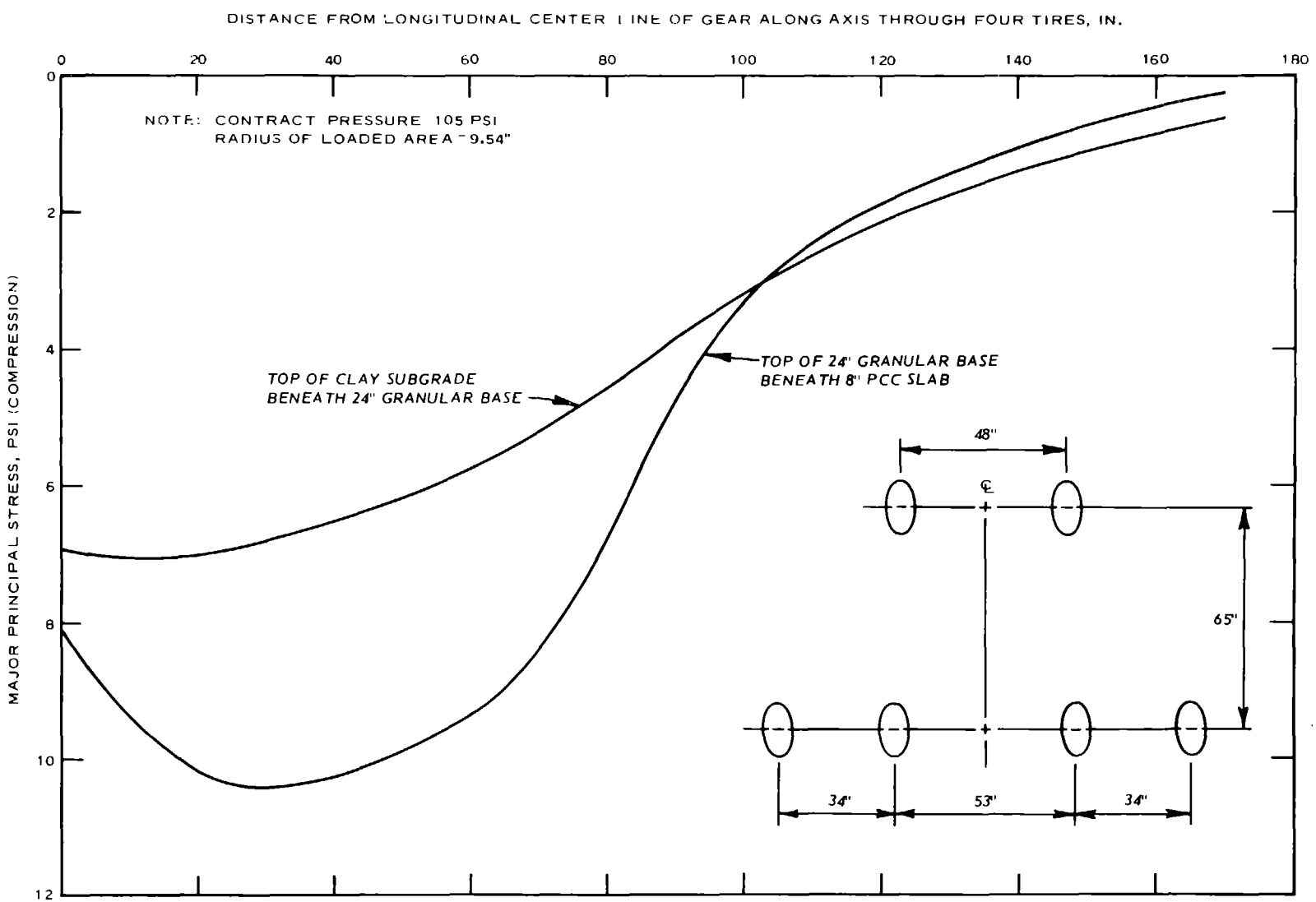


Figure A30. Variation of major principal stress at the top of the granular base and at the top of the subgrade

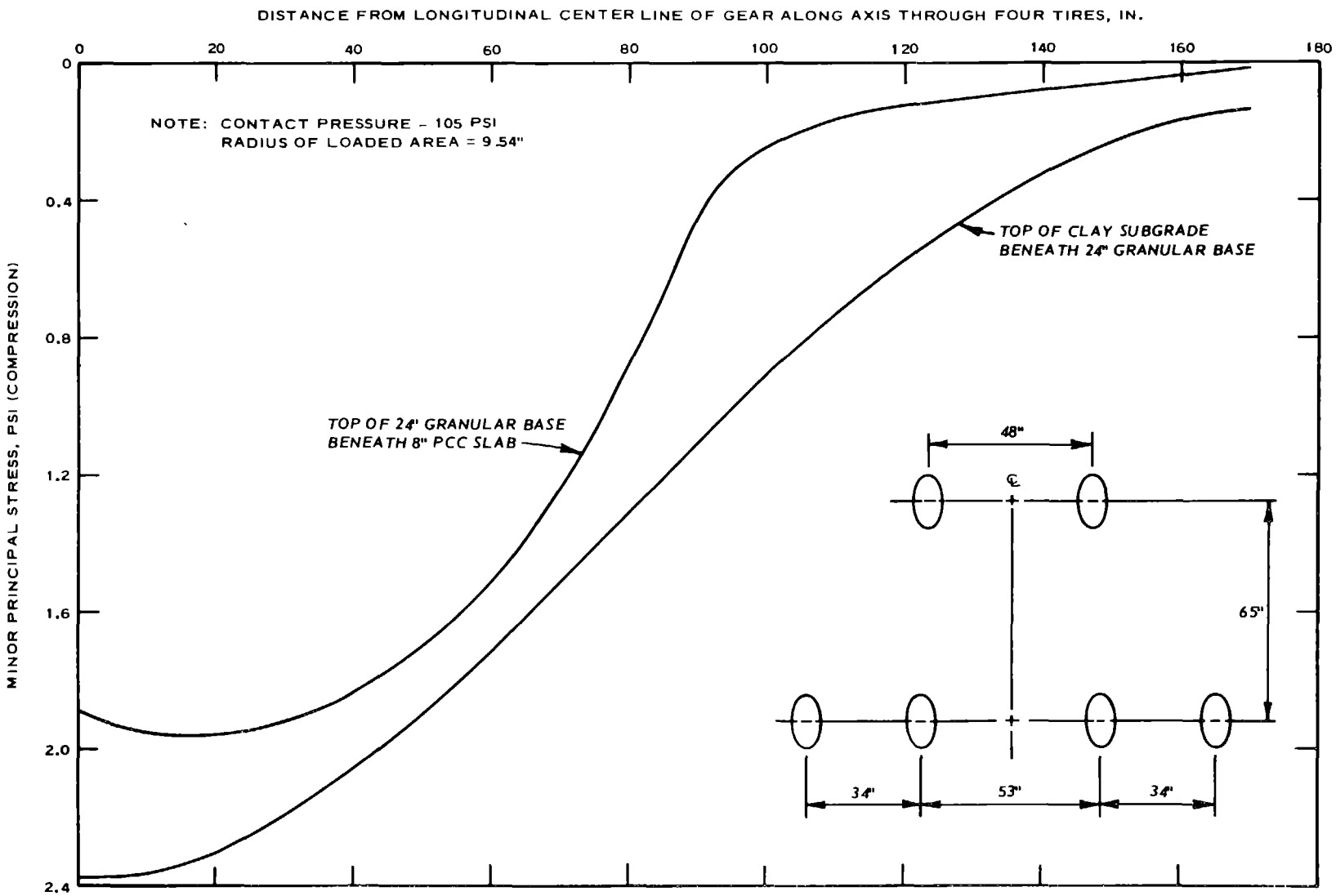


Figure A31. Variation of minor principal stress at the top of the granular base and at the top of the subgrade

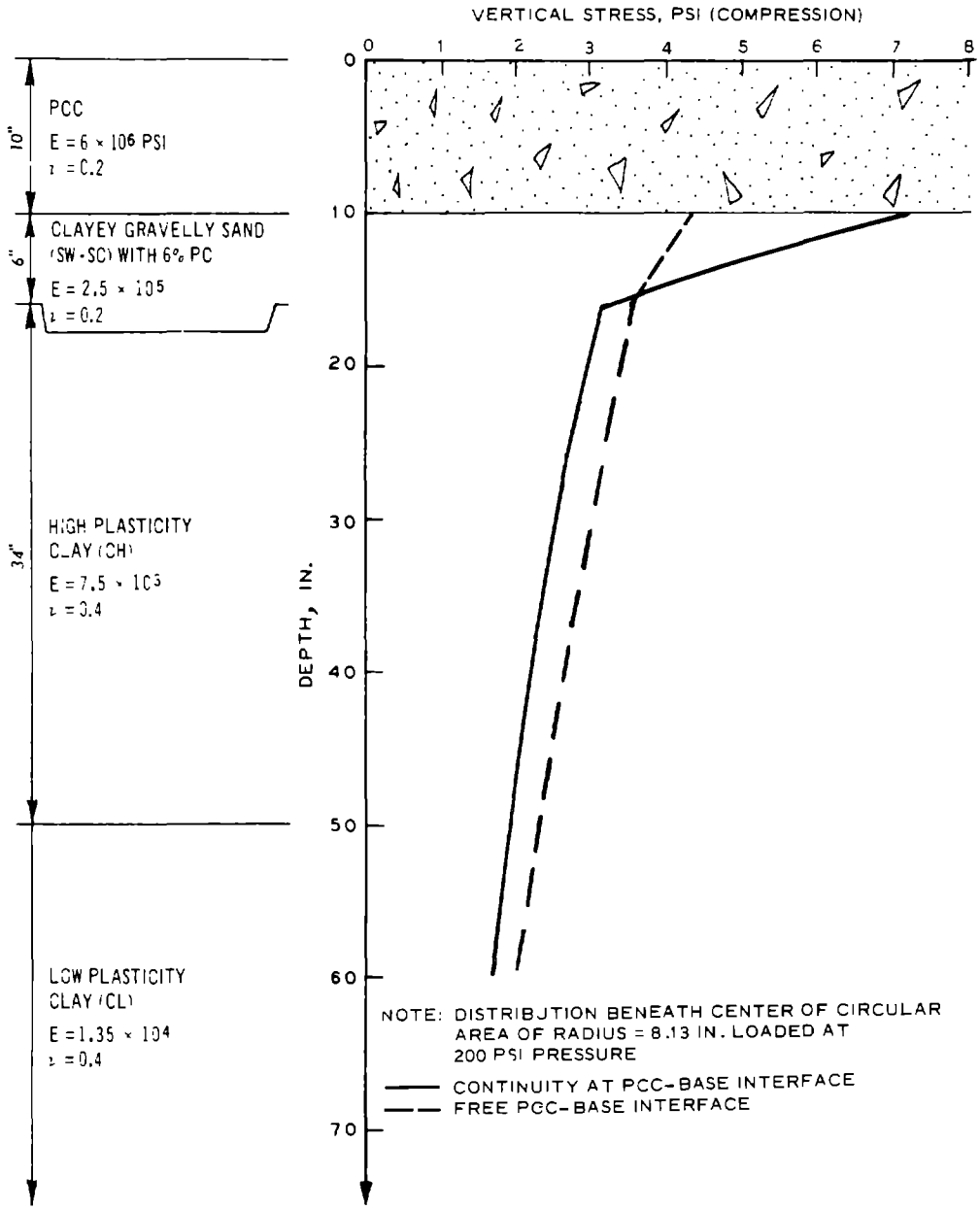


Figure A32. Vertical stress distribution illustrating effect of interface condition between PCC slab and base

to distribute the load over a wider area and to reduce the curvature, thereby increasing the possibility of tensile stress in the top of the second layer. This may be illustrated by comparing the stresses in Figures A17 and A19. However, the difference in moduli of the base course materials also contributed to the differences in the distributions. If the interface were completely frictionless, the two layers would respond as separate layers and compression would exist in the top of the second layer (Figure A23).

From a practical standpoint, there is not much that can be done to improve the representation of layer interface conditions. Although more complicated computational models are available that permit any degree of continuity at the interfaces, no definitive data exist to accurately quantify interface conditions. The conditions will probably be intermediate between full continuity and fully frictionless. In terms of material characterization, the difference may not be as important as it appears. For instance, if a completely frictionless interface were used between the PCC layer and the bound base, the stress in the top of the bound base would be compressive rather than tensile (Figure A19). However, the stresses in the bottom of the layer would be tensile, and the basic loading mode would still be bending.

To summarize this section on selection of states of stress, it can be stated that the stresses for a pavement vary with vertical location, or for a fixed location the state of stress will vary as a moving load passes. Results from the elastic layered model are only approximations, but the general response patterns are correct and the approximations are reasonable. Based on these observations, the procedures employed to select a state of stress for characterizing paving materials and subgrades will be to select a value of the critical response parameter that will be representative for a practical range of conditions.

Portland Cement Concrete. In rigid pavements, PCC and bound bases experience flexural loading. Certainly, this is two-dimensional loading and is more complex than that experienced by a simply supported beam that has been selected to characterize these materials. However,

of the types of tests available, the flexural tests were considered the most practical and usable.

There are several discrepancies between the state of stress in a PCC slab or a bound base layer and a simply supported beam that is tested to characterize the material. In a beam, a plane stress condition exists, whereas in a pavement slab, the stresses are three-dimensional (if the vertical stresses are considered). Certainly, the horizontal components of stress due to the bending are the largest components of stress, but the vertical support provided by the underlying material affects the response of the slabs. Forrest, Katona, and Griffin,[61] recognizing that the conditions in a slab are different from those in a beam, have suggested a series of tests to better define these differences. Because of differences in the deformed shape of slabs, they suggest that strain in the slab rather than stress may be the critical parameter. Nevertheless, the use of a flexural test on a simply supported unconfined beam loaded with essentially point loads was considered adequate for determining the modulus of elasticity and strength of PCC.

Bound Bases (Subbases). The same type test is used to characterize bound bases, although the conditions may be less representative than for PCC slabs. The vertical stresses will be distributed over the top and bottom of the layer, rather than just on the bottom as it is for the PCC surface layer. Certainly, stresses in the bound layer will be different from those in a simply supported beam loaded with essentially point loads. The shape the bound base layer takes may also be much different. In a simply supported beam, there will be compressive bending stresses in the top of the beam, but in the bound base layer, there may be tensile stresses throughout the layer. This results from the composite action of the PCC slab and base (Figures A19 and A25). The magnitude of the stresses will depend on the bond between the PCC and bound base layers and the modulus ratios between the two layers. However, the general trend is that the loaded and support conditions result in stress conditions different from those for a simply supported beam.

For the test pavements (described in Appendix B) that have contained bound base layers, there is no apparent relationship between the state of stress and the traffic that the pavement will sustain. Figure A33 shows a plot of minor principal stress directly below the PCC slabs versus the traffic level at failure, and Figure A34 shows the minimum first stress invariant plotted versus the traffic level at failure in the layer directly beneath the PCC slab for the test pavements described in Appendix B. In the pavements with no base, the stress would be in the top of the subgrade, and in the pavements with bases (bound or granular), the stress would be in the top of the base layer. The open circles are for pavements with bound bases, and no relationship with traffic is apparent in either figure.

Another interesting aspect of Figures A33 and A34 is the difference between the pavements with bound bases and those with granular or no bases. The stresses were computed with layered elastic theory that assumes full continuity between layers. The moduli for the bound base materials were larger than those for the granular base materials, which were in turn larger than the subgrade. The three distinct groupings of data are thought to be a result of the effects of the assumption of full continuity and modulus ratio between the PCC and the second material. Nevertheless, the general trend, indicating that the basic loading mode in the bound base layer was flexural, is valid. This is indicated by the fact that the minor principal stresses (horizontal stress essentially) and the first stress invariants are both tensile, whereas for the pavements with no base or granular bases, the values are compressive. This condition is representative for a large area in the base layer since the average values for the several computational points (as illustrated on the sketches in Figures A14-A29) are approximately equal to the minimum, or maximum, as the case may be, value. As an example, consider the six pavements with bound bases. If the minimum first stress invariant is selected from the stresses computed at the locations shown in Figures A14-A29, then the average of these values is -17.71 psi; whereas, if values from all computational points are used, the average is -16.12 psi. For the minor

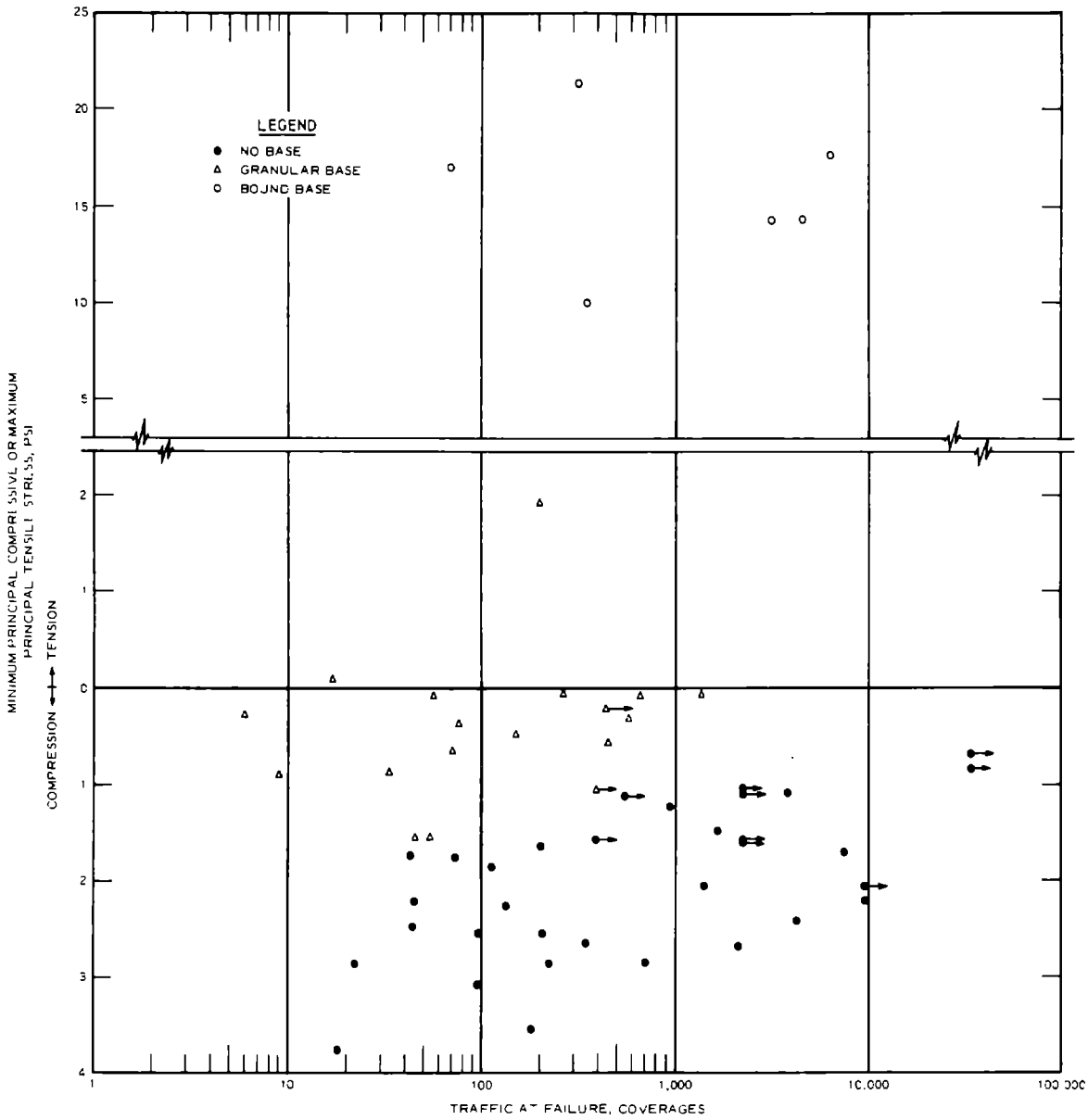


Figure A33. Minimum minor principal stress directly beneath the PCC slab versus traffic to failure

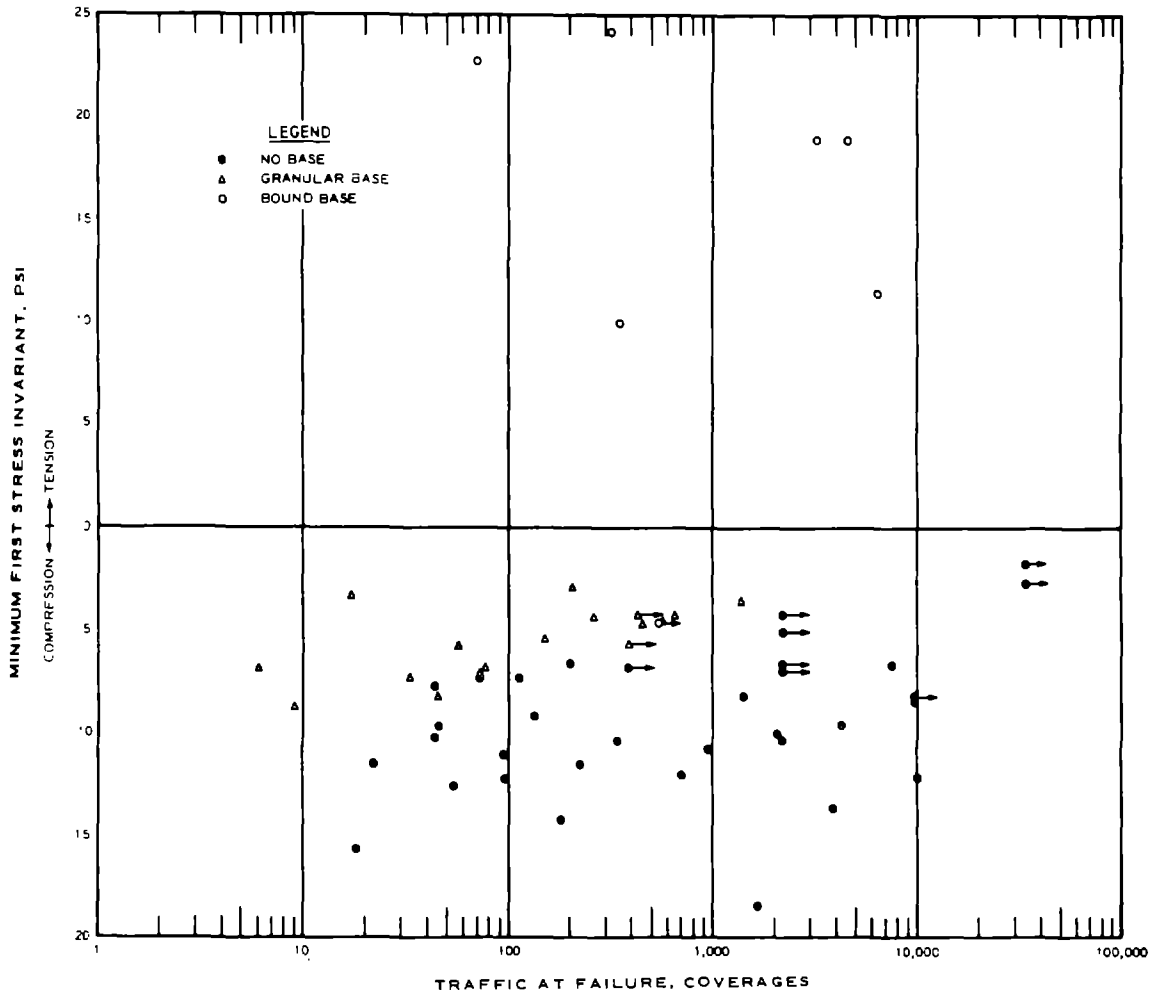


Figure A34. Minimum first stress invariant directly beneath the PCC slab versus traffic to failure

principal stress, the average of the minimum values is -15.81 psi, and the average of all values is -13.32 psi.

The references to minimum values of minor principal stress and first stress invariant are not actually correct. These stresses should be referred to as the minimum values within the area bounded by the wheel loads of a gear. Stresses were computed at the locations shown in Figures A14-A29. The minimum values referred to herein are the minimum of the stresses computed at the horizontal locations indicated in Figures A14-A29. At points some distance from the gear, stresses may be smaller than indicated; however, maximum values of principal stress, deviator stress, and first stress invariant computed at the indicated locations will be the maximum (or very close to the maximum) that will occur at the particular depth within the pavement.

The vertical stress, which is essentially equal to the major principal stress, appears to be only slightly larger for bound bases than for pavements with no bases or granular bases. There also appears to be no definite relationship between this stress component and traffic. Figure A35 shows the maximum major principal stress directly beneath the PCC slab plotted versus traffic for the test pavements described in Appendix B. The open circles are for pavements with bound base layers. The average of the maximum values of the major principal stress for the bound bases is 10.40 psi, for the granular base 6.87 psi, and for the subgrade 6.15 psi. This is fairly representative of the area beneath the gears since the corresponding average of all values computed for bound bases was 7.54 psi as compared with 10.40 psi for the average of the maximum values.

The absolute magnitudes of the compressive stresses in the bound base layers are less than the tensile stresses. Implications are that the flexural tests are more appropriate for bound bases than a compression test.

A simply supported unconfined beam loaded at the third point with essentially point loads is recommended for PCC and bound bases (subbases). For PCC, the beam is loaded to failure to determine the flexural strength, and the modulus of elasticity is computed from the

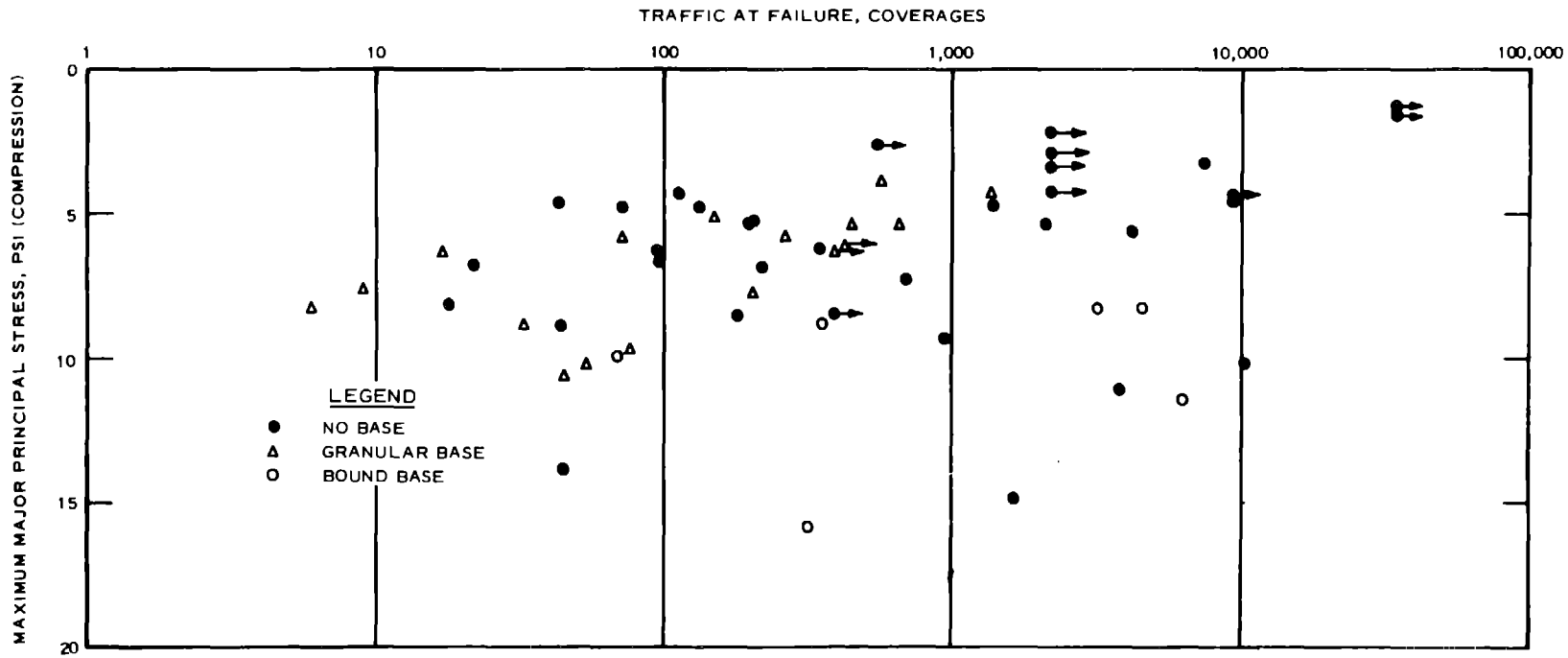


Figure A35. Maximum major principal stress directly beneath the PCC slab versus traffic to failure

slope of the straight-line portion of the load-deflection curve. For chemically stabilized bound bases, the ultimate load is determined, loads of 0.4, 0.6, and 0.8 times the ultimate load are applied repetitively, and the modulus is computed from the load-deflection curves. The modulus used should be the average of that obtained for the three loadings. For bituminous-stabilized materials, the definition of an ultimate load will be dependent on the rate of application of load and the temperature. Several loads should be selected that will result in stresses in the outer fibers of the beam, which are less than the values shown in Table A4. One test should be conducted at about 50 psi.

Table A4

Recommended Maximum Stress Levels at Which to Test Bituminous-Stabilized Materials

Temperature Range, °F	Maximum Stress Level in Extreme Fibers, psi
40-60	450
60-80	300
80-100	200

Granular Bases (Subbases). Load-deformation properties of granular base material are highly sensitive to the state of stress. Unfortunately, the state of stress within a granular base layer is also highly variable, which makes selection of representative conditions difficult. Figures A16, A17, A22, and A26-A29 illustrate the distribution of stress within a granular layer.

Triaxial compression tests will be used to characterize granular materials. The two parameters that significantly affect the load-deformation properties of granular materials are the confining pressure and the first stress invariant. A measure of the minimum confinement is the minor principal stress plotted in Figures A17, A22, A27, and A29. The compressive values at the top of the layer are low, and the tensile values are at the bottom of the layer. The magnitudes of these stresses

are affected by the assumption of full continuity at the interface between the PCC slab and the granular layer. For real conditions where some slip is permitted, the confinement would probably be somewhat greater than indicated. The inability of the material to sustain tensile stresses should also contribute to a buildup of confinement by a redistribution of stresses.

Additional factors not accounted for in the stress computations are overburden stresses and residual horizontal stresses, induced during compaction, which remain in the material. The effect of these stresses would be to increase the confinement on the material. There are also equipment limitations and lack of precision when testing at low stress levels. As a result, the states of stress selected for characterizing granular materials will be somewhat different from those indicated by the computed values of load-induced stresses.

The open triangles in Figure A33 are for pavements with granular bases and thus represent the stress at the top of the granular layer. There is no apparent relationship with traffic, but only a limited range of traffic is available. The values are low, indicating low confinement. For the pavements with granular bases, the average of the minimum values was only 0.38 psi (the average of all computed values was 0.50 psi, indicating relatively uniform conditions in the vicinity of the load). The effects of the modulus ratio between the PCC slab and the material directly beneath the slab is illustrated by the three groupings of the points in Figure A33, i.e., groups for pavements with bound bases, with granular bases, and with no bases. The larger the modulus ratio, the larger the confinement.

The open triangles in Figure A34 are data points for pavements with granular bases. The trends are the same as previously discussed for the minor principal stress, i.e., no relationship with traffic, three distinct groups of data, effects of modulus ratio, and uniformity beneath loads. The average of the computed minimum values is 5.96 psi, and the average of all computed values is 7.33 psi.

Figure A35 shows a plot of maximum major principal (essentially vertical) stress versus traffic. For the pavements with granular bases,

there is no apparent relationship with traffic. In addition, there is not much difference between the three types of sections, although the points for bound bases, denoted by the open circles, are higher than for the other two conditions. The average of the maximum values of major principal stress for the pavements with no base was 6.15 psi, for pavements with granular base 6.87 psi, and for pavements with bound bases 10.40 psi.

For characterizing granular bases, triaxial compression tests should be conducted at confining pressures of 2, 4, 6, and 10 psi. Axial stresses should be applied that result in ratios with confining stresses (σ_1/σ_3) of 2, 3, 4, and 5. Plots of resilient modulus versus first stress invariant, similar to the plot shown in Figure A28, should be prepared and an average relationship established. From this relationship, a value of resilient modulus at a first stress invariant of 10 psi should be selected.

No well-defined relationships exist for Poisson's ratio. However, plots of Poisson's ratio versus ratio of axial to confining stress (σ_1/σ_3) should be made, and representative values selected.

SUBGRADE SOILS

Subgrade soils beneath rigid pavements are subjected primarily to compressive stresses. Figures A14-A19 illustrate the distribution of major and minor principal stresses in the subgrade of rigid pavements with no base, with a granular base layer, and with a bound base layer. The stresses within the subgrade are always compressive.

Cohesive soils, which will be the predominate type encountered, are sensitive to the deviator stress, i.e., the difference between the major and minor principal stress. Figure A20 contains distributions of deviator stress with depth for a PCC slab directly on a subgrade loaded with a dual-tandem gear. Differences between the major (Figures A16 and A18) and the minor principal stresses (Figures A17 and A19) in the subgrade for pavements with granular and bound bases are similar to those shown in Figure A20.

Cohesionless subgrade soils are similar to granular base material

in that they will be sensitive to the confining stress and the total state of stress as represented by the first stress invariant. Figures A15, A17, A19, A27, and A29 show distributions of the minor principal stress in the subgrade for the three types of pavements considered. Considering that the loads are applied vertically and that the major principal stresses act essentially in the vertical direction, the minor principal stresses that act essentially in the horizontal direction are confining stresses. Within the subgrade, the values are always compressive but are small in magnitude. Distribution of the first stress invariant is shown in Figures A22 and A25, respectively, for the pavements with granular and bound bases. Within the subgrade, the values of the first stress invariant indicate that the loading is essentially compressive.

The distributions shown were computed with material properties of the subgrade obtained from tests on the cohesive subgrade soils. The modulus of elasticity would probably be higher for cohesionless soils than for cohesive soils, and the Poisson's ratio would be lower. However, the general trends illustrated by the computations, i.e., low compressive confining stresses and compressive first stress invariants, should be applicable to cohesionless subgrade soils.

Triaxial compression tests will be used to characterize subgrade soils. The deviator stress in the triaxial tests will be the difference between axial stress applied to the specimen and the confining pressure in the triaxial chamber. For cohesive soils, this should approximate as closely as possible conditions in the subgrade. The maximum deviator stress is considered appropriate for characterizing cohesive materials, since the general trends indicated in Figures A2 and A3 have been found to hold for a wide range of materials, i.e., the resilient modulus decreases as deviator stress increases.

The maximum deviator stress at the top of the subgrade versus the traffic to failure in Figure A36 applies to the test pavements described in Appendix B. There are no apparent relationships with traffic and no easily discernible differences between pavements without bases, with granular bases, or with bound bases. The average value for pavements

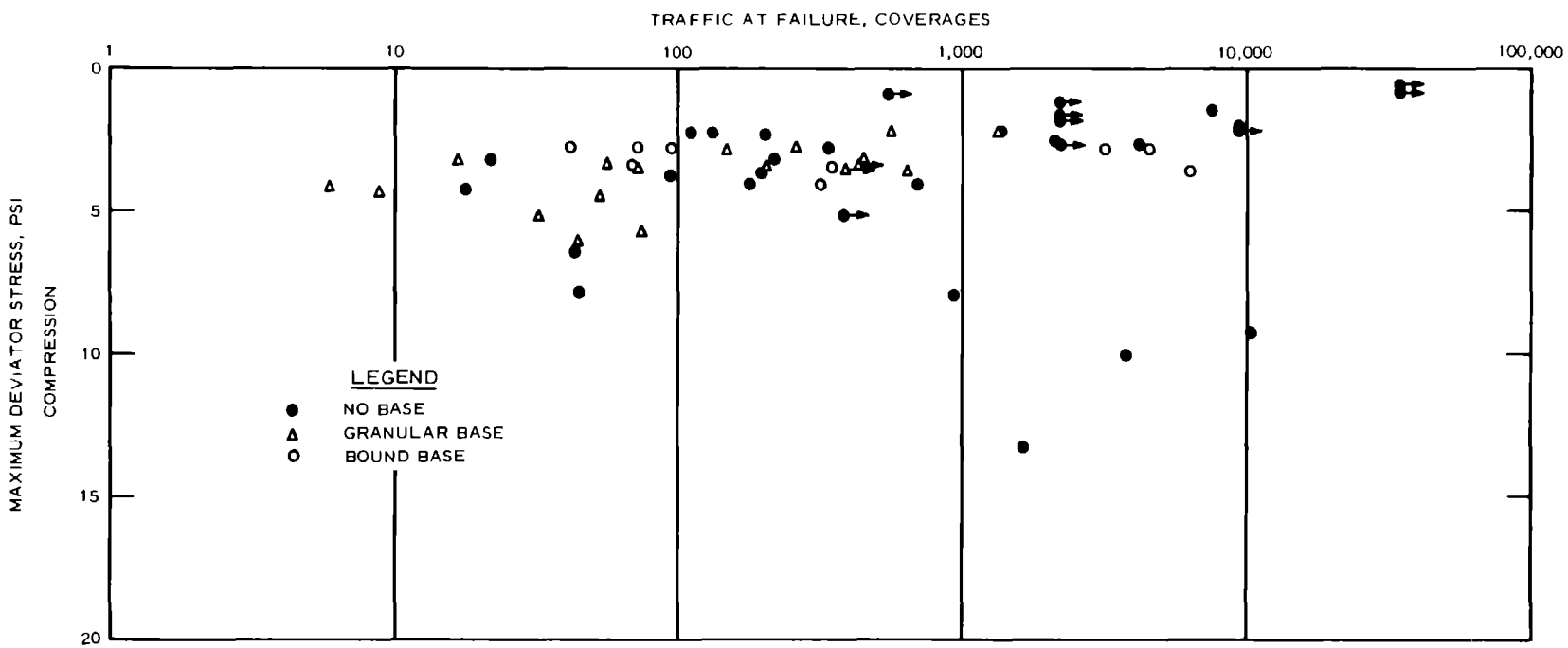


Figure A36. Maximum deviator stress at the top of the subgrade versus traffic to failure

without bases is 3.69 psi, with granular bases, 3.79 psi, and with bound bases, 3.46 psi; the overall average is 3.70 psi. Stresses due to the overburden and residual stresses remaining after compaction were not considered in the computation of these stresses.

For characterizing cohesive materials, the triaxial tests should be conducted at a range of stress conditions and a composite curve established in Figures A2 and A3. Tests should be conducted at confining stresses of 2, 4, and 6 psi, and at axial stresses applied that will result in a range of deviator stress from about 2 to 16 psi. From the composite curve, the resilient modulus used to represent the material should be selected at a deviator stress of 5 psi. No well-defined relationships exist for Poisson's ratio, but similar plots should be made and a representative value selected.

For cohesionless soils, the confining stress in the triaxial tests should approximate conditions in the subgrade. The minor principal stress in the subgrade is a measure of the confinement. For cohesionless subgrade soils, it is considered appropriate to select properties at minimum values of the first stress invariant and confining stress, since the general trends illustrated in Figures A12 and A13 are applicable for cohesionless subgrade soils, i.e., as the confining stress and the first stress invariant decreases, the resilient modulus decreases.

The minor principal stress and minimum first stress invariant at the top of the subgrade versus the traffic to failure in Figures A37 and A38, respectively, apply to the test pavements described in Appendix B. As previously noted, these are the minimum values in the vicinity of the load or loads, and smaller values (in fact, zero) will exist at locations far removed from the loads. There are no obvious relationships with traffic for either of the parameters. There are some apparent differences in the stresses for the different type pavements. The average of the minor principal stress at the top of the subgrade is 1.99 psi for pavements with no base, 1.25 psi for pavements with granular bases, and 2.54 psi for pavements with bound bases. The overall average is 1.82 psi. The average of the minimum first stress

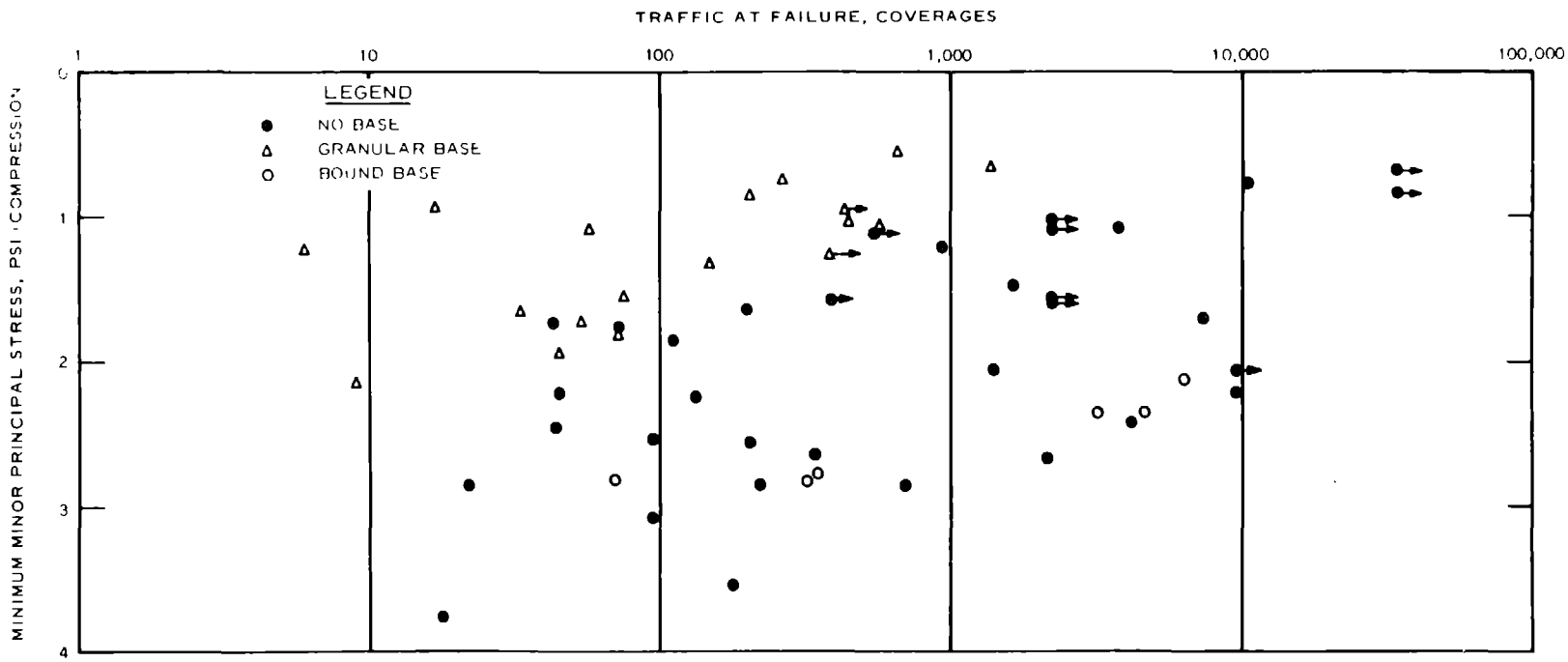


Figure A37. Minimum minor principal stress at the top of the subgrade versus traffic at failure

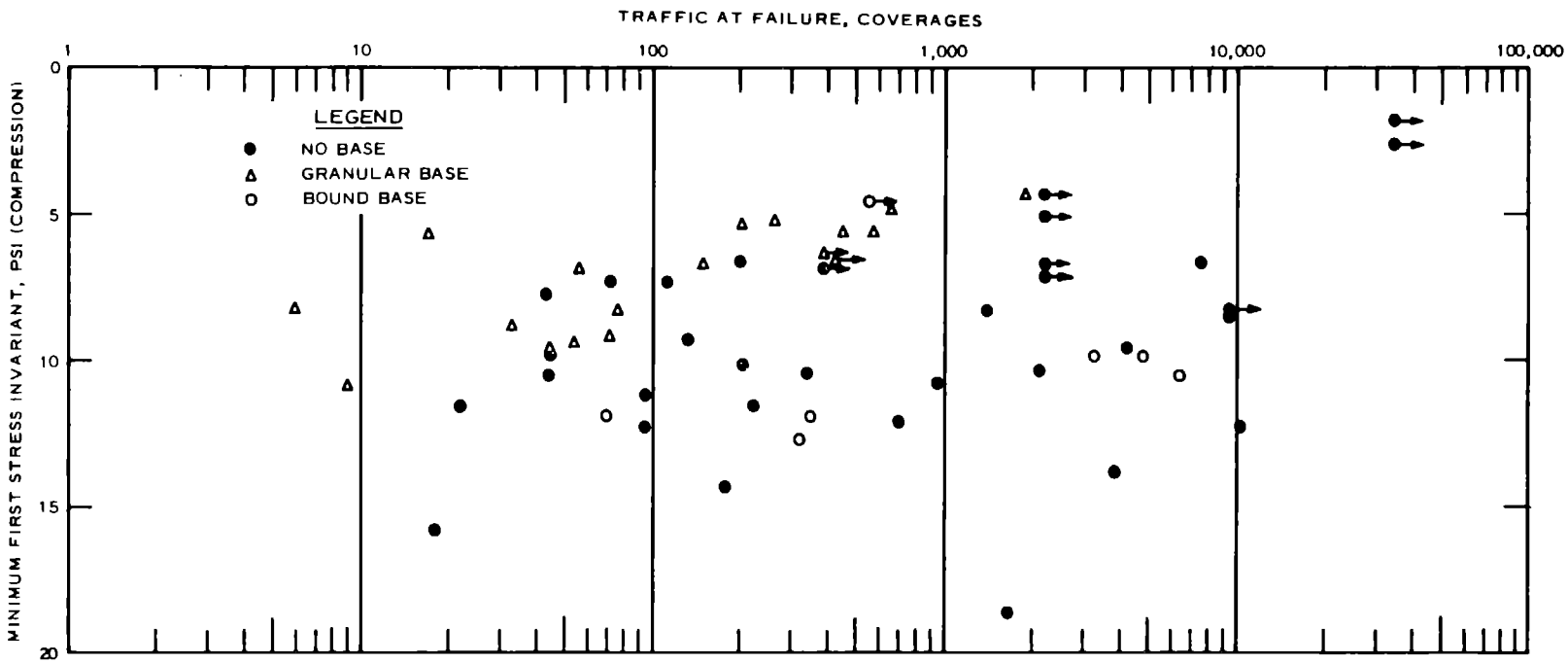


Figure A38. Minimum first stress invariant at the top of the subgrade versus traffic at failure

invariant at the top of the subgrade is 9.27 psi for no base, 6.59 psi for granular bases, and 11.15 psi for bound base. The overall average is 8.63 psi. Stresses due to the overburden and residual stresses that may exist in the subgrade due to the compaction process were not considered in the computation of these stresses. The effect of these stresses on material characterization would be to increase the confinement and the first stress invariant.

Basically, the same stresses should be used in the triaxial tests for characterizing cohesionless material as are used for granular bases. Confining pressures of 2, 4, 6, and 10 psi and axial stresses that result in principal stress ratios (σ_1/σ_3) of 2, 3, 4, and 5 should be applied. From the average relationship of resilient modulus versus first stress invariant, a representative modulus value should be selected at a first stress invariant of 10 psi. A representative value of Poisson's ratio should be selected from a composite plot of Poisson's ratio versus principal stress ratio.

SUMMARY FOR CHARACTERIZING MATERIALS

It is recommended that modulus of elasticity and flexural strength of PCC be determined from static flexural tests of beams having a cross-sectional area of 6 by 6 in. The recommended procedures are widely accepted and extensively used for determining the properties of PCC. The test procedure for determining flexural strength is ASTM Standard Method of Test C 78-75.[57] This test is also designated CRD-C 16-66 in the CE Handbook for Concrete and Cement.[56] There is no ASTM standard test for determining the modulus of elasticity of PCC from flexural tests. The recommended procedure is contained in the CE Handbook for Concrete and Cement[56] and is designated CRD-C 21-58. No procedures are provided for determining Poisson's ratio of PCC. It is recommended that a value of 0.2 be assigned for all PCC.

It is recommended that the modulus of elasticity of bound base material be determined from cyclic flexural tests of beams. The recommended test procedures have not been standardized but are described in detail in Appendix C. There are differences in the procedures for

chemically stabilized materials and those stabilized with bituminous binders. These differences are necessary because of the sensitivity of bituminous-stabilized bases to rates of loading and temperature. No procedures are provided for determining Poisson's ratio of bound base material. It is recommended that the following values extracted from Reference 10 be used.

Material	Poisson's Ratio
Bituminous-stabilized	0.5 for E < 500,000 psi 0.3 for E > 500,000 psi
Chemically stabilized	0.2

It is recommended that properties of granular bases (subbases) be determined from cyclic triaxial tests on prepared samples. The recommended test procedure is outlined in Appendix D. The outputs from the test procedure are measures of modulus of elasticity and Poisson's ratio.

There is concern among some engineers as to the accuracy with which the results from laboratory tests on granular materials represent field conditions. This is the result of such factors as the sensitivity to the state of stress, sensitivity to the degree of compaction, inability to take undisturbed samples, which necessitates laboratory preparation of specimens, difficulty in measuring parameters needed to compute material properties, and the apparent existence of tensile stresses in granular layers, which would result in redistributions of stress within the pavement system. In addition, there is also the feeling that the properties of granular materials meeting requirements for base or subbase material will not vary over a wide range. These factors have led to the use of various methods for selecting representative properties of granular bases and subbases. Barker and Brabston recommend a Poisson's ratio of 0.3 for granular base or subbase materials and provide a procedure (Appendix G[10]) for determining representative values of moduli based on layer thickness and the modulus of the foundation upon which the layer was compacted. This procedure appears to produce reasonable results. However, it is recommended that it be

used in conjunction with test results to determine a representative modulus rather than as the sole method. The use of a value for Poisson's ratio of 0.3 is acceptable unless there is reason to believe that it is significantly different for the material in question.

It is recommended that the modulus of elasticity and Poisson's ratio of subgrade soils be determined from cyclic triaxial tests on undisturbed samples when possible or on samples prepared as close as possible to field conditions when fill is involved. The recommended test procedures are outlined in Appendix E. The procedures are similar to those used for granular base (subbase) materials. There are differences in details of the test procedures and presentation of results for cohesive and cohesionless materials. These differences are necessary because of the sensitivity of cohesive soils to moisture and the differences in the behavior as a function of the state of stress. As with other materials, determination of a representative Poisson's ratio is difficult, and the values of 0.4 for cohesive and 0.3 for cohesionless materials suggested by Barker and Brabston[10] may be used if test results prove unreliable.

APPENDIX B: DESCRIPTION OF TEST PAVEMENTS

The pavements described herein were part of eight test tracks. These test tracks will be referred to as Lockbourne No. 1, Lockbourne No. 2, Lockbourne No. 3, Sharonville Channelized Traffic, Sharonville Heavy Load, Multiple-Wheel Heavy Gear Load (MWHGL), Keyed Longitudinal Joint Study (KLJS), and Soil Stabilization Pavement Study (SSPS). The pavements were constructed and tested by the U. S. Army Corps of Engineers during the period from 1943 to 1973. The pavements were constructed under controlled conditions, and simulated aircraft traffic was applied in an accelerated manner.

LOCKBOURNE NO. 1

The Lockbourne No. 1 Test Track was constructed between August and November 1943 at the Lockbourne Army Air Base near Columbus, Ohio. The construction, testing, and analyses of the data from this test track are discussed in References 40-42. This test track had two continuous traffic lanes 20 ft wide, composed of adjacent 20- by 20-ft slabs. The concrete test slabs varied in thicknesses from 5 to 10 in. and were placed with and without base courses. The type base material varied and the thickness ranged from 6 to 12 in. Transition slabs between the traffic slabs and turnaround sections at each end combined to form a continuous track, which was subjected to single-wheel loadings of 20, 37, and 60 kips. Between 5 June and 10 July 1944 some sections of the test track were reconstructed due to early failure caused by the traffic loading tests.

Table B1 summarizes the flexural strength of the PCC concrete and the traffic applied to each pavement. Figure B1 illustrates the properties of the pavements used in this study; Figure B2, the loads used to traffic the test pavements. The two tires in Figure B2 were on one axle of the load cart but were far enough apart so that the overlap in zones of influence was small.

Table B1

Summary of Load, PCC Flexural Strength, and Traffic--Lockbourne No. 1 Test Track

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
A	20	740	390+	First crack
A	37	780	45	First crack
B	20	740	187	First crack
B	37	780	35	First crack
C	20	740	200	First crack
C	37	780	44	First crack
D	20	740	450	First crack
D	37	780	33	First crack
E	20	740	430+	First crack
E	37	780	77	First crack
F	20	740	550+	No failures
F	37	780	111	First crack
K	37	780	700	First crack
K	60	735	72	First crack
N	37	780	150	First crack
N	60	735	9	First crack
O	37	780	573	First crack
O	60	735	72	First crack
P	37	780	262	First crack
P	60	735	6	First crack
Q	37	780	1390	First crack
Q	60	735	57	First crack
U	37	780	88	First crack
U	60	735	1.5	First crack
A (Reconstr)	37	725	658	First crack

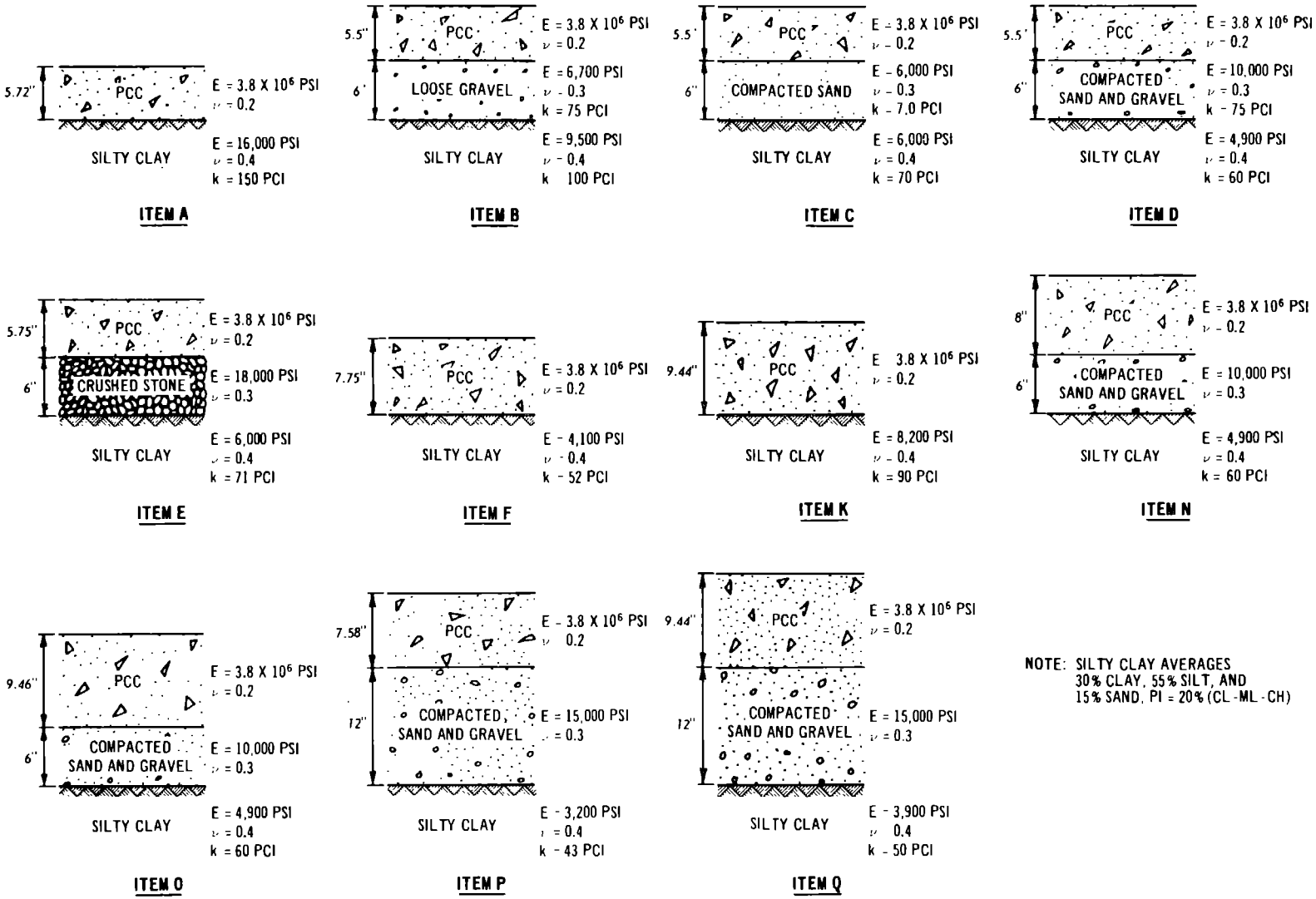


Figure B1. Description of test pavements in Lockbourne No. 1 Test Track (sheet 1 of 2)

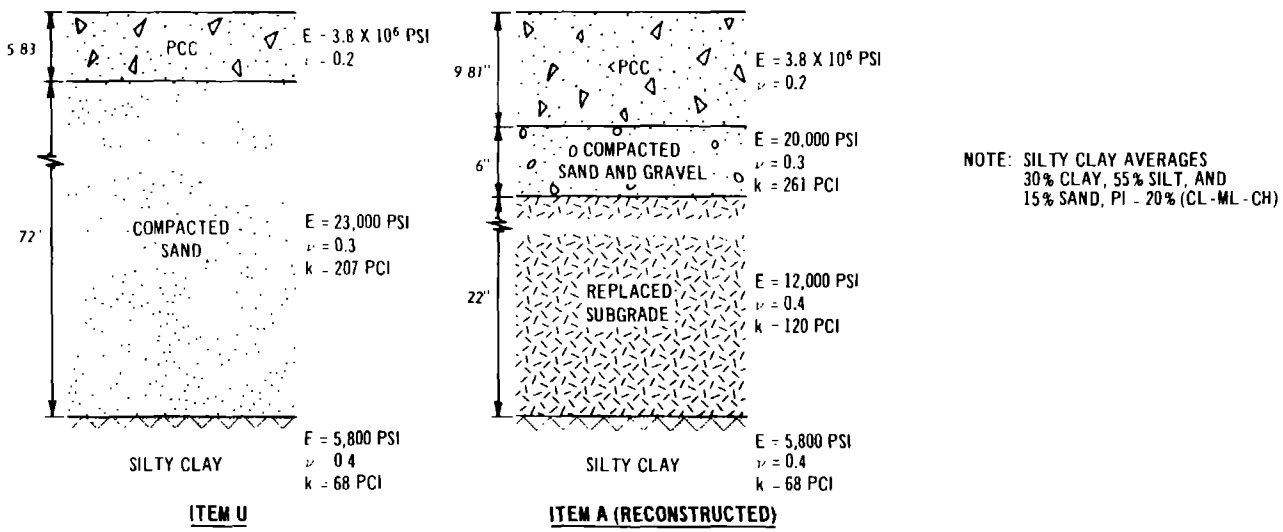


Figure B1 (sheet 2 of 2)

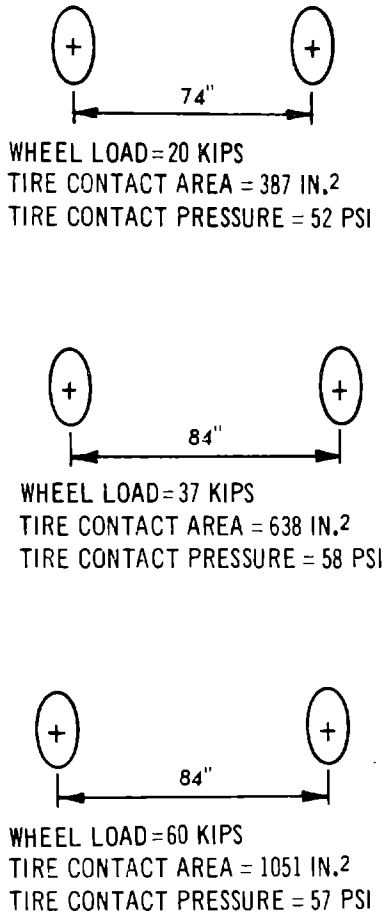


Figure B2. Schematic representation of loads used in Lockbourne No. 1 Test Track

LOCKBOURNE NO. 2

The Experimental Mat test section designated as Lockbourne No. 2 was rectangular in shape and constructed adjacent to the Lockbourne No. 1 Test Track between September 1944 and April 1945. A discussion of the construction, testing, and analyses of data from this test section can be found in References 43 and 44. The concrete test pavements varied from 15 to 24 in. in thickness and consisted of both 25- by 25- and 25- by 50-ft slabs. Perimeter slabs varying in thickness completely surrounded the test pavements. Their purpose was to provide a maneuver area for the outrigger wheels of the 150-kip single-wheel load rig.

The Lockbourne No. 2 Modification test section was an extension of the Experimental Mat and was constructed between August and October 1946. A discussion of construction, testing, and analyses of data can be found in References 45 and 46. This test section consisted of 12-, 15-, and 20-in. plain concrete pavement placed directly on the subgrade. The test slabs were arranged to form three 25-ft lanes with transition slabs separating the three design thicknesses. Traffic was applied with a special loading device that produced a 150-kip load on four wheels.

Table B2 summarizes the flexural strengths of the concrete test pavements and the traffic applied to each item. The pavement properties used in this study are for the Experimental Mat section (Figure B3) and for the Multiple-Wheel Modification section (Figure B4). The load assembly in Figure B5 is for both the Experimental Mat and the Modification sections.

Table B2

Summary of Load, PCC Flexural Strength, and Traffic--Lockbourne No. 2 Test Track

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
E1	150	725	97	First crack
E2	150	680	942	First crack
E3	150	710	17	First crack
E4	150	680	203	First crack
E5	150	695	43	First crack
E6	150	700	2204+	No failure
E7	150	760	2204+	No failure
M1	150	725	134	First crack
M2	150	725	2204+	No failure
M3	150	725	2204+	No failure

Note: Items E1-E7 (Experimental Mat) loaded with single wheel.
Items M1-M3 (Modification) loaded with dual-tandem gear.

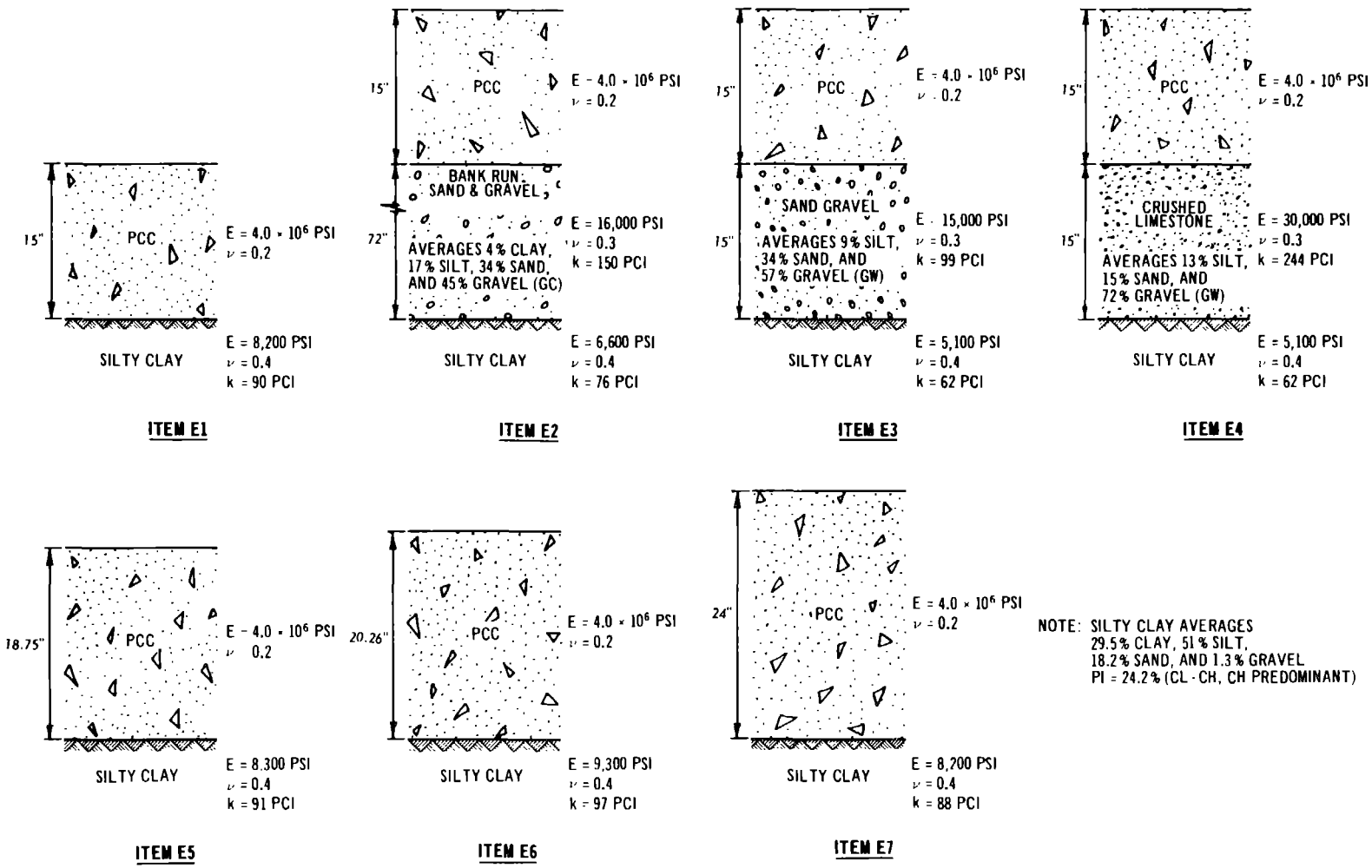


Figure B3. Description of test pavements in Lockbourne No. 2 Experimental Mat test section

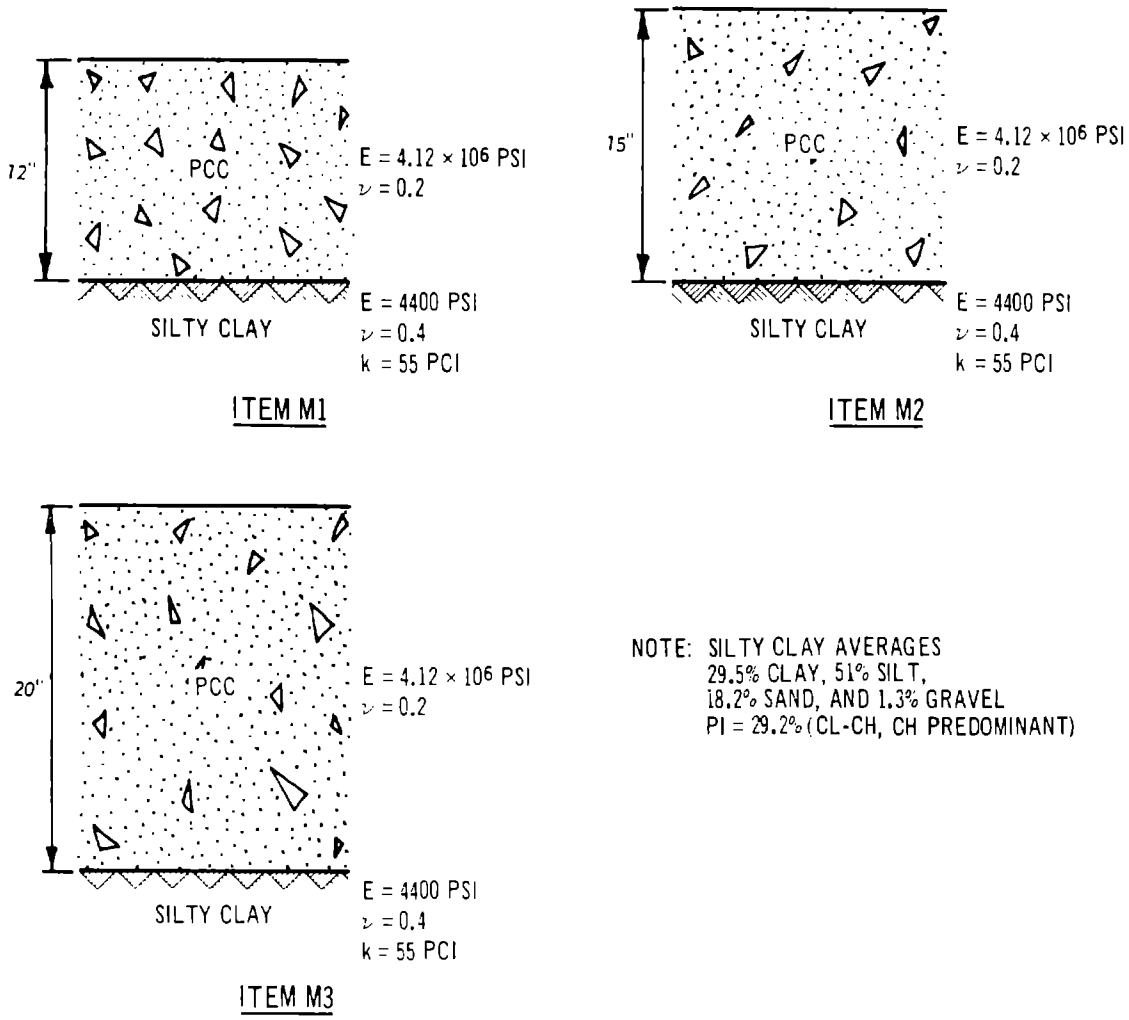


Figure B4. Description of test pavements in Lockbourne No. 2 Multiple-Wheel Modification test section

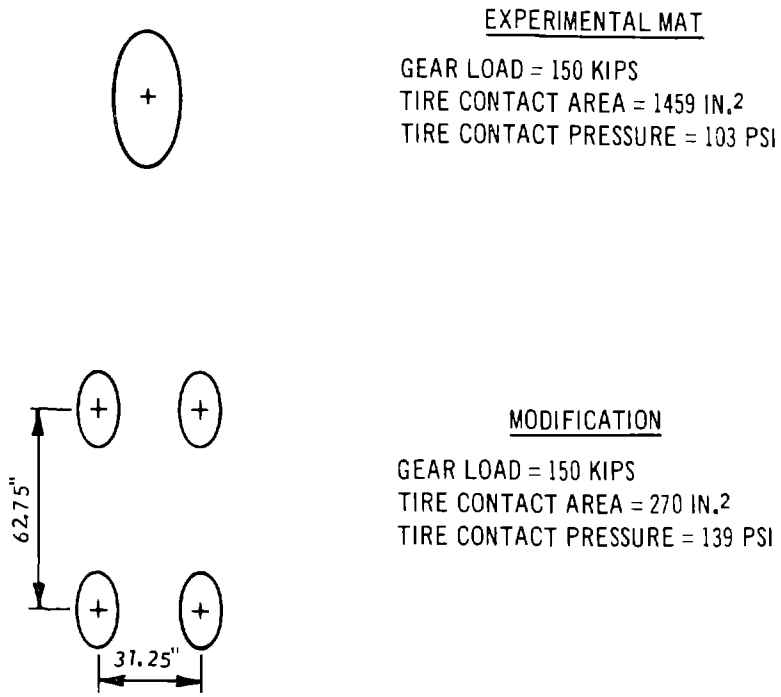


Figure B5. Schematic representation of loads used in Lockbourne No. 2 Test Track

LOCKBOURNE NO. 3

The Lockbourne No. 3 Overlay Mat test section was constructed in the area encompassed by the inner boundaries of the Lockbourne No. 1 Test Track. Construction of the test section was begun 1 August 1946 and was completed 26 October 1946. The construction, testing, and analyses of the data from this overlay mat test section are discussed in References 45 and 46. The concrete pavement comprising part of the Lockbourne No. 3 Test Track was divided into nine test items. With the exception of item 1, each was overlain with a flexible overlay. The overlay thickness varied as well as the type of overlay material and ranged between 3 to 9 in. Item 1 consisted of 6-in. plain concrete and was divided into four lanes; three of these lanes were 25 ft wide and one was 10 ft wide.

Traffic was applied to the test items with a rig producing a 60-kip load on a dual-wheel assembly. Figure B6 lists the properties of the pavement used in this study including the applied load. The PCC flexural strength was 800 psi, and the first crack failure occurred at 18 coverages.

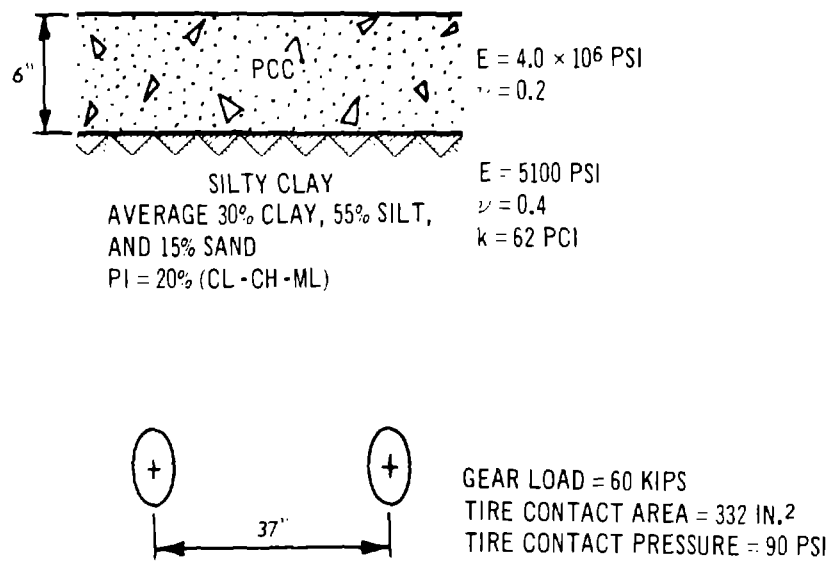


Figure B6. Summary of data from Lockbourne No. 3 Test Track

SHARONVILLE CHANNELIZED TRAFFIC

Two channelized traffic test sections, one on a high bearing-capacity foundation and the other on a low bearing-capacity foundation, were constructed at Sharonville, Ohio, between November 1955 and February 1956. The construction, testing, and analyses of data from this test track are discussed in References 34 and 47. The two channelized traffic test sections, designated as Parts 1 and 2, were 25 ft wide and about 600 ft long, and contained no longitudinal joints. The test items in each section varied from 50 to 65 ft in length. Each item was separated by a heavily reinforced concrete transition slab 10 ft long. Traffic was applied to both sections by using a load rig which produced a 100-kip load on a dual gear.

Table B3 summarizes the flexural strength of the test items and the amount of traffic applied to each. Figure B7 shows the pavement properties used in this study; Figure B8, the load used to traffic the Sharonville Channelized test pavement.

Table B3

Summary of Load, PCC Flexural Strength, and Traffic--Sharonville Channelized Test Track

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
57	100	740	34,650+	No failure
58	100	740	34,650+	No failure
59	100	730	7,600	First crack
60	100	730	1,674	First crack
61	100	730	3,867	First crack
62	100	730	10,082	First crack

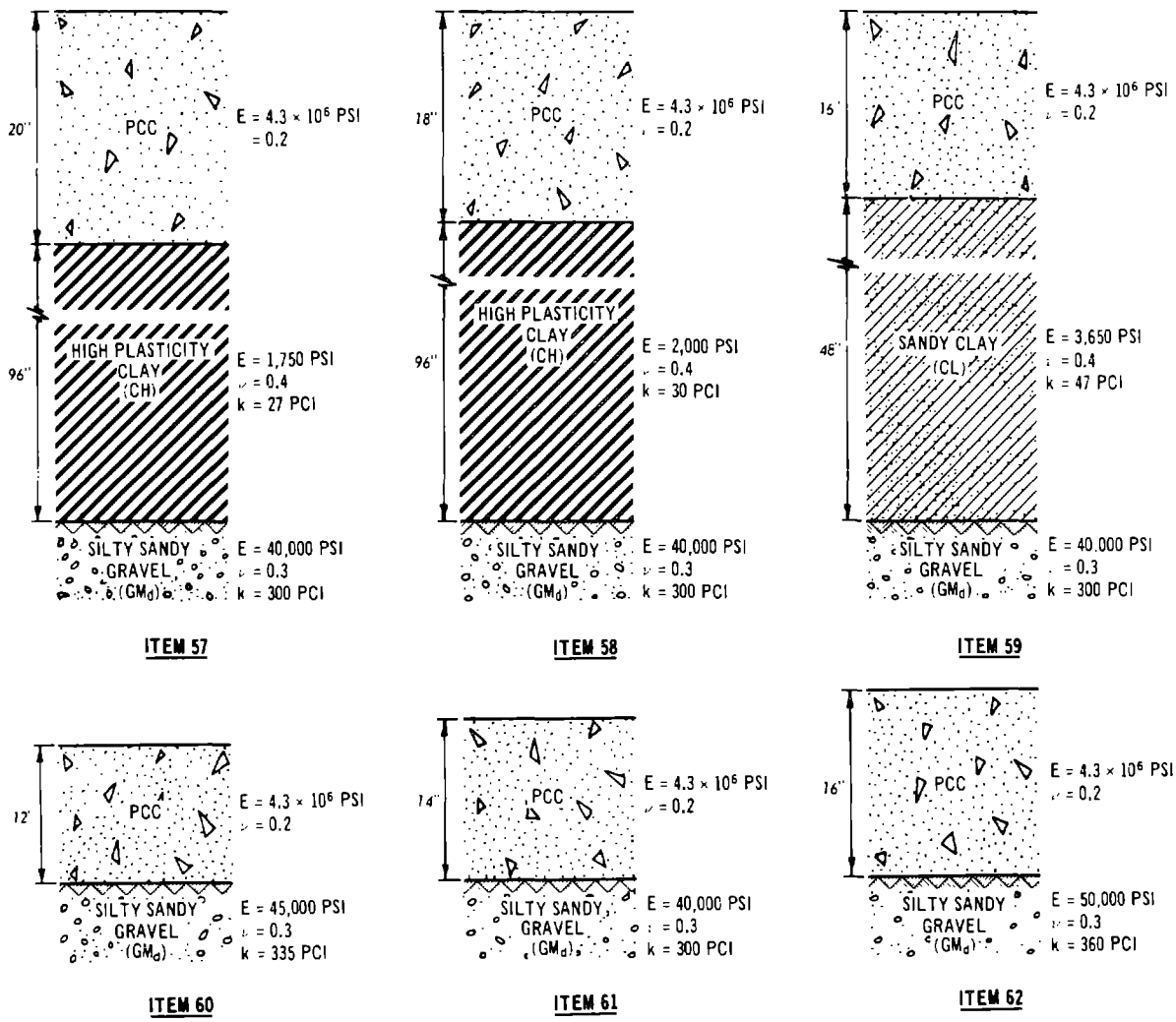


Figure B7. Description of test pavements in Sharonville Channelized Traffic Test Track

Figure B8. Schematic representation of loads used in Sharonville Channelized Traffic Test Track

SHARONVILLE HEAVY LOAD

The Heavy Load Test Track was constructed between July and November 1957 at the Sharonville test site just north of Cincinnati, Ohio. The construction of the test track is discussed in Reference 48. Testing and analysis of the data have not been published. Test Track A, used in this study, was 50 ft wide and 525 ft long. An extra 25-ft length of pavement was provided at each end of the track to provide a maneuver area for the traffic rig. Track A was referred to as the "plain concrete" test track. Traffic tests on all pavements were accomplished with a 325-kip load on a dual-tandem aircraft gear.

Table B4 summarizes the PCC flexural strength and the traffic applied to the various test items that were used in this study. Figure B9 shows the pavement properties of the test items used in this study; Figure B10, the load used to traffic the pavements.

Table B4

Summary of Load, PCC Flexural Strength, and Traffic--Sharonville Heavy Load Test Track

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
71	325	800	9680+	No failure
72	325	800	9680	First crack
73	325	800	2115	First crack

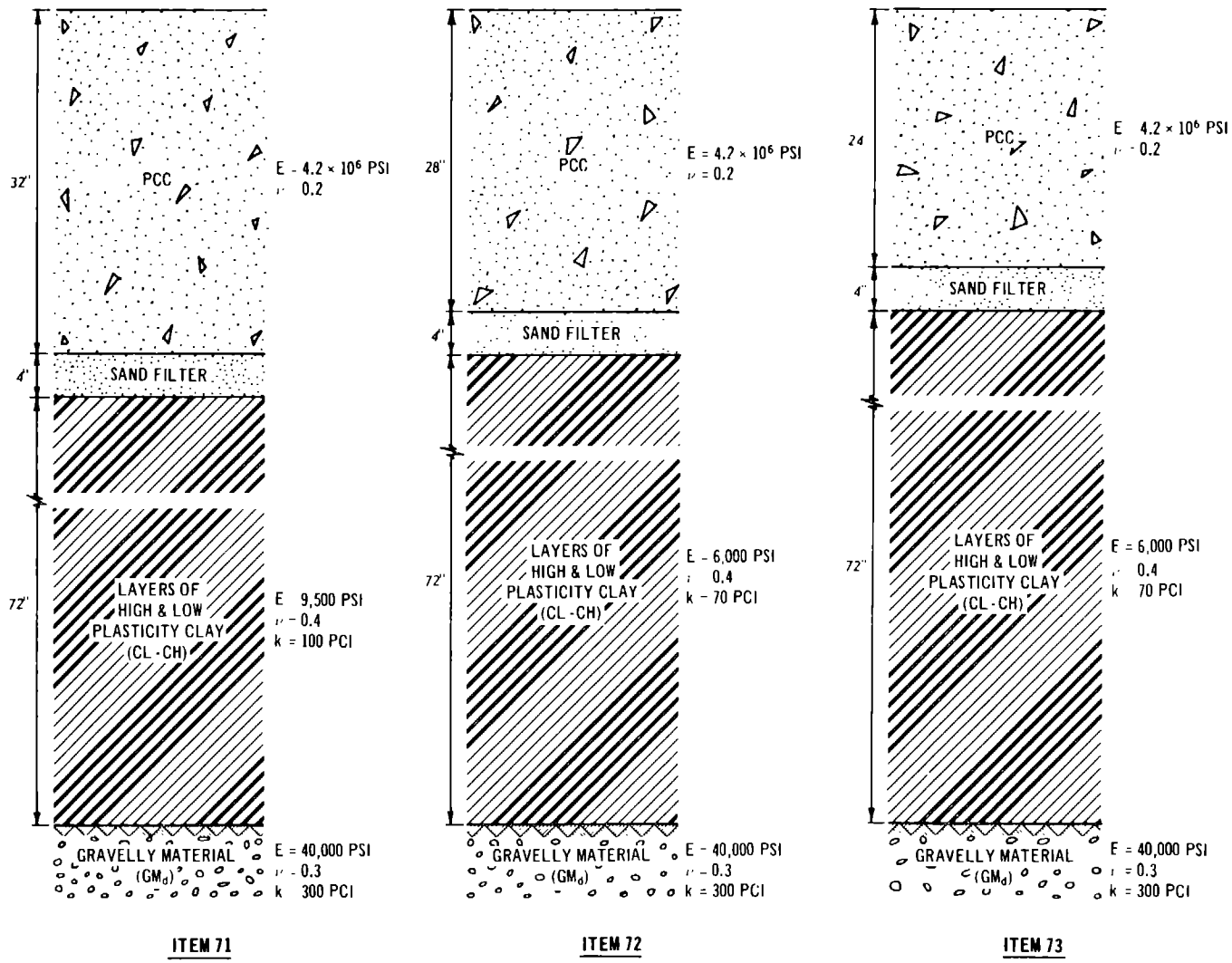


Figure B9. Description of test pavements in Sharonville Heavy Load Test Track

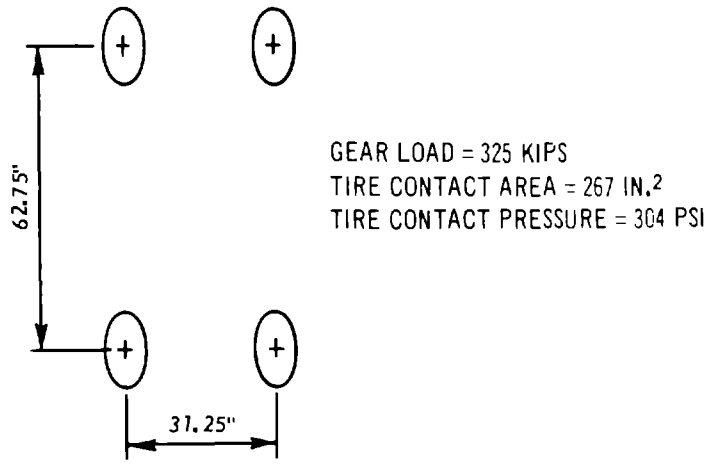


Figure B10. Schematic representation of loads used in Sharonville Heavy Load Test Track

MULTIPLE-WHEEL HEAVY GEAR LOAD (MWHGL)

The MWHGL pavement tests were conducted at the test site located at the U. S. Army Engineer Waterways Experiment Station in Vicksburg, Miss. The construction of this test track was initiated in July 1968, and the rigid pavement portion was completed in December 1968. The construction, testing, and analyses of the data from this test track are discussed in Reference 35. The rigid pavement test items were each 50 ft square and composed of four 25-ft-square slabs separated by a longitudinal construction joint and a transverse contraction joint. The items were separated by 25-ft-long by 50-ft-wide transition slabs, which were heavily reinforced to prevent the migration of cracks from one test item to another. The simulated traffic portion of the tests was run in two parts: the first part consisted of trafficking the south paving lane with a 12-wheel assembly (C-5A) loaded to 30,000 lb per wheel; the second part of the test program consisted of trafficking the north paving lane with a dual-tandem assembly (B-747) loaded to 41,500 lb per wheel.

Figure B11 shows the properties of the test pavements used in this study; Figure B12, the load assemblies used to traffic the test items. Table B5 summarizes the wheel loads, PCC flexural strengths, and the traffic applied to the test pavements.

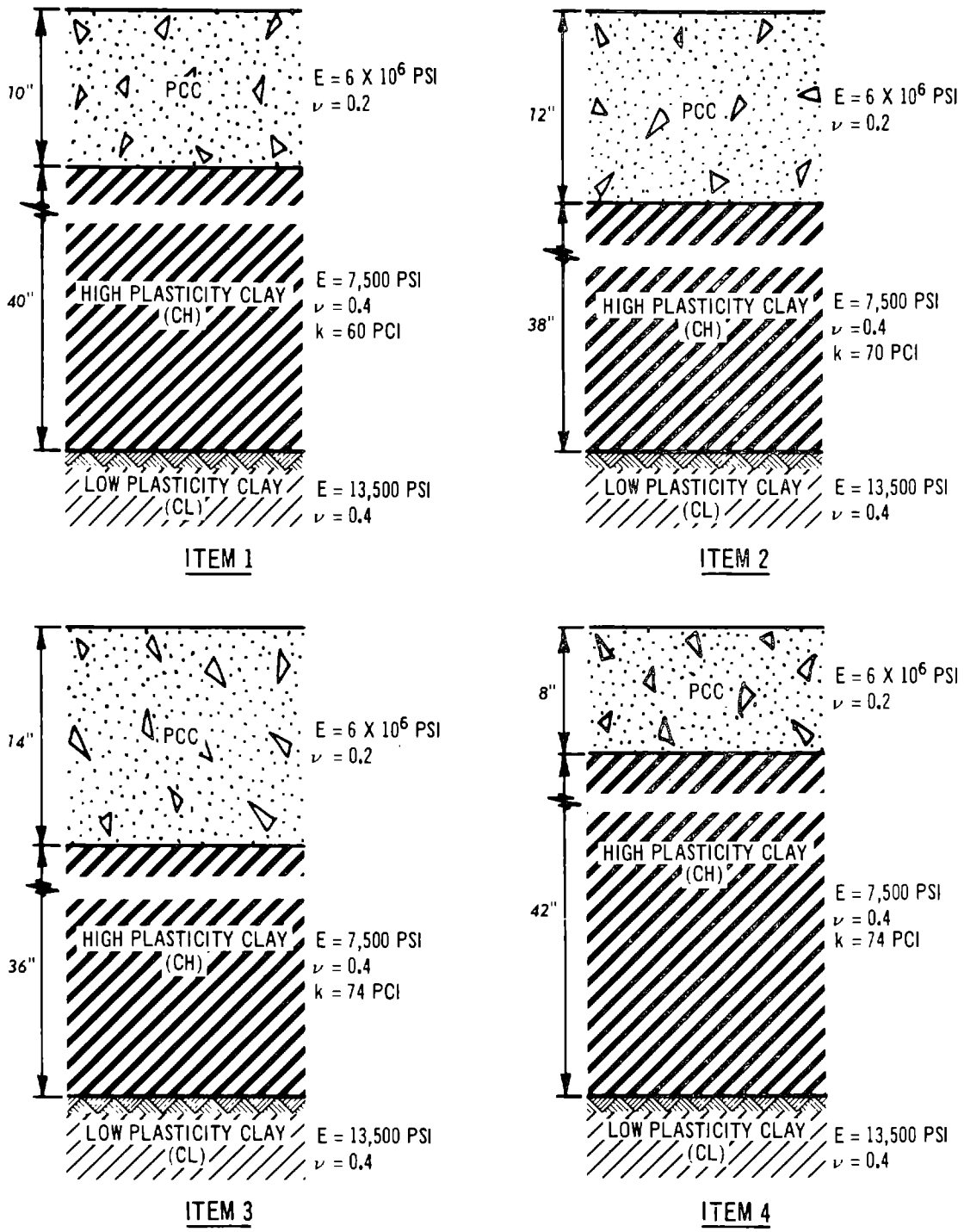


Figure B11. Description of test pavements in Multiple-Wheel Heavy Gear Load Test Track

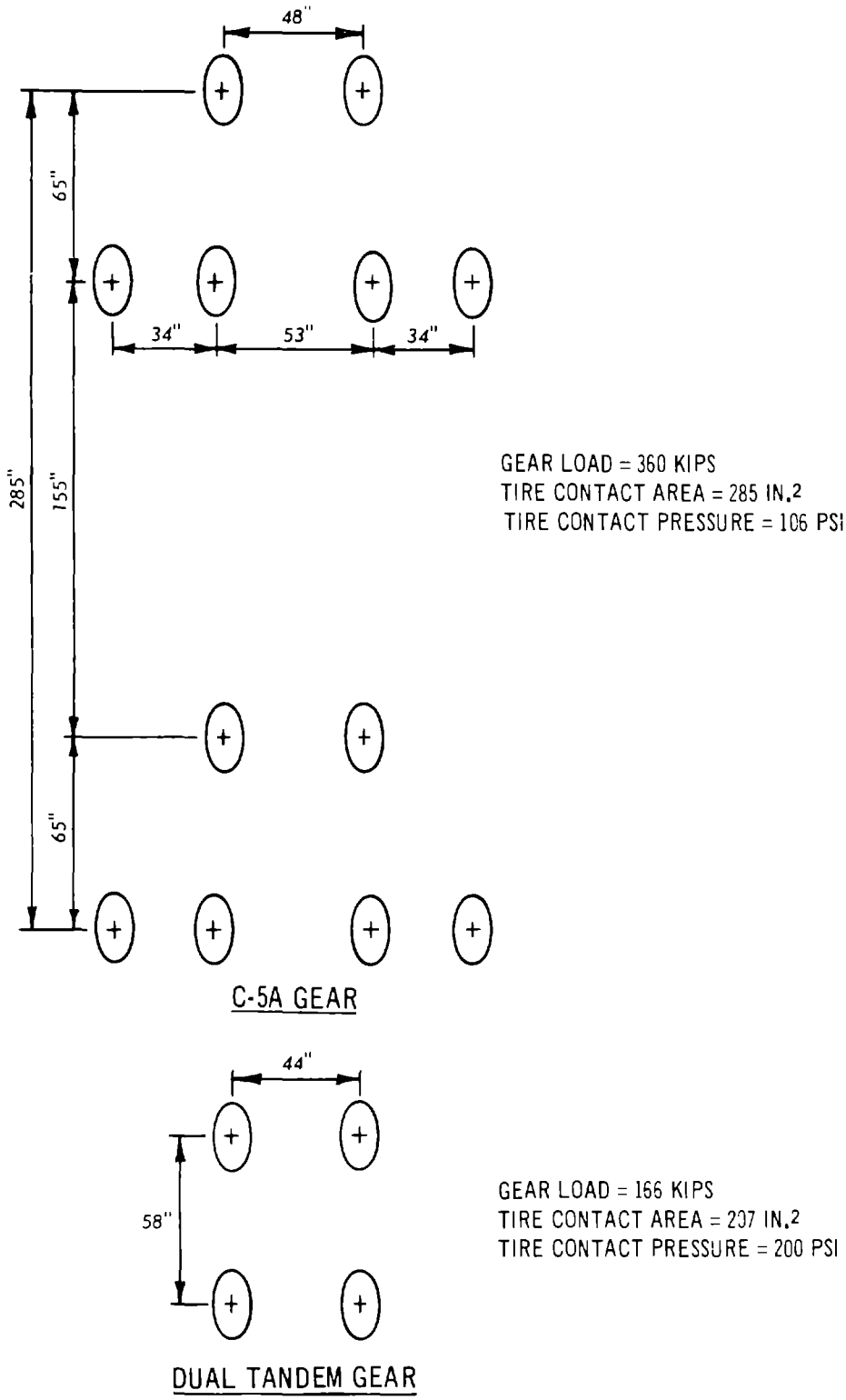


Figure B12. Schematic representation of loads used in Multiple-Wheel Heavy Gear Load Test Track

Table B5

Summary of Load, PCC Flexural Strength, and Traffic--
Multiple-Wheel Heavy Gear Load Test Track

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
1	360*	725	221	First crack
2	360*	800	4230	First crack
2	166**	700	95	First crack
3	166**	660	205	First crack
3	360*	700	1400	First crack
4	360*	775	180	First crack

* C-5A gear.
** Dual-tandem gear.

KEYED LONGITUDINAL
JOINT STUDY (KLJS)

The excavation, construction of the foundation materials, and final concrete paving phases of the KLJS test section occurred during the period April-May 1971. The construction, testing, and analysis of the data from the KLJS test pavements are discussed in Reference 38. The pavement thicknesses, foundation materials, and types of longitudinal construction joints varied in the four test items. The rigid pavement thicknesses of items 1 and 2 were 8 and 11 in., respectively; items 3 and 4 were both 10 in. thick. Each test item contained four 25-ft-square concrete slabs at uniformed thickness, two in the north lane and two in the south lane. The transition slabs that had been constructed for the MWHGL were left in place and used in this study. The gear configurations used for trafficking the KLJS test items were the same that were used in the MWHGL study (Figure B12).

Figure B13 shows the pavement properties used in this study. Table B6 summarizes the wheel loads, concrete flexural strengths, and traffic coverages applied to the test items.

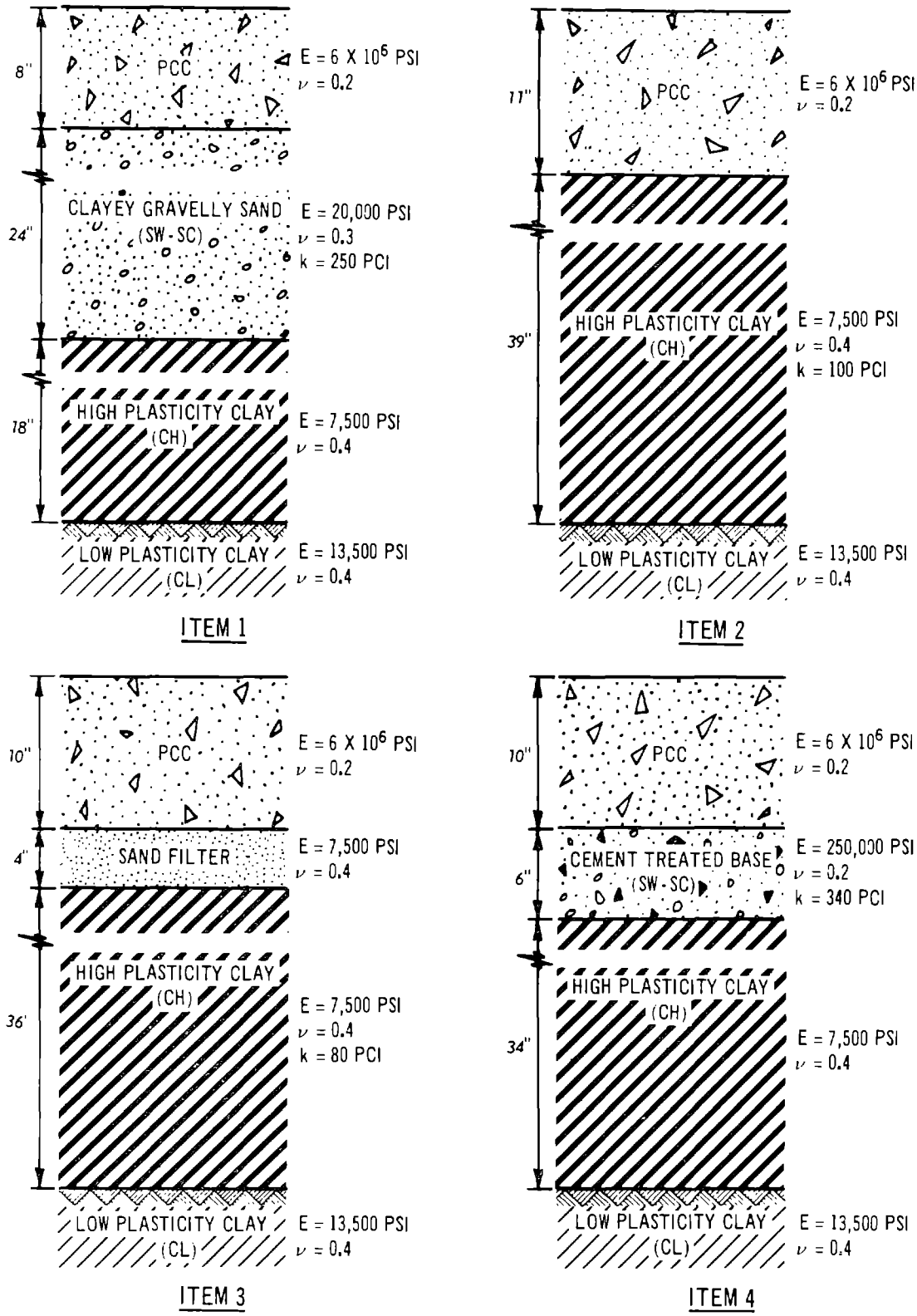


Figure B13. Description of test pavements in Keyed Longitudinal Joint Study

Table B6

Summary of Load, PCC Flexural Strengths, and Traffic--
Keyed Longitudinal Joint Study

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
1	360*	905	54	First crack
2	360*	730	344	First crack
3	360*	810	22	First crack
4	360*	860	6336	First crack
4	166**	860	320	First crack

* C-5A gear.
** Dual-tandem gear.

SOIL STABILIZATION
PAVEMENT STUDY (SSPS)

This final test section, designated the SSPS, was constructed during the period March-August 1972. This area had been used previously for the MWHGL and KLJS tests. The construction, testing, and analyses of the data from this test section are discussed in Reference 39. The test section was 290 ft long and 50 ft wide and consisted of five test items, each 50 ft square and separated by 10-ft-wide transition slabs. The concrete in each item was first placed in the north paving lane (25- by 50-ft sections) between the transition slabs. Only two items from this test track were used in this study. These were 15-in.-thick slabs on 6-in.-thick bound base course layers. Simulated aircraft traffic was applied to the test items using a dual-tandem assembly. A net weight of 200 kips (50 kips/wheel) was used for trafficking lane 1 and 240 kips (60 kips/wheel) for trafficking lane 2. The contact area for both loads was maintained at 267 sq in. by using inflation pressures of 190 and 250 psi for the 200- and 240-kip loads, respectively.

Figure B14 shows the pavement properties used in this study; Figure B15, the loads used to traffic the test pavements. Table B7 summarizes the concrete flexural strengths and the traffic applied to each test item.

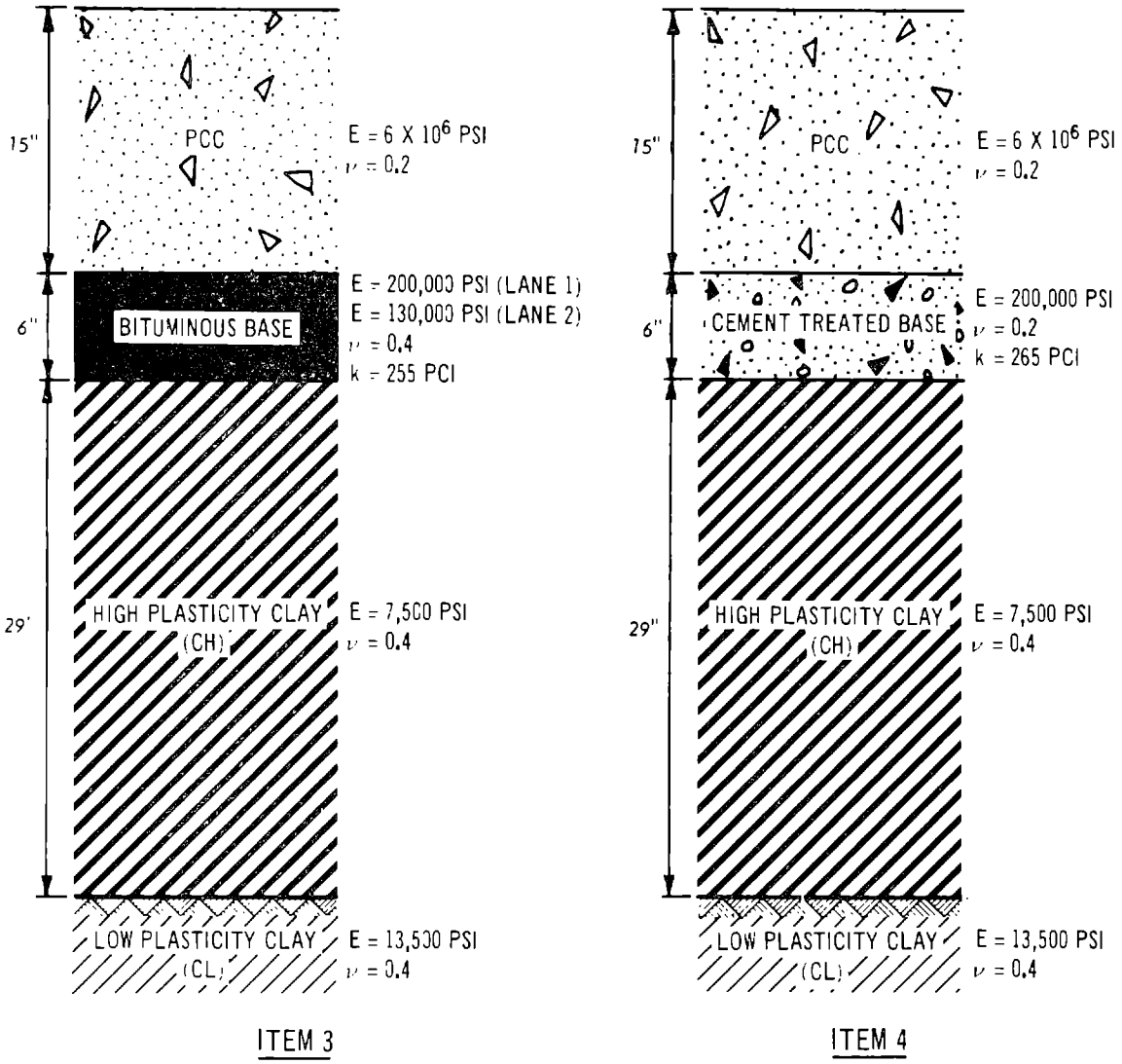
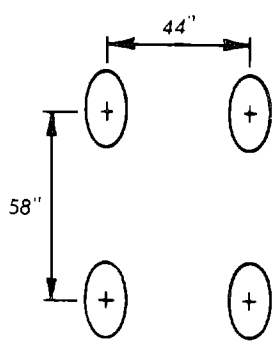


Figure B14. Description of test pavements in Soil Stabilization Pavement Study



	200-KIP GEAR LOAD	240-KIP GEAR LOAD
TIRE CONTACT AREA, IN.^{2}	267	267
TIRE CONTACT PRESSURE, PSI	187	225

Figure B15. Schematic representation of loads used in Soil Stabilization Pavement Study

Table B7

Summary of Load, PCC Flexural Strength, and Traffic--
Soil Stabilization Pavement Study

Item	Load, kips	Flexural Strength psi	Traffic Coverages	Remarks
3	200	900	3215	First crack
3	240	900	350	First crack
4	200	870	4660	First crack
4	240	870	70	First crack

APPENDIX C: LABORATORY PROCEDURE FOR DETERMINING THE FLEXURAL MODULUS OF BOUND BASES

The procedure contained herein involves application of a repetitive loading to a beam specimen under controlled stress conditions. Applied load and deflection along the neutral axis and at the lower surface are monitored, and the results are used to determine the flexural modulus. Because of the sensitivity of bituminous-stabilized materials to temperature, rate of loading, and repeated loading, some differences are noted with the procedure for chemically stabilized materials.

SPECIMEN PREPARATION

CHEMICALLY STABILIZED MATERIAL

Beam specimens should be prepared following the general procedures outlined in ASTM Standard Procedure Designation: D 1632-63.[65] This method describes procedures for molding 3- by 3- by 11-1/4-in. specimens; however, any size mold may be used for the test. For soils containing aggregate particles larger than 3/4 in., it is recommended that molds on the order of 4 by 4 to 6 by 6 in. be used. In general, specimens should have an approximately square cross-sectional configuration and a length adequate to accommodate an effective test span equal to three times the height or width. Specimens should be molded to the stabilizer treatment level, moisture content, and density expected in the field structures. Specimens should be moist-cured for 28 days.

BITUMINOUS-STABILIZED MATERIAL

Beam specimens should be prepared following the general procedures outlined in ASTM Standard Procedure Designation: D 3202-73.[66] If there is undue movement of the mixture under the compactor foot during beam compaction, the temperature, foot pressure, and number of tamping blows should be reduced. Similar modifications to compaction procedures should be made if specimens with less density are desired.

A diamond-blade masonry saw is used to cut 3-in.- (or slightly less) deep by 3-in.- (or slightly less) wide test specimens from the 15-ft-long beams. Specimens with suitable dimensions can also be cut from pavement samples. The widths and depths of the specimens are measured to the nearest 0.01 in. at the center and at 2 in. from both sides of the center. Mean values are determined and used for subsequent calculations.

EQUIPMENT

The following equipment is required:

a. Loading frame capable of receiving specimen for third-point loading test.

b. A 3000-lb capacity electrohydraulic testing machine capable of applying static and repeated tension-compression loads in the form of haversine waves.

c. Load cell (approximately 3000-lb capacity).

d. Two linear variable differential transformers (LVDT's).

e. Recording equipment for monitoring deflection, strain, and load.

f. Miscellaneous pins and yokes, as described in the equipment setup below, for mounting the LVDT's.

g. Controlled-temperature cabinet capable of controlling temperature within $\pm 1^{\circ}$F.

EQUIPMENT SETUP

CHEMICALLY STABILIZED
MATERIALS

Figures C1-C3 present the details of the equipment. The beam should be positioned so that the molding laminations are horizontal. The three yokes are positioned over the top of the beam and held in place by threaded pins positioned along the neutral axis. The end pins, A and C, are positioned directly over the end reaction points, and the middle pin B is positioned at the center of the beam. A metal bar rests on top of the pins. At the A position, the bar is equipped with a lower vertical tab having a hold that slips loosely over the pin. A nut is placed on the end of the pin to prevent the bar from slipping. At the center or B position, the bar is equipped with a vertical tab onto which

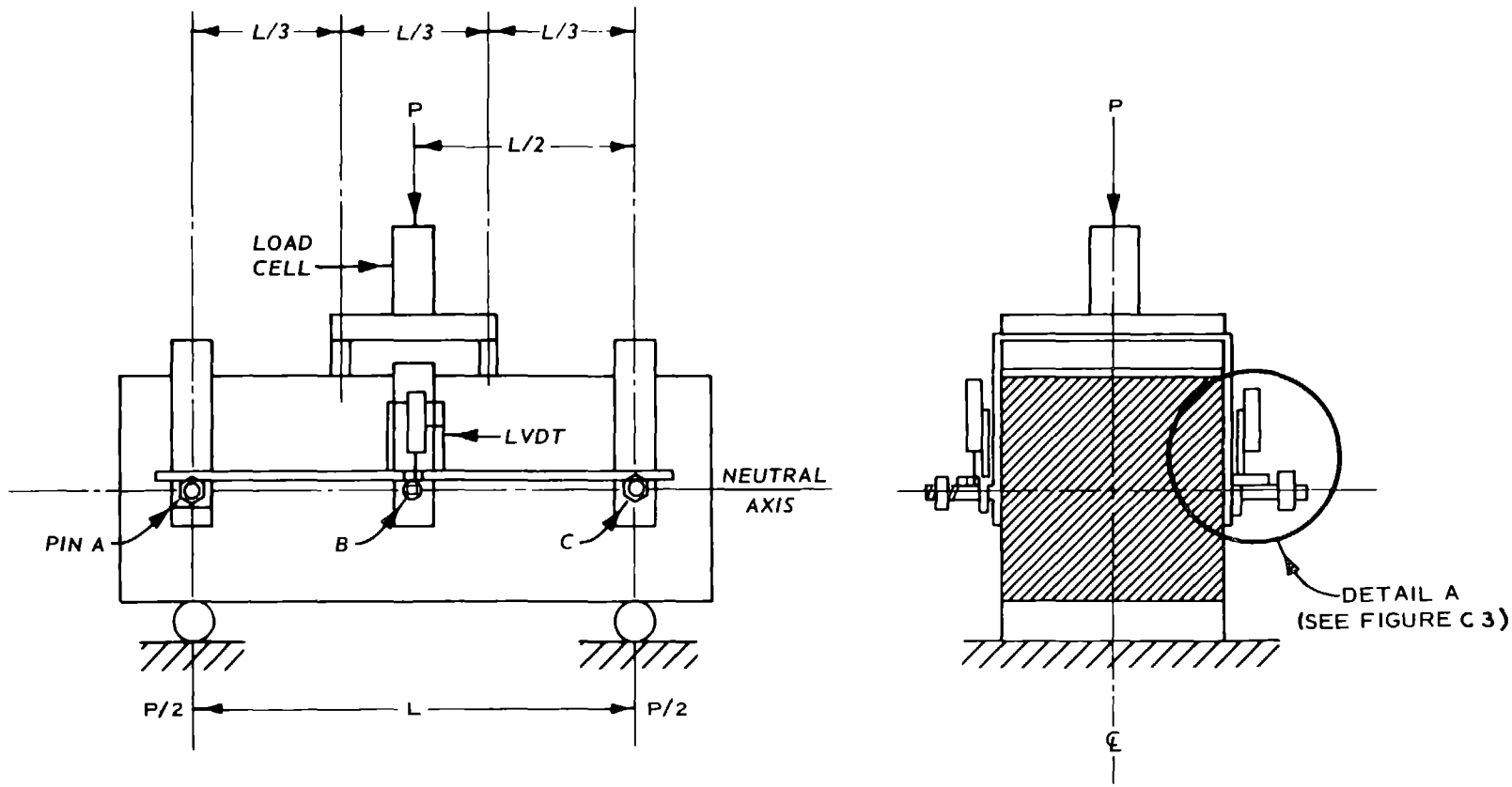


Figure C1. General view of equipment setup for one-way loading

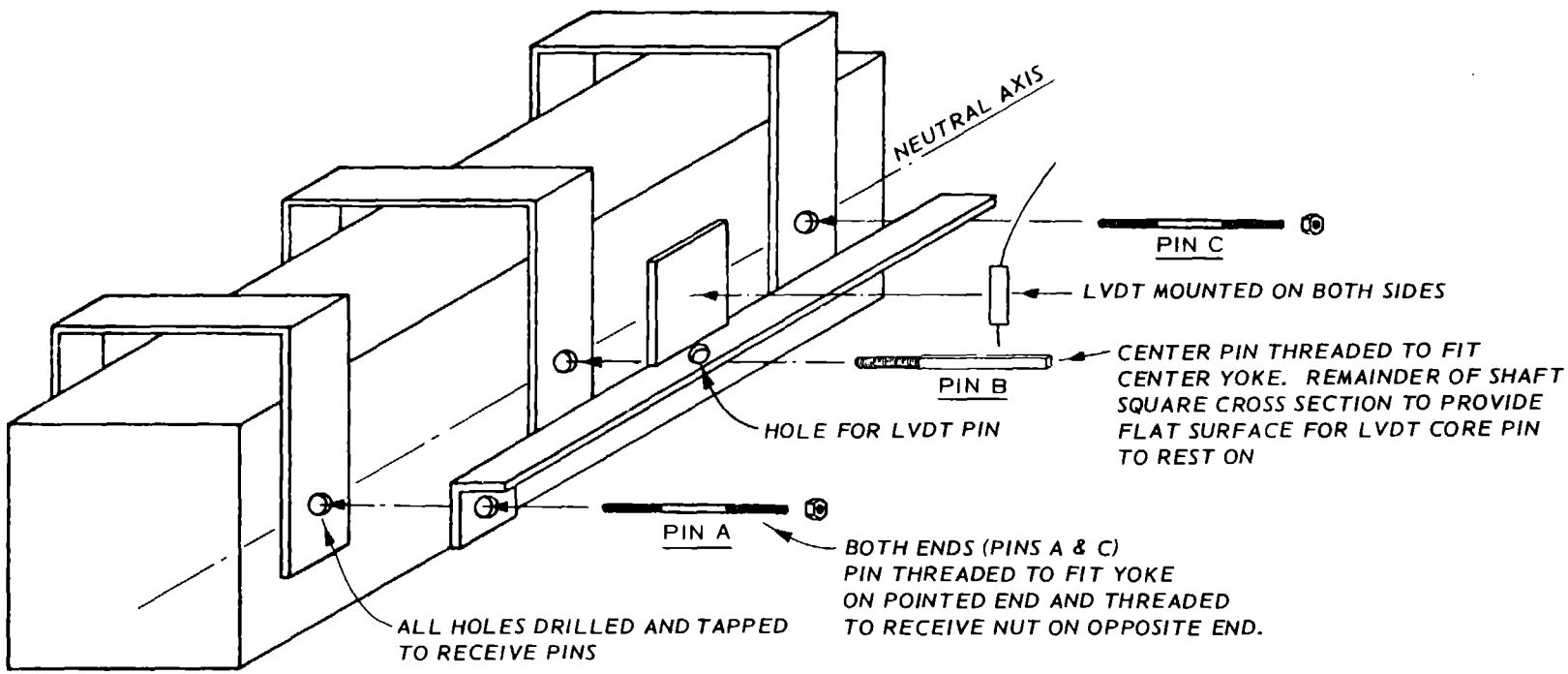
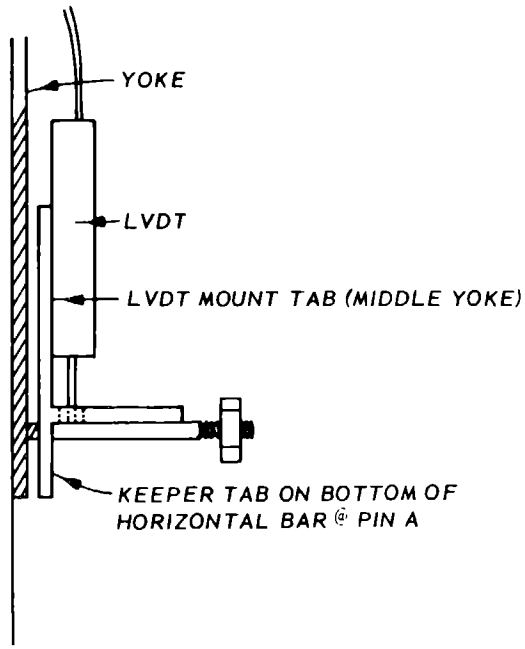
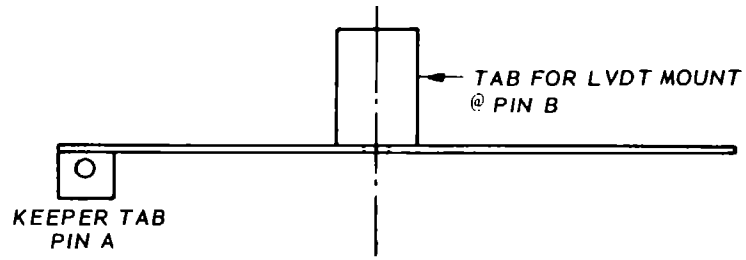


Figure C2. Details of equipment setup

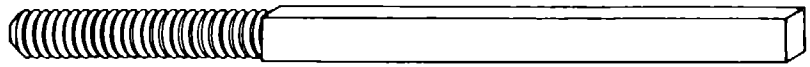


DETAIL A (FROM FIGURE C1)



DETAIL OF HORIZONTAL BAR

NOTE: PIN B IS ALWAYS POSITIONED SO THAT FLAT SURFACE IS HORIZONTAL



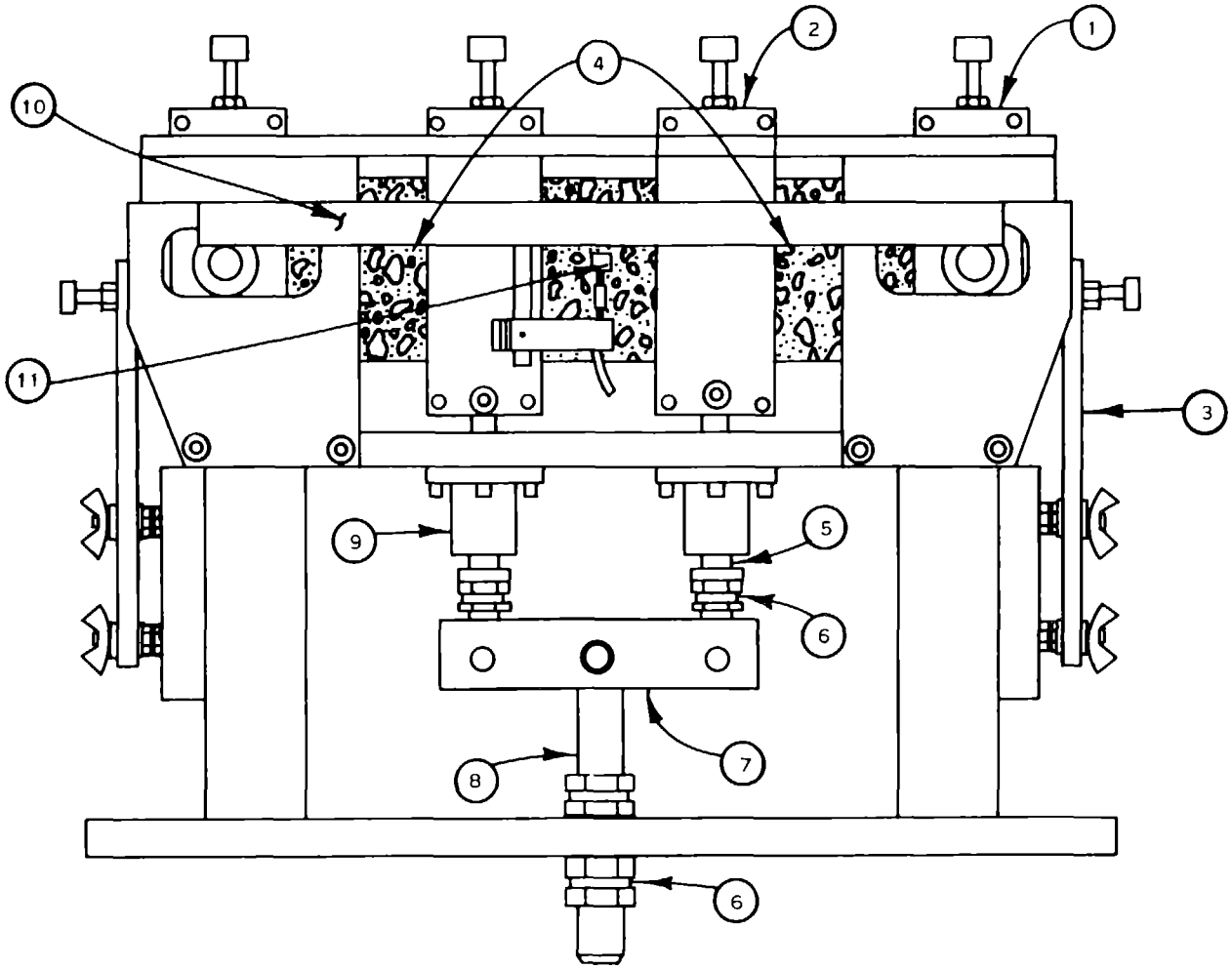
DETAIL OF PIN B

Figure C3. Details of pins A and B and horizontal bar

an LVDT is cemented in a vertical position. At this position on the bar, there is a hole through which the LVDT core pin falls to rest on the B pin. This pin must be fabricated with flat sides on the shaft to provide a horizontal surface on which the LVDT core pin rests. At the C position, the end of the bar simply rests on the unthreaded portion of the C pin. A nut is placed on the end of the C pin to prevent excessive side movement of the bar end. This type of bar, pin, and LVDT arrangement is provided on both sides of the beam. Although no dimensions are provided in Figures C1-C3, this type of equipment can easily be dimensioned and fabricated to fit any size beam. Either steel or aluminum may be used. The beam should be positioned and arranged to accommodate third-point loading as indicated in Figure C2. As the beam bends under loading, deflection at the center is measured by determining the movement of the LVDT stems from their original positions. The LVDT's are connected to the monitoring system to give an average deflection reading.

BITUMINOUS-STABILIZED
MATERIALS

Bituminous-stabilized materials are affected by temperature, rate of loading, and repeated load applications. The tests should be conducted in a controlled-temperature cabinet capable of controlling temperature within $\pm 1^{\circ}F$. The beam will tend to deform permanently under repeated load applications or its own weight due to the viscous nature of the binder. Therefore, a loading device capable of transmitting both upward and downward to the specimen is required. The arrangement in Figures C1-C3 is capable of applying only a downward force. A loading device similar to that in Figure C4 is needed. This device permits both upward and downward forces to be applied to the specimen. A sufficient load, approximately 10 percent of the load deflecting the beam upward, is applied in the opposite direction forcing the beam to return to its original horizontal position and holding it at that position during the rest period. Adjustable stop nuts installed on the flexure apparatus loading rod prevent the beam



KEY

1. REACTION CLAMP
2. LOAD CLAMP
3. RESTRAINER
4. SPECIMEN
5. LOADING ROD
6. STOP NUTS
7. LOAD BAR
8. PISTON ROD
9. THOMPSON BALL BUSHING
10. LVDT HOLDER
11. LVDT

Figure C4. General view of equipment setup for two-way loading

from bending below the initial horizontal position during the rest period. With the device presented in Figure C4, the LVDT cores are normally attached to a nut bonded to the center of the specimen. However, a mechanical clamp, as in Figure C3, may be used. The specimen is clamped in the fixture using a jig to position the centers of the two loading clamps. Double layers of Teflon sheets are placed between the specimen and the loading clamps to reduce friction and longitudinal restraint caused by the clamps.

TEST PROCEDURE

CHEMICALLY STABILIZED MATERIALS

The flexural beam test is a stress-controlled test. Therefore, an initial specimen should be statically loaded to failure. The repetitive load test should be conducted using a haversine wave form, a loading duration of 0.5 sec, and a frequency of about 1 Hz. It is recommended that tests be conducted at 40, 60, and 80 percent of the maximum rupture value; however, stress levels can be varied to higher or lower levels. About 400 load repetitions should be applied to condition the specimen, and all gages reset. The load and deflection along the neutral axis should be monitored at 100, 1,000, and 10,000 load repetitions.

BITUMINOUS-STABILIZED MATERIALS

The beam is positioned in the loading device, placed in the controlled-temperature cabinet, and brought to the desired temperature. Specimens should be tested at 40, 70, and 100°F. A dummy specimen, with a thermocouple in the center, may be used to determine when the specimen has reached the desired temperature. The repetitive load tests should be conducted using a haversine wave form, a loading duration of 0.2 sec, and a frequency of about 2 Hz. The test is stress-controlled. Thus, loads that will result in reasonable stresses in the specimen should be used. One test should be run with a load that will

result in stresses in the outer fibers of the beam of about 50 psi, and subsequent loads should be applied that will result in stress less than the following:

Temperature Range, °F	Maximum Stress, psi
40-60	450
60-80	300
80-100	200

Should excessive deflections occur the load should be reduced. About 500 conditioning load repetitions should be applied, and all gages reset. The load and deflections should be monitored at 100, 1,000, and 10,000 load repetitions.

REPORTING OF TEST RESULTS

The flexural modulus should be determined at 100, 1,000, and 10,000 load repetitions or at failure. This value may be determined from load and deflection data monitored at these repetition levels using the expression

$$E = \frac{23PL^3}{1296dI}\left[1 + 2.11\left(\frac{h}{L}\right)^2\right] \qquad (C1)$$

where

E = flexural modulus, psi

P = maximum load amplitude, lb

L = specimen length, in.

d = deflection at the neutral axis, in.

I = moment of inertia, $in.^4$

h = specimen height, in.

For chemically stabilized materials, the value to be used for E is the arithmetic mean of all values obtained during the test. For bituminous-stabilized materials, the average should be determined for each temperature at which tests were conducted and a relationship between modulus of elasticity and temperature was established.

APPENDIX D: LABORATORY PROCEDURE FOR DETERMINING THE RESILIENT MODULUS OF GRANULAR BASE MATERIALS

This procedure is designed to determine resilient properties of granular base (subbase) materials. The test is similar to a standard triaxial compression test, the primary exception being that the deviator stress is applied repetitively at several stress levels. The procedure allows testing under a repetitive stress state similar to that encountered in a base (subbase) course layer in a pavement under a moving wheel load.

DEFINITIONS

The following symbols and terms are used in the description of this procedure:

a. σ_1 = total axial stress.

b. σ_3 = total radial stress, i.e., confining pressure in the triaxial test.

c. σ_d = deviator stress ($\sigma_1 - \sigma_3$), i.e., the repeated axial stress in this procedure.

d. ε_1 = total axial strain due to σ_d .

e. ε_R = resilient axial strain due to σ_d .

f. ε_ℓ = resilient lateral strain due to σ_d .

g. M_R = the resilient modulus = σ_d/ε_R .

h. ν_R = the resilient Poisson's ratio = $\varepsilon_\ell/\varepsilon_R$.

i. θ = sum of the principal stresses in the triaxial state of stress ($\sigma_1 + 2\sigma_3 = \sigma_d + 3\sigma_3$).

j. σ_1/σ_3 = principal stress ratio.

k. Load duration = time interval during which the sample is subjected to a stress deviator.

l. Cycle duration = time interval between successive applications of the deviator stress.

SPECIMENS

For base course materials, 6-in.-diam specimens are generally required with the maximum particle size being limited to 1 in. The specimen height should be at least twice the diameter. Methods for

preparation of specimens are set forth in Engineer Manual EM 1110-2-1906.[67]

EQUIPMENT

TRIAXIAL TEST CELL

The triaxial cell shown schematically in Figure D1 is suitable for use in resilient testing of soils. The equipment is similar to most standard cells. However, there are a few specialized criteria that must be met to provide acceptable test results. Generally, the equipment is slightly larger than most standard cells to accommodate the 6-in.-diam specimens and the internally mounted load and deformation measuring equipment. Additional outlets for the electrical leads from these measuring devices are required.

Cell pressures of 80 psi are generally sufficient to duplicate the maximum confining pressures under aircraft loadings. Compressed air is generally used as the confining fluid to avoid detrimental effects of water on the internally mounted electronic measuring equipment.

END PLATENS

End platens should be "frictionless," as "barrelling" caused by end restraint jeopardizes resilient Poisson's ratio values by causing lateral deformations to be concentrated in the middle of the specimen. Furthermore, nonuniform displacements can create problems with axial strain measurements due to realignment of the LVDT clamps. Whereas "frictionless" platens (Figure D2) may not be entirely frictionless under short-term repetitive loadings, they constitute an improvement over conventional end platens. The essential features of "frictionless" end platens are (a) hard polished end plates, (b) coated by high-vacuum silicone grease, (c) covered by a thin rubber sheet. If externally mounted axial deformation measuring devices, such as an LVDT or potentiometer mounted on the loading piston, or devices measuring the total specimen displacements are used, the use of frictionless caps and bases with grease invalidates any measurements. In this case, the deformation due to the grease and rubber sheet or Teflon probably exceeds the actual

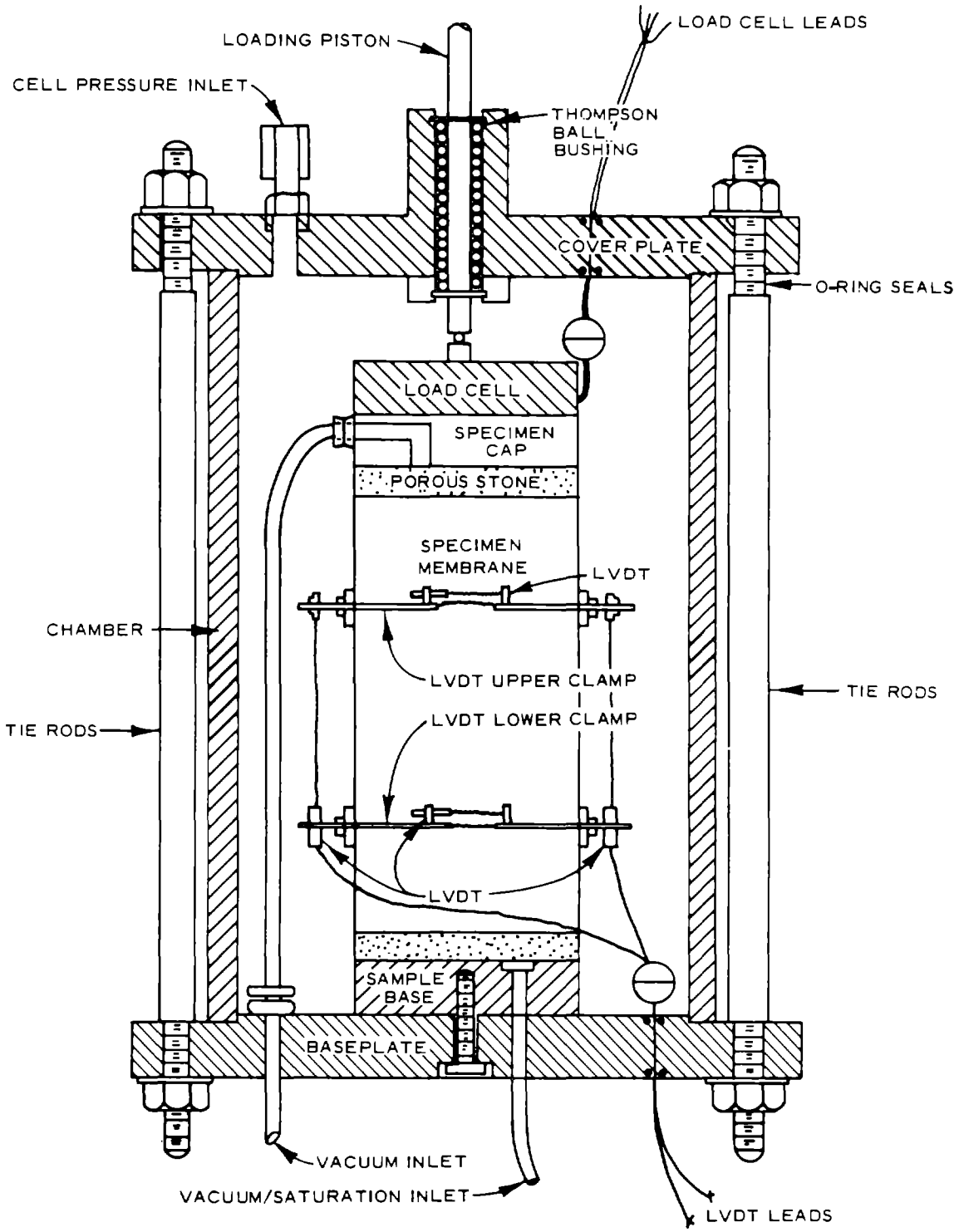


Figure D1. Triaxial cell used in resilience testing of granular base material

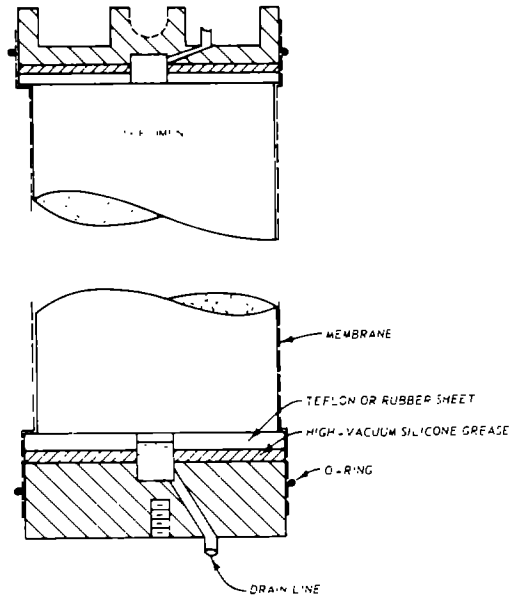


Figure D2. Schematic of frictionless cap and base

deformation of the specimen. Hence, frictionless caps and bases are restricted to use with internally mounted deformation sensors.

REPETITIVE LOADING EQUIPMENT

The external loading source may be any device capable of providing a variable load of fixed cycle and load duration, ranging from simple switch control of static weights or air pistons to a closed-loop electrohydraulic system. A load duration of 0.1 to 0.2 sec and a cycle duration of 3 sec have been found satisfactory for most applications. A haversine wave form is recommended; however, a rectangular wave form can be used.

DEFORMATION AND LOAD MEASURING EQUIPMENT

The deformation measuring equipment consists of four LVDT's attached to the soil specimen with a pair of clamps, as shown in Figure D1. Two LVDT's are used to measure axial deformations, and two are used to measure lateral deformations. Figures D3 and D4 show the details of the clamps for attaching the LVDT's to the soil specimens. Only a-c transducers that have a minimum sensitivity of

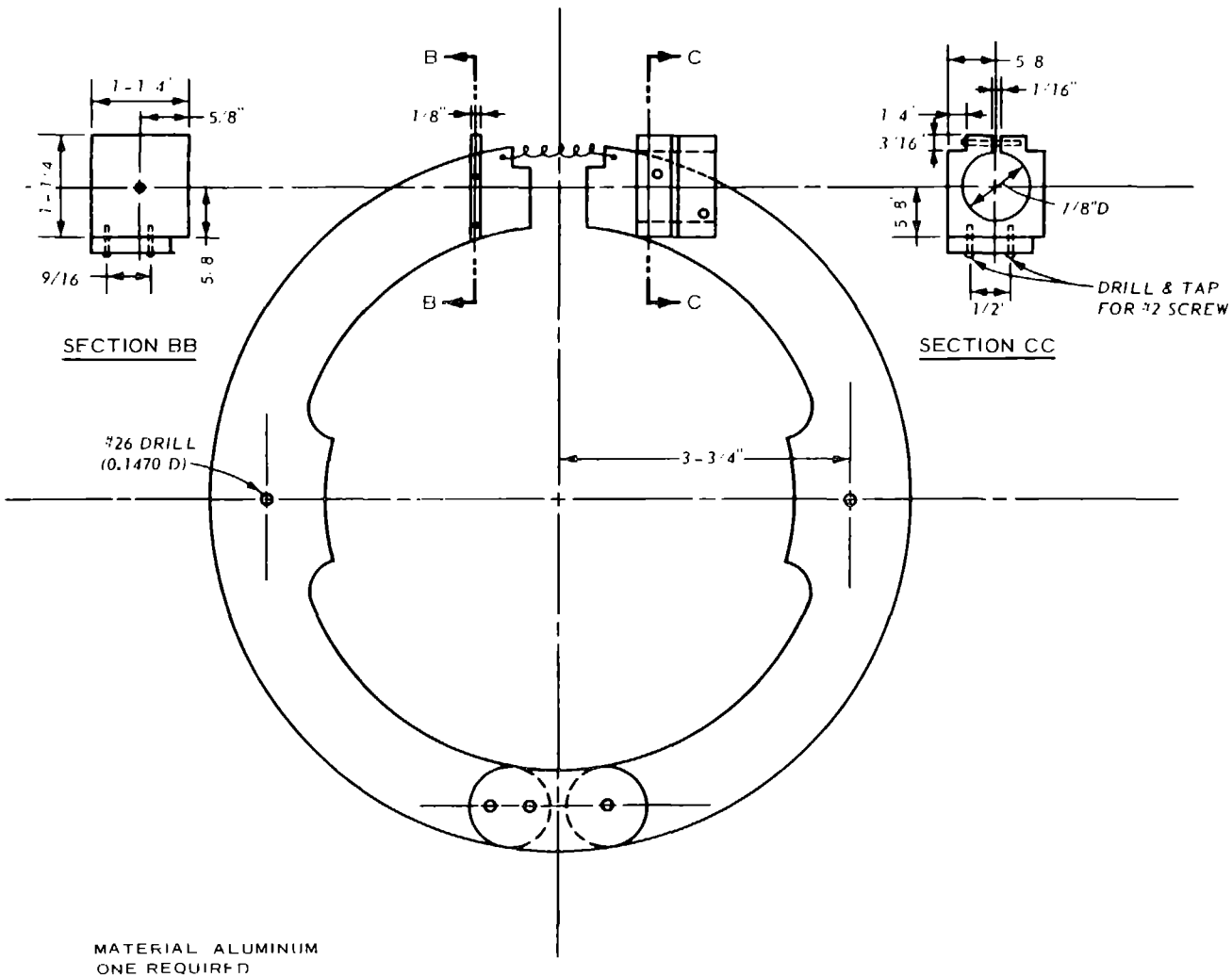


Figure D3. Details of top LVDT ring clamp

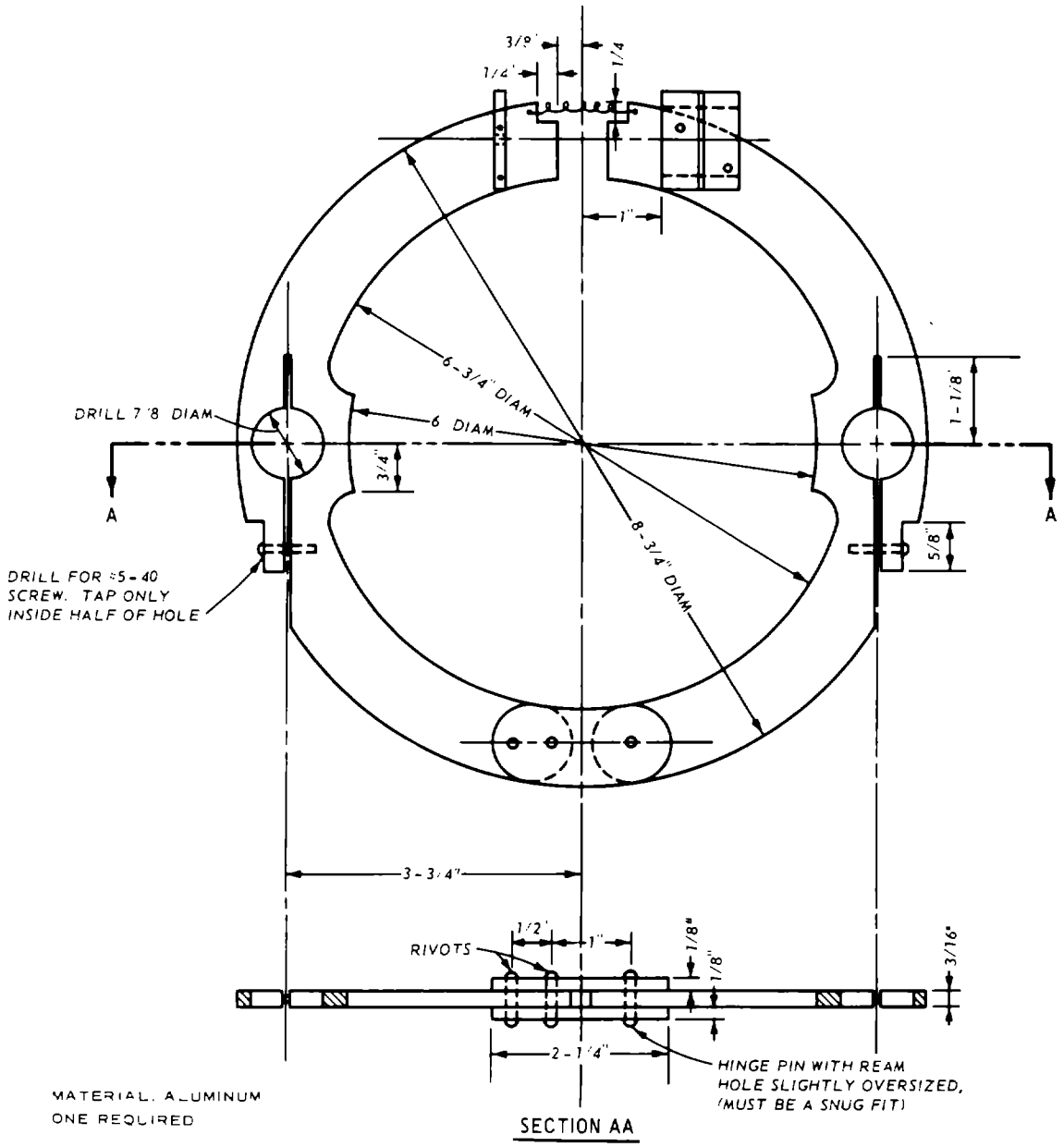


Figure D4. Details of bottom LVDT ring clamp

0.2 mv/0.001 in./v should be used. Load is measured with an internally mounted load cell that is sufficiently lightweight so as not to provide any significant inertia forces. It should have a capacity no greater than 2 to 3 times that of the maximum applied load and a minimum sensitivity of 2 mv/v.

ADDITIONAL EQUIPMENT

In addition to the equipment described above, the following items are also used:

a. Calipers, a micrometer gage, and a steel rule (calibrated to 0.01 in.).

b. Rubber membranes (0.012 to 0.025 in. thick) and a membrane stretcher.

c. Rubber O-rings.

d. Guide rods for positioning LVDT clamps.

e. Epoxy for cementing clamps to membrane.

f. A vacuum source with a bubble chamber (optional) and regulator.

g. Specimen forming jacket.

It is also necessary to have a fast recording system for accurate testing. It is recommended, for analog recording equipment, that the resolution of the parameter being controlled be better than 1.5 percent of the maximum value of the parameter being measured and that any variable amplitude signals be changed from high to low resolution as required during the test. If multichannel recorders are not available, by introducing switching and balancing units, a single-channel recorder can be used.

PREPARATION OF SPECIMENS AND
PLACEMENT IN TRIAXIAL CELL

Specimen preparation procedures are governed by the criteria set forth in Appendix X of Engineer Manual EM 1110-2-1906.[67] The following procedures describe a step-by-step account for preparing remolded specimens. Generally, for base course materials, 6-in.-diam specimens are required with the maximum particle size being limited to 1 in. in diameter.

MATERIAL PREPARATION

The material should be air-dried and subsequently sufficient water added to bring the material to the desired compaction water content (usually field condition). Sealing the material in a container for 24 hr prior to compaction will allow the moisture to equilibrate. For well-graded materials, it may be necessary to break the material down into several sieve sizes and recombine for each layer to prevent serious segregation of material in the specimen. If the compaction effort required to duplicate the desired testing water content and density is known, sufficient material for several specimens may have to be prepared. The compaction effort required will then be established on a trial-and-error basis.

SPECIMEN COMPACTION

Generally, base course materials are compacted on the triaxial cell baseplate using a split mold. If the particles are angular, two membranes may be required: one used during compaction and the second placed after compaction to seal any holes punctured in the membrane. A successful procedure has been to use a Teflon-lined mold and a thin sheet of wrapping paper instead of a membrane. Often the density is sufficiently high and the water content such that effective cohesion will permit a free-standing specimen to be prepared. In this case, the wrapping paper is carefully removed and a membrane substituted. In most cases, impact or kneading compaction is used. Although EM 1110-2-1906[67] mentions vibratory compaction, vibratory compaction is only permitted on uniform materials where segregation is not a problem. The specimens should be compacted in layers, the height of which exceeds the maximum particle size.

It may be necessary to place a thin layer of fine sand in the bottom layer to provide a smooth bearing surface. Likewise, after compacting and trimming the topmost layer (it may be necessary to remove large particles from this layer), fine sand can be sieved on the surface to fill in the voids and provide a smooth bearing surface for the top cap.

Center the top cap and lightly tap the cap to level and ensure a good smooth contact of the cap on the specimen. A level placed on top of the cap is used to check leveling. The forming mold is then removed, the membrane placed using a membrane stretcher and sealed with O-rings or a hose clamp, and a vacuum applied. Check for leakage by using a bubble chamber or closing the vacuum line and observing if a vacuum is maintained in the specimen. Specimen dimensions should be measured to determine density conditions. A π-tape has been found most useful for diametrical measurements.

PLACEMENT OF LVDT
MEASUREMENT CLAMPS

Measure the diameter as accurately as possible at the location of the LVDT clamps for calculation of radial strains. Place the lower LVDT clamp in the specimen at approximately the lower third point of the specimen. A "jig" or gage rods have been used successfully to assist in placing the clamps. The lower LVDT clamp generally holds the LVDT body. Repeat the procedure for the upper clamp, being careful to align the clamps so the LVDT core matches the LVDT body. It is essential that the clamps lie in a horizontal plane and their spacing be precisely known for calculating the axial strain. Again, gage rods or a "jig" in conjunction with a small level have been used successfully for this operation. With the clamps in position and secured by the springs, a small amount of epoxy (a "5-min" epoxy has been used; rubber cement was found unacceptable) is placed on top of the four contact points and allowed to dry.

Install the LVDT's and connect the recording unit. Generally, ±0.040-in. LVDT's are used for radial deformations, while ±0.100-in. LVDT's are used for axial deformations. Balance the vertical spacing between LVDT clamps or check gage rods for secure contact, and record LVDT readings and spacing. Remove gage rods and assemble triaxial chamber. Any shifting of LVDT clamps during chamber assembly will be noted by LVDT reading changes and can be accounted for.

RESILIENT TESTING

The resilient properties of granular materials are dependent primarily upon confining pressure and to a lesser extent upon cyclic deviator stress. Therefore, it is necessary to conduct the tests for a range of confining pressures and deviator stress values. Generally, chamber pressure values of 2, 4, 6, and 10 psi are suitable. Ratios of σ_1/σ_3 of 2, 3, 4, and 5 are typically used for the cyclic deviator stress. Tests should be conducted in an undrained condition with excess pressures relieved after application of each stress state. The testing procedure is as follows:

a. Balance the recorders and recording bridges and record calibration steps.

b. Apply about 2-psi axial load σ_d as a seating load simulating the weight of the pavement and ensuring contact is maintained between the loading piston and top cap during testing.

c. Condition the specimen by applying 500 to 1000 load repetitions with drainage lines open. This conditioning stress should be the maximum stress expected to be applied to the specimen in the field by traffic. If this is unknown, a chamber pressure of 5 or 10 psi and a deviator stress ($\sigma_1 - \sigma_3$) twice the chamber pressure can be used.

d. Decrease the chamber pressure to the lowest value to be used. Apply 200 load repetitions of the smallest deviator stress under undrained conditions, recording the resilient deformations and load at or near the 200th repetition. After 200 load repetitions, relieve any pore pressures, increase the deviator stress to the next highest value, and repeat procedure over the range of deviator stresses to be used.

e. After completing the stress states for the initial confining pressure, repeat for each succeedingly higher chamber pressure.

f. After completion of the loading, remove the axial load, apply a vacuum to the specimen, release the confining pressure, and disassemble the triaxial chamber.

g. Check the calibration of the LVDT's and load cell.

h. Dry the entire specimen for determination of the water content.

COMPUTATIONS AND PRESENTATION OF RESULTS

COMPUTATIONS

The computations consist of the following:

a. From the measured dimensions and weights, compute and record the initial dry density, degree of saturation, and water content using the equations in Appendix II, EM 1110-2-1906.[67]

b. The resilient modulus is computed and recorded for each stress state using the following formulas:

(1) Resilient axial strain $\varepsilon_R = \Delta H_r / H_i$.

(2) Resilient lateral strain $\varepsilon_\ell = \Delta D_r / D_i$.

(3) Deviator stress $\sigma_d = \Delta P / A_o$.

(4) Resilient modulus $M_R = \sigma_d / \varepsilon_R$.

(5) Resilient Poisson's ratio $\nu_R = \varepsilon_\ell / \varepsilon_R$.

where

ΔH_r = resilient change in gage height (distance between LVDT clamps) after specified number of load repetitions

H_i = instantaneous gage height after specified number of load repetitions. Can be calculated from $H_o - \Delta H$. If ΔH is small, H_o can be used.

H_o = initial gage height or distance between LVDT's less adjustments occurring during triaxial chamber assembly

ΔH = permanent change in gage height

ΔP = change in axial load, maximum axial load minus surcharge load

A_o = original cross-sectional area of specimen

ΔD_r = resilient change in diameter after specified number of load repetitions

D_i = instantaneous diameter after specified number of load repetitions. Can be calculated from $D_o + \Delta D$.

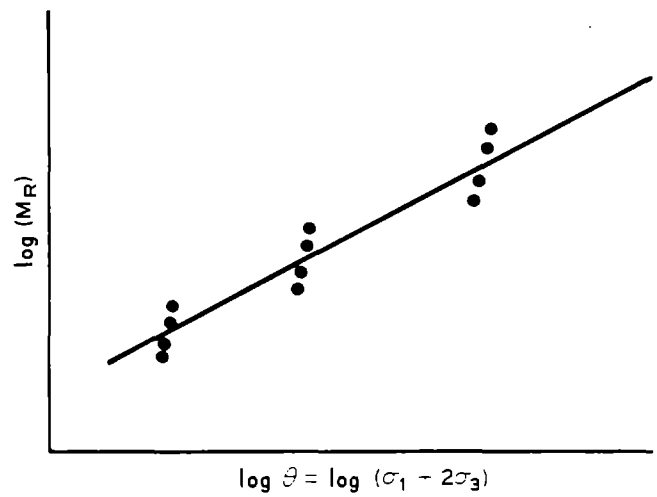
D_o = initial specimen diameter

ΔD = permanent change in specimen diameter

PRESENTATION OF RESULTS

Test results should be presented in the form of plots of log M_R versus log of the sum of the principal stresses and ν_R versus the

principal stress ratio (Figure D5). The equation of the line for resilient modulus is $M_R = K_1(\theta)^{K_2}$ where K_1 is the intercept when $\theta = 1$ psi and K_2 is the slope of the line.



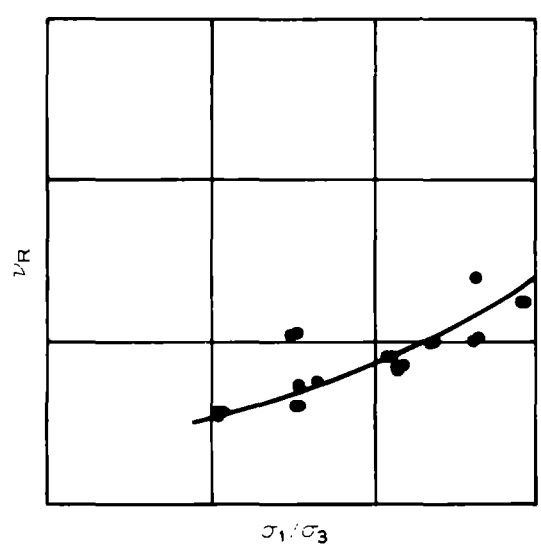


Figure D5. Representation of results of resilience test on cohesionless soils

APPENDIX E: LABORATORY PROCEDURE FOR DETERMINING THE RESILIENT MODULUS OF SUBGRADE SOILS

The objective of this test procedure is to determine a modulus value for subgrade soils by means of resilient triaxial techniques. The test is similar to a standard triaxial compression test, the primary exception being that the deviator stress is applied repetitively and at several stress levels. This procedure allows testing of soil specimens in a repetitive stress state similar to that encountered by a soil in a pavement under a moving wheel load.

DEFINITIONS

The following symbols and terms are used in the description of this procedure:

a. σ_1 = total axial stress.

b. σ_3 = total radial stress; i.e., confining pressure in the triaxial test chamber.

c. $\sigma_d = \sigma_1 - \sigma_3$ = deviator stress; i.e., the repeated axial stress in this procedure.

d. ε_1 = total axial strain due to σ_d .

e. ε_R = resilient or recoverable axial strain due to σ_d .

f. ε_ℓ = resilient or recoverable axial strain due to σ_d .

g. $M_R = \sigma_d/\varepsilon_R$ = resilient modulus.

h. $\nu_R = \varepsilon_R/\varepsilon_\ell$ = resilient Poisson's ratio.

i. $\theta = \sigma_1 + 2\sigma_3 = \sigma_d + 3\sigma_3$ = sum of the principal stresses in the triaxial state of stress.

j. σ_1/σ_3 = principal stress ratio.

k. Load duration = time interval over which the specimen is subjected to a deviator stress.

l. Cycle duration = time interval between successive applications of a deviator stress.

SPECIMENS

Various diameter soil specimens may be used in this test, but the recommended specimen diameter is 2.5 to 3.0 in., or at least four times maximum particle size for granular materials. The specimen height should

be at least twice the diameter. Undisturbed or laboratory molded specimens can be used. Procedures for obtaining undisturbed soil specimens are given in Engineer Manual EM 1110-2-1907, "Soil Sampling."[68] Methods for laboratory preparation of molded specimens and for back-pressure saturation of specimens, if required, are presented in EM 1110-2-1906, "Laboratory Soils Testing."[67]

EQUIPMENT

TRIAXIAL TEST CELL

The triaxial cell in Figure E1 is suitable for use in resilience testing of soils. This equipment is similar to most standard cells, with the exceptions of being somewhat larger to facilitate the internally mounted load and deformation measuring equipment and having additional outlets for the electrical leads from the measuring devices. For the type of equipment shown, air or nitrogen is used as the cell fluid.

REPETITIVE LOADING EQUIPMENT

The external loading source may be any device capable of providing a variable load of fixed cycle and load duration, ranging from simple cam-and-switch control of static weights of air pistons to a closed-loop electrohydraulic system. A load duration of 0.2 sec and a cycle duration of 3 sec have been found to be satisfactory for most applications. A haversine wave form is recommended; however, a rectangular wave form can be used.

DEFORMATION AND LOAD MEASURING EQUIPMENT

The deformation measuring equipment consists of linear variable differential transducers (LVDT's) attached to the soil specimen by a pair of clamps. The transducers should have a high resolution and a small range to measure the extremely small resilient deformations. Two LVDT's are used for the measurement of axial deformation, and two for the measurement of lateral deformation. The clamps and LVDT's are shown in position on a soil specimen in Figure E1, and the details of the

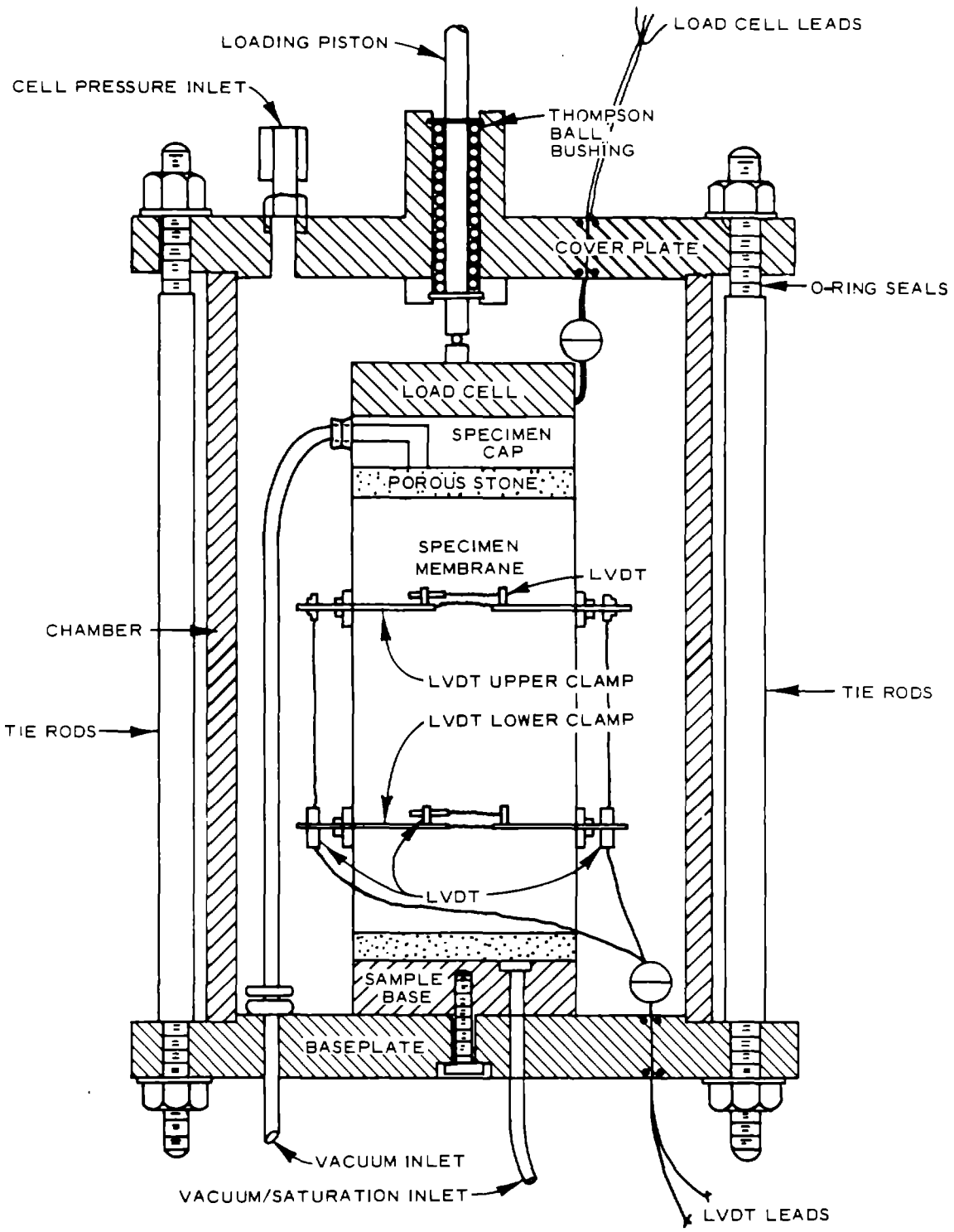


Figure E1. Triaxial cell

clamps in Figure E2. Load is measured by placing a load cell between the specimen cap and the loading piston (Figure E1). The load cell must be small enough so that no significant inertial forces are developed, and the capacity should be no greater than 2 to 3 times the maximum applied load.

Use of the type of measuring equipment described above offers several advantages:

a. It is not necessary to reference deformations to the equipment, which deforms during loading.

b. The effect of end-cap restraint on soil response is virtually eliminated.

c. Any effects of piston friction are eliminated by measuring loads inside the triaxial cell.

ADDITIONAL EQUIPMENT

In addition to the equipment described above, the following items are also used:

a. A 10- to 30-ton-capacity loading machine.

b. Calipers, a micrometer gage, and a steel rule (calibrated to 0.01 in.).

c. Rubber membranes, 0.01 to 0.025 in. thick.

d. Rubber O-rings.

e. A vacuum source with a bubble chamber and regulator.

f. A back-pressure chamber with pressure transducers.

g. A membrane stretcher.

h. Porous stones.

It is also necessary to maintain suitable recording equipment. It is desirable to have simultaneous recording of load and deformation. The number of recording channels can be reduced by wiring the leads from the LVDT's so that only the average signal from each pair is recorded. The introduction of switching and balancing units permits use of a single-chamber recorder. However, this will not permit simultaneous recording.

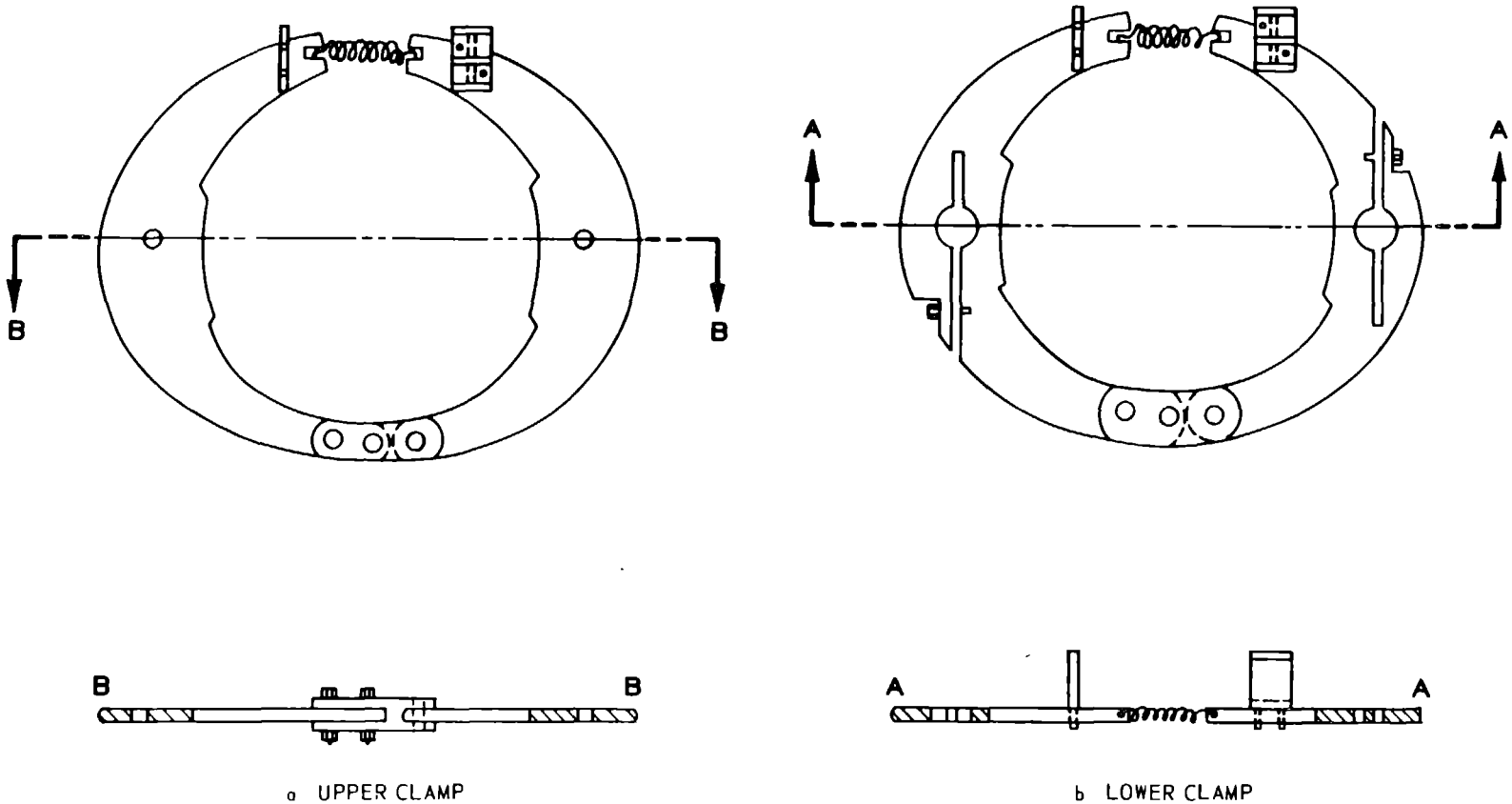


Figure E2. LVDT clamps

PREPARATION OF SPECIMENS AND PLACEMENT IN TRIAXIAL CELL

The following procedures should be followed in preparing and placing specimens:

a. In accordance with procedures specified in EM 1110-2-1906,[67] prepare the specimen and place it on the baseplate complete with porous stones, cap, and base and equipped with a rubber membrane secured with O-rings. Check for leakage. If back-pressure saturation is anticipated for cohesive soils, procedures indicated in Appendix X to EM 1110-2-1906 for the Q-type triaxial tests should be followed. For purely noncohesive soils, it will be necessary to maintain the vacuum during placement of the LVDT's. The specimen is now ready to receive the LVDT's.

b. Extend the lower LVDT clamp and slide it carefully down over the specimen to approximately the lower third point of the specimen.

c. Repeat this step for the upper clamp, placing it at the upper third point. Ensure that both clamps lie in horizontal planes.

d. Connect the LVDT's to the recording unit, and balance the recording bridges. This step will require recorder adjustments and adjustment of the LVDT stems. When a recording bridge balance has been obtained, determine (to the nearest 0.01 in.) the vertical spacing between the LVDT clamps and record this value.

e. Place the triaxial chamber in position. Set the load cell in place on the specimen.

f. Place the cover plate on the chamber. Insert the loading piston, and obtain a firm connection with the load cell.

g. Tighten the tie rods firmly.

h. Slide the assembled apparatus into position under the axial loading device. Bring the loading device to a position in which it nearly contacts the loading piston.

i. If the specimen is to be back-pressure saturated, proceed in accordance with EM 1110-2-1906.

j. After saturation has been completed, rebalance the recorder bridge to the load cell and LVDT's.

RESILIENCE TESTING OF COHESIVE SOILS

The resilient properties of cohesive soils are only slightly affected by the magnitude of the confining pressure σ_3. For most

applications, this effect can be disregarded. When back-pressure saturation is not used, the confining pressures used should approximate the expected in situ horizontal stresses. Chamber pressures of 2, 4, and 6 psi should be used. If back-pressure saturation is used, the chamber pressure will depend on the required saturation pressure.

Resilient properties are highly dependent on the magnitude of the deviator stress σ_d. It is therefore necessary to conduct the tests for a range in deviator stress values. Deviator stresses ranging from 2 to 16 psi are recommended. The following procedure should be followed:

a. If back-pressure saturation is not used, connect the chamber pressure supply line and apply the confining pressure (equal to the chamber pressure). If back-pressure saturation is used, the chamber pressure will already have been established.

b. Rebalance the recording bridges for the LVDT's, and balance the load cell recording bridge.

c. Begin the test by applying 500 to 1000 repetitions of a deviator stress of not more than one-half the unconfined compressive strength.

d. Decrease the deviator load to the lowest value to be used. Apply 200 repetitions of load, recording the recovered vertical deformation at or near the last repetition.

e. Increase the deviator load, recording deformations as in Step d. Repeat over the range of deviator stresses to be used.

f. At the completion of the loading, reduce the chamber pressure to zero. Remove the chamber LVDT's and load cell. Use the entire specimen for the purpose of determining the moisture content.

COMPUTATIONS AND PRESENTATION OF RESULTS FOR COHESIVE SOILS

Computations consist of the following:

a. From the measured dimensions and weights, compute and record the initial dry density, degree of saturation, and water content using the equations in Appendix II, EM 1110-2-1906.[67]

b. The resilient modulus is computed and recorded for each stress state using the following formulas:

(1) Resilient axial strain $\varepsilon_R = \Delta H_r / H_i$.

(2) Resilient lateral strain $\varepsilon_\ell = \Delta D_r/D_i$.

(3) Deviator stress $\sigma_d = \Delta P/A_o$.

(4) Resilient modulus $M_R = \sigma_d/\varepsilon_r$.

where

ΔH_r = resilient change in gage height (distance between LVDT clamps) after specified number of load repetitions.

H_i = instantaneous gage height after specified number of load repetitions. Can be calculated from $H_o - \Delta H$. If ΔH is small, H_o can be used.

H_o = initial gage height or distance between LVDT's less adjustments occurring during triaxial chamber assembly

ΔH = permanent change in gage height

ΔP = change in axial load, maximum axial load minus surcharge load

A_o = original cross-sectional area of specimen

ΔD_r = resilient change in diameter after specified number of load repetitions

D_i = instantaneous diameter after specified number of load repetitions. Can be calculated from $D_o + \Delta D$.

D_o = initial specimen diameter

ΔD = permanent change in specimen diameter

The results of the resilience tests can be presented in the form of a summary table, such as Table E1, and graphically as shown in Figure E3 for the resilient modulus.

RESILIENCE TESTING OF COHESIONLESS SOILS

The resilient modulus of cohesionless soils M_R is dependent upon the magnitude of the confining pressure σ_3 and is nearly independent of the magnitude of the repeated axial stress. Therefore, it is necessary to test cohesionless materials over a range of confining and axial stresses. (The confining pressure is equal to the chamber pressure less the back pressure for saturated specimens.) The following procedures should be used for this type of test:

a. Use confining pressures of 2, 4, 6, and 10 psi. At each confining pressure, test at four values of the principal

Table E1

Example Data Form for Recording Results of Resilience Tests of Cohesive Soils

SOIL SAMPLE _____

LOCATION _____

SAMPLE NO. _____

SPECIFIC GRAVITY G_s _____

SOIL SPECIMEN MEASUREMENTS

DIAMETER in.
- TOP _____
- MIDDLE _____
- BOTTOM _____
- AVERAGE _____

MEMBRANE THICKNESS, in. _____

NET DIAMETER, in. _____

SPECIMEN + CAP + BASE HEIGHT, in. _____

CAP + BASE HEIGHT, in. _____

INITIAL LENGTH L_0, in. _____

SOIL SPECIMEN WEIGHT

INITIAL WEIGHT OF CONTAINER + WET SOIL, g _____

FINAL WEIGHT OF CONTAINER + WET SOIL, g _____

WEIGHT OF WET SOIL USED, g _____

SOIL SPECIMEN VOLUME

INITIAL AREA A_0 $(in.)^2$ _____

VOLUME A_0L_0 $(in.)^3$ _____

WET DENSITY, pcf _____

WATER CONTENT, % _____

SATURATION, % _____

DRY DENSITY, pcf _____

DATE _____

COMPACTION METHOD _____

VERTICAL SPACING BETWEEN LVDT CLAMPS, in. _____

CHAMBER PRESSURE, psi _____

CONSTANTS

VERTICAL LVDT _____

LOAD CELL _____

COMMENTS _____

LOAD CELL CHART READING	DEVIATOR LOAD lb	σ_d $\sigma_1 - \sigma_3$ psi	VERTICAL LVDT CHART READING	VERTICAL DEFORMATION in.	ε_R in./in.	$M_R = \sigma_d/\varepsilon_R$ psi	ε_f in./in.	$\varepsilon'_R = \varepsilon_p/\varepsilon_R$ in./in.

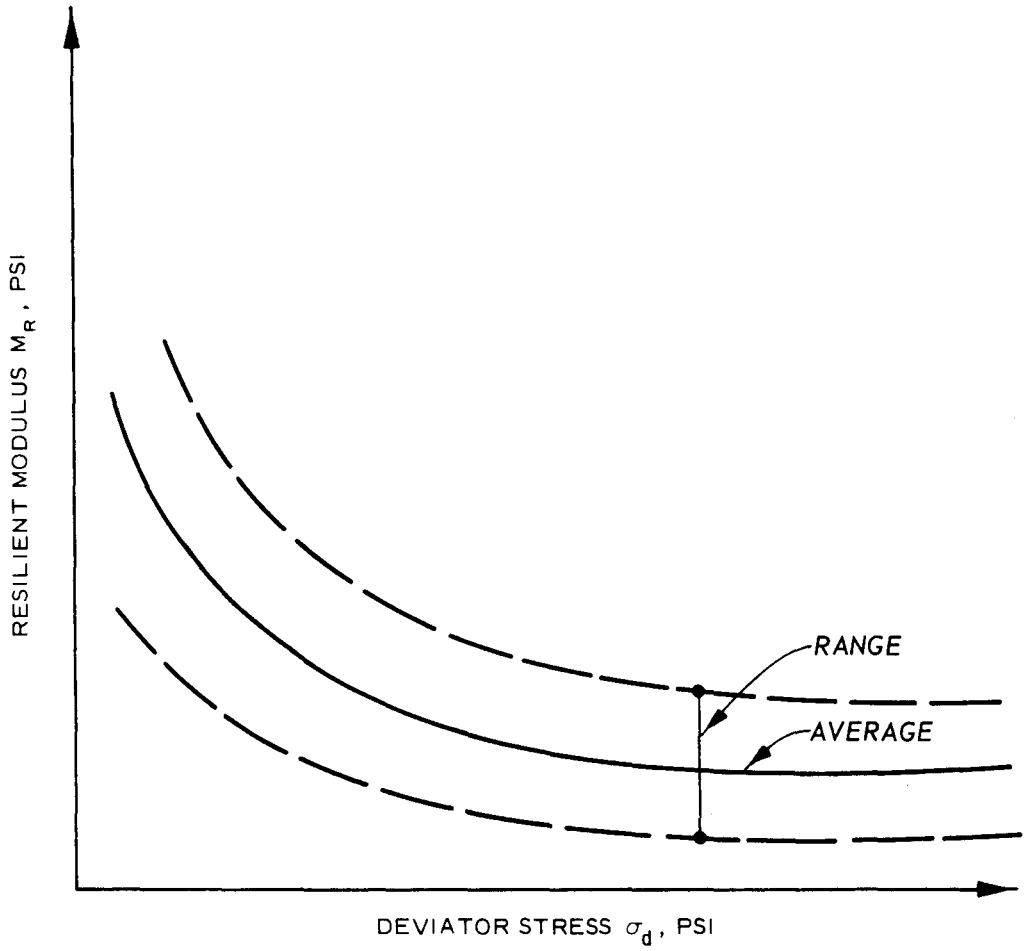


Figure E3. Presentation of results of resilience tests on cohesive soils

stress difference corresponding to multiples (2, 3, 4, 15) of the cell pressure.

b. Before beginning to record deformations, apply a series of conditioning stresses to the material to eliminate initial loading effects. The greatest amount of volume change occurs during the application of the conditioning stresses. Simulation of field conditions suggests that drainage of saturated specimens should be permitted during the application of these loads but that the test loading (beginning in Step f below) should be conducted in an undrained state.

c. Set the axial load generator to apply a deviator stress of 10 psi (i.e., a stress ratio equal to 3). Activate the load generator and apply 200 repetitions of this load. Stop the loading.

d. Set the axial load generator to apply a deviator stress of 200 psi (i.e., a stress ratio equal to 5). Activate the load generator and apply 200 repetitions of this load. Stop the loading.

e. Repeat as in Step d above maintaining a stress ratio equal to 6 and using the following order and magnitude of confining pressures: 10, 20, 10, 5, 3, and 1 psi.

f. Begin the record test using a confining pressure of 2 psi and an equal value of deviator stress. Record the resilient deformation after 200 repetitions. Increase the deviator stress to twice the confining pressure and record the resilient deformation after 200 repetitions. Repeat until a deviator stress of 4 times the confining pressure is reached (stress ratio of 5).

g. Repeat as in Step f above for each value of confining pressure.

h. When the test is completed, decrease the back pressure to zero, reduce the chamber pressure to zero, and dismantle the cell. Remove the LVDT clamps, etc. Remove the soil specimen, and use the entire amount of soil to determine the moisture content.

COMPUTATIONS AND PRESENTATION OF RESULTS FOR COHESIONLESS SOILS

Computations are similar to those for cohesive soils. The ratio of axial and confining stress and the first stress invariant are added. Tests results can be presented in the form of a summary table, such as Table E2, or a plot of log M_R versus log of the sum of the principal stresses (Figure E4).

Table E2

Example Data Form for Recording Results of Resilience Tests of Cohesive Soils

SOIL SAMPLE _____

LOCATION _____

SAMPLE NO. _____

SPECIFIC GRAVITY G_s _____

SOIL SPECIMEN MEASUREMENTS

DIAMETER, in.
- TOP _____
- MIDDLE _____
- BOTTOM _____
- AVERAGE _____

MEMBRANE THICKNESS, in. _____

NET DIAMETER, in. _____

SPECIMEN + CAP + BASE HEIGHT, in. _____

CAP + BASE HEIGHT, in. _____

INITIAL LENGTH L_0, in. _____

SOIL SPECIMEN WEIGHT

INITIAL WEIGHT OF CONTAINER + WET SOIL, g _____

FINAL WEIGHT OF CONTAINER + WET SOIL, g _____

WEIGHT OF WET SOIL USED, g _____

SOIL SPECIMEN VOLUME

INITIAL AREA A_0 $(in.)^2$ _____

VOLUME A_0L_0 $(in.)^3$ _____

WET DENSITY, pcf _____

WATER CONTENT, % _____

SATURATION, % _____

DRY DENSITY, pcf _____

VOID RATIO e _____

DATE _____

COMPACTION METHOD _____

VERTICAL SPACING BETWEEN LVDT CLAMPS, in. _____

CONSTANTS

VERTICAL LVDT _____

LOAD CELL _____

COMMENTS _____

CONFINING PRESSURE σ_3 psi	LOAD CELL CHART READING	DEVIATOR LOAD lb	$\sigma_d = \sigma_1 - \sigma_3$ psi	$\sigma_1 + \sigma_3$ psi	τ psi	VERTICAL LVDT CHART READING	VERTICAL DEFORMATION in.	ϵ_R in./in.	$M_R = \sigma_d/\epsilon_R$ psi	ϵ_f in./in.	ϵ_R/ϵ_f in./in.

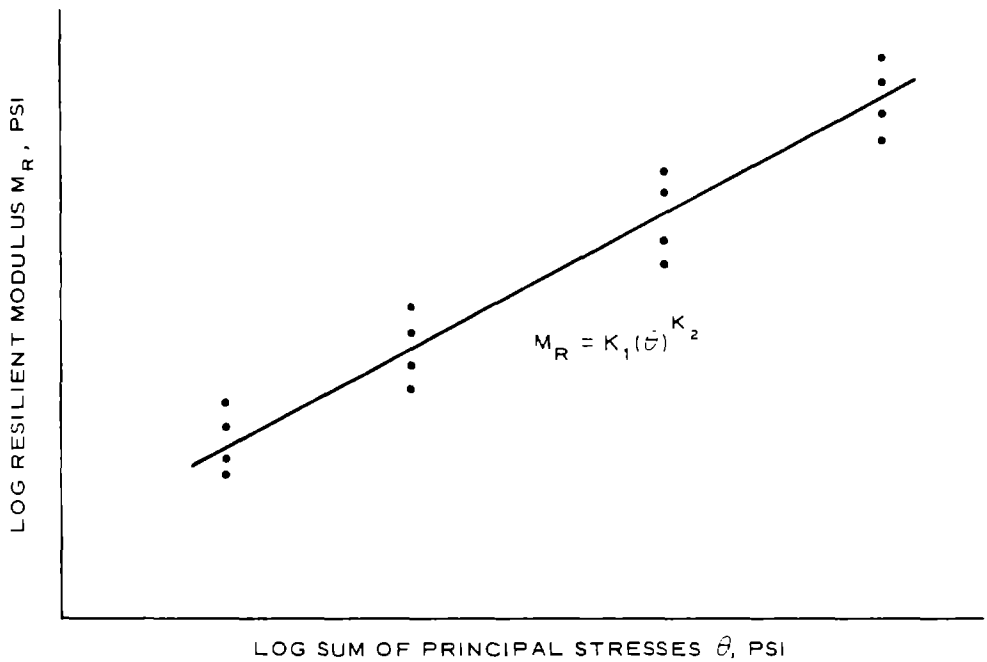


Figure E4. Presentation of results of resilience tests on cohesionless soils

APPENDIX F: USER INFORMATION FOR THE BISAR COMPUTER PROGRAM

INTRODUCTION

The copyright of much of the information provided herein is vested in Shell Research N.V. The information is published with the expressed permission of the Shell Oil Company and Koninklijke/Shell Laboratorium, Amsterdam (KSLA). Use of the BISAR computer program in any design procedure other than the procedure contained in this document is prohibited without written permission from KSLA. It may not be included in a library of programs maintained for commercial purposes by sellers of computer services nor used in any other way which results in payment of fees to the possessor of the program solely for the usage of the program. When reference is made to the results obtained with the program, acknowledgement should be given to KSLA.

The information provided herein is as follows:

a. Program listing.

b. Input guide.

c. Example problem with coded data and program output.

The listing provided is identical, except for a few minor modifications, to that for the program received by the WES from Shell in 1974. The modifications made at the WES are primarily changes in input format and changes necessary for the program to run on a Honeywell 6000 series computer. The information provided with the input guide was extracted from the User's Manual[23] for the BISAR computer program. The data for the example problem were developed at the WES, and the output given is from a Honeywell 6000 computer located there.

PROGRAM LISTING

A complete listing of the computer program is presented on the following pages.

```
C                 COMPUTATION OF STRESSES,STRAINS AND        MAIN0010
C                 DISPLACEMENTS IN LAYERED ELASTIC SYSTEMS   MAIN0020
C                 THIS PROGRAM CALCULATES THE FOLLOWING      MAIN0030
C                 STRESSES,STRAINS AND DISPLACEMENTS         MAIN0040
C                  1)                RADIAL      DISPLACEMENT MAIN0050
C                  2)                TANGENTIAL DISPLACEMENT MAIN0060
C                  3)                VERTICAL    DISPLACEMENTSMAIN0070
C                  4)                RADIAL      STRESS       MAIN0080
C                  5)                TANGENTIAL STRESS        MAIN0090
C                  6)                VERTICAL    STRESS       MAIN0100
C                  7)     RADIAL AND TANGENTIAL STRESS        MAIN0110
C                  8)     RADIAL AND VERTICAL   STRESS        MAIN0120
C                  9)TANGENTIAL AND VERTICAL    STRESS        MAIN0130
C                 10)                RADIAL      STRAIN       MAIN0140
C                 11)                TANGENTIAL STRAIN        MAIN0150
C                 12)                VERTICAL    STRAIN       MAIN0160
C                 13)     RADIAL AND TANGENTIAL STRAIN        MAIN0170
C                 14)     RADIAL AND VERTICAL   STRAIN        MAIN0180
C                 15)TANGENTIAL AND VERTICAL    STRAIN        MAIN0190
C                                                             MAIN0200
C                 MASTERPROGRAM                               MAIN0210
C                                                             MAIN0220
C                 PURPOSE                                     MAIN0230
C                                                             MAIN0240
C                   THIS MASTERPROGRAM READS DATA WHICH       MAIN0250
C                   DETERMINE THE PHYSICAL BEHAVIOUR OF       MAIN0260
C                   THE SYSTEM OF LAYERS AND WHICH            MAIN0270
C                   DESCRIBE THE CONFIGURATION OF THE LOADS.MAIN0280
C                   FOR EACH SYSTEM THE REQUIRED STRESSES     MAIN0290
C                   STRAINS AND DISPLACEMENTS ARE READ IN,    MAIN0300
C                   THEN THE COORDINATES OF EACH POSITION     MAIN0310
C                   ARE READ.                                 MAIN0320
C                   FOR A COMPLETE INPUT-DESCRIPTION SEE      MAIN0330
C                   GROUP EXTERNAL REPORT AMSR.     .72       MAIN0340
C                 SYSTEM DATA ARE OUTPUTTED BY                MAIN0350
C                           1) SYSTEM                         MAIN0360
C                 AFTER SUBSEQUENT CALLING IN OF"             MAIN0370
C                           2) MACON1                         MAIN0380
C                           3) CONSYS                         MAIN0390
C                           4) MA2CON                         MAIN0400
C                           5) CONPNT                         MAIN0410
C                           6) ASYMPT                         MAIN0420
C                           7) GENDAT                         MAIN0430
C                           8) INGRAL                         MAIN0440
C                 THE STRESSES,STRAINS AND DISPLACEMENTS      MAIN0450
C                 ARE CALCULATED AND AFTER SUBSEQUENT         MAIN0460
C                 CALLING IN OF"                              MAIN0470
C                           9) CALC                           MAIN0480
C                           10)OUTPUT                         MAIN0490
C                           11)JACOBI                         MAIN0500
C                           12)ESORT                          MAIN0510
C                 MAIN OUTPUTS OR HAS ALREADY OUTPUTTED"      MAIN0520
C                 -ALL STRESSES,STRAINS AND DISPLACEMENTS,    MAIN0530
C                  INDUCED BY EACH LOAD SEPARATELY AND        MAIN0540
C                  EXPRESSED IN CYLINDRICAL COMPONENTS.       MAIN0550
C                 -ALL TOTAL STRESSES STRAINS AND DISPLACE-   MAIN0560
C                  MENTS EXPRESSED IN CARTESIAN COMPONENTS.   MAIN0570
C                 -ALL PRINCIPAL TOTAL STRESSES AND STRAINS,MAIN0580
C                  WITH THEIR PRINCIPLE DIRECTIONS.           MAIN0590
C                 -ALL MAXIMUM TOTAL SHEAR STRESSES AND       MAIN0600
C                  STRAINS,WITH THEIR PRINCIPLE DIRECTIONS    MAIN0610
```

```
C                                 -THE MIDPOINTS OF THE THREE ACCOMPANYING  MAIN0620
C                                  MOHR'S CIRCLES.                          MAIN0630
C                                 -THE TOTAL STRAIN ENERGY AND STRAIN       MAIN0640
C                                  ENERGY OF DISTORTION.                    MAIN0650
C-----------------------------------------------------------------------MAIN0660
      LOGICAL STRESS,EPS,RLOW,AID(27),N,L,N2,L2,NZEP,NZEQ                  MAIN0670
      INTEGER REQUEST(27),IQ(3),DATE(3),ISTRSS(27),INTV(10),IVER1(7),      MAIN0680
     +IVER2(10)                                                            MAIN0690
      REAL NU,K5,MU,LDSTRS(10),HOSTR(10),LOAD,INT(17),V(15),X(10),Y(10),MAIN0700
     +A(3,3),HH(3,3),R(3),C(39),B(3,3),TEXT(20),ACCUR(3),PSI(10),AK(9),  MAIN0710
     +ALK(9)                                                               MAIN0711
      DOUBLE PRECISION CZ,ELLE,ELLK                                        MAIN0720
      COMMON/ASCT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,HOSTRS,NZEROS,H(9)MAIN0730
     +,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                              MAIN0740
      COMMON/STRDTA/STRESS(27),EPS(17),RLOW,ST,CT,L,ACC                    MAIN0750
      COMMON/CONST/CZ,ELLE,ELLK,ALMBDA                                     MAIN0760
      COMMON/CNTING/F10M1,F100,F101,F11M2,F11M1,F110,F111                  MAIN0770
      COMMON/TAPE/NOUT                                                     MAIN0780
      DATA NBLANK,ISTRSS,IREF1,IREF2/                                      MAIN0790
     +"    ","UR ","UT ","UZ ","SRR","STT","SZZ","SRT","SRZ","STZ","ERR"MAIN0800
     +,"ETT","EZZ","ERT","ERZ","ETZ","UX ","UY ","SXX","SXY","SXZ","SYY"MAIN0810
     +,"SYZ","EXX","EXY","EXZ","EYY","EY","LOAD","STRS"/                   MAIN0820
      DATA REQEST/"UR ","UT ","UZ ","SRR","STT","SZZ","SRT","SRZ","STZ",
     1"ERR",
     2"ETT","EZZ","ERT","ERZ","ETZ","UX ","UY ","SXX","SXY","SXZ","SYY",
     3"SYZ","EXX","EXY","EXZ","EYY","EY "/
      DATA IVER1,IVER2/1,2,3,6,7,13,14,4,5,8,9,10,11,12,15,16,17/          MAIN0830
      DATA IDENT/"LOAD"/
C-----------------------------------------------------------------------MAIN0840
C                                THESE ARE THREE ACCURACIES                MAIN0850
C                                ACCUR(1) IS USED FOR TESTING SEVERAL      MAIN0860
C                                         VARIABLES AGAINST EACH OTHER.    MAIN0870
C                                ACCUR(2) IS USED FOR ABSOLUTE ACCURACY    MAIN0880
C                                         OF THE INTEGRATION PROCEDURE     MAIN0890
C                                ACCUR(3) IS USED FOR RELATIVE ACCURACY    MAIN0900
C                                         OF THE INTEGRATION PROCEDURE     MAIN0910
C                                NIN ,                                     MAIN0920
C                                NOUT     ARE SYMBOLIC NAMES FOR INPUT AND MAIN0930
C                                         OUTPUT MEDIA RESP.               MAIN0940
C-----------------------------------------------------------------------MAIN0950
      ACCUR(1)=1.0E-04                                                     MAIN0960
      ACCUR(2)=1.0E-4                                                      MAIN0970
      ACCUR(3)=1.0E-3                                                      MAIN0980
      ACC=ACCUR(1)                                                         MAIN0990
      NIN=5                                                                MAIN1000
      NOUT=6                                                               MAIN1010
      V2=1.414214                                                          MAIN1020
      WRITE(NOUT,9000)                                                     MAIN1030
C-----------------------------------------------------------------------MAIN1040
C                                READ TEXT AND DATE CARD                   MAIN1050
C-----------------------------------------------------------------------MAIN1060
      READ(NIN,9010)TEXT
      WRITE(NOUT,9020)TEXT
C-----------------------------------------------------------------------MAIN1090
C                                READ NUMBER OF SYSTEMS AND SET LOOP       MAIN1100
C-----------------------------------------------------------------------MAIN1110
      READ(NIN,9030)NSYS
 800  FORMAT(V)
      DO 460 ISYS=1,NSYS                                                   MAIN1130
C-----------------------------------------------------------------------MAIN1140
C                                READ NUMBER OF LAYERS AND THEIR PARAMETERSMAIN1150
```

```
C-----------------------------------------------------------------------MAIN1160
      READ(NIN,9030)NLAYS,ISMO,IRED
         IF(NLAYS.EQ.1)  GO TO 10                                       MAIN1180
         M=NLAYS-1                                                      MAIN1190
      DO 313 I=1,M
 313  READ(NIN,9050)E(I),NU(I),THICK(I),AK(I)
 10   READ(NIN,9050)E(NLAYS),NU(NLAYS)
C-----------------------------------------------------------------------MAIN1220
C                              READ NUMBER OF LOADS AND THEIR PARAMETERS MAIN1230
C-----------------------------------------------------------------------MAIN1240
      READ(NIN,9030)NLOAD
         NZEP = .FALSE.                                                 MAIN1260
         NZEQ = .FALSE.                                                 MAIN1270
         DO 30 I=1,NLOAD                                                MAIN1280
      READ(NIN,9050)LDSTRS(I),RADIUS(I),X(I),Y(I),HOSTR(I),PSI(I)
            PSI(I)=.0174533*PSI(I)                                      MAIN1310
            IF(LDSTRS(I).GT.ACCUR(1))  NZEP = .TRUE.                    MAIN1320
            IF(HOSTR(I).GT.ACCUR(1))  NZEQ = .TRUE.                     MAIN1330
            IF(IDENT.EQ.IREF1)  GO TO 20                                MAIN1340
            IF(IDENT.NE.IREF2)  WRITE(NOUT,9040) LDSTRS(I),HOSTR(I)     MAIN1350
            GO TO 30                                                    MAIN1360
   20       LDSTRS(I) = LDSTRS(I)/(3.14159*RADIUS(I)*RADIUS(I))         MAIN1370
            HOSTR(I) = HOSTR(I)/(3.14159*RADIUS(I)*RADIUS(I))           MAIN1380
   30    CONTINUE                                                       MAIN1390
C-----------------------------------------------------------------------MAIN1410
C                              TEST ON OBVIOUS MISTAKES IN SYSTEM'S DATA-MAIN1411
C                              CARDS.                                   MAIN1412
C                              WHEN IRED % 0 THE REDUCED SPRINGCOMPLIAN- MAIN1413
C                              CE WAS READ.                             MAIN1414
C                              A NON-VANISHING SLIPRESISTANCE IS SUBSTI- MAIN1420
C                              TUTED TO PREVENT RIGID-BODY MOTION OF THE MAIN1430
C                              TOPLAYERS                                MAIN1440
C-----------------------------------------------------------------------MAIN1450
         DO 50 J = 1,NLAYS                                              MAIN1460
            IF((1.0-NU(J)).LT.ACCUR(1)) GO TO 410                       MAIN1461
            IF(E(J).LT.ACCUR(1)) GO TO 420                              MAIN1462
            IF(J.EQ.NLAYS) GO TO 50                                     MAIN1463
            IF(IRED.EQ.0) GO TO 40                                      MAIN1464
            ALK(J) = AK(J)                                              MAIN1465
            IF(ALK(J).LT.1000.0.OR..NOT.NZEQ) GO TO 50                  MAIN1466
            ALK(J) = 1000.0                                             MAIN1467
            AK(J) = 1000.0                                              MAIN1468
            GO TO 50                                                    MAIN1470
   40       ALK(J) = AK(J)*E(J)/(1.0+NU(J))                             MAIN1471
            IF(ALK(J).LT.1000.0.OR..NOT.NZEQ) GO TO 50                  MAIN1472
            ALK(J) = 1000.0                                             MAIN1473
            AK(J) = ALK(J)*(1.0+NU(J))/E(J)                             MAIN1474
   50    CONTINUE                                                       MAIN1480
C-----------------------------------------------------------------------MAIN1490
C                              OUTPUT OF ALL PHYSICAL DATA OF SYSTEM    MAIN1500
C                              AND LOADS BY CALLING IN SYSTEM.          MAIN1510
C-----------------------------------------------------------------------MAIN1520
         CALL SYSTEM(ISYS,E,NU,THICK,AK,NLAYS,M,NLOAD,LDSTRS,HOSTR,ALK, MAIN1530
     +   RADIUS,X,Y,PSI,ISMO,IRED)                                      MAIN1540
         IF(.NOT.NZEP.AND..NOT.NZEQ)  GO TO 430                         MAIN1550
C-----------------------------------------------------------------------MAIN1560
C                              CALCULATION OF CONSTANTS USED IN SUBROU- MAIN1570
C                              TINE MATRIX TO BUILT UP VARIOUS MATRICES MAIN1580
C                              BY CALLING IN MACON1.                    MAIN1590
C-----------------------------------------------------------------------MAIN1600
         CALL MACON1(ISMO,ALK,NE*SYS)                                   MAIN1610
```

```
   60 IF(NEWSYS.EQ.0)  GO TO 70                                          MAIN1630
         CALL SYSTEM(ISYS,E,NU,THICK,AK,NLAYS,M,NLOAD,LDSTRS,HOSTR,ALK,  MAIN1640
     +   RADIUS,X,Y,PSI,ISMU,IRED)                                        MAIN1650
C-----------------------------------------------------------------------MAIN1660
C                              READ STRESSES,STRAINS AND DISPLACEMENTS   MAIN1670
C                              TO BE CALCULATED.                         MAIN1680
C-----------------------------------------------------------------------MAIN1690
 70    CONTINUE
         DO 90 I=1,27                                                    MAIN1710
            IF(REQEST(I).EQ.NBLANK)  GO TO 80                            MAIN1720
            IF(REQEST(I).NE.ISTRSS(I))  WRITE(NOUT,9070) ISTRSS(I)       MAIN1730
            AID(I)=.TRUE.                                                MAIN1740
            GO TO 90                                                     MAIN1750
   80       AID(I)=.FALSE.                                               MAIN1760
   90    CONTINUE                                                        MAIN1770
C-----------------------------------------------------------------------MAIN1780
C                              CONSYS DETERMINES FOR EACH SYSTEM WHICH   MAIN1790
C                              STRESSES,STRAINS AND DISPLACEMENTS WILL   MAIN1800
C                              BE CALCULATED.                            MAIN1810
C-----------------------------------------------------------------------MAIN1820
         CALL CONSYS(AID,NZEP,NZEQ,N,L)                                  MAIN1830
C-----------------------------------------------------------------------MAIN1840
C                              READ NUMBER OF POSITIONS AND SET LOOP     MAIN1850
C-----------------------------------------------------------------------MAIN1860
 100  READ(NIN,9030)NPOS
         DO 400 IPOS=1,NPOS                                              MAIN1880
            N2 = N                                                       MAIN1890
            L2 = L                                                       MAIN1900
            DO 110 I=1,3                                                 MAIN1910
            DO 110 J=1,3                                                 MAIN1920
  110       A(I,J)=0.0                                                   MAIN1930
C-----------------------------------------------------------------------MAIN1940
C                              READ POINT COORDINATES AND LAYERNUMBER.   MAIN1950
C-----------------------------------------------------------------------MAIN1960
      READ(NIN,9060)LAYER,AX,AY,DEPTH,ETA
            ETA=.0174533*ETA                                             MAIN1980
            IF(NLAYS.EQ.1)  LAYER=1                                      MAIN1990
            WRITE(NOUT,9090) IPOS,LAYER,AX,AY,DEPTH                      MAIN2000
            TMIN=1.0E+10                                                 MAIN2010
            IF(NLAYS.EQ.1)  GO TO 130                                    MAIN2020
            J=LAYER+1                                                    MAIN2030
            J=MIN0(J,M)                                                  MAIN2040
            DO 120 I=1,J                                                 MAIN2050
               IF(THICK(I).LT.TMIN)  TMIN=THICK(I)                       MAIN2060
  120       CONTINUE                                                     MAIN2070
  130       UX=0.0                                                       MAIN2080
            UY=0.0                                                       MAIN2090
            UZ=0.0                                                       MAIN2100
            MU=NU(LAYER)                                                 MAIN2110
            FT=(1.0+MU)/E(LAYER)                                         MAIN2120
C-----------------------------------------------------------------------MAIN2130
C                              SET LOOP FOR NUMBER OF LOADS.             MAIN2140
C-----------------------------------------------------------------------MAIN2150
            DO 330 I=1,NLOAD                                             MAIN2160
               DO 140 J=1,17                                             MAIN2170
  140          INT(J)=0.0                                                MAIN2180
               DO 150 J=1,27                                             MAIN2190
  150          STRESS(J)=AID(J)                                          MAIN2200
               IF(NLAYS.EQ.1)  GO TO 160                                 MAIN2210
C-----------------------------------------------------------------------MAIN2220
C                              CALCULATION OF CONSTANTS NEEDED FOR THE   MAIN2230
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C                                EVALUATION OF THE CHARACTERISTIC FUNCTI-  MAIN2240
C                                ONS IN MATRIX BY CALLING IN MA2CON,       MAIN2250
C                                MA2CON,                                   MAIN2260
C-----------------------------------------------------------------------MAIN2270
                 CALL MA2CON(TMIN,I,ISMU,ALK)                              MAIN2280
C-----------------------------------------------------------------------MAIN2290
C                                DETERMINATION OF POINT COORDINATES IN THE MAIN2300
C                                CYLINDRICAL COORDINATE SYSTEM WITH LOAD-  MAIN2310
C                                AXIS AS AXIS OF SYMMETRY,                 MAIN2320
C-----------------------------------------------------------------------MAIN2330
  160            IF(X(I).EQ.AX.AND.Y(I).EQ.AY)  GO TO 170                  MAIN2340
                 THETA=ATAN2((AY-Y(I)),(AX-X(I)))-PSI(I)                   MAIN2350
                 GO TO 180                                                 MAIN2360
  170            THETA=ETA-PSI(I)                                          MAIN2370
  180            RADDIS=SQRT((AX-X(I))**2+(AY-Y(I))**2)                    MAIN2380
                 WRITE(NOUT,9100) I,RADDIS,THETA                           MAIN2390
                 R=RADDIS/RADIUS(I)                                        MAIN2400
                 Z=DEPTH/RADIUS(I)                                         MAIN2410
                 IF(NLAYS.EQ.1)  GO TO 230                                 MAIN2420
  190            IF(LAYER.GT.1)  GO TO 210                                 MAIN2430
                 IF(Z.GT.-ACCUR(1).AND.Z.LT.(H(1)+ACCUR(1)))  GO TO 230    MAIN2440
  200            WRITE(NOUT,9110)                                          MAIN2450
                 GO TO 400                                                 MAIN2460
  210            IF(LAYER.LT.NLAYS)  GO TO 220                             MAIN2470
                 IF(Z.GT.(H(M)-ACCUR(1)))  GO TO 230                       MAIN2480
                 GO TO 200                                                 MAIN2490
  220            IF(Z.GT.(H(LAYER-1)-ACCUR(1)).AND.Z.LT.(H(LAYER)+ACCUR(1)MAIN2500
     +           ))  GO TO 230                                             MAIN2510
                 GO TO 200                                                 MAIN2520
  230            RADI=RADIUS(I)                                            MAIN2530
                 LOAD=LDSTRS(I)                                            MAIN2540
                 HOSTRS=HOSTR(I)                                           MAIN2550
                 RLOW=R.LT.ACCUR(1)                                        MAIN2560
                 ST=SIN(THETA)                                             MAIN2570
                 CT=COS(THETA)                                             MAIN2580
C-----------------------------------------------------------------------MAIN2590
C                                CONPNT DETERMINES FOR EACH POINT-LOAD     MAIN2600
C                                CONFIGURATION WHICH INTEGRALS HAVE TO BE  MAIN2610
C                                CALCULATED.                               MAIN2620
C-----------------------------------------------------------------------MAIN2630
                 CALL CONPNT(R,HOSTRS,LOAD,Z,N2,L2)                        MAIN2640
                 IF(LAYER.NE.1)  GO TO 250                                 MAIN2650
                 CZ = DBLE(Z)                                              MAIN2660
                 IF(Z.LT.ACCUR(1).AND.ABS(R-1.0).LT.ACCUR(1))  GO TO 240   MAIN2670
C-----------------------------------------------------------------------MAIN2680
C                                ASYMPT DETERMINES THE LIPSCHITZ-HANKEL    MAIN2690
C                                INTEGRALS NEEDED FOR THE ASYMPTOTIC PART  MAIN2700
C                                OF THE INTEGRALS,FOR POINTS IN THE TOP-   MAIN2710
C                                LAYER ONLY.                               MAIN2720
C-----------------------------------------------------------------------MAIN2730
                 CALL ASYMPT(R,ACCUR(1))                                   MAIN2740
                 GO TO 250                                                 MAIN2750
C-----------------------------------------------------------------------MAIN2760
C                                FOR POINTS AT THE RIM OF THE LOAD THE     MAIN2770
C                                LIPSCHITZ-HANKEL INTEGRALS CAN BE GIVEN   MAIN2780
C                                DIRECTLY.                                 MAIN2790
C-----------------------------------------------------------------------MAIN2800
  240            F10M1 = 0.63662                                           MAIN2810
                 F100 = 0.5                                                MAIN2820
                 F11M1 = 0.5                                               MAIN2830
                 F11M2 = 0.424413                                          MAIN2840
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      F101 = 0.0                                                        MAIN2850
      F110 = 0.0                                                        MAIN2860
      F111 = 0.0                                                        MAIN2870
C-----------------------------------------------------------------------MAIN2880
C                          COMPUTATION OF THE REQUIRED INTEGRALS BY     MAIN2890
C                          CALLING IN GENDAT AND INGRAL                 MAIN2900
C-----------------------------------------------------------------------MAIN2910
  250 INTT = 0                                                          MAIN2920
      DO 260 J = 1,17                                                   MAIN2930
         INT(J) = 0.0                                                   MAIN2940
  260 CONTINUE                                                          MAIN2950
      DO 270 J = 1,10                                                   MAIN2960
         INTV(J) = 0                                                    MAIN2970
         K = IVER2(J)                                                   MAIN2980
         IF(.NOT.EPS(K))  GO TO 270                                     MAIN2990
         INTV(J) = K                                                    MAIN3000
         INTT = INTT+1                                                  MAIN3010
  270 CONTINUE                                                          MAIN3020
      IF(INTT.EQ.0)  GO TO 280                                          MAIN3030
      IF(NLAYS.NE.1)  CALL GENDAT(1,NZEROS,R,ACC)                       MAIN3040
      CALL INGRAL(2,INTV,INTT,INT)                                      MAIN3050
  280 INTT = 0                                                          MAIN3060
      DO 290 J = 1,7                                                    MAIN3070
         INTV(J) = 0                                                    MAIN3080
         K = IVER1(J)                                                   MAIN3090
         IF(.NOT.EPS(K))  GO TO 290                                     MAIN3100
         INTV(J) = K                                                    MAIN3110
      INTT = INTT+1                                                     MAIN3120
  290 CONTINUE                                                          MAIN3130
      IF(INTT.EQ.0)  GO TO 300                                          MAIN3140
      IF(NLAYS.NE.1)  CALL GENDAT(0,NZEROS,R,ACC)                       MAIN3150
      CALL INGRAL(1,INTV,INTT,INT)                                      MAIN3160
  300 PSI0 = PSI(I)                                                     MAIN3170
C-----------------------------------------------------------------------MAIN3180
C                          CALC COMPUTES AND OUTPUTS THE STRESSES,      MAIN3190
C                          STRAINS AND DISPLACEMENTS,INDUCED BY EACH    MAIN3200
C                          LOAD SEPARATELY.                             MAIN3210
C-----------------------------------------------------------------------MAIN3220
      CALL CALC(INT,V,R,MU,RADI,FI,LOAD,HOSTRS,PSI0,Z)                  MAIN3230
      IF(.NOT.N2)  GO TO 330                                            MAIN3240
C-----------------------------------------------------------------------MAIN3250
C                          COMPUTATION AND SUMMATION OF CARTESIAN       MAIN3260
C                          COORDINATES,THE USED COORDINATE SYSTEM IS    MAIN3270
C                          THE ONE WHEREIN POINTCOORDINATES WERE        MAIN3280
C                          STATED.                                      MAIN3290
C-----------------------------------------------------------------------MAIN3300
      UZ     =UZ+V(3)                                                   MAIN3310
      IF(ABS(RADDIS).LT.ACCUR(1))  GO TO 310                            MAIN3320
      CT     = (AX-X(I))/RADDIS                                         MAIN3330
      ST     = (AY-Y(I))/RADDIS                                         MAIN3340
      GO TO 320                                                         MAIN3350
  310 CT     =COS(ETA)                                                  MAIN3360
      ST     =SIN(ETA)                                                  MAIN3370
  320 CT2    =CT*CT                                                     MAIN3380
      ST2    =ST*ST                                                     MAIN3390
      STCT   =ST*CT                                                     MAIN3400
      A(1,1)=A(1,1)+V(4)*CT2+V(5)*ST2-2.0*V(7)*STCT                     MAIN3410
      A(1,2)=A(1,2)+V(7)*(CT2-ST2)+(V(4)-V(5))*STCT                     MAIN3420
      A(1,3)=A(1,3)+V(8)*CT-V(9)*ST                                     MAIN3430
      A(2,1)=A(1,2)                                                     MAIN3440
      A(2,2)=A(2,2)+V(4)*ST2+V(5)*CT2+2.0*V(7)*STCT                     MAIN3450
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            A(2,3)=A(2,3)+V(8)*ST+V(9)*CT                                MAIN3460
            A(3,1)=A(1,3)                                                MAIN3470
            A(3,2)=A(2,3)                                                MAIN3480
            A(3,3)=A(3,3)+V(6)                                           MAIN3490
            UX     =UX+V(1)*CT-V(2)*ST                                   MAIN3500
            UY     =UY+V(1)*ST+V(2)*CT                                   MAIN3510
  330     CONTINUE                                                       MAIN3520
          TRACE=A(1,1)+A(2,2)+A(3,3)                                     MAIN3530
          AB    =(1.0+MU)/E(LAYER)                                       MAIN3540
          AC    =MU*TRACE/E(LAYER)                                       MAIN3550
          DO 350 I=1,3                                                   MAIN3560
            DO 340 J=1,3                                                 MAIN3570
              B(I,J)=AB*A(I,J)                                           MAIN3580
              IF(I.NE.J)  GO TO 340                                      MAIN3590
              B(I,J)=B(I,J)-AC                                           MAIN3600
  340       CONTINUE                                                     MAIN3610
  350     CONTINUE                                                       MAIN3620
C-----------------------------------------------------------------------MAIN3630
C                                OUTPUT FOR TOTAL STRESSES,STRAINS AND  MAIN3640
C                                DISPLACEMENTS BY THREE TIMES CALLING IN MAIN3650
C                                OUTPUT.                                MAIN3660
C-----------------------------------------------------------------------MAIN3670
          WRITE(NOUT,9120)                                               MAIN3680
          EPS(1)=STRESS(18)                                              MAIN3690
          EPS(2)=STRESS(21)                                              MAIN3700
          EPS(3)=STRESS( 6)                                              MAIN3710
          EPS(4)=STRESS(22)                                              MAIN3720
          EPS(5)=STRESS(20)                                              MAIN3730
          EPS(6)=STRESS(19)                                              MAIN3740
            C(1)=A(1,1)                                                  MAIN3750
            C(2)=A(2,2)                                                  MAIN3760
            C(3)=A(3,3)                                                  MAIN3770
            C(4)=A(2,3)                                                  MAIN3780
            C(5)=A(1,3)                                                  MAIN3790
            C(6)=A(1,2)                                                  MAIN3800
          CALL OUTPUT(EPS,C,6,1)                                         MAIN3810
          EPS(1)=STRESS(23)                                              MAIN3820
          EPS(2)=STRESS(26)                                              MAIN3830
          EPS(3)=STRESS(12)                                              MAIN3840
          EPS(4)=STRESS(27)                                              MAIN3850
          EPS(5)=STRESS(25)                                              MAIN3860
          EPS(6)=STRESS(24)                                              MAIN3870
            C(1)=B(1,1)                                                  MAIN3880
            C(2)=B(2,2)                                                  MAIN3890
            C(3)=B(3,3)                                                  MAIN3900
            C(4)=B(2,3)                                                  MAIN3910
            C(5)=B(1,3)                                                  MAIN3920
            C(6)=B(1,2)                                                  MAIN3930
          CALL OUTPUT(EPS,C,6,2)                                         MAIN3940
          EPS(1)=STRESS(16)                                              MAIN3950
          EPS(2)=STRESS(17)                                              MAIN3960
          EPS(3)=STRESS( 3)                                              MAIN3970
            C(1)=UX                                                      MAIN3980
            C(2)=UY                                                      MAIN3990
            C(3)=UZ                                                      MAIN4000
          CALL OUTPUT(EPS,C,3,3)                                         MAIN4010
  360     IF(.NOT.L2)  GO TO 400                                         MAIN4020
C-----------------------------------------------------------------------MAIN4030
C                                JACOBI COMPUTES PRINCIPAL VALUES AND   MAIN4040
C                                DIRECTIONS OF TOTAL STRESSES AND STRAINS, MAIN4050
C                                THE PRINCIPAL VALUES ARE SORTED ACCORDING MAIN4060
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C                                TO MAGNITUDE BY CALLING IN ESORT,          MAIN4070
C-----------------------------------------------------------------------MAIN4080
            CALL JACOBI(A,HH,3,3,1,N,IQ)                                   MAIN4090
            CALL ESORT(A,HH,3,3,1,N,IQ)                                    MAIN4100
C-----------------------------------------------------------------------MAIN4110
C                                DETERMINATION OF MAX.SHEAR STRESSES AND    MAIN4120
C                                STRAINS WITH THEIR DIRECTIONS AND DETERMI-MAIN4130
C                                NATION OF MIDPOINTS OF THE MOHR"S CIRCLE.  MAIN4140
C-----------------------------------------------------------------------MAIN4150
            DO 370 J=1,3                                                   MAIN4160
               C(J    )=AB*A(J,J)-AC                                       MAIN4170
               C(J+ 5)=(HH(J,1)-HH(J,3))/V2                                MAIN4180
               C(J+ 9)=(HH(J,1)+HH(J,3))/V2                                MAIN4190
               C(J+14)=(HH(J,1)-HH(J,2))/V2                                MAIN4200
               C(J+18)=(HH(J,1)+HH(J,2))/V2                                MAIN4210
               C(J+23)=(HH(J,2)-HH(J,3))/V2                                MAIN4220
               C(J+27)=(HH(J,2)+HH(J,3))/V2                                MAIN4230
  370       CONTINUE                                                       MAIN4240
            C( 4)=0.5*(A(1,1)-A(3,3))                                      MAIN4250
            C( 9)=0.5*(A(1,1)+A(3,3))                                      MAIN4260
            C(13)=0.5*(A(1,1)-A(2,2))                                      MAIN4270
            C(18)=0.5*(A(1,1)+A(2,2))                                      MAIN4280
            C(22)=0.5*(A(2,2)-A(3,3))                                      MAIN4290
            C(27)=0.5*(A(2,2)+A(3,3))                                      MAIN4300
            C( 5)=0.5*(C(1)-C(3))                                          MAIN4310
            C(14)=0.5*(C(1)-C(2))                                          MAIN4320
            C(23)=0.5*(C(2)-C(3))                                          MAIN4330
            IF(C(13).GT.C(22))  GO TO 390                                  MAIN4340
            DO 380 I=1,9                                                   MAIN4350
               C(I+30)=C(I+12)                                             MAIN4360
               C(I+12)=C(I+21)                                             MAIN4370
               C(I+21)=C(I+30)                                             MAIN4380
  380       CONTINUE                                                       MAIN4390
C-----------------------------------------------------------------------MAIN4400
C                                OUTPUT FOR PRINCIPAL STRESSES,ETC,MAXIMUM MAIN4410
C                                SHEAR STRESSES,ETC AND STRAIN ENERGIES.    MAIN4420
C-----------------------------------------------------------------------MAIN4430
  390       WRITE(NOUT,9130)A(1,1),C(1),HH(1,1),HH(2,1),HH(3,1),           MAIN4440
     +                      A(2,2),C(2),HH(1,2),HH(2,2),HH(3,2),           MAIN4450
     +                      A(3,3),C(3),HH(1,3),HH(2,3),HH(3,3),           MAIN4460
     +                      (C(I),I=4,30)                                  MAIN4470
            BX = (A(1,1)+C(1)+A(2,2)+C(2)+A(3,3)+C(3))*0.5                 MAIN4480
            BY = 0.666667*AB*(C(4)*C(4)+C(13)*C(13)+C(22)*C(22))           MAIN4490
            WRITE(NOUT,9200) BX,BY                                         MAIN4500
  400     CONTINUE                                                         MAIN4510
          GO TO 460                                                        MAIN4520
  410     WRITE(NOUT,9140) J                                               MAIN4530
          GO TO 440                                                        MAIN4540
  420     WRITE(NOUT,9180) J                                               MAIN4550
          GO TO 440                                                        MAIN4560
  430     WRITE (NOUT,9190)                                                MAIN4570
C-----------------------------------------------------------------------MAIN4580
C                                FOR SYSTEMS FOR WHICH IT IS CLEAR THAT     MAIN4590
C                                MISTAKES OCCUR IN THE INPUTCARDS,THE       MAIN4600
C                                REQUEST AND POINT INPUT CARDS ARE SKIPPED.MAIN4610
C                                PROGRAM PROCEEDS BY TAKING NEXT SYSTEM.    MAIN4620
C-----------------------------------------------------------------------MAIN4630
  440     READ (NIN,9150)                                                  MAIN4640
          READ(NIN,9030) NPOS                                              MAIN4650
          DO 450 I=1,NPOS                                                  MAIN4660
  450     READ (NIN ,9150)                                                 MAIN4670
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  460 CONTINUE                                                          MAIN4680
      WRITE(NOUT,9160)                                                  MAIN4690
      STOP                                                              MAIN4700
 9000 FORMAT(1H1,17X,11("B"),5X,"III",5X,11("S"),6X,9("A"),5X,11("R")/  MAIN4710
     +            18X,12("B"),4X,"III",4X,12("S"),5X,11("A"),4X,12("R")/ MAIN4720
     +18X,"BB",8X,"BBB   III   SSS",14X,"AAA",7X,"AAA   RR",8X,"RRR"/   MAIN4730
     +18X,"BB",9X,"BB   III   SS",15X,"AA",9X,"AA   RR",9X,"RR"/        MAIN4740
     +18X,"BB",7X,"BBB    III   SSS",14X,"AA",9X,"AA   RR",8X,"RR"/     MAIN4750
     +18X,11("B"),5X,"III",4X,11("S"),5X,13("A"),3X,12("R")/            MAIN4760
     +18X,11("B"),5X,"III",5X,11("S"),4X,13("A"),3X,11("R")/            MAIN4770
     +18X,"BB",7X,"BBB    III",14X,"SSS   AA",9X,"AA   RR",5X,"RR"/     MAIN4780
     +18X,"BB",8X,"BB    III",15X,"SS   AA",9X,"AA   RR",6X,"RR"/       MAIN4790
     +18X,"BB",7X,"BBB    III",14X,"SSS   AA",9X,"AA   RR",7X,"RR"/     MAIN4800
     +18X,12("B"),4X,"III",3X,13("S"),4X,"AA",9X,"AA   RR",8X,"RR"/     MAIN4810
     +18X,11("B"),5X,"III",3X,12("S"),5X,"AA",9X,"AA   RR",9X,"RR"////  MAIN4820
     +75X,"THIS ""BISAR"" PROGRAM HAS BEEN OBTAINED FROM"/89X,"SHELL RESMAIN4830
     +EARCH B.V."/89X,"FOR THE SOLE USE OF"//76X,                       MAIN4840
     +"SHELL OIL COMPANY"/76X,"HOUSTON, TEXAS"                          MAIN4850
     +                                //76X,"ALL RIGHTS ARE RESERVED,MAIN4860
     + USE OF THIS PROGRAM"/76X,"BY UNAUTHORIZED PERSONS IS PROHIBITED")MAIN4870
 9010 FORMAT(20A4)
 9020 FORMAT(1H1,25(/),15X,20A4)
 9030 FORMAT(I2,I3,I1)                                                  MAIN4900
 9040 FORMAT(" NOTE THAT ",E12.6," AND ",E12.6," WILL BE CONSIDERED TO BMAIN4910
     +E LOADS IN STRESS UNITS")                                         MAIN4920
 9050 FORMAT(6F10.0)
 9060 FORMAT(I2,8X,4F10.0)
 9070 FORMAT(" NOTE THAT INCORRECT SPELLING HAS NOT STOPPED THE EVALUATIMAIN4950
     +ON OF STRESS",4X,A3)                                              MAIN4960
 9080 FORMAT(I2,4E12.6)                                                 MAIN4970
 9090 FORMAT(1H1,///52X,"POSITION NUMBER ",I2//54X,"LAYER NUMBER  ",I2//MAIN4980
     +55X,"COORDINATES"//46X,"X",11X,"Y",11X,"Z"/40X,3E12.4)            MAIN4990
 9100 FORMAT(/21X,"DISTANCE TO LOAD-AXIS(",I2,")",34X,"THETA"/25X,E12.4,MAIN5000
     +41X,E12.4/)                                                       MAIN5010
 9110 FORMAT(//,30X,"THIS POSITION HAS BEEN OMITTED SINCE THE LAYER NUMBMAIN5020
     +ER IS INCORRECT")                                                 MAIN5030
 9120 FORMAT(/30X,"XX",10X,"YY",10X,"ZZ",10X,"YZ",10X,"XZ",10X,"XY",10X,MAIN5040
     +"UX",10X,"UY",10X,"UZ")                                           MAIN5050
 9130 FORMAT(/" P R I N C I P A L  V A L U E S  A N D  D I R E C T I O NMAIN5060
     + S  O F  T O T A L  S T R E S S E S  A N D  S T R A I N S"/15X,"NOMAIN5070
     +RMAL",9X,"NORMAL",9X,"SHEAR",10X,"SHEAR",13X,"X",14X,"Y",14X,"Z"/1MAIN5080
     +5X,"STRESS",9X,"STRAIN",9X,"STRESS",9X,"STRAIN",9X,"COMPONENT",6X,MAIN5090
     +"COMPONENT",6X,"COMPONENT"/" MAXIMUM",2E15.3,30X,3F15.3/" MINIMAX"MAIN5100
     +,2E15.3,30X,3F15.3/" MINIMUM",2E15.3,30X,3F15.3/" MAXIMUM",30X,2E1MAIN5110
     +5.3,3F15.3/8X,E15.3,45X,3F15.3/" MINIMAX",30X,2E15.3,3F15.3/8X,   MAIN5120
     +E15.3,45X,3F15.3/" MINIMUM",30X,2E15.3,3F15.3/8X,E15.3,45X,3F15.3)MAIN5130
 9140 FORMAT(" THE PROBLEM CANNOT BE SOLVED,NU(",I2,") EQUALS ONE")     MAIN5140
 9150 FORMAT(/)                                                         MAIN5150
 9160 FORMAT(1H1)                                                       MAIN5160
 9170 FORMAT(A4,6X,6F10.0)
 9180 FORMAT(" THE PROBLEM CANNOT BE SOLVED,E(",I2,") EQUALS ZERO")     MAIN5180
 9190 FORMAT(" SYSTEM SKIPPED NO LOADS")                                MAIN5190
 9200 FORMAT(1H0,13X," STRAIN ENERGY",E11.4/" STRAIN ENERGY OF DISTORTIOMAIN5200
     +N",E11.4)                                                         MAIN5210
      END                                                               MAIN5220
      SUBROUTINE SYSTEM(ISYS,E,NU,THICK,AK,NLAYS,M,NLOAD,LDSTRS,HOSTR,  SYST0010
     1ALK,RADIUS,X,Y,PSI,ISMO,IKED)                                     SYST0020
C-----------------------------------------------------------------------SYST0030
C                              THIS SUBROUTINE OUTPUTS ALL PHYSICAL DATA SYST0040
C                              OF THE MULTI-LAYERED SYSTEM AND ALL DATA  SYST0050
C                              ON CONFIGURATION AND MAGNITUDE OF THE     SYST0060
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C                                LOADS.                                  SYST0070
C-----------------------------------------------------------------------SYST0080
      INTEGER ROUGH(2),SMOOTH(2),ISMTH(2)                                SYST0090
      REAL E(10),NU(10),THICK(9),AK(9),ALK(9),LDSTRS(10),HOSTR(10),      SYST0100
     1RADIUS(10),X(10),Y(10),PSI(10)                                     SYST0110
      COMMON/TAPE/NOUT                                                   SYST0120
      DATA ROUGH,SMOOTH/"ROU","GH ","SMO","OTH"/                         SYST0130
      WRITE(NOUT,1001) ISYS                                              SYST0140
      IF(IRED.EQ.0) WRITE(NOUT,1002)                                     SYST0150
      IF(IRED.NE.0) WRITE(NOUT,1007)                                     SYST0151
      IF(NLAYS.EQ.1) GO TO 40                                            SYST0160
      DO 30 I=1,M                                                        SYST0170
         IF(ISMO.EQ.1) GO TO 10                                          SYST0180
         ISMTH(1) = ROUGH(1)                                             SYST0190
         ISMTH(2) = ROUGH(2)                                             SYST0200
         IF(ALK(I).LT.100.0) GO TO 20                                    SYST0210
   10    ISMTH(1) = SMOOTH(1)                                            SYST0220
         ISMTH(2) = SMOOTH(2)                                            SYST0230
   20    WRITE(NOUT,1003) I,ISMTH(1),ISMTH(2),E(I),NU(I),THICK(I),AK(I) SYST0240
   30 CONTINUE                                                           SYST0250
   40 WRITE(NOUT,1004) NLAYS,E(NLAYS),NU(NLAYS)                          SYST0260
      WRITE(NOUT,1005)                                                   SYST0270
      DO 50 I = 1,NLOAD                                                  SYST0280
   50 WRITE(NOUT,1006) I,LDSTRS(I),HOSTR(I),RADIUS(I),X(I),Y(I),PSI(I)   SYST0290
 1001 FORMAT(1H1,10(/),52X,"SYSTEM NUMBER",3X,I2)                        SYST0300
 1002 FORMAT(5(/),8X,"LAYER",4X,"CALCULATION",2X,"YOUNG""S",4X,"POISSON"SYST0310
     1"S",3X,"THICKNESS",3X,"INTERFACE"/8X,"NUMBER",3X,"METHOD",7X,"MODUSYST0320
     2LUS",4X,"RATIO",18X,"SPRINGCOMPL"/)                                SYST0330
 1003 FORMAT(10X,I2,5X,2A3,3X,4E12.4)                                    SYST0340
 1004 FORMAT(10X,I2,14X,2E12.4)                                          SYST0350
 1005 FORMAT(///8X,"LOAD",5X,"NORMAL",7X,"SHEAR",5X,"RADIUS OF",7X,"LOADSYST0360
     1 - POSITION",6X,"SHEAR"/8X,"NUMBER",3X,"STRESS",7X,"STRESS",4X,    SYST0370
     2"LOADED AREA",6X,"X",11X,"Y",7X,"DIRECTION"/)                      SYST0380
 1006 FORMAT(10X,I2,2X,6E12.4)                                           SYST0390
 1007 FORMAT(5(/),8X,"LAYER",4X,"CALCULATION",2X,"YOUNG""S",4X,"POISSON"SYST0400
     1"S",3X,"THICKNESS",3X," REDUCED" /8X,"NUMBER",3X,"METHOD",7X,"MODUSYST0410
     2LUS",4X,"RATIO",18X,"SPRINGCOMPL"/)                                SYST0420
      RETURN                                                             SYST0430
      END                                                                SYST0440
      SUBROUTINE MACON1(ISMO,ALK,NSYS)                                   MAC00010
C-----------------------------------------------------------------------MAC00020
C                                THIS SUBROUTINE CALCULATES CONSTANTS USED MAC00030
C                                IN SUBROUTINE MATRIX TO BUILD UP VARIOUS  MAC00040
C                                MATRICES.                                 MAC00050
C                                THE CONSTANTS ARE STORED IN               MAC00060
C                                COMMON/INDATA/.                           MAC00070
C                                NUMERICAL STABILITY OF SOLUTIONPROCEDURE  MAC00080
C                                FOR THE SYSTEM IS TESTED BY CALLING IN"   MAC00090
C                                          MA2CON,                         MAC00100
C                                          MATRIX                          MAC00110
C                                WHEN INSTABILITY HAS TO BE EXPECTED THE   MAC00120
C                                SMOOTH CALCULATION PROCEDURE IS CHOSEN BY MAC00130
C                                TAKING ISMO = 1 AND NSYS IS SET EQUAL 1 . MAC00140
C-----------------------------------------------------------------------MAC00150
      REAL K1,K2,K3(10),K4(10),K5,K6,NU,II,LOAD,ACCUR(3),ALK(9)          MAC00160
      COMMON/ASDT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,HOSTRS,NZEROS,H(9)MAC00170
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                            MAC00180
      COMMON/INDATA/XMAX,  A1(9),B1(9),C1(9),D(9),EE(9),F(9),G(9),H1(9),MAC00190
     1I1(9),K1(9),K2(9),K6(10),BE(9),BU(9),BUU(9),BMU(9),B2U(9),B2UU(9),MAC00200
     2J2(9),J1,T2(10),SS(2,10),G012(9),G021(9),G022(9),G122(9),          MAC00210
     3H012(9),H022(9),H122(9),D012(9),D022(9),C011(9),C012(9),E012(9),   MAC00220
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     4F012(9),F112(9),F022(9),CC(4,2,9),DD(2,2,9),FF(2,2,9),GG(2,2,10),
     5HH(2,2,10),KR(4,2,10),DD2(9),G20(9),G21(9),H20(9),H021(9),GG2(10),
     6HH2(10),Q011(9),Q111(9),Q012(9),Q112(9),Q212(9),Q022(9),Q122(9),
     7QF0(9),QF1(9),Z011,Z111,Z211,Z012,Z112,Z212,Z312,Z021,Z121,Z022,
     8Z122,Z222,K4
      COMMON/TAPE/NOUT
      NSYS = 0
      IF(NLAYS.EQ.1) GO TO 10
      GG(1,1,1) = -1.0
      GG(2,1,1) = 1.0
      GG(1,2,1) = 1.0-2.0*NU(1)
      GG(2,2,1) = 2.0*NU(1)
      HH(1,1,1) = 1.0
      HH(1,2,1) = GG(1,2,1)
      HH(2,1,1) = 1.0
      HH(2,2,1) = -GG(2,2,1)
      RR(1,1,NLAYS) = 0.0
      RR(1,2,NLAYS) = 0.0
      RR(2,1,NLAYS) = 0.0
      RR(2,2,NLAYS) = 0.0
      RR(3,1,NLAYS) = 1.0
      RR(3,2,NLAYS) = 0.0
      RR(4,1,NLAYS) = 0.0
      RR(4,2,NLAYS) = 1.0
      SS(1,NLAYS) = 0.0
      SS(2,NLAYS) = 1.0
      GG2(1) = 1.0
      HH2(1) =-1.0
   10 K5(1)=1.0-2.0*NU(1)
      IF(NLAYS.EQ.1) GO TO 70
      K = 0
      K6(1) = 4.0*(1.0-NU(1))
      DO 30 J=1,M
         K1(J)=(1.0+NU(J+1))*E(J)/((1.0+NU(J))*E(J+1))
         K2(J)=1.0-K1(J)
         K3(J)=NU(J+1)-NU(J)*K1(J)
         K4(J)=8.0*NU(J)*NU(J+1)
         K5(J)=1.0-2.0*NU(J)
         K6(J+1) = 4.0*(1.0-NU(J+1))
         A1(J)=     K6(J)-K2(J)
         B1(J)=     K2(J)+K1(J)*K6(J+1)
         C1(J)=2.0*K2(J)
         D (J)=     K2(J)*(1.0-4.0*NU(J))
         EE(J)=     K2(J)*(1.0+K4(J))-6.0*K3(J)
         F (J)=     A1(J)-B1(J)
         G (J)=     K2(J)*(1.0-K4(J))+2.0*K3(J)
         H1(J)=4.0*K2(J)*(NU(J+1)-NU(J))
         II(J)=     D(J)-H1(J)
   30 CONTINUE
      K5(M+1)=1.0-2.0*NU(M+1)
      IF(ISMO.EQ.1) GO TO 70
      DO 40 I = 1,M
         IF(ALK(I).LT.100.0) GO TO 50
   40 CONTINUE
      GO TO 70
C-----------------------------------------------------------------------
C                              CALCULATION OF CONSTANTS ONLY NEEDED IN
C                              MATRIX FOR STABILITY TEST.
C-----------------------------------------------------------------------
   50 TMIN = 1.0E+10
      NTELL = 2
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      IF(AID(10).AND..NOT.NZEQ) AID( 4)=.TRUE.                          CONS0300
      IF(AID( 6).AND..NOT.NZEP) AID(12)=.TRUE.                          CONS0310
      IF(AID( 4)) CALL LOGSET(JARG(1, 6),AID)                           CONS0320
      IF(AID(10)) AID(11)=.TRUE.                                        CONS0330
      IF(AID( 5)) AID(11)=.TRUE.                                        CONS0340
      IF(AID(12)) AID( 6)=.TRUE.                                        CONS0350
      IF(AID( 8)) AID(14)=.TRUE.                                        CONS0360
      IF(AID(14)) AID( 8)=.TRUE.                                        CONS0370
      IF(AID( 5).AND.AID( 6)) CALL LOGSET(JARG(1, 7),AID)               CONS0380
      IF(AID(11).AND.AID(12)) CALL LOGSET(JARG(1, 8),AID)               CONS0390
      IF(.NOT.NZEQ) GO TO 20                                            CONS0400
      IF(AID( 7)) AID(13)=.TRUE.                                        CONS0410
      IF(AID(13)) AID( 7)=.TRUE.                                        CONS0420
      IF(AID( 9)) AID(15)=.TRUE.                                        CONS0430
      IF(AID(15)) AID( 9)=.TRUE.                                        CONS0440
      IF(AID( 5).AND.AID(10)) CALL LOGSET(JARG(1, 9),AID)               CONS0450
      IF(AID( 6).AND.AID(10)) CALL LOGSET(JARG(1,10),AID)               CONS0460
      IF(AID( 1).AND.AID( 2)) CALL LOGSET(JARG(1,11),AID)               CONS0470
      IF(AID( 4).AND.AID( 7)) CALL LOGSET(JARG(1,12),AID)               CONS0480
      IF(AID( 7).AND.AID(10)) CALL LOGSET(JARG(1,13),AID)               CONS0490
      IF(AID( 8).AND.AID( 9)) CALL LOGSET(JARG(1,14),AID)               CONS0500
      GO TO 30                                                          CONS0510
   20 IF(AID( 1)) CALL LOGSET(JARG(1,11),AID)                           CONS0520
      IF(AID( 4)) CALL LOGSET(JARG(1,12),AID)                           CONS0530
      IF(AID( 8)) CALL LOGSET(JARG(1,14),AID)                           CONS0540
   30 N = .FALSE.                                                       CONS0550
      L = .TRUE.                                                        CONS0560
      IF(AID(3).OR.AID(6).OR.AID(12).OR.AID(16).OR.AID(17)) N=.TRUE.    CONS0570
      DO 50 I = 18,27                                                   CONS0580
         IF(AID(I)) GO TO 40                                            CONS0590
         L = .FALSE.                                                    CONS0600
         GO TO 50                                                       CONS0610
   40 N = .TRUE.                                                        CONS0620
   50 CONTINUE                                                          CONS0630
      RETURN                                                            CONS0640
      END                                                               CONS0650
      SUBROUTINE LOGSET(I,LOG)                                          LOGS0010
C-----------------------------------------------------------------------LOGS0020
C                              THIS SUBROUTINE,CALLED IN BY CONSYS AND  LOGS0030
C                              CONPNT,SETS THE LOGICAL VARIABLES LOG(K) LOGS0040
C                              TRUE FOR THE K-VALUES,STORED IN THE ARGU- LOGS0050
C                              MENT I.                                  LOGS0060
C-----------------------------------------------------------------------LOGS0070
      LOGICAL LOG(1)                                                    LOGS0080
      INTEGER I(1)                                                      LOGS0090
      DO 10 L=1,6                                                       LOGS0100
         IF(I(L).EQ.0)            GO TO 20                              LOGS0110
         K=I(L)                                                         LOGS0120
         LOG(K)=.TRUE.                                                  LOGS0130
   10 CONTINUE                                                          LOGS0140
   20 RETURN                                                            LOGS0150
      END                                                               LOGS0160
      SUBROUTINE MA2CON(TMIN,I,ISMD,ALK)                                MA2C0010
C-----------------------------------------------------------------------MA2C0020
C                              THIS SUBROUTINE CALCULATES CONSTANTS USED MA2C0030
C                              IN SUBROUTINE MATRIX TO BUILD UP VARIOUS  MA2C0040
C                              MATRICES, THESE CONSTANTS ALL DEPENDENT   MA2C0050
C                              ON ALK(J) AND / OR RADIUS(I), ARE STORED  MA2C0060
C                              IN COMMON/INDATA/.                        MA2C0070
C-----------------------------------------------------------------------MA2C0080
      REAL K1,K2,K4(10),K5,K6,K11,K12,NU,II,LOAD,ACCUR(3),ALK(9)        MA2C0090
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      DUMMY = 0.0                                                       MAC00840
      LAYER = NLAYS                                                     MAC00850
      T2(NLAYS) = 0.0                                                   MAC00860
      DO 60 K = 1,M                                                     MAC00870
         IF(THICK(K).LT.TMIN) TMIN = THICK(K)                           MAC00880
         DUMMY = DUMMY+THICK(K)                                         MAC00890
         T2(K) = 2.0*THICK(K)/RADIUS(1)                                 MAC00900
         H(K) = DUMMY/RADIUS(1)                                         MAC00910
   60 CONTINUE                                                          MAC00920
      CALL MA2CON(TMIN,1,ISMO,ALK)                                      MAC00930
      TX = 6.6*RADIUS(1)/TMIN                                           MAC00940
      XMAX = TX+1.0                                                     MAC00950
C-----------------------------------------------------------------------MAC00960
C                              TEST ON NUMERICAL STABILITY OF THE SOLU- MAC00970
C                              TION-PROCEDURE TO BE FOLLOWED FOR THIS   MAC00980
C                              SYSTEM BY CALLING IN THE MATRIX SUBROUTI- MAC00990
C                              NE WITH NTELL = 2 .                      MAC01000
C                              AFTER TEST THE SMOOTH OR ROUGH CALCULATI- MAC01010
C                              ON PROCEDURE IS CHOSEN.                  MAC01020
C                              TEST IS ONLY NECESSARY IF NOT DIRECTLY   MAC01030
C                              THE SMOOTH CALCULATION PROCEDURE HAS BEEN MAC01040
C                              CHOSEN BY ISMO=1.                        MAC01050
C-----------------------------------------------------------------------MAC01060
      CALL MATRIX(TX,1,NTELL)                                           MAC01070
      IF(NTELL.EQ.2) GO TO 70                                           MAC01080
      ISMO = 1                                                          MAC01090
      NSYS = 1                                                          MAC01100
      WRITE(NOUT,1001)                                                  MAC01110
   70 RETURN                                                            MAC01120
 1001 FORMAT(" THE MORE STABLE SMOOTH CALCULATION PROCEDURE HAS BEEN CHOMAC01130
     1SEN.")                                                            MAC01140
      END                                                               MAC01150
      SUBROUTINE CONSYS(AID,NZEP,NZEQ,N,L)                              CONS0010
C-----------------------------------------------------------------------CONS0020
C                              THIS SUBROUTINE DETERMINES FOR EACH SYS- CONS0030
C                              TEM THE CYLINDRICAL COMPONENTS NEEDED FOR CONS0040
C                              COMPUTATION OF THE REQUIRED CARTESIAN    CONS0050
C                              COMPONENTS OF STRESSES,STRAINS AND DISPLA-CONS0060
C                              CEMENT. GIVEN THIS SET OF COMPONENTS A   CONS0070
C                              FURTHER SELECTION IS PERFORMED ON THE    CONS0080
C                              COMPONENTS THAT CAN BE COMPUTED WITH THE CONS0090
C                              INTEGRALS.                               CONS0100
C                              CONSYS CALLS IN SUBROUTINE LOGSET        CONS0110
C-----------------------------------------------------------------------CONS0120
      LOGICAL AID(27),NZEP,NZEQ,EPS(5),N,L                              CONS0130
      INTEGER JARG(6,14)                                                CONS0140
      DATA JARG/                                                        CONS0150
     1 4, 5, 7,18,19,21,      8, 9,20,22, 0, 0,     10,11,13,23,24,26,  CONS0160
     214,15,25,27, 0, 0,      1, 2,16,17, 0, 0,      5,10,12, 0, 0, 0,  CONS0170
     3 4,10,12, 0, 0, 0,      4, 5,10, 0, 0, 0,      4, 6,12, 0, 0, 0,  CONS0180
     4 4, 5,12, 0, 0, 0,     16,17, 0, 0, 0, 0,     18,19,21,23,24,26,  CONS0190
     523,24,26, 0, 0, 0,     20,22,25,27, 0, 0/                         CONS0200
      EPS(1) = AID(18).OR.AID(19).OR.AID(21)                            CONS0210
      EPS(2) = AID(20).OR.AID(22)                                       CONS0220
      EPS(3) = AID(23).OR.AID(24).OR.AID(26)                            CONS0230
      EPS(4) = AID(25).OR.AID(27)                                       CONS0240
      EPS(5) = AID(16).OR.AID(17)                                       CONS0250
      DO 10 I = 1,5                                                     CONS0260
         IF(.NOT.EPS(I)) GO TO 10                                       CONS0270
         CALL LOGSET(JARG(1,I),AID)                                     CONS0280
   10 CONTINUE                                                          CONS0290
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      COMMON/ASDT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,MOSTRS,NZEROS,H(9)MA2C0100
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                          MA2C0110
      COMMON/INDATA/XMAX,  A1(9),B1(9),C1(9),D(9),EE(9),F(9),G(9),H1(9),MA2C0120
     1II(9),K1(9),K2(9),K6(10),BE(9),BU(9),BUU(9),BMU(9),B2U(9),B2UU(9),MA2C0130
     2J2(9),J1,T2(10),SS(2,10),G012(9),G021(9),G022(9),G122(9),        MA2C0140
     3H012(9),H022(9),H122(9),D012(9),D022(9),C011(9),C012(9),E012(9),  MA2C0150
     4F012(9),F112(9),F022(9),CC(4,2,9),DD(2,2,9),FF(2,2,9),GG(2,2,10), MA2C0160
     5HH(2,2,10),RR(4,2,10),DD2(9),G20(9),G21(9),H20(9),H021(9),GG2(10),MA2C0170
     6HH2(10),Q011(9),Q111(9),Q012(9),Q112(9),Q212(9),Q022(9),Q122(9),  MA2C0180
     7QF0(9),QF1(9),Z011,Z111,Z211,Z012,Z112,Z212,Z312,Z021,Z121,Z022,  MA2C0190
     8Z122,Z222,K4                                                     MA2C0200
      XMAX =6.5*RADIUS(I)/TMIN                                         MA2C0210
      K = 0                                                            MA2C0220
      DO 30 J = 1,M                                                    MA2C0230
         AL(J) = ALK(J)/(RADIUS(I)+ALK(J))                             MA2C0240
         K12 = 1.0-AL(J)                                               MA2C0250
         Q011(J) = K12*B1(J)                                           MA2C0260
         Q111(J) = 2.0*AL(J)*NU(J+1)                                   MA2C0270
         Q012(J) =-K12*EE(J)                                           MA2C0280
         Q022(J) = K12*A1(J)                                           MA2C0290
         Q122(J) = AL(J)*K5(J)                                         MA2C0300
         QF0(J) = K12*A1(J)*B1(J)                                      MA2C0310
         QF1(J) = 2.0*AL(J)*(1.0-NU(J)+(1.0-NU(J+1))*K1(J))            MA2C0320
         IF(ISMO.EQ.1) GO TO 20                                        MA2C0330
         IF(ALK(J).GE.100.0) GO TO 20                                  MA2C0340
         BE(J) = -AL(J)/(1.0-AL(J))                                    MA2C0350
         BU(J) = BE(J)*2.0*NU(J+1)                                     MA2C0360
         BUU(J) = BU(J)*K5(J)                                          MA2C0370
         BMU(J) = BE(J)*K5(J)                                          MA2C0380
         B2U(J) = BE(J)*(K5(J)-2.0*NU(J+1))                            MA2C0390
         B2UU(J) = BE(J)*(K5(J)+2.0*NU(J+1))                           MA2C0400
         GO TO 30                                                      MA2C0410
20       K11 = 2.0*(NU(J)-NU(J+1))                                     MA2C0420
         K = K+1                                                       MA2C0430
         J2(K) = J                                                     MA2C0440
         GG(1,1,K+1) = K2(J)                                           MA2C0450
         G012(K) = K11-K2(J)*(2.0-4.0*NU(J+1))                         MA2C0460
         G021(K) =-K12*K2(J)                                           MA2C0470
         G022(K) = (K11-K2(J))*K12                                     MA2C0480
         G122(K) =-2.0*NU(J+1)*AL(J)                                   MA2C0490
         HH(1,1,K+1) =-3.0+4.0*NU(J)-K1(J)                             MA2C0500
         H012(K) =-2.0+2.0*NU(J)+6.0*NU(J+1)-K4(J)-(2.0-4.0*NU(J+1))*  MA2C0510
     1             K1(J)                                               MA2C0520
         H021(K) = HH(1,1,K+1)*K12                                     MA2C0530
         H022(K) =-K12*(1.0-2.0*NU(J)-6.0*NU(J+1)+K4(J)-K1(J))         MA2C0540
         H122(K) = 2.0*AL(J)*NU(J+1)                                   MA2C0550
         DD(1,1,K) =-K6(J)                                             MA2C0560
         D012(K) =-K6(J)*K5(J)                                         MA2C0570
         DD(2,1,K) =-K12*K6(J)                                         MA2C0580
         D022(K) =-NU(J)*2.0*DD(2,1,K)                                 MA2C0590
         C011(K) =-1.0+4.0*NU(J)                                       MA2C0600
         C012(K) = 4.0*NU(J)*K5(J)                                     MA2C0610
         CC(2,1,K) =-2.0                                               MA2C0620
         E012(K) =-K11                                                 MA2C0630
         F012(K) = K4(J)-2.0*(NU(J)+NU(J+1))                           MA2C0640
         F112(K) = 2.0*E012(K)                                         MA2C0650
         F022(K) = 1.0-4.0*NU(J+1)                                     MA2C0660
         FF(2,1,K) = 2.0                                               MA2C0670
         DD2(K) = 2.0*K12/K1(J)                                        MA2C0680
         G20(K) = K12*(1.0-1.0/K1(J))                                  MA2C0690
         G21(K) = AL(J)*0.5/K1(J)                                      MA2C0700
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      H20(K) = K12*(1.0+1.0/K1(J))                                      MA2C071U
   30 CONTINUE                                                          MA2C072U
      J1 = K                                                            MA2C073U
      DUMMY=0.0                                                         MA2C074U
      T2(NLAYS) = 0.0                                                   MA2C075U
      DO 40 K = 1,M                                                     MA2C076U
         DUMMY=DUMMY+THICK(K)                                           MA2C077U
         T2(K)=2.0*THICK(K)/RADIUS(I)                                   MA2C078U
          H(K)=DUMMY/RADIUS(I)                                          MA2C079U
         K12 = 1.0-AL(K)                                                MA2C080U
         Q112(K) = -K12*F(K)*H(K)+Q111(K)*K5(K)                         MA2C081U
         Q212(K) = AL(K)*H(K)*(2.0*NU(K+1)-K5(K))                       MA2C082U
   40 CONTINUE                                                          MA2C083U
      IF(LAYER.EQ.NLAYS) GO TO 50                                       MA2C084U
C-----------------------------------------------------------------------MA2C085U
C                                THESE CONSTANTS ARE USED FOR THE ASYMPTO- MA2C086U
C                                TIC EVALUATION OF THE CHARACTERISTIC     MA2C087U
C                                FUNCTIONS IN MATRIX.                     MA2C088U
C-----------------------------------------------------------------------MA2C089U
      J = LAYER                                                         MA2C090U
      RK1 = 2.0*NU(J+1)*C1(J)                                           MA2C091U
      RK2 = 2.0*NU(J+1)*A1(J)                                           MA2C092U
      RK3 = 2.0*NU(J+1)*D(J)                                            MA2C093U
      Z021 = Q011(J)*C1(J)                                              MA2C094U
      RK4 = Z021*H(J)                                                   MA2C095U
      K12 = 1.0-AL(J)                                                   MA2C096U
      Z011 = Q011(J)*D(J)                                               MA2C097U
      Z111 = AL(J)*(RK3-G(J)-K5(J)*B1(J))-RK4                           MA2C098U
      Z211 = AL(J)*H(J)*(B1(J)-II(J)-RK1)                               MA2C099U
      Z012 = -K12*(D(J)*EE(J)+A1(J)*G(J))                               MA2C100U
      Z112 = Q122(J)*(RK2-G(J)+RK3+EE(J))+K12*H(J)*(A1(J)*              MA2C101U
     1       H1(J)+C1(J)*EE(J)-D(J)*F(J))                               MA2C102U
      Z212 = -AL(J)*H(J)*(K5(J)*(RK1+B1(J)+II(J))-RK3+EE(J)+            MA2C103U
     1        RK2+G(J))-RK4*H(J)                                        MA2C104U
      Z312 = AL(J)*H(J)*H(J)*(B1(J)-RK1-II(J))                          MA2C105U
      Z121 = AL(J)*(RK1-B1(J)+II(J))                                    MA2C106U
      Z022 = K12*(A1(J)*II(J)-EE(J)*C1(J))                              MA2C107U
      Z122 = AL(J)*(RK2+EE(J)+K5(J)*(RK1+II(J)))+RK4                    MA2C108U
      Z222 = Z121*H(J)                                                  MA2C109U
   50 RETURN                                                            MA2C110U
      END                                                               MA2C111U
      SUBROUTINE CONPNT(R,HOSTRS,LOAD,Z,N2,L2)                          CONP001U
C-----------------------------------------------------------------------CONP002U
C                                THIS SUBROUTINE DETERMINES FOR EACH POINT-CONP003U
C                                LOAD CONFIGURATION SEPARATELY THE        CONP004U
C                                INTEGRALS NEEDED FOR COMPUTATION OF THE  CONP005U
C                                DESIRED COMPONENTS OF STRESS,ETC.        CONP006U
C                                FOR POINTS AT THE RIM OF THE LOAD SOME   CONP007U
C                                COMPONENTS CANNOT BE CALCULATED BECAUSE  CONP008U
C                                OF SINGULAR BEHAVIOUR, A MESSAGE IS      CONP009U
C                                PRINTED.                                 CONP010U
C-----------------------------------------------------------------------CONP011U
      LOGICAL STRESS,EPS,RLOW,N2,L2                                     CONP012U
      REAL LOAD                                                         CONP013U
      INTEGER IARG(6,12),KARG(6,4),JJ(12,15)                            CONP014U
      COMMON/STRDTA/STRESS(27),EPS(17),RLOW,ST,CT,L,ACC                 CONP015U
      COMMON/TAPE/NOUT                                                  CONP016U
      DATA IARG/                                                        CONP017U
     1 7,12,17, 0, 0, 0,        12,14,17, 0, 0, 0,      10,11, 0, 0, 0, 0,CONP018U
     2 7, 8, 9,12,14,17,         7, 9,12,14,17, 0,       8, 9, 0, 0, 0, 0,CONP019U
     3 7,12,14,15,17, 0,         6,10,16, 0, 0, 0,      10,13,16, 0, 0, 0,CONP020U
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     4 7, 8,12,14,17, 0,          7,12,14,17, 0, 0,       8, 9, 0, 0, 0, 0/CONP021U
      DATA KARG/                                                          CONP022U
     1 1, 2, 4, 0, 0, 0,          2, 4, 0, 0, 0, 0,       1, 2, 0, 0, 0, 0,CONP023U
     2 4, 5,10,11,12, 0/                                                  CONP024U
      DATA JJ/                                                            CONP025U
     1 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,                                CONP026U
     2 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,                                CONP027U
     3 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,                                CONP028U
     4 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0,                                CONP029U
     5 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,                                CONP030U
     6 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0,                                CONP031U
     7 1, 0, 1,-1,-1, 0, 0, 0, 0,-1, 1,-1,                                CONP032U
     8 0, 1, 0, 0, 0, 0,-1, 0, 0, 0, 0, 0,                                CONP033U
     9 1, 1, 1,-1,-1, 0,-1, 0, 0,-1, 1,-1,                                CONP034U
     T 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1,                                CONP035U
     1 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,                                CONP036U
     2 1, 1, 1, 1, 1, 0, 1, 0, 0, 1, 1, 1,                                CONP037U
     3 1, 0, 1, 1, 1, 1, 0, 1, 0, 1, 1, 1,                                CONP038U
     4 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0,                                CONP039U
     5 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1/                                CONP040U
      NERR = 0                                                            CONP041U
      DO 10 I = 1,17                                                      CONP042U
         EPS(I) = .FALSE.                                                 CONP043U
   10 CONTINUE                                                            CONP044U
      J=3                                                                 CONP045U
      IF(ABS(ST).LT.ACC)          J=1                                     CONP046U
      IF(ABS(CT).LT.ACC)          J=2                                     CONP047U
      IF(HOSTPS.LT.ACC)           GO TO 20                                CONP048U
      IF(RLO=)                    GO TO 30                                CONP049U
      IF(Z.LT.ACC)                GO TO 40                                CONP050U
      I=J+14                                                              CONP051U
      GO TO 50                                                            CONP052U
   20 I=2                                                                 CONP053U
      IF(RLO=)                    I=1                                     CONP054U
      GO TO 50                                                            CONP055U
   30 I=J+5                                                               CONP056U
      IF(Z.LT.ACC)                I=I-3                                   CONP057U
      GO TO 50                                                            CONP058U
   40 I=J+11                                                              CONP059U
      IF(ABS(R-1.0).LT.ACC)       I=I-3                                   CONP060U
   50 IF(STRESS( 4).OR.STRESS(10))CALL LOGSET(KARG(1,1),EPS)              CONP061U
      IF(STRESS( 5))              CALL LOGSET(KARG(1,2),EPS)              CONP062U
      IF(STRESS( 3))              EPS( 3)=.TRUE.                          CONP063U
      IF(STRESS(11))              EPS( 4)=.TRUE.                          CONP064U
      IF(STRESS(12))              CALL LOGSET(KARG(1,3),EPS)              CONP065U
      IF(STRESS( 6).AND.(Z.GT.ACC)) EPS( 1)=.TRUE.                        CONP066U
      IF(.NOT.STRESS( 8))         GO TO 60                                CONP067U
      IF(Z.LT.ACC)                GO TO 60                                CONP068U
      IF(R.GT.ACC)                EPS(5)=.TRUE.                           CONP069U
   60 IF(I.LT.3) GO TO 180                                                CONP070U
      DO 90 J = 1,12                                                      CONP071U
         IF(.NOT.STRESS(J))       GO TO 90                                CONP072U
         IF(JJ(J,I-2))            70,90,80                                CONP073U
   70    NERR = 1                                                        CONP074U
         STRESS(J) = .FALSE.                                             CONP075U
         L2 = .FALSE.                                                    CONP076U
         GO TO 90                                                        CONP077U
   80    CALL LOGSET(IARG(1,J),EPS)                                      CONP078U
   90 CONTINUE                                                            CONP079U
      IF(NERR) 160,160,100                                                CONP080U
  100 IF(I-10) 110,130,120                                                CONP081U
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  110 WRITE(NOUT,9000)                                                  CONP0820
      GO TO 140                                                         CONP0830
  120 WRITE(NOUT,9020)                                                  CONP0840
      STRESS(13) = .FALSE.                                              CONP0850
      GO TO 140                                                         CONP0860
  130 WRITE(NOUT,9010)                                                  CONP0870
      STRESS(13) = .FALSE.                                              CONP0880
      IF(STRESS(12)) GO TO 150                                          CONP0890
  140 IF(STRESS(3).OR.STRESS(6)) GO TO 150                              CONP0900
      IF(STRESS(16)) GO TO 150                                          CONP0910
      IF(STRESS(17)) GO TO 150                                          CONP0920
      IF(STRESS(20)) GO TO 150                                          CONP0930
      IF(STRESS(22)) GO TO 150                                          CONP0940
      IF(STRESS(25)) GO TO 150                                          CONP0950
      IF(STRESS(27)) GO TO 150                                          CONP0960
      N2 = .FALSE.                                                      CONP0970
  150 STRESS(18) = .FALSE.                                              CONP0980
      STRESS(19) = .FALSE.                                              CONP0990
      STRESS(21) = .FALSE.                                              CONP1000
      STRESS(23) = .FALSE.                                              CONP1010
      STRESS(24) = .FALSE.                                              CONP1020
      STRESS(26) = .FALSE.                                              CONP1030
  160 IF(LOAD.GT.ACC) GO TO 180                                         CONP1040
      DO 170 J = 1,5                                                    CONP1050
  170    EPS(J) = .FALSE.                                               CONP1060
  180 RETURN                                                            CONP1070
 9000 FORMAT(" AT THIS POINT SRR,STT,ERR AND EZZ HAVE A LOGARITHMIC SINGCONP1080
     1ULARITY")                                                         CONP1090
 9010 FORMAT(" AT THIS POINT SRT AND ERT HAVE A LOGARITHMIC SINGULARITY"CONP1100
     1)                                                                 CONP1110
 9020 FORMAT(" AT THIS POINT SRR,STT,SRT,ERR,EZZ AND ERT HAVE A LOGARITHCONP1120
     1MIC SINGULARITY")                                                 CONP1130
      END                                                               CONP1140
      SUBROUTINE GENDAT(N,NZEROS,R,ACC)                                 GEND0010
C-----------------------------------------------------------------------GEND0020
C                              THIS SUBROUTINE GIVES THE ZEROS OF THE   GEND0030
C                              PRODUCTS J0(XR)*J1(X) AND J1(XR)*J1(X) IN GEND0040
C                              THE RIGHT ORDER. THE SUBSEQUENT ZEROS ARE GEND0050
C                              STORED IN ZEROS FOR USING THEM IN INGRAL. GEND0060
C                              THE ZEROS OF J0 AND J1 ARE STORED AS     GEND0070
C                              BZEROS IN THE BLOCK DATA.                GEND0080
C-----------------------------------------------------------------------GEND0090
      COMMON/GEDATA/BZEROS(149,2),ZEROS(298)                            GEND0100
      IF(R.LT.ACC.OR.ABS(R-1.0).LT.ACC) GO TO 40                        GEND0110
      I=1                                                               GEND0120
      J=1                                                               GEND0130
      DO 20 K=1,298                                                     GEND0140
         IF(I.GT.149) GO TO 30                                          GEND0150
         IF(J.GT.149) GO TO 30                                          GEND0160
         IF(BZEROS(I,2).LT.BZEROS(J,N+1)/R) GO TO 10                    GEND0170
         ZEROS(K) = BZEROS(J,N+1)/R                                     GEND0180
         J=J+1                                                          GEND0190
         GO TO 20                                                       GEND0200
   10    ZEROS(K)=BZEROS(I,2)                                           GEND0210
         I=I+1                                                          GEND0220
   20 CONTINUE                                                          GEND0230
   30 NZEROS = K-1                                                      GEND0240
      RETURN                                                            GEND0250
   40 IF(R.GT.ACC) GO TO 70                                             GEND0260
   50 DO 60 I=1,149                                                     GEND0270
         ZEROS(I)=BZEROS(I,2)                                           GEND0280
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   60 CONTINUE                                                          GEND0290
      NZEROS=149                                                        GEND0300
      RETURN                                                            GEND0310
   70 IF(N.EQ.1) GO TO 50                                               GEND0320
      DO 80 K=1,149                                                     GEND0330
         ZEROS(2*K-1)=BZEROS(K,1)                                       GEND0340
         ZEROS(2*K  )=BZEROS(K,2)                                       GEND0350
   80 CONTINUE                                                          GEND0360
      NZEROS=298                                                        GEND0370
      RETURN                                                            GEND0380
      END                                                               GEND0390
      SUBROUTINE ASYMPT(R,ACC)                                          ASYM0010
C-----------------------------------------------------------------------ASYM0020
C                              THIS SUBROUTINE ORGANIZES THE COMPUTATION ASYM0030
C                              OF THE ASYMPTOTIC PART OF THE INTEGRALS   ASYM0040
C                              AS USED FOR THE TOP-LAYER ONLY.           ASYM0050
C                              ASYMPT CALLS IN SUBROUTINE   ASS          ASYM0060
C                              ASYMPT CALLS IN FUNCTIONS   FLLE          ASYM0070
C                                                          FLLK          ASYM0080
C                                                          FLMBDA        ASYM0090
C-----------------------------------------------------------------------ASYM0100
      DOUBLE PRECISION DR,KACC2,C,ELLE,ELLK,FLLE,FLLK                   ASYM0110
      COMMON/CONST/C,ELLE,ELLK,ALMBDA                                   ASYM0120
      IF(R.LT.ACC) GO TO 10                                             ASYM0130
      DR=DBLE(R)                                                        ASYM0140
      KACC2=((1.0D0-DR)*(1.0D0-DR)+C*C)/((1.0D0+DR)*(1.0D0+DR)+C*C)     ASYM0150
      ELLK=FLLK(KACC2)                                                  ASYM0160
      ELLE=FLLE(KACC2)                                                  ASYM0170
      ALMBDA = FLMBDA(DR,C,ELLK,ELLE,KACC2)                             ASYM0180
   10 CALL ASS(ACC,R)                                                   ASYM0190
      RETURN                                                            ASYM0200
      END                                                               ASYM0210
      DOUBLE PRECISION FUNCTION FLLK(KACC2)                             FLLK0010
C-----------------------------------------------------------------------FLLK0020
C                              THIS FUNCTIONSUBROUTINE EVALUATES THE     FLLK0030
C                              COMPLETE ELLIPTIC INTEGRAL OF THE FIRST   FLLK0040
C                              KIND FROM A SERIES-EXPANSION ACCORDING TO FLLK0050
C                              BYRD AND FRIEDMAN,HANDBOOK OF ELLIPTIC    FLLK0060
C                              INTEGRALS FOR ENGINEERS AND PHYSICISTS,   FLLK0070
C                              FORMULA  900.00 FOR KACC2.GE.0.5          FLLK0080
C                              FORMULA  900.06 FOR KACC2.LT.0.5          FLLK0090
C-----------------------------------------------------------------------FLLK0100
      DOUBLE PRECISION KACC2,KA,M1,KACC                                 FLLK0110
      KA = 1.0D0-KACC2                                                  FLLK0120
      IF(KA.GT.0.5D0) GO TO 10                                          FLLK0130
      FLLK=1.0D0+KA*(0.25D0+KA*(0.140625D0+KA*(0.09765625D0+KA*(0.074768FLLK0140
     10664D0+KA*(0.060562133 8D0+KA*(0.050889015 0D0+KA*(0.0438767937D0+KAFLLK0150
     2*(0.0385653465D0+KA*(0.0343993364D0+KA*(0.0310454012D0+KA*(0.02828FLLK0160
     372353D0+KA*(0.0259790743D0+KA*(0.0240191152D0+KA*(0.0223341012D0+KFLLK0170
     4A*0.0208699768D0)))))))))))))))                                   FLLK0180
      GO TO 30                                                          FLLK0190
   10 KACC=DSQRT(KACC2)                                                 FLLK0200
      IF(KACC.LT.1.0D-04) GO TO 20                                      FLLK0210
      M1=-DLOG(KACC)                                                    FLLK0220
      FLLK=M1*(1.0D0+KACC2*(0.25D0+KACC2*(0.140625D0+KACC2*(0.09765625D0FLLK0230
     1+KACC2*(0.0747680664D0+KACC2*(0.0605621338D0+KACC2*(0.050889015  DFLLK0240
     20+KACC2*(0.0438767937 D0+KACC2*(0.0385653465 D0+KACC2*(0.0343993 36FLLK0250
     34 D0+KACC2*(0.0310454012 D0+KACC2*(0.0282872353 D0+KACC2*(0.025979FLLK0260
     40743 D0+KACC2*(0.0240191152 D0+KACC2*0.0223341012 D0)))))))))))))))FLLK0270
     5+1.38629436D0+KACC2*(0.0965735903D0+KACC2*(0.0308851445D0+KACC2*(0FLLK0280
     6.0149376004D0+KACC2*(0.0087663122D0+KACC2*(0.0057548877D0+KACC2*(0FLLK0290
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     7,0040646585D0+KACC2*(0,0030225465D0+KACC2*(0,0023351572D0+KACC2*(0FLLK0300
     8,0018580703D0+KACC2*(0,0015135116D0+KACC2*(0,0012565911D0+KACC2*(0FLLK0310
     9,0010599297D0+KACC2*(0,0009060596D0+KACC2*0,0007834118D0)))))))))FLLK0320
     T)))                                                              FLLK0330
      FLLK = 2,0D0*FLLK/3,141592653D0                                  FLLK0340
      GO TO 30                                                         FLLK0350
   20 FLLK = 0,0D0                                                     FLLK0360
   30 RETURN                                                           FLLK0370
      END                                                              FLLK0380
      DOUBLE PRECISION FUNCTION FLLE(KACC2)                            FLLE0010
C-----------------------------------------------------------------------FLLE0020
C                              THIS FUNCTIONSUBROUTINE EVALUATES THE   FLLE0030
C                              COMPLETE ELLEPTIC INTEGRAL OF THE SECOND FLLE0040
C                              KIND FROM A SERIES-EXPANSION ACCORDING TO FLLE0050
C                              BYRD AND FRIEDMAN,HANDBOOK OF ELLIPTIC  FLLE0060
C                              INTEGRALS FOR ENGINEERS AND PHYSICISTS, FLLE0070
C                              FORMULA  900,07 FOR KACC2.GE.0,5        FLLE0080
C                              FORMULA  900,10 FOR KACC2.LT.0,5        FLLE0090
C-----------------------------------------------------------------------FLLE0100
      DOUBLE PRECISION KACC2,KACC,KA                                   FLLE0110
      KA = 1,0D0-KACC2                                                 FLLE0120
      IF(KA.GT.0,5D0) GO TO 10                                         FLLE0130
      FLLE=1,0D0-KA*(0,25D0+KA*(0,046875D0+KA*(0,01953125D0+KA*(0,01068FLLE0140
     11523D0+KA*(0,006729126D0+KA*(0,004626274 1D0+KA*(0,003375291 8D0+KAFLLE0150
     2*(0,002571023 1D0+KA*(0,002023490 4D0+KA*(0,001633968 5D0+KA*0,001347FLLE0160
     30112D0))))))))))                                                 FLLE0170
      GO TO 30                                                         FLLE0180
   10 KACC=DSQRT(KACC2)                                                FLLE0190
      IF(KACC.LT.1,0D-04)        GO TO 20                              FLLE0200
      FLLE=1,0D0-0,5D0*KACC2*DLOG(KACC)*(1,0D0+KACC2*(0,375D0+KACC2*(0 FLLE0210
     1,234375D0+KACC2*(0,170898437 5D0+KACC2*(0,134582519 6D0+KACC2*(0,111FLLE0220
     2030578 6D0+KACC2*(0,094508171 1D0+KACC2*(0,082272738 2D0+KACC2*(0,072FLLE0230
     3845653 6D0+KACC2*(0,065358739 3D0+KACC2*(0,059268493 1D0+KACC2*(0,054FLLE0240
     4217201 1D0+KACC2*(0,049959760 6D0+KACC2*(0,046322580 2D0+KACC2*0,0431FLLE0250
     579262 3D0))))))))))))))))+0,25D0*KACC2*(1,772588722D0+KACC2*(0,227220FLLE0260
     6770 7D0+KACC2*(0,087325481 7D0+KACC2*(0,046178085 6D0+KACC2*(0,028568FLLE0270
     7001 2D0+KACC2*(0,019418973 3D0+KACC2*(0,014058751 8D0+KACC2*(0,010648FLLE0280
     8943 4D0+KACC2*(0,008345589 5D0+KACC2*(0,006716673 7D0+KACC2*(0,005522FLLE0290
     9288 8D0+KACC2*(0,004620493 6D0+KACC2*(0,003922930 4D0+KACC2*(0,003372FLLE0300
     T2549D0+KACC2*0,002929929 8D0))))))))))))))                       FLLE0310
      FLLE = 2,0D0*FLLE/3,141592653D0                                  FLLE0320
      GO TO 30                                                         FLLE0330
   20 FLLE = 2,0D0/3,141592653D0                                       FLLE0340
   30 RETURN                                                           FLLE0350
      END                                                              FLLE0360
      FUNCTION FLMBDA(DR,C,ELLK,ELLE,KACC2)                            FLMB0010
C-----------------------------------------------------------------------FLMB0020
C                              THIS FUNCTIONSUBROUTINE EVALUATES THE   FLMB0030
C                              HEUMAN"S-LAMBDA FUNCTION FROM A SERIES- FLMB0040
C                              EXPANSION ACCORDING TO                  FLMB0050
C                              BYRD AND FRIEDMAN,HANDBOOK OF ELLIPTIC  FLMB0060
C                              INTEGRALS FOR ENGINEERS AND PHYSICISTS, FLMB0070
C                              FORMULA  904,00                         FLMB0080
C                              USE IS MADE OF THE COMPLETE ELLIPTIC INTE-FLMB0090
C                              GRALS OF THE FIRST AND SECOND KIND ELLK FLMB0100
C                              AND ELLE EVALUATED BY FLLK AND FLLE,    FLMB0110
C-----------------------------------------------------------------------FLMB0120
      DOUBLE PRECISION DR,DASIN,SUM,PHI,DS,DC,A,T,AI,KACC2,TMAI,DAR,ELLKFLMB0130
     1,ELLE,E,K,C                                                      FLMB0140
      DAR = DABS(1,0D0-DR)                                             FLMB0150
      IF(C.LT.DAR) GO TO 10                                            FLMB0160
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      DASIN = DAR/C                                                     FLMB0170
      PHI = 1.57079632680D0-DATAN(DASIN)                                FLMB0180
      GO TO 30                                                          FLMB0190
   10 DASIN = C/DAR                                                     FLMB0200
      IF(C.LT.(0.1D-05*DAR)) GO TO 20                                   FLMB0210
      PHI = DATAN(DASIN)                                                FLMB0220
      GO TO 30                                                          FLMB0230
   20 PHI = DASIN                                                       FLMB0240
   30 IF(DABS(PHI-1.57079632680D0).GT.1.0D-6) GO TO 40                  FLMB0250
      FLMBDA=1.0                                                        FLMB0260
      GO TO 60                                                          FLMB0270
   40 DS=DSIN(PHI)                                                      FLMB0280
      DC=DCOS(PHI)                                                      FLMB0290
      E = ELLE                                                          FLMB0300
      K = ELLK                                                          FLMB0310
      FLMBDA=PHI*E                                                      FLMB0320
      T=0.5D0*(PHI-DS*DC)                                               FLMB0330
      A=0.5D0*KACC2                                                     FLMB0340
      SUM=A*T*(2.0D0*K-E)                                               FLMB0350
      IF(SUM.LT.1.0D-07) GO TO 60                                       FLMB0360
      I=1                                                               FLMB0370
   50 FLMBDA = FLMBDA-SNGL(SUM)                                         FLMB0380
      I=I+1                                                             FLMB0390
      AI=I                                                              FLMB0400
      TMAI=2.0D0*AI-1.0D0                                               FLMB0410
      T=0.5D0*T*TMAI/AI-0.5D0*DC*(DS**TMAI)/AI                          FLMB0420
      A=0.5D0*A*(TMAI-2.0D0)*KACC2/AI                                   FLMB0430
      SUM=A*T*(2.0D0*AI*K-TMAI*E)                                       FLMB0440
      IF(SUM.GT.1.0D-07) GO TO 50                                       FLMB0450
   60 RETURN                                                            FLMB0460
      END                                                               FLMB0470
      SUBROUTINE ASS(ACC,R)                                             ASS 0010
C-----------------------------------------------------------------------ASS 0020
C                                THIS SUBROUTINE COMPUTES THE LIPSCHITZ-  ASS 0030
C                                HANKEL INTEGRALS I(I,J,K) FROM EXPRESSI- ASS 0040
C                                ONS IN EARLIER EVALUATED ELLIPTIC FUNCTI- ASS 0050
C                                ONS OF THE FIRST AND SECOND KIND,ELLK AND ASS 0060
C                                ELLE,AND HEUMAN'S-LAMBDA FUNCTION,ALMBDA, ASS 0070
C                                REFERENCE                                ASS 0080
C                                EASON,NOBLE AND SNEDDON,CERTAIN INTEGRALS ASS 0090
C                                OF LIPSCHITZ-HANKEL TYPE INVOLVING PRO-  ASS 0100
C                                DUCTS OF BESSEL FUNCTIONS,PHILOSOPHICAL  ASS 0110
C                                TRANSACTIONS,VOL 247,SERIES A935,APRIL   ASS 0120
C                                1955,PP 529-546.                         ASS 0130
C                                          F10M1=I(1,0?-1)                ASS 0140
C                                          F100 =I(1,0?0)                 ASS 0150
C                                          F101 =I(1,0?1)                 ASS 0160
C                                          F11M2=I(1,1?-2)                ASS 0170
C                                          F11M1=I(1,1?-1)                ASS 0180
C                                          F110 =I(1,1?0)                 ASS 0190
C                                          F111 =I(1,1?1)                 ASS 0200
C-----------------------------------------------------------------------ASS 0210
      COMMON/CONST/C,ELLE,ELLK,ALMBDA                                   ASS 0220
      COMMON/CNTING/F10M1,F100,F101,F11M2,F11M1,F110,F111               ASS 0230
      DOUBLE PRECISION DR,C,DEPR,DEMR,DC2,DRT2,DRT,DAD,DR2,DRC2,DRRT,   ASS 0240
     1DEMRR,ELLE,ELLK                                                   ASS 0250
      EC = SNGL(C)                                                      ASS 0260
      IF(R.LT.ACC) GO TO 20                                             ASS 0270
      EMR = 1.0-R                                                       ASS 0280
      EPR = 1.0+R                                                       ASS 0290
      C2 = EC*EC                                                        ASS 0300
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      RT2 = C2+EPR*EPR                                                  ASS 0310
      RT = SQRT(RT2)                                                    ASS 0320
      R2 = R*R                                                          ASS 0330
      EMRR = 1.0-R2                                                     ASS 0340
      DR = DBLE(R)                                                      ASS 0350
      DEPR = 1.0D0+DR                                                   ASS 0360
      DEMR = 1.0D0-DR                                                   ASS 0370
      DC2 = C*C                                                         ASS 0380
      DRT2 = DC2+DEPR*DEPR                                              ASS 0390
      DRT = DSQRT(DRT2)                                                 ASS 0400
      DAD = DC2+DEMR*DEMR                                               ASS 0410
      DR2 = DR*DR                                                       ASS 0420
      DRC2 = DR2+DC2                                                    ASS 0430
      DRRT = DR*DRT                                                     ASS 0440
      DEMRR = 1.0D0-DR2                                                 ASS 0450
      F101 = 0.5D0*(ELLE*(1.0D0-DRC2)/(DAD*DRT)+ELLK/DRT)               ASS 0460
      F110 = DRT*(ELLK*(1.0D0+DRC2)/DRT2-ELLE)/(2.0D0*DR)               ASS 0470
      F111 = C*(ELLE*(1.0D0+DRC2)/DAD-ELLK)/(2.0D0*DRRT)                ASS 0480
      F10M1 = 0.5D0*ELLE*DRT                                            ASS 0490
      F100 = -0.5D0*C*ELLK/DRT                                          ASS 0500
      F11M2 = -C*(C*ELLE*DRT/(4.0D0*DR)-C*ELLK*(1.0D0+DR2+0.5D0*DC2)/   ASS 0510
     1(2.0D0*DRRT))/3.0D0+DR*(DRT*ELLE/2.0D0+DEMRR*ELLK/(2.0D0*DRT))/   ASS 0520
     23.0D0+(ELLE*DRT/(2.0D0*DR)-DEMRR*ELLK/(2.0D0*DRRT))/3.0D0         ASS 0530
      F11M1 = 0.5D0*(0.5D0*ELLE*C*DRT/DR-ELLK*C*(1.0D0+DR2+0.5D0*DC2)/  ASS 0540
     1DRRT)                                                             ASS 0550
      MLP = R                                                           ASS 0560
      IF(R.GT.1.0) MLP=1.0/R                                            ASS 0570
      IF(ABS(EMR).LT.ACC) GO TO 10                                      ASS 0580
      F10M1 = F10M1+0.5*(SNGL(ELLK)*EMRR/RT+SIGN(EC*ALMBDA,EMR))        ASS 0590
      F100 = F100+0.5*SIGN(ALMBDA,-EMR)                                 ASS 0600
      F11M2 = F11M2-EC*ALMBDA*SIGN(1.0,EMR)*EMRR/(4.0*R)                ASS 0610
      F11M2 = F11M2-EC*(MLP/2.0+MLP)/3.0                                ASS 0620
      F11M1 = F11M1+SIGN(0.25,EMR)*EMRR*ALMBDA/R+0.5*MLP                ASS 0630
      IF(R.GT.1.0) GO TO 30                                             ASS 0640
      F10M1 = F10M1-EC                                                  ASS 0650
      F100 = F100+1.0                                                   ASS 0660
      GO TO 30                                                          ASS 0670
   10 F10M1 = F10M1-0.5*EC                                              ASS 0680
      F100 = F100+0.5                                                   ASS 0690
      F11M2 = F11M2-0.5*EC                                              ASS 0700
      F11M1 = F11M1+0.5*MLP                                             ASS 0710
      GO TO 30                                                          ASS 0720
   20 AD = 1.0+EC*EC                                                    ASS 0730
      RT = SQRT(AD)                                                     ASS 0740
      ADRT = AD*RT                                                      ASS 0750
      F101 = 1.0/ADRT                                                   ASS 0760
      F110 = 0.5/ADRT                                                   ASS 0770
      F111 = 1.5*EC/(AD*ADRT)                                           ASS 0780
      F10M1 = RT-EC                                                     ASS 0790
      F100 = 1.0-EC/RT                                                  ASS 0800
      F11M2 = 0.5*(RT-EC)                                               ASS 0810
      F11M1 = 0.5*(1.0-EC/RT)                                           ASS 0820
   30 RETURN                                                            ASS 0830
      END                                                               ASS 0840
      SUBROUTINE INGRAL(IL,INTV,INTT,INT)                               INGR0010
C-----------------------------------------------------------------------INGR0020
C                              THIS SUBROUTINE CONTROLS THE SIMULTANEOUS INGR0030
C                              COMPUTATION OF A GROUP OUT OF THE 17 INTE-INGR0040
C                              GRALS.                                   INGR0050
C                              IL=1,THE GROUP WITH J0(XR)J1(X) IN INTEGR,INGR0060
C                              IL=2,THE GROUP WITH J1(XR)J1(X) IN INTEGR,INGR0070
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C                                THE ELEMENTS OF INTV ARE THE NUMBERS OF    INGR0080
C                                REQUIRED INTEGRALS OF THE GROUP,           INGR0090
C                                INTT IS THE TOTAL NUMBER OF REQUIRED INTE-INGR0100
C                                GRALS IN THE GROUP,                        INGR0110
C                                THE SET OF COMPUTED INTEGRALS IS DELIVE-   INGR0120
C                                RED IN INT,                                INGR0130
C                                ACTUAL INTEGRATION BY MEANS OF A GAUSS-    INGR0140
C                                QUADRATURE IS PERFORMED BY CALLING QUAD,   INGR0150
C                                INTEGRATION PROCEEDS BY QUADRATURE OVER    INGR0160
C                                INTERVALS FROM ONE BESSEL ZERO TO THE      INGR0170
C                                NEXT, FROM THE ORIGIN TO THE FIRST BESSEL-INGR0180
C                                ZERO A LEGENDRE-GAUSS QUADRATURE OF ORDER INGR0190
C                                R,OBTAINING DESIRED ACCURACY BY SUBSE-     INGR0200
C                                QUENT SUBDIVISION OF THE INTERVAL,         INGR0210
C                                FROM THE FIRST BESSELZERO ON A JACOBI-     INGR0220
C                                GAUSS QUADRATURE,OBTAINING DESIRED ACCU-   INGR0230
C                                RACY BY SUBSEQUENT RAISING THE ORDER       INGR0240
C                                STARTING WITH THE 4TH ORDER,               INGR0250
C                                INTEGRATION STOPS AS SOON AS TWO SUBSE-    INGR0260
C                                QUENT INTERVALS  DO NOT CONTRIBUTE         INGR0270
C                                SIGNIFICANTLY,                             INGR0280
C                                INTEGRATION STOPS PREMATURELY IF*          INGR0290
C                                -IN THE FIRST INTERVAL MORE THAN 30 SUB-   INGR0300
C                                 DIVISIONS ARE NEEDED,                     INGR0310
C                                -IN THE FOLLOWING INTERVALS EVEN THE 15TH INGR0320
C                                 ORDER IS NOT ACCURATE ENOUGH,             INGR0330
C                                -EVEN THE 149-TH(298 TH)INTERVAL DOES      INGR0340
C                                 GIVE A NON-NEGIGIBLE CONTRIBUTION,        INGR0350
C-----------------------------------------------------------------------INGR0360
      INTEGER ALFA,ORDER,INTV(10),INTV2(10),INTV3(10),KK(10),BETA          INGR0370
      REAL MIDPNT,LOWER,LOAD,NU,ACCUR(3),K5,COMP(10),FIRST(10),            INGR0380
     1SECOND(10),INT(17),RES(10)                                           INGR0390
      COMMON/ASUT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,MOSTRS,NZEROS,H(9)INGR0400
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                              INGR0410
      COMMON/GEDATA/BZEROS(149,2),ZEROS(298)                               INGR0420
      COMMON/CNTING/F10M1,F100,F101,F11M2,F11M1,F110,F111                  INGR0430
      COMMON/TAPE/NOUT                                                     INGR0440
      NTELL = 0                                                            INGR0450
      NINT = 7                                                             INGR0460
      IF(IL.EQ.2) NINT = 10                                                INGR0470
      DO 1000 I = 1,NINT                                                   INGR0480
         KK(I) = 0                                                         INGR0490
 1000 CONTINUE                                                             INGR0500
      IF(LAYER.NE.1) GO TO 2000                                            INGR0510
C-----------------------------------------------------------------------INGR0520
C                                CALCULATION OF THE ASYMPTOTIC PART OF THE INGR0530
C                                INTEGRALS,FOR POINTS IN THE TOPLAYER ONLY.INGR0540
C-----------------------------------------------------------------------INGR0550
      DO 1190 I = 1,NINT                                                   INGR0560
         K = INTV(I)                                                       INGR0570
         IF(K.EQ.0) GO TO 1190                                             INGR0580
         GO TO (1010,1020,1030,1040,1050,1060,1070,1080,1090,1100,1110,    INGR0590
     1   1120,1130,1140,1150,1160,1170),K                                  INGR0600
 1010    INT(K) = F100+Z*F101                                              INGR0610
         GO TO 1190                                                        INGR0620
 1020    INT(K) = F100                                                     INGR0630
         GO TO 1190                                                        INGR0640
 1030    INT(K) = -2.0*(1.0-NU(1))*F10M1-Z*F100                            INGR0650
         GO TO 1190                                                        INGR0660
 1040    INT(K) = (1.0-2.0*NU(1))*F11M1-Z*F110                             INGR0670
         GO TO 1180                                                        INGR0680
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      IF(INTV3(J).EQ.0) GO TO 2090                                      INGR1300
      IF(ABS(COMP(J)).LT.ACCUR(2)) GO TO 2080                           INGR1310
      IF(ABS((COMP(J)-FIRST(J)-SECOND(J))/COMP(J)).LT.ACCUR(3))         INGR1320
     1 GO TO 2080                                                       INGR1330
      GO TO 2090                                                        INGR1340
 2080 INTT3 = INTT3-1                                                   INGR1350
      IF(LOWER.GT.ACCUR(1)) GO TO 2090                                  INGR1360
      INTT2 = INTT2-1                                                   INGR1370
      INTV3(J) = 0                                                      INGR1380
 2090 CONTINUE                                                          INGR1390
      IF(INTT3.EQ.0) GO TO 2110                                         INGR1400
      ALFA = 0                                                          INGR1410
      LOWER = MIDPNT                                                    INGR1420
      DELTA = 0.5*DELTA                                                 INGR1430
      BETA = BETA+1                                                     INGR1440
      IF(BETA.GT.30) GO TO 2150                                         INGR1450
C-----------------------------------------------------------------------INGR1460
C                    ARRIVAL HERE MEANS THAT THE INTEGRAND IS TOO       INGR1470
C                    IRREGULAR TO GET INTEGRATED OVER THE REGION FROM   INGR1480
C                    THE ORIGIN TO THE FIRST BESSEL ZERO.               INGR1490
C-----------------------------------------------------------------------INGR1500
      IRES = 1                                                          INGR1510
      DO 2100 J = 1,NINT                                                INGR1520
         COMP(J) = SECOND(J)                                            INGR1530
         RES(J) = FIRST(J)                                              INGR1540
 2100 CONTINUE                                                          INGR1550
      INTT3 = INTT2                                                     INGR1560
      GO TO 2070                                                        INGR1570
 2110 DO 2120 J = 1,NINT                                                INGR1580
         K = INTV2(J)                                                   INGR1590
         IF(K.EQ.0) GO TO 2120                                          INGR1600
         INT(K) = INT(K)+FIRST(J)+SECOND(J)                             INGR1610
         IF(INTV3(J).NE.0) GO TO 2120                                   INGR1620
         INTV2(J) = 0                                                   INGR1630
 2120 CONTINUE                                                          INGR1640
      UPPER = LOWER                                                     INGR1650
      INTT3 = INTT2                                                     INGR1660
      IF(ALFA) 3000,2140,2130                                           INGR1670
 2130 DELTA = DELTA*2.0                                                 INGR1680
      BETA = BETA-1                                                     INGR1690
 2140 ALFA = ALFA+1                                                     INGR1700
      GO TO 2020                                                        INGR1710
 2150 WRITE(NOUT,9040)                                                  INGR1720
      GO TO 3180                                                        INGR1730
C-----------------------------------------------------------------------INGR1740
C                              INTEGRATION OVER THE REMAINING INTERVALS, INGR1750
C-----------------------------------------------------------------------INGR1760
 3000 IFIN = NZEROS-1                                                   INGR1770
      DO 3010 J = 1,NINT                                                INGR1780
         INTV2(J) = INTV(J)                                             INGR1790
 3010 CONTINUE                                                          INGR1800
      INTT2 = INTT                                                      INGR1810
      DO 3130 IBESS = 1,IFIN                                            INGR1820
         DO 3020 J = 1,NINT                                             INGR1830
            INTV3(J) = INTV2(J)                                         INGR1840
            FIRST(J) = 0.0                                              INGR1850
 3020    CONTINUE                                                       INGR1860
         INTT3 = INTT2                                                  INGR1870
         DO 3070 ORDER = 4,15                                           INGR1880
            CALL QUAD(IL,INTV3,ZEROS(IBESS),ZEROS(IBESS+1),ORDER,SECOND,INGR1890
     1      NTELL)                                                      INGR1900
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 1050     INT(K) = Z*F111                                                INGR0690
          GO TO 1190                                                     INGR0700
 1060     INT(K) = F100-Z*F101                                           INGR0710
          GO TO 1190                                                     INGR0720
 1070     INT(K) = -2.0*(1.0-NU(1))*F10M1+Z*F100                         INGR0730
          GO TO 1190                                                     INGR0740
 1080     INT(K) = -2.0*(1.0-NU(1))*F110+Z*F111                          INGR0750
          GO TO 1190                                                     INGR0760
 1090     INT(K) = -F110                                                 INGR0770
          GO TO 1190                                                     INGR0780
 1100     INT(K) = F11M1-Z*F110                                          INGR0790
          GO TO 1180                                                     INGR0800
 1110     INT(K) = F11M1                                                 INGR0810
          GO TO 1180                                                     INGR0820
 1120     INT(K) = -2.0*(1.0-NU(1))*F11M2+Z*F11M1                        INGR0830
          GO TO 1180                                                     INGR0840
 1130     INT(K) = -F100                                                 INGR0850
          GO TO 1190                                                     INGR0860
 1140     INT(K) = F10M1                                                 INGR0870
          GO TO 1190                                                     INGR0880
 1150     INT(K) = F110                                                  INGR0890
          GO TO 1190                                                     INGR0900
 1160     INT(K) = -F11M1                                                INGR0910
          GO TO 1180                                                     INGR0920
 1170     INT(K) = F11M2                                                 INGR0930
 1180     IF(R.GE.ACCUR(1)) INT(K) = INT(K)/R                            INGR0940
 1190 CONTINUE                                                           INGR0950
      IF(NLAYS.EQ.1) GO TO 3140                                          INGR0960
C-----------------------------------------------------------------------INGR0970
C                                INTEGRATION FROM THE ORIGIN TO THE FIRST  INGR0980
C                                BESSELZERO.                             INGR0990
C-----------------------------------------------------------------------INGR1000
 2000 INTT2 = INTT                                                       INGR1010
      INTT3 = INTT                                                       INGR1020
      DO 2010 J = 1,NINT                                                 INGR1030
         INTV2(J) = INTV(J)                                              INGR1040
         INTV3(J) = INTV(J)                                              INGR1050
 2010 CONTINUE                                                           INGR1060
      UPPER = ZEROS(1)                                                   INGR1070
      BETA = 0                                                           INGR1090
      ALFA = 0                                                           INGR1080
      IRES = 0                                                           INGR1100
      DELTA = 0.5*ZEROS(1)                                               INGR1110
 2020 LOWER = UPPER-DELTA                                                INGR1120
      IF(LOWER-ACCUR(1)) 2030,2030,2040                                  INGR1130
 2030 ALFA = -1                                                          INGR1140
      LOWER = 0.0                                                        INGR1150
 2040 IF(IRES.EQ.1) GO TO 2050                                           INGR1160
      CALL QUAD(IL,INTV3,LOWER,UPPER,16,COMP,NTELL)                      INGR1170
      IF(NTELL.NE.0) GO TO 3180                                          INGR1180
      GO TO 2070                                                         INGR1190
 2050 DO 2060 J = 1,NINT                                                 INGR1200
         COMP(J) = RES(J)                                                INGR1210
 2060 CONTINUE                                                           INGR1220
      IRES = 0                                                           INGR1230
 2070 MIDPNT = 0.5*(LOWER+UPPER)                                         INGR1240
      CALL QUAD(IL,INTV3,LOWER,MIDPNT,16,FIRST,NTELL)                    INGR1250
      IF(NTELL.NE.0) GO TO 3180                                          INGR1260
      CALL QUAD(IL,INTV3,MIDPNT,UPPER,16,SECOND,NTELL)                   INGR1270
      IF(NTELL.NE.0) GO TO 3180                                          INGR1280
      DO 2090 J = 1,NINT                                                 INGR1290
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            IF(NTELL.NE.0) GO TO 3180                                   INGR1910
            DO 3060 J = 1,NINT                                          INGR1920
               K = INTV3(J)                                             INGR1930
               IF(K.EQ.0) GO TO 3060                                    INGR1940
               IF(ABS(INT(K)).LT.0.01) GO TO 3030                       INGR1950
               IF(ABS((FIRST(J)-SECOND(J))/INT(K)).LT.0.1*ACCUR(3))     INGR1960
     1         GO TO 3040                                               INGR1970
               GO TO 3050                                               INGR1980
 3030          IF(ABS(FIRST(J)-SECOND(J)).GE.0.1*ACCUR(2)) GO TO 3050   INGR1990
 3040          INTV3(J) = 0                                             INGR2000
               INTT3 = INTT3-1                                          INGR2010
               GO TO 3060                                               INGR2020
 3050          FIRST(J) = SECOND(J)                                     INGR2030
 3060       CONTINUE                                                    INGR2040
            IF(INTT3.EQ.0) GO TO 3080                                   INGR2050
 3070    CONTINUE                                                       INGR2060
         WRITE(NOUT,9020)                                               INGR2070
         WRITE(NOUT,9050) ZEROS(IBESS)                                  INGR2080
C-----------------------------------------------------------------------INGR2090
C                    ARRIVAL HERE MEANS THAT THE DESIRED ACCURACY CANNOT INGR2100
C                    BE MET BY MEANS OF THE AVAILABLE GAUSS-JACOBI      INGR2110
C                    POLYNOMINALS.                                      INGR2120
C-----------------------------------------------------------------------INGR2130
         GO TO 3180                                                     INGR2140
 3080    DO 3120 J = 1,NINT                                             INGR2150
            K = INTV2(J)                                                INGR2160
            IF(K.EQ.0) GO TO 3120                                       INGR2170
            INT(K) = INT(K)+SECOND(J)                                   INGR2180
            IF(ABS(INT(K)).LT.0.01) GO TO 3090                          INGR2190
            IF(ABS(SECOND(J)/INT(K)).LT.0.1*ACCUR(3)) GO TO 3110        INGR2200
            GO TO 3100                                                  INGR2210
 3090       IF(ABS(SECOND(J)).LT.0.1*ACCUR(2)) GO TO 3110               INGR2220
 3100       KK(J) = 0                                                   INGR2230
            GO TO 3120                                                  INGR2240
 3110       KK(J) = KK(J)+1                                             INGR2250
            IF(KK(J).LT.2) GO TO 3120                                   INGR2260
            INTV2(J) = 0                                                INGR2270
            INTT2 = INTT2-1                                             INGR2280
 3120    CONTINUE                                                       INGR2290
         IF(INTT2.EQ.0) GO TO 3140                                      INGR2300
 3130 CONTINUE                                                          INGR2310
      WRITE(NOUT,9030)                                                  INGR2320
      WRITE(NOUT,9050) ZEROS(IFIN)                                      INGR2330
C-----------------------------------------------------------------------INGR2340
C                    ARRIVAL HERE MEANS THAT ALL AVAILABLE BESSEL ZEROS INGR2350
C                    HAVE BEEN EXHAUSTED BECAUSE OF ILL CONVERGENCE OF  INGR2360
C                    THE INTEGRALS.                                     INGR2370
C-----------------------------------------------------------------------INGR2380
      GO TO 3180                                                        INGR2390
 3140 DO 3170 J = 1,NINT                                                INGR2400
         K = INTV(J)                                                    INGR2410
         IF(K.EQ.0) GO TO 3170                                          INGR2420
         IF(K-5) 3150,3150,3160                                         INGR2430
 3150    INT(K) = INT(K)*LOAD                                           INGR2440
         GO TO 3170                                                     INGR2450
 3160    INT(K) = INT(K)*MOSTRS                                         INGR2460
 3170 CONTINUE                                                          INGR2470
      RETURN                                                            INGR2480
 3180 WRITE(NOUT,9010) (INTV3(J),J=1,NINT)                              INGR2490
      GO TO 3140                                                        INGR2500
 9010 FORMAT(" DURING CALCULATION OF INTEGRALS",10I3)                   INGR2510
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 9020 FORMAT(" SUSPEND PROGRAM GAUSS POLYS EXHAUSTED")                  INGR2520
 9030 FORMAT(" SUSPEND PROGRAM BESSEL ZEROS EXHAUSTED").                INGR2530
 9040 FORMAT(" SUSPEND PROGRAM STEPSIZE FIRST INTERVAL TOO SMALL")      INGR2540
 9050 FORMAT(" AT THE VALUE ",E11.4," FOR THE INTEGRATION VARIABLE")    INGR2550
      END                                                               INGR2560
      SUBROUTINE QUAD (IL,INTV,ALO,UP,NGAUSS,FSC,NTELL)                 QUAD0010
C- ------------------------------------  ------------ ---------------QUAD0020
C                                THIS SUBROUTINE CALCULATES FOR THE SET   QUAD0030
C                                INTV THE INTEGRALS OF THE CORRESPONDING  QUAD0040
C                                FUNCTIONS IGRAND BETWEEN THE LIMITS ALO  QUAD0050
C                                AND UP BY USING A GAUSS QUADRATURE OF    QUAD0060
C                                ORDER NGAUSS.                            QUAD0070
C                                -FOR NGAUSS=16 A LEGENDRE-GAUSS QUADRATU- QUAD0080
C                                 RE OF ORDER  .                          QUAD0090
C                                -FOR NGAUSS.LT.16 A JACOBI-GAUSSQUADRATU- QUAD0100
C                                 RE.                                     QUAD0110
C                                THE ABSCISSAE AND WEIGHTS OF BOTH ARE    QUAD0120
C                                STORED AS AGAUSS AND WGAUSS IN THE BLOCK QUAD0130
C                                DATA.                                    QUAD0140
C                                THE SET OF INTEGRANDS IS COMPUTED DURING QUAD0150
C                                SUBSEQUENT CALLING IN OF"                QUAD0160
C                                          SUBROUTINES  MATRIX            QUAD0170
C                                                       FPIGRA            QUAD0180
C                                         AND FUNCTION  IGRAND            QUAD0190
C                                THE SET OF RESULTING INTEGRALS IS        QUAD0200
C                                DELIVERED IN FSC                         QUAD0210
C-----------------------------------------------------------------------QUAD0220
      INTEGER INTV(10)                                                  QUAD0230
      REAL IGRAND,FSC(10)                                               QUAD0240
      COMMON/GAUSS/AGAUSS(16,16),WGAUSS(16,16)                          QUAD0250
      NINT = 7                                                          QUAD0260
      IF(IL.EQ.2) NINT = 10                                             QUAD0270
      DO 10 J = 1,NINT                                                  QUAD0280
         K = INTV(J)                                                    QUAD0290
         IF(K.EQ.0) GO TO 10                                            QUAD0300
         FSC(J) = 0.0                                                   QUAD0310
   10 CONTINUE                                                          QUAD0320
      LABEL = 0                                                         QUAD0330
      IF(IL.EQ.2) GO TO 20                                              QUAD0340
      IF((INTV(1)+INTV(2)+INTV(3)).GT.0) LABEL = LABEL+1                QUAD0350
      IF((INTV(4)+INTV(5)).GT.0) LABEL = LABEL+2                        QUAD0360
      IF((INTV(6)+INTV(7)).GT.0) LABEL = LABEL+4                        QUAD0370
      GO TO 30                                                          QUAD0380
   20 IF((INTV(1)+INTV(2)).GT.0) LABEL = LABEL+1                        QUAD0390
      IF((INTV(3)+INTV(4)+INTV(5)+INTV(6)+INTV(7)).GT.0) LABEL = LABEL+2QUAD0400
      IF((INTV(8)+INTV(9)+INTV(10)).GT.0) LABEL = LABEL+4               QUAD0410
   30 F1 = 0.5*(UP-ALO)                                                 QUAD0420
      F2 = 0.5*(UP+ALO)                                                 QUAD0430
      IGAUSS = NGAUSS                                                   QUAD0440
      IF(NGAUSS.EQ.16) IGAUSS=8                                         QUAD0450
      DO 50 I = 1,IGAUSS                                                QUAD0460
         X = F1*AGAUSS(I,NGAUSS)+F2                                     QUAD0470
         CALL MATRIX (X,LABEL,NTELL)                                    QUAD0480
         IF(NTELL.EQ.1) RETURN                                          QUAD0490
         CALL FPIGRA (IL,X)                                             QUAD0500
         DO 40 J = 1,NINT                                               QUAD0510
            K = INTV(J)                                                 QUAD0520
            IF(K.EQ.0) GO TO 40                                         QUAD0530
            FSC(J) = FSC(J)+WGAUSS(I,NGAUSS)*IGRAND(X,K)                QUAD0540
   40    CONTINUE                                                       QUAD0550
   50 CONTINUE                                                          QUAD0560
```

```
      DO 60 J = 1,NINT                                                  QUAD0570
         K = INTV(J)                                                    QUAD0580
         IF(K.EQ.0) GO TO 60                                            QUAD0590
         FSC(J) = FSC(J)*F1                                             QUAD0600
   60 CONTINUE                                                          QUAD0610
   70 RETURN                                                            QUAD0620
      END                                                               QUAD0630
      SUBROUTINE MATRIX (X,LABL,NTELL)                                  MATR0010
C-----------------------------------------------------------------------MATR0020
C                                THIS SUBROUTINE COMPUTES THE SET OF CHA-  MATR0030
C                                RACTERISTIC-FUNCTIONS T0,V0,S0,U0,T1,V1,  MATR0040
C                                S1,U1,TQ1 AND SQ1 FOR THE VALUE X OF THE  MATR0050
C                                INTEGRATION-PARAMETER. USE IS MADE OF     MATR0060
C                                CONSTANTS CALCULATED IN MACON1 AND MA2CON,MATR0070
C                                THEY WERE STORED IN COMMON/INDATA/.       MATR0080
C                                CHARACTERISTIC-FUNCTION VALUES ARE DELIVE-MATR0090
C                                RED IN COMMON/IGRAN/.                     MATR0100
C                                LABL DETERMINES WHICH CHARACTERISTIC-     MATR0110
C                                FUNCTIONS ARE NEEDED"                     MATR0120
C                                -LABL=1"T0,V0,S0,U0                       MATR0130
C                                -LABL=2"T1,V1,S1,U1                       MATR0140
C                                -LABL=3"T0,V0,S0,U0,T1,V1,S1,U1           MATR0150
C                                -LABL=4"TQ1,SQ1                           MATR0160
C                                -LABL=5"T0,V0,S0,U0,TQ1,SQ1               MATR0170
C                                -LABL=6"T1,V1,S1,U1,TQ1,SQ1               MATR0180
C                                -LABL=7"T0,V0,S0,U0,T1,V1,S1,U1,TQ1,SQ1   MATR0190
C                                SUBROUTINE IS INTERRUPTED AND RETURNED    MATR0200
C                                WITH NTELL=1 WHEN SOLUTION BECOMES TOO    MATR0210
C                                INACCURATE BECAUSE OF ILL MATRIX-CONDI-   MATR0220
C                                ON DURING INVERSION.                      MATR0230
C-----------------------------------------------------------------------MATR0240
      REAL LOAD,NU,W(4,4,9),P(4,2),PP(2,2),K1,K2,K5,K6,II,NJ(2,2,9),KK6,MATR0250
     1ACCUR(3),NP(2,10),NJ2(9),P3(2),NP2(10),K4(10)                     MATR0260
      COMMON/ASDT/LAYER,NLAYS,M,F,Z,NU(10),ACCUR,LOAD,HOSTRS,NZEROS,H(9)MATR0270
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                           MATR0280
      COMMON/INDATA/XMAX,  A1(9),H1(9),C1(9),D(9),EE(9),F(9),G(9),H1(9),MATR0290
     1II(9),K1(9),K2(9),K6(10),BE(9),BU(9),BUU(9),BMU(9),B2U(9),B2UU(9),MATR0300
     2J2(9),J1,T2(10),SS(2,10),G012(9),G021(9),G022(9),G122(9),         MATR0310
     3H012(9),H022(9),H122(9),D012(9),D022(9),C011(9),C012(9),E012(9),  MATR0320
     4F012(9),F112(9),F022(9),CC(4,2,9),DD(2,2,9),FF(2,2,9),GG(2,2,10), MATR0330
     5HH(2,2,10),RR(4,2,10),DD2(9),G20(9),G21(9),H20(9),H021(9),GG2(10),MATR0340
     6HH2(10),Q011(9),Q111(9),Q012(9),Q112(9),Q212(9),Q022(9),Q122(9),  MATR0350
     7QF0(9),QF1(9),Z011,Z111,Z211,Z012,Z112,Z212,Z312,Z021,Z121,Z022,  MATR0360
     8Z122,Z222,K4                                                      MATR0370
      COMMON/IGRAN/T0,V0,S0,U0,T1,V1,S1,U1,TQ1,SQ1,FPIGR,EX1,EX2        MATR0380
      COMMON/TAPE/NOUT                                                  MATR0390
      LABEL = LABL                                                      MATR0400
      IF(LABEL.LT.4) GO TO 1000                                         MATR0410
      IF(X.LT.XMAX) GO TO 100                                           MATR0420
C-----------------------------------------------------------------------MATR0430
C                                ASYMPTOTIC EVALUATION OF TQ1 AND SQ1      MATR0440
C                                FOR X.GE.XMAX.                            MATR0450
C-----------------------------------------------------------------------MATR0460
      TQ1 = 1.0                                                         MATR0470
      IF(LAYER.EQ.1) GO TO 30                                           MATR0480
      J = LAYER-1                                                       MATR0490
      DO 20 K = 1,J                                                     MATR0500
         TQ1=TQ1*2.0*(1.0-AL(K))/((1.0-AL(K))*(1.0+K1(K))+0.5*AL(K)*X)  MATR0510
   20 CONTINUE                                                          MATR0520
   30 SQ1=TQ1*(0.5*AL(LAYER)*X-(1.0-AL(LAYER))*K2(LAYER))/((1.0-AL(LAYERMATR0530
     1    )) *(1.0+K1(LAYER))+0.5*AL(LAYER)*X)                          MATR0540
```

```
      LABEL = LABEL-4                                                   MATR0550
      GO TO 1000                                                        MATR0560
C-----------------------------------------------------------------------MATR0570
C                              CALCULATION OF TNI AND SNI FOR X.LT.XMAX  MATR0580
C-----------------------------------------------------------------------MATR0590
  100 IF(J1.EQ.0) GO TO 120                                             MATR0600
      DO 110 J = 1,J1                                                   MATR0610
         GG2(J+1) = G20(J)-G21(J)*X                                     MATR0620
         HH2(J+1) = H20(J)+G21(J)*X                                     MATR0630
  110 CONTINUE                                                          MATR0640
  120 DO 150 K = 1,M                                                    MATR0650
         IF(J1.EQ.0) GO TO 140                                          MATR0660
         DO 130 I = 1,J1                                                MATR0670
            IF(J2(I).EQ.K) GO TO 150                                    MATR0680
  130    CONTINUE                                                       MATR0690
  140    W1 = 0.5*(1.0+K1(K))                                           MATR0700
         W2 =-0.5*K2(K)                                                 MATR0710
         W3 = BE(K)*0.25*X                                              MATR0720
         NJ(1,1,K) = W1+W3                                              MATR0730
         NJ(1,2,K) = W2-W3                                              MATR0740
         NJ(2,1,K) = W2+W3                                              MATR0750
         NJ(2,2,K) = W1-W3                                              MATR0760
  150 CONTINUE                                                          MATR0770
      J5 = J1+1                                                         MATR0780
      DO 300 MM = 1,J5                                                  MATR0790
         N = J5+1-MM                                                    MATR0800
         IF(N-1) 160,160,170                                            MATR0810
  160    J3 = 1                                                         MATR0820
         GO TO 180                                                      MATR0830
  170    J3 = J2(N-1)+1                                                 MATR0840
  180    IF(J5-N) 190,190,200                                           MATR0850
  190    J4 = M                                                         MATR0860
         GO TO 210                                                      MATR0870
  200    J4 = J2(N)-1                                                   MATR0880
  210    IF(J3.GT.J4) GO TO 240                                         MATR0890
         DO 230 IJ = J3,J4                                              MATR0900
         IK = J4+J3-IJ                                                  MATR0910
         IL = IK+1                                                      MATR0920
      EXPO=-X*T2(IL)                                                    MATR0930
      IF(EXPO.LT.-70.0)GO TO 212                                        MATR0931
      EXP1=EXP(EXPO)*SS(1,IL)                                           MATR0932
      GO TO 214                                                         MATR0933
 212  EXP1=0.0                                                          MATR0934
 214  DO 220 I=1,2                                                      MATR0940
 220         SS(I,IK) = NJ(I,1,IK)*EXP1+NJ(I,2,IK)*SS(2,IL)             MATR0950
 230    CONTINUE                                                        MATR0960
 240    NN = N-1                                                        MATR0970
      EXPO=-X*T2(J3)                                                    MATR0971
      IF(EXPO.LT.-70.0)GO TO 242                                        MATR0972
      EXP2=EXP(EXPO)                                                    MATR0973
      GO TO 244                                                         MATR0974
 242  EXP2=0.0                                                          MATR0975
 244  PROD=GG2(N)*SS(1,J3)*EXP2                                         MATR0980
         P2 = PROD+HH2(N)*SS(2,J3)                                      MATR0990
C-----------------------------------------------------------------------MATR1000
C                              TEST MATRIX-CONDITION BEFORE INVERSION,  MATR1010
C-----------------------------------------------------------------------MATR1020
         IF(ABS(P2).LT.1.0E-7*ABS(PROD)/ACCUM(3)) GO TO 2000            MATR1030
         PP2 = 1.0/P2                                                   MATR1040
         IF(N.EQ.1) GO TO 310                                           MATR1050
         NJ2(NN) = PP2*DD2(NN)                                          MATR1060
```

```
      DO 270 I = 1,2                                                    MATR1070
  270    P3(I) = SS(I,J3)*NJ2(NN)                                       MATR1080
      PP2=P3(1)*EXP2-P3(2)                                              MATR1090
      SS(1,J3-1) = PP2+1.0                                              MATR1100
      SS(2,J3-1) = 1.0                                                  MATR1110
  300 CONTINUE                                                          MATR1120
  310 IF(NTELL.EQ.2) RETURN                                             MATR1130
      NP2(1) = -PP2                                                     MATR1131
      IF(J1.EQ.0) GO TO 390                                             MATR1140
      DO 350 I = 1,J1                                                   MATR1150
         J = J1+1-I                                                     MATR1160
         IF(LAYER.GT.J2(J)) GO TO 360                                   MATR1170
  350 CONTINUE                                                          MATR1180
      J5 = 1                                                            MATR1190
      GO TO 390                                                         MATR1200
  360 DO 380 I = 1,J                                                    MATR1210
         NP2(I+1) = NJ2(I)*NP2(1)                                       MATR1220
  380 CONTINUE                                                          MATR1230
      J5 = J+1                                                          MATR1240
  390 J = LAYER                                                         MATR1250
      SQ1 = SS(1,J)*NP2(J5)                                             MATR1260
      TQ1 = SS(2,J)*NP2(J5)                                             MATR1270
      LABEL = LABEL-4                                                   MATR1280
C-----------------------------------------------------------------------MATR1290
C                                ASYMPTOTIC EVALUATION OF T0,V0,S0,U0,T1, MATR1300
C                                V1,S1 AND U1 FOR X.GE.XMAX.            MATR1310
C-----------------------------------------------------------------------MATR1320
 1000 IF(LABEL.EQ.0) RETURN                                             MATR1330
      IF(X.LT.XMAX) GO TO 1100                                          MATR1340
      L = LAYER                                                         MATR1350
      X2 = X*X                                                          MATR1360
      X3 = X2*X                                                         MATR1370
      IF(L.EQ.NLAYS) GO TO 1010                                         MATR1380
      Z11 = Z011+X*Z111+X2*Z211                                         MATR1390
      Z12 = Z012+X*Z112+X2*Z212+X3*Z312                                 MATR1400
      Z21 = Z021+X*Z121                                                 MATR1410
      Z22 = Z022+X*Z122+X2*Z222                                         MATR1420
 1010 IF(LABEL.GT.1) GO TO 1030                                         MATR1430
 1020 IF(LABEL.EQ.0) RETURN                                             MATR1440
      NP(1,1) = 2.0*NU(1)                                               MATR1450
      NP(2,1) = 1.0                                                     MATR1460
      GO TO 1040                                                        MATR1470
 1030 NP(1,1) = 1.0-2.0*NU(1)                                           MATR1480
      NP(2,1) = -1.0                                                    MATR1490
 1040 PQF = 1.0                                                         MATR1500
      IF(L.EQ.1) GO TO 1060                                             MATR1510
      DO 1050 K = 2,L                                                   MATR1520
         J = K-1                                                        MATR1530
         PQF = PQF*K6(J)/(QF0(J)+QF1(J)*X)                              MATR1540
         W1 = -AL(J)*X                                                  MATR1550
         W9 = H(J)*X                                                    MATR1560
         NP(1,K) = NP(1,J)*(Q011(J)+Q111(J)*X+W1*W9)+NP(2,J)*(Q012(J)   MATR1570
     1   +Q112(J)*X+Q212(J)*X2+W1*W9*W9)                                 MATR1580
 1050    NP(2,K) = -W1*NP(1,J)+NP(2,J)*(Q022(J)+Q122(J)*X-W1*W9)        MATR1590
      IF(L.NE.NLAYS) GO TO 1060                                         MATR1600
      S = 0.0                                                           MATR1610
      U = 0.0                                                           MATR1620
      GO TO 1070                                                        MATR1630
 1060 S = (NP(1,L)*Z11+NP(2,L)*Z12)*PQF/(QF0(L)+QF1(L)*X)               MATR1640
      U = (NP(1,L)*Z21+NP(2,L)*Z22)*PQF/(QF0(L)+QF1(L)*X)               MATR1650
 1070 T = NP(1,L)*PQF                                                   MATR1660
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      V = NP(2,L)*PUF                                                   MATR1670
      IF(LABEL.GT.1) GO TO 1080                                         MATR1680
      S0 = S                                                            MATR1690
      U0 = U                                                            MATR1700
      T0 = T                                                            MATR1710
      V0 = V                                                            MATR1720
      RETURN                                                            MATR1730
 1080 S1 = S                                                            MATR1740
      U1 = U                                                            MATR1750
      T1 = T                                                            MATR1760
      V1 = V                                                            MATR1770
      LABEL = LABEL-2                                                   MATR1780
      GO TO 1020                                                        MATR1790
C-----------------------------------------------------------------------MATR1800
C                                CALCULATION OF T0,V0,S0,U0,T1,V1,S1 AND  MATR1810
C                                U1 FOR X.LT.XMAX.                      MATR1820
C-----------------------------------------------------------------------MATR1830
 1100 IF(J1.EQ.0) GO TO 1120                                            MATR1840
      DO 1110 J = 1,J1                                                  MATR1850
         K = J2(J)                                                      MATR1860
         W1 =-AL(K)*X                                                   MATR1870
         W9 = M(K)*X                                                    MATR1880
         CC(1,1,J) = C011(J)+2.0*W9                                     MATR1890
         CC(1,2,J) = C012(J)+2.0*W9*W9                                  MATR1900
         CC(2,2,J) = C011(J)-2.0*W9                                     MATR1910
         DD(1,2,J) = D012(J)+DD(1,1,J)*W9                               MATR1920
         DD(2,2,J) = D022(J)+DD(2,1,J)*W9                               MATR1930
         FF(1,1,J) = -C011(J)-2.0*W9                                    MATR1940
         FF(1,2,J) = F012(J)+F112(J)*W9-2.0*W9*W9                       MATR1950
         FF(2,2,J) = F022(J)+2.0*W9                                     MATR1960
         GG(1,2,J+1) = G012(J)+GG(1,1,J+1)*W9                           MATR1970
         GG(2,1,J+1) = G021(J)+W1                                       MATR1980
         GG(2,2,J+1) = G022(J)+(G021(J)*M(K)+G122(J))*X+W1*W9           MATR1990
         HH(1,2,J+1) = H012(J)+HH(1,1,J+1)*W9                           MATR2000
         HH(2,1,J+1) = H021(J)+W1                                       MATR2010
         HH(2,2,J+1) = H022(J)+H021(J)*W9+H122(J)*X+W1*W9               MATR2020
 1110 CONTINUE                                                          MATR2030
 1120 DO 1150 K=1,M                                                     MATR2040
         IF(J1.EQ.0) GO TO 1140                                         MATR2050
         DO 1130 I = 1,J1                                               MATR2060
            IF(J2(I).EQ.K) GO TO 1150                                   MATR2070
 1130    CONTINUE                                                       MATR2080
 1140    W1 = BMU(K)*X                                                  MATR2090
         W9 = M(K)*X                                                    MATR2100
         W10 = W9*X                                                     MATR2110
         W2 = W10*BE(K)                                                 MATR2120
         W11 = W2*W9                                                    MATR2130
         W3 = W9*C1(K)                                                  MATR2140
         W4 = BE(K)*X                                                   MATR2150
         W5 = BU(K)*X                                                   MATR2160
         W8 = BUU(K)*X                                                  MATR2170
         W7 = C1(K)*W9*W9                                               MATR2180
         W(1,1,K) = A1(K)+W1-W2                                         MATR2190
         W(1,2,K) = -EE(K)+F(K)*W9+W8+B2U(K)*W10-W11                    MATR2200
         W(1,3,K) = D(K)-W3+W1-W2                                       MATR2210
         W(1,4,K) = -G(K)+H1(K)*W9-BUU(K)*X-W7+B2UU(K)*W10-W11          MATR2220
         W(2,1,K) = W4                                                  MATR2230
         W(2,2,K) = B1(K)+W5+W2                                         MATR2240
         W(2,3,K) = C1(K)+W4                                            MATR2250
         W(2,4,K) = I1(K)+W3-W5+W2                                      MATR2260
         W(3,1,K) = D(K)+W3-W1-W2                                       MATR2270
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      W(3,2,K) = G(K)+H1(K)*W9-W8+W7-B2UU(K)*W10-W11                    MATR2280
      W(3,3,K) = A1(K)-W1-W2                                            MATR2290
      W(3,4,K) = EE(K)+F(K)*W9+W6-B2U(K)*W10-W11                        MATR2300
      W(4,1,K) = -C1(K)+W4                                              MATR2310
      W(4,2,K) = II(K)-W3+W5+W2                                         MATR2320
      W(4,3,K) = W4                                                     MATR2330
      W(4,4,K) = B1(K)-W5+W2                                            MATR2340
 1150 CONTINUE                                                          MATR2350
      J5 = J1+1                                                         MATR2360
      PKK6 = 1.0                                                        MATR2370
      DO 1300 MM = 1,J5                                                 MATR2380
         KK6 = 1.0                                                      MATR2390
         N = J5+1-MM                                                    MATR2400
         IF(N-1) 1160,1160,1170                                         MATR2410
 1160    J3 = 1                                                         MATR2420
         GO TO 1180                                                     MATR2430
 1170    J3 = J2(N-1)+1                                                 MATR2440
 1180    IF(J5-N) 1190,1190,1200                                        MATR2450
 1190    J4 = M                                                         MATR2460
         GO TO 1210                                                     MATR2470
 1200    J4 = J2(N)-1                                                   MATR2480
 1210    IF(J3.GT.J4) GO TO 1240                                        MATR2490
         DO 1230 IJ = J3,J4                                             MATR2500
            IK = J4+J3-IJ                                               MATR2510
            IL = IK+1                                                   MATR2520
            KK6 = KK6*W6(IK)                                            MATR2530
            IF(IK.EQ.LAYER) PKK6 = KK6                                  MATR2540
         EXPO=-X*T2(IL)                                                 MATR2541
         IF(EXPO.LT.-70.0)GO TO 1212                                    MATR2542
         EXP1=EXP(EXPO)                                                 MATR2543
         GO TO 1214                                                     MATR2544
 1212    EXP1=0.0                                                       MATR2545
 1214    DO 1220 I=1,4                                                  MATR2550
               DO 1220 K = 1,2                                          MATR2560
 1220             RR(I,K,IK)=(W(I,1,IK)*RR(1,K,IL)+W(I,2,IK)*RR(2,K,IL))MATR2570
     1       *EXP1+W(I,3,IK)*RR(3,K,IL)+W(I,4,IK)*RR(4,K,IL)            MATR2580
 1230    CONTINUE                                                       MATR2600
 1240    NN = N-1                                                       MATR2610
      EXPO=-X*T2(J3)                                                    MATR2611
      IF(EXPO.LT.-70.0)GO TO 1242                                       MATR2612
      EXP2=EXP(EXPO)                                                    MATR2613
      GO TO 1244                                                        MATR2614
1242  EXP2=0.0                                                          MATR2615
1244  DO 1250 I=1,2                                                     MATR2620
            DO 1250 K = 1,2                                             MATR2630
 1250          P(I,K) = (GG(I,1,N)*RR(1,K,J3)+GG(I,2,N)*RR(2,K,J3))    MATR2640
     1       *EXP2+HH(I,1,N)*RR(3,K,J3)+HH(I,2,N)*RR(4,K,J3)            MATR2650
         PROD = P(1,1)*P(2,2)                                           MATR2660
         DET = PROD-P(1,2)*P(2,1)                                       MATR2670
C-----------------------------------------------------------------------MATR2680
C                              TEST MATRIX,CONDITION BEFORE INVERSION.  MATR2690
C-----------------------------------------------------------------------MATR2700
         IF(ABS(DET).LT.1.0E-7*ABS(PROD)/ACCUR(3)) GO TO 2000           MATR2710
         QKK6 = KK6/DET                                                 MATR2720
         PP(1,1) = P(2,2)*QKK6                                          MATR2730
         PP(1,2) = -P(1,2)*QKK6                                         MATR2740
         PP(2,1) = -P(2,1)*QKK6                                         MATR2750
         PP(2,2) = P(1,1)*QKK6                                          MATR2760
         IF(N.EQ.1) GO TO 1310                                          MATR2770
         DO 1260 I = 1,2                                                MATR2780
            DO 1260 K = 1,2                                             MATR2790
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```
 1260          NJ(I,K,NN)=PP(I,1)*DD(1,K,NN)+PP(I,2)*DD(2,K,NN)          MATR2800
         DO 1270 I = 1,4                                                 MATR2810
            DO 1270 K = 1,2                                              MATR2820
 1270          P(I,K) =(RR(I,1,J3)*NJ(1,K,NN)+RR(I,2,J3)*NJ(2,K,NN))/KK6MATR2830
         DO 1280 I = 1,2                                                 MATR2840
         PP(1,I)=(P(1,I)+E012(NN)*P(2,I))*EXP2+FF(1,1,NN)                MATR2850
     1      *P(3,I)+FF(1,2,NN)*P(4,I)                                    MATR2860
 1280    PP(2,I)=P(2,I)*EXP2+FF(2,1,NN)*P(3,I)+FF(2,2,NN)*P(4,I)         MATR2870
         DO 1290 I = 1,2                                                 MATR2890
            DO 1290 K = 1,2                                              MATR2900
 1290          RR(I,K,J3-1) = CC(I,K,NN)+PP(I,K)                         MATR2910
         RR(3,1,J3-1) = 1.0                                              MATR2920
         RR(3,2,J3-1) = 0.0                                              MATR2930
         RR(4,1,J3-1) = 0.0                                              MATR2940
         RR(4,2,J3-1) = 1.0                                              MATR2950
 1300 CONTINUE                                                           MATR2960
 1310 IF(NTELL.EQ.2) GO TO 100                                           MATR2970
      IF(LABEL.GT.1) GO TO 1330                                          MATR2971
 1320 IF(LABEL.EQ.0) RETURN                                              MATR2980
      NP(1,1) = PP(1,1)                                                  MATR2990
      NP(2,1) = PP(2,1)                                                  MATR3000
      GO TO 1340                                                         MATR3010
 1330 NP(1,1) = PP(1,2)                                                  MATR3020
      NP(2,1) = PP(2,2)                                                  MATR3030
 1340 IF(J1.EQ.0) GO TO 1390                                             MATR3040
      DO 1350 I = 1,J1                                                   MATR3050
         J = J1+1-I                                                      MATR3060
         IF(LAYER.GT.J2(J)) GO TO 1360                                   MATR3070
 1350 CONTINUE                                                           MATR3080
      J5 = 1                                                             MAT-30 0
      GO TO 1390                                                         MATR3100
 1360 DO 1380 I = 1,J                                                    MATR3110
         IM = I+1                                                        MATR3120
         DO 1370 K = 1,2                                                 MATR3130
 1370       NP(K,IM) = NJ(K,1,I)*NP(1,I)+NJ(K,2,I)*NP(2,I)               MATR3140
 1380 CONTINUE                                                           MATR3150
      J5 = J+1                                                           MATR3160
 1390 J = LAYER                                                          MATR3170
      S =(RR(1,1,J)*NP(1,J5)+RR(1,2,J)*NP(2,J5))/PKK6                    MATR3180
      U =(RR(2,1,J)*NP(1,J5)+RR(2,2,J)*NP(2,J5))/PKK6                    MATR3190
      T =(RR(3,1,J)*NP(1,J5)+RR(3,2,J)*NP(2,J5))/PKK6                    MATR3200
      V =(RR(4,1,J)*NP(1,J5)+RR(4,2,J)*NP(2,J5))/PKK6                    MATR3210
      IF(LABEL.GT.1) GO TO 1400                                          MATR3220
      T0 = T                                                             MATR3230
      S0 = S                                                             MATR3240
      U0 = U                                                             MATR3250
      V0 = V                                                             MATR3260
      RETURN                                                             MATR3270
 1400 S1 = S                                                             MATR3280
      T1 = T                                                             MATR3290
      U1 = U                                                             MATR3300
      V1 = V                                                             MATR3310
      LABEL = LABEL-2                                                    MATR3320
      GO TO 1320                                                         MATR3330
C-----------------------------------------------------------------------MATR3340
C                                ARRIVAL HERE MEANS THAT SOLUTION OF THE  MATR3350
C                                CHARACTERISTIC FUNCTIONS HAS BEEN STOPPED MATR3360
C                                PREMATURELY BECAUSE OF ILL MATRIX CONDI- MATR3370
C                                TION MET DURING SOLUTION PROCESS.        MATR3380
C-----------------------------------------------------------------------MATR3390
 2000 WRITE(NOUT,9000)X
```

```
      NTELL = 1                                                         MATR3410
      RETURN                                                            MATR3420
 9000 FORMAT(" ILL=CONDITIONED DETERMINANT FOR X=",E15.7)               MATR3430
      END                                                               MATR3440
      SUBROUTINE FPIGRA (IL,X)                                          FPIG0010
C-----------------------------------------------------------------------FPIG0020
C                               THIS SUBROUTINE COMPUTES THE BESSELFUNC-  FPIG0030
C                               TION-PART OF THE INTEGRANDS FOR THE       FPIG0040
C                               INTEGRALS COMPUTED IN INGRAL,             FPIG0050
C                               FOR IL=1 THIS PART IS" J0(XR)*J1(X)       FPIG0060
C                               FOR IL=2 THIS PART IS" J1(XR)*J1(X)       FPIG0070
C                               COMPUTED RESULTS ARE DELIVERED AS FPIGR,  FPIG0080
C                               EXP1 AND EXP2 IN COMMON/IGRAN/            FPIG0090
C                               THE SUBROUTINE CALLS IN FUNCTION BESS,    FPIG0100
C-----------------------------------------------------------------------FPIG0110
      REAL LOAD,NU,ACCUR(3),K5                                          FPIG0120
      COMMON/ASDT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,HOSTRS,NZEROS,H(9)FPIG0130
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                           FPIG0140
      COMMON/IGRAN/T0,V0,S0,U0,T1,V1,S1,U1,TQ1,SQ1,FPIGR,EXP1,EXP2      FPIG0150
      IF(LAYER.NE.1) GO TO 20                                           FPIG0160
      T0 = T0-2.0*NU(1)                                                 FPIG0170
      V0 = V0-1.0                                                       FPIG0180
      T1 = T1-1.0+2.0*NU(1)                                             FPIG0190
      V1 = V1+1.0                                                       FPIG0200
      TQ1 = TQ1-1.0                                                     FPIG0210
   20 IF(H.LT.ACCUR(1)) GO TO 40                                        FPIG0220
      IF(IL.EQ.2) GO TO 30                                              FPIG0230
      FPIGR = BESS(0,X*R)*BESS(1,X)/X                                   FPIG0240
      GO TO 60                                                          FPIG0250
   30 FPIGR = BESS(1,X*R)*BESS(1,X)/(X*R)                               FPIG0260
      GO TO 60                                                          FPIG0270
   40 IF(IL.EQ.2) GO TO 50                                              FPIG0280
      FPIGR = BESS(1,X)/X                                               FPIG0290
      GO TO 60                                                          FPIG0300
   50 FPIGR = 0.5*BESS(1,X)                                             FPIG0310
   60 IF(NLAYS.EQ.LAYER) GO TO 70                                       FPIG0320
      IF(ABS(X*(2.0*H(LAYER)-Z)).GT.70.0)GO TO 70                       FPIG0330
      EXP1 = EXP(-X*(2.0*H(LAYER)-Z))                                   FPIG0340
      IF((X*Z).GT.70.0)GO TO 90                                         FPIG0350
      EXP2 = EXP(-X*Z)                                                  FPIG0360
      GO TO 100                                                         FPIG0370
 70   IF((X*Z).GT.70.0)GO TO 80                                         FPIG0380
      EXP1 = 0.0                                                        FPIG0390
      EXP2 = EXP(-X*Z)                                                  FPIG0400
      GO TO 100                                                         FPIG0410
   80 EXP1 = 0.0                                                        FPIG0420
   90 EXP2 = 0.0                                                        FPIG0430
  100 RETURN                                                            FPIG0440
      END                                                               FPIG0450
      FUNCTION BESS(N,X)                                                BESS0010
C-----------------------------------------------------------------------BESS0020
C                               THE BESSEL FUNCTIONS J0(X) AND J1(X) ARE  BESS0030
C                               EVALUATED FROM THEIR CHEBYSHEV SERIES,    BESS0040
C                               (SEE CLENSHAW,MATH. TABLES-VOL.5,         BESS0050
C                               CHEBYSHEV SERIES FOR MATH. FUNCTIONS      BESS0060
C                               NPL-DSIR).                                BESS0070
C                               THIS PROGRAM SELECTS THE APPROPRIATE      BESS0080
C                               CHEBYSHEV CONSTANTS ACCORDING TO WHETHER  BESS0090
C                               N=0 OR N=1 AND WHETHER X IS GREATER OR    BESS0100
C                               LESS THAN 8.0 AND CALLS IN FUNCTION CHEB  BESS0110
C                               TO SUM THE SERIES.                        BESS0120
```

```
C-----------------------------------------------------------------------BESS0130
      DOUBLE PRECISION B(12,2),BP(5,2),BQ(5,2),Z                         BESS0140
      DATA B /-.3D-8,.76D-7,-.1762D-5,.32460D-4,-.4606260D-3,.4819180D-2,BESS0150
     1-.34893769D-1,.158067102D+0,-.370094994D+0,.265178613D+0,          BESS0160
     2-.8723442D-2,.315455943D+0,-.1D-8,.29D-7,-.762D-6,.15887D-4,       BESS0170
     3-.260444D-3,.3240270D-2,-.29175525D-1,.177709117D+0,-.661443934D+0BESS0180
     4,1.28799410D+0,-1.19180116D+0,1.29671754D+0/                      BESS0190
      DATA BP/.2D-8,-.52D-7,.3075D-5,-.536522D-3,1.99692070D+0,         BESS0200
     1-.2D-8,.62D-7,-.3987D-5,.898990D-3,2.00180608D+0/                 BESS0210
      DATA BQ/-.1D-8,.18D-7,-.741D-6,.68385D-4,-.311117709D-1,          BESS0220
     1.1D-8,-.21D-7,.914D-6,-.96277D-4,.93555574D-1/                    BESS0230
      M = N+1                                                           BESS0240
      IF(X-8.0) 1,1,2                                                   BESS0250
    1 Z = X*X*0.0625-2.0                                                BESS0260
      BESS = CHEB(B(1,M),12,Z)                                          BESS0270
      IF(N.EQ.1) BESS = 0.125*X*BESS                                    BESS0280
      RETURN                                                            BESS0290
    2 Z = 256.0/(X*X)-2.0                                               BESS0300
      XI = X-0.78539816                                                 BESS0310
      IF(N.EQ.1) XI = XI-1.5707963                                      BESS0320
      BESS = (0.79768456/SQRT(X))*(CHEB(BP(1,M),5,Z)*COS(XI)-8.0*       BESS0330
     1        CHEB(BQ(1,M),5,Z)*SIN(XI)/X)                              BESS0340
      RETURN                                                            BESS0350
      END                                                               BESS0360
      FUNCTION CHEB(A,N,Z)                                              CHEB0010
C-----------------------------------------------------------------------CHEB0020
C                                THIS SUBPROGRAM EVALUATES THE CHEBYSHEV  CHEB0030
C                                SERIES USING THE RECURRENCE RELATION     CHEB0040
C                                TECHNIQUE (SEE CLENSHAW NPL MATH. TABLES CHEB0050
C                                VOLUME 5 PAGE 9).                        CHEB0060
C-----------------------------------------------------------------------CHEB0070
      DOUBLE PRECISION A(1),B(14),Z                                     CHEB0080
      B(1)=0.0D+0                                                       CHEB0090
      B(2)=0.0D+0                                                       CHEB0100
      DO 1 I=1,N                                                        CHEB0110
         B(I+2)=Z*B(I+1)-B(I)+A(I)                                      CHEB0120
    1 CONTINUE                                                          CHEB0130
      CHEB = 0.5D0*(B(N+2)-B(N))                                        CHEB0140
      RETURN                                                            CHEB0150
      END                                                               CHEB0160
      REAL FUNCTION IGRAND (X,LABEL)                                    IGRA0010
C-----------------------------------------------------------------------IGRA0020
C                                THIS SUBROUTINE COMPUTES THE INTEGRANDS  IGRA0030
C                                FOR THE INTEGRALS COMPUTED IN INGRAL.    IGRA0040
C                                USE IS MADE OF THE RESULTS OF FPIGRA AND IGRA0050
C                                MATRIX STORED IN COMMON/IGRAN/.          IGRA0060
C-----------------------------------------------------------------------IGRA0070
      REAL LOAD,NU,ACCUR(3),K5                                          IGRA0080
      COMMON/ASDT/LAYER,NLAYS,M,R,Z,NU(10),ACCUR,LOAD,HOSTRS,NZEROS,H(9)IGRA0090
     1,K5(10),E(10),AL(9),THICK(9),RADIUS(10)                           IGRA0100
      COMMON/IGRAN/T0,V0,S0,U0,T1,V1,S1,U1,T01,S01,FPIGR,EXP1,EXP2      IGRA0110
      GO TO (10,20,30,40,50,60,70,80,90,100,110,120,130,140,150,160,170)IGRA0120
     1,LABEL                                                            IGRA0130
   10 IGRAND =FPIGR*X*((U0*(K5(LAYER)-X*Z)-S0)*EXP1+(T0+V0*(K5(LAYER)+X*IGRA0140
     1Z))*EXP2)                                                         IGRA0150
      RETURN                                                            IGRA0160
   20 IGRAND =FPIGR*X*(U0*EXP1+V0*EXP2)                                 IGRA0170
      RETURN                                                            IGRA0180
   30 IGRAND =FPIGR*((U0*(2.0*K5(LAYER)-X*Z)-S0)*EXP1-(T0+V0*(2.0*K5(LAYIGRA0190
     1ER)+X*Z))*EXP2)                                                   IGRA0200
      RETURN                                                            IGRA0210
```

```
   40 IGRAND =FPIGR*((S0+U0*(1.0+X*Z))*EXP1+(V0*(1.0-X*Z)-T0)*EXP2)      IGRA0220
      RETURN                                                            IGRA0230
   50 IGRAND =FPIGR*X*R*((S0+U0*(2.*NU(LAYER)+X*Z))*EXP1+(T0+V0*(X*Z-2.*IGRA0240
     1NU(LAYER)))*EXP2)                                                 IGRA0250
      RETURN                                                            IGRA0260
   60 IGRAND =FPIGR*X*((S1+U1*(2.0*NU(LAYER)+X*Z))*EXP1+(T1+V1*(X*Z-2.0*IGRA0270
     1NU(LAYER)))*EXP2)                                                 IGRA0280
      RETURN                                                            IGRA0290
   70 IGRAND =FPIGR*((S1+U1*(1.0+X*Z))*EXP1+(V1*(1.0-X*Z)-T1)*EXP2)      IGRA0300
      RETURN                                                            IGRA0310
   80 IGRAND =FPIGR*X*R*((S1+U1*(1.0+X*Z))*EXP1+(V1*(1.0-X*Z)-T1)*EXP2) IGRA0320
      RETURN                                                            IGRA0330
   90 IGRAND =FPIGR*X*R*(U1*EXP1+V1*EXP2)                               IGRA0340
      RETURN                                                            IGRA0350
  100 IGRAND =FPIGR*((S1+U1*(2.*NU(LAYER)+X*Z))*EXP1+(T1+V1*(X*Z-2.*NU(LIGRA0360
     1AYER)))*EXP2)                                                     IGRA0370
      RETURN                                                            IGRA0380
  110 IGRAND =FPIGR*(U1*EXP1-V1*EXP2)                                   IGRA0390
      RETURN                                                            IGRA0400
  120 IGRAND =FPIGR*((S1+U1*(1.+X*Z))*EXP1+(V1*(1.-X*Z)-T1)*EXP2)/X     IGRA0410
      RETURN                                                            IGRA0420
  130 IGRAND =FPIGR*X*(SQ1*EXP1-TQ1*EXP2)                               IGRA0430
      RETURN                                                            IGRA0440
  140 IGRAND =FPIGR*(SQ1*EXP1+TQ1*EXP2)                                 IGRA0450
      RETURN                                                            IGRA0460
  150 IGRAND =FPIGR*X*R*(SQ1*EXP1+TQ1*EXP2)                             IGRA0470
      RETURN                                                            IGRA0480
  160 IGRAND =FPIGR*(SQ1*EXP1-TQ1*EXP2)                                 IGRA0490
      RETURN                                                            IGRA0500
  170 IGRAND =FPIGR*(SQ1*EXP1+TQ1*EXP2)/X                               IGRA0510
      RETURN                                                            IGRA0520
      END                                                               IGRA0530
      SUBROUTINE CALC(INT,V,R,MU,RADI,FT,LOAD,HOSTRS,PSI0,Z)            CALC0010
C-----------------------------------------------------------------------CALC0020
C                                COMPONENTS OF THE STRESSES,STRAINS AND CALC0040
C                                THIS SUBROUTINE COMPUTES THE CYLINDRICAL CALC0030
C                                DISPLACEMENTS FROM THE 17 INTEGRALS STO- CALC0050
C                                RED IN INT. THESE CALCULATED COMPONENTS CALC0060
C                                ARE STORED IN V AND OUTPUTTED.          CALC0070
C-----------------------------------------------------------------------CALC0080
      REAL INT(17),V(15),MU,LOAD,C(6)                                   CALC0090
      INTEGER FM(19),FMT(5),T(12)                                       CALC0100
      LOGICAL STRESS,EPS,RLOW                                           CALC0110
      COMMON/STRDTA/STRESS(27),EPS(17),RLOW,ST,CT,L,ACC                 CALC0120
      COMMON/TAPE/NOUT                                                  CALC0130
      DATA FM(1),FMT,FM(19),T/                                          CALC0140
     +"(1X ",",E12",".4,1",".0X  ",",12X","  ,1","X)  ",                CALC0150
     +"DISP","LACE","MENT","S   ",                                      CALC0160
     +"   S","TRES","SES ","    ",                                      CALC0170
     +"   S","TRAI","NS  ","    "/                                      CALC0180
      DO 10 I=1,15                                                      CALC0190
   10 V(I)=0.0                                                          CALC0200
      IF((STRESS( 4).OR.STRESS( 5).OR.STRESS( 7).OR.STRESS(10).OR.      CALC0210
     +    STRESS(11)).AND.(.NOT.RLOW)) FCT=(2.0*INT(12)-INT( 7)-2.0*INT(CALC0220
     + 14)+4.0*INT(17))/R                                               CALC0230
      IF(.NOT.STRESS( 1))  GO TO 20                                     CALC0240
      V( 1)=FT*RADI*CT*(2.0*INT(17)+INT(12)-INT( 7))                    CALC0250
      IF(RLOW)             GO TO 20                                     CALC0260
      V( 1)=V( 1)-FT*R*RADI*INT( 4)                                     CALC0270
   20 IF(STRESS( 2))  V( 2)=FT*RADI*ST*(2.0*(INT(17)-INT(14))+INT(12))  CALC0280
      IF(.NOT.STRESS( 3))  GO TO 30                                     CALC0290
```

```
      V( 3)=-FT*RADI*INT( 3)                                            CALC0300
      IF(RLOW)                GO TO 30                                  CALC0310
      V( 3)=V( 3)+FT*R*RADI*CT*((2.0-2.0*MU)*INT(11)-INT(10))           CALC0320
   30 IF(.NOT.STRESS( 4))  GO TO 40                                     CALC0330
      V( 4)=CT*(INT( 8)+2.0*MU*INT( 9))+INT( 1)+INT( 4)-2.0*INT( 2)     CALC0340
      IF(RLOW)                GO TO 40                                  CALC0350
      V( 4)=V( 4)-CT*FCT                                                CALC0360
   40 IF(.NOT.STRESS( 5))  GO TO 50                                     CALC0370
      V( 5)=CT*2.0*MU*INT( 9)-2.0*MU*INT( 2)-INT( 4)                    CALC0380
      IF(RLOW)                GO TO 50                                  CALC0390
      V( 5)=V( 5)+CT*FCT                                                CALC0400
   50 IF(.NOT.STRESS( 7))  GO TO 60                                     CALC0410
      V( 7)=ST*INT(15)                                                  CALC0420
      IF(RLOW)                GO TO 60                                  CALC0430
      V( 7)=V( 7)-ST*FCT                                                CALC0440
   60 IF(.NOT.STRESS(10))  GO TO 70                                     CALC0450
      V(10)=FT*(CT*INT( 8)+INT( 1)+INT( 4)-(2.0-2.0*MU)*INT( 2))        CALC0460
      IF(RLOW)                GO TO 70                                  CALC0470
      V(10)=V(10)-FT*CT*FCT                                             CALC0480
   70 IF(.NOT.STRESS(11))  GO TO 80                                     CALC0490
      V(11)=-FT*INT( 4)                                                 CALC0500
      IF(RLOW)                GO TO 80                                  CALC0510
      V(11)=V(11)+FT*CT*FCT                                             CALC0520
   80 IF(STRESS(12))  V(12)=FT*(CT*((2.0-4.0*MU)*INT( 9)-INT( 8))+      CALC0530
     +                         2.0*MU*INT( 2)-INT( 1))                  CALC0540
      IF(Z.LT.ACC)  GO TO 90                                            CALC0550
      IF(STRESS( 6))  V( 6)=CT*((2.0-2.0*MU)*INT( 9)-INT( 8))-INT( 1)   CALC0560
      IF(STRESS( 8))  V( 8)=CT*(INT(16)+INT(10)-INT( 6))-INT( 5)        CALC0570
      IF(STRESS( 9))  V( 9)=ST*(INT(16)-INT(13)+INT(10))                CALC0580
      GO TO 110                                                         CALC0590
   90 IF(ABS(R-1.0).LT.ACC)  GO TO 100                                  CALC0600
      IF(R.GT.1.0)            GO TO 110                                 CALC0610
      IF(STRESS( 6))  V( 6)=-LOAD                                       CALC0620
      IF(STRESS( 8))  V( 8)=-HOSTRS*COS(PSI0)                           CALC0630
      IF(STRESS( 9))  V( 9)=HOSTRS*SIN(PSI0)                            CALC0640
      GO TO 110                                                         CALC0650
  100 IF(STRESS( 6))  V( 6)=-0.5*LOAD                                   CALC0660
      IF(STRESS( 8))  V( 8)=-0.5*HOSTRS*COS(PSI0)                       CALC0670
      IF(STRESS( 9))  V( 9)= 0.5*HOSTRS*SIN(PSI0)                       CALC0680
  110 IF(STRESS(13))  V(13)=FT*V( 7)                                    CALC0690
      IF(STRESS(14))  V(14)=FT*V( 8)                                    CALC0700
      IF(STRESS(15))  V(15)=FT*V( 9)                                    CALC0710
      DO 120 I=2,18                                                     CALC0720
  120 FM(I)=T(8)                                                        CALC0730
      DO 130 I=4,16,3                                                   CALC0740
  130 FM(I)=FMT(3)                                                      CALC0750
      K=0                                                               CALC0760
      J=0                                                               CALC0770
      DO 210 I=1,15                                                     CALC0780
         J=J+1                                                          CALC0790
         IF(I-4)  190,160,140                                           CALC0800
  140    IF(I-10)  190,150,190                                          CALC0810
  150    IF(K.EQ.0)   GO TO 180                                         CALC0820
         WRITE(NOUT,9010) (T(J),J=5,8)                                  CALC0830
         WRITE(NOUT,FM) (C(J),J=1,K)                                    CALC0840
         GO TO 170                                                      CALC0850
  160    IF(K.EQ.0)   GO TO 180                                         CALC0860
         WRITE(NOUT,9000) (T(J),J=1,4)                                  CALC0870
         WRITE(NOUT,FM) (C(J),J=1,K)                                    CALC0880
  170    K=0                                                            CALC0890
  180    J=1                                                            CALC0900
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  190     M=3*J                                                         CALC0910
          IF(.NOT.STRESS(I))  GO TO 200                                 CALC0920
          K=K+1                                                         CALC0930
          C(K)=V(I)                                                     CALC0940
          FM(M-1)=FMT(1)                                                CALC0950
          FM(M  )=FMT(2)                                                CALC0960
          GO TO 210                                                     CALC0970
  200     FM(M-1)=FMT(4)                                                CALC0980
          FM(M  )=FMT(5)                                                CALC0990
  210 CONTINUE                                                          CALC1000
      IF(K.EQ.0)  RETURN                                                CALC1010
      WRITE(NOUT,9010) (T(J),J=9,12)                                    CALC1020
      WRITE(NOUT,FM) (C(J),J=1,K)                                       CALC1030
      RETURN                                                            CALC1040
 9000 FORMAT(1X,4A4/5X,"RADIAL",12X,"TANGENTIAL",14X,"VERTICAL")        CALC1050
 9010 FORMAT(1X,4A4/5X,"RADIAL",12X,"TANGENTIAL",14X,"VERTICAL",12X,"RADCALC1060
     +./TANG.",11X,"RAD./VERT.",12X,"TANG./VERT.")                      CALC1070
      END                                                               CALC1080
      SUBROUTINE OUTPUT(EPS,C,K,L)                                      OUTP0010
C-----------------------------------------------------------------------OUTP0020
C                                THIS SUBROUTINE OUTPUTS BY MEANS OF THREE OUTP0030
C                                SUBSEQUENT CALLS FROM THE MAIN PROGRAM  OUTP0040
C                                THE TOTAL STRESSES,STRAINS AND SISPLACE- OUTP0050
C                                MENTS.                                  OUTP0060
C-----------------------------------------------------------------------OUTP0070
      INTEGER FM(16),FMT(8)                                             OUTP0080
      LOGICAL EPS(6)                                                    OUTP0090
      DIMENSION C(6),TKST(6,4)                                          OUTP0100
      COMMON/TAPE/NOUT                                                  OUTP0110
      DATA TKST/                                                        OUTP0120
     1" T O","  T A","  L  ","S T ","R E ","S S ",                       OUTP0130
     2" T O","  T A","  L  ","S T ","R A ","I N ",                       OUTP0140
     3" T O","  T A","  L  ","D I ","S P ","L A ",                       OUTP0150
     4"C E ","M E ","N T ","    ","    ","    "/                         OUTP0160
      DATA FMT,FM(16)/                                                  OUTP0170
     1"(6A4",",12X","    ",",E12",".3  ","(12A","4,48",".X   ",")   "/  OUTP0180
      IF(L.NE.3) GO TO 10                                               OUTP0190
      FM(1)=FMT(6)                                                      OUTP0200
      FM(2)=FMT(7)                                                      OUTP0210
      FM(3)=FMT(8)                                                      OUTP0220
      GO TO 20                                                          OUTP0230
   10 FM(1)=FMT(1)                                                      OUTP0240
      FM(2)=FMT(3)                                                      OUTP0250
      FM(3)=FMT(3)                                                      OUTP0260
   20 N=0                                                               OUTP0270
      M=2*K+2                                                           OUTP0280
      DO 40 I=4,M,2                                                     OUTP0290
         J=I/2-1                                                        OUTP0300
         IF(.NOT.EPS(J)) GO TO 30                                       OUTP0310
         FM(I)=FMT(4)                                                   OUTP0320
         FM(I+1)=FMT(5)                                                 OUTP0330
         N=N+1                                                          OUTP0340
         C(N)=C(J)                                                      OUTP0350
         GO TO 40                                                       OUTP0360
   30    FM(I)=FMT(2)                                                   OUTP0370
         FM(I+1)=FMT(3)                                                 OUTP0380
   40 CONTINUE                                                          OUTP0390
      IF(L.EQ.3) GO TO 60                                               OUTP0400
      IF(N.EQ.0) GO TO 50                                               OUTP0410
      WRITE(NOUT,FM) (TKST(I,L),I=1,6),(C(I),I=1,N)                     OUTP0420
      RETURN                                                            OUTP0430
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   50 WRITE(NOUT,FM) (TKST(I,L),I=1,6)                                  OUTP0440
      RETURN                                                            OUTP0450
   60 DO 70 I=10,15                                                     OUTP0460
   70 FM(I)=FMT(3)                                                      OUTP0470
      IF(N.EQ.0) GO TO 80                                               OUTP0480
      WRITE(NOUT,FM) (TKST(I,3),I=1,6),(TKST(I,4),I=1,6),(C(I),I=1,N)   OUTP0490
      RETURN                                                            OUTP0500
   80 WRITE(NOUT,FM) (TKST(I,3),I=1,6),(TKST(I,4),I=1,6)                OUTP0510
      RETURN                                                            OUTP0520
      END                                                               OUTP0530
      SUBROUTINE JACOBI (H,U,ND,N,IVEC,W,IQ)                            JAC00010
C-----------------------------------------------------------------------JAC00020
C                               SUBROUTINE JACOBI TO COMPUTE EIGENVALUES JAC00030
C                               AND EIGENVECTORS OF A SYMMETRIC MATRIX,  JAC00040
C                               H IS THE GIVEN MATRIX,THE DIAGONAL OF    JAC00050
C                               WHICH CONTAINS AFTER THE ITERATION THE   JAC00060
C                               EIGENVALUES OF H.                        JAC00070
C                               U IS THE MATRIX,THE COLUMNS OF WHICH ARE JAC00080
C                               THE EIGENVECTORS OF H.                   JAC00090
C                               N AND ND ARE THE DIMENSIONS OF THE ACTUAL JAC00100
C                               MATRIX AND THE ONE USED IN THE DIMENSION- JAC00110
C                               STATEMENT OF THE CALLINGPROGRAM          JAC00120
C                               RESPECTIVELY.                            JAC00130
C                               IVEC=0 IF NO EIGENVECTORS ARE REQUIRED,  JAC00140
C                               IVEC=1 IF THE EIGENVECTORS SHOULD BE     JAC00150
C                               CALCULATED.                              JAC00160
C                               THE ACCURACY OF THE EIGENVALUES IS ABOUT JAC00170
C                               1.0E-6,THE ACCURACY OF AN EIGENVECTOR IS JAC00180
C                               ABOUT 1.0E-6/D,WHERE D IS THE MINIMUM-   JAC00190
C                               DISTANCE OF THE CORRESPONDING EIGENVALUE JAC00200
C                               FROM THE OTHER EIGENVALUES.              JAC00210
C                               W AND IQ ARE WORKINGSPACES,WHICH SHOULD BEJAC00220
C                               DIMENSIONED IN THE CALLING PROGRAM.      JAC00230
C-----------------------------------------------------------------------JAC00240
      REAL H(ND,ND),U(ND,ND),W(ND)                                      JAC00250
      INTEGER IQ(ND)                                                    JAC00260
      DOUBLE PRECISION TA,SI,CO,Z,Y,HTE,UTE                             JAC00270
      AN   =N                                                           JAC00280
      NMI1=N-1                                                          JAC00290
      IF(IVEC-1) 60,10,60                                               JAC00300
   10 DO  50 I=1,N                                                      JAC00310
         DO  40 J=1,N                                                   JAC00320
            IF(I-J) 30,20,30                                            JAC00330
   20       U(I,J)=1.0                                                  JAC00340
            GO TO 40                                                    JAC00350
   30       U(I,J)=0.0                                                  JAC00360
   40    CONTINUE                                                       JAC00370
   50 CONTINUE                                                          JAC00380
   60 DO  90 I=1,NMI1                                                   JAC00390
         W(I)=0.0                                                       JAC00400
         IPL1=I+1                                                       JAC00410
         DO  80 J=IPL1,N                                                JAC00420
            IF(W(I)-ABS(H(I,J))) 70,70,80                               JAC00430
   70       W(I)=ABS(H(I,J))                                            JAC00440
            IQ(I)=J                                                     JAC00450
   80    CONTINUE                                                       JAC00460
   90 CONTINUE                                                          JAC00470
  100 DO 120 I=1,NMI1                                                   JAC00480
         IF(I.EQ.1) GO TO 110                                           JAC00490
         IF(XMAX.GE.W(I)) GO TO 120                                     JAC00495
  110    XMAX=W(I)                                                      JAC00500
```

```
         IPIV=I                                                         JAC00510
         JPIV=IQ(I)                                                     JAC00520
  120 CONTINUE                                                          JAC00530
      IF(XMAX-1.E-12/AN) 170,170,130                                    JAC00540
  130 Z  =H(IPIV,IPIV)-H(JPIV,JPIV)                                     JAC00550
      Y = 2.0D0*DBLE(H(IPIV,JPIV))                                      JAC00560
      TA =Y/(DABS(Z)+DSQRT(Z*Z+Y*Y))                                    JAC00570
      IF(Z.LT.0.0D0)  TA=-TA                                            JAC00580
      CO =1.D0/DSQRT(1.D0+TA*TA)                                        JAC00590
      SI =TA*CO                                                         JAC00600
      HII=H(IPIV,IPIV)                                                  JAC00610
      HJJ=H(JPIV,JPIV)                                                  JAC00620
      HIJ=H(IPIV,JPIV)                                                  JAC00630
      DO 140 K=1,N                                                      JAC00640
         HTE=H(K,IPIV)                                                  JAC00650
         H(K,IPIV)=DBLE(H(K,IPIV))*CO+DBLE(H(K,JPIV))*SI               JAC00660
         H(K,JPIV)=DBLE(H(K,JPIV))*CO-HTE*SI                           JAC00670
         H(IPIV,K)=H(K,IPIV)                                            JAC00680
         H(JPIV,K)=H(K,JPIV)                                            JAC00690
  140 CONTINUE                                                          JAC00700
      H(IPIV,JPIV)=0.0                                                  JAC00710
      H(JPIV,IPIV)=0.0                                                  JAC00720
      AA=DBLE(HIJ)*TA                                                   JAC00730
      H(IPIV,IPIV)=HII+AA                                               JAC00740
      H(JPIV,JPIV)=HJJ-AA                                               JAC00750
      IF(IVEC)  60,60,150                                               JAC00760
  150 DO 160 K=1,N                                                      JAC00770
         UTE=U(K,IPIV)                                                  JAC00780
         U(K,IPIV)=DBLE(U(K,IPIV))*CO+DBLE(U(K,JPIV))*SI                JAC00790
         U(K,JPIV)=DBLE(U(K,JPIV))*CO-UTE*SI                            JAC00800
  160 CONTINUE                                                          JAC00810
      GO TO  60                                                         JAC00820
  170 RETURN                                                            JAC00830
      END                                                               JAC00840
      SUBROUTINE ESORT (H,U,ND,N,IVEC,W,IQ)                             ESOR0010
C-----------------------------------------------------------------------ESOR0020
C                                SUBROUTINE ESORT.                      ESOR0030
C                                THIS ROUTINE SORTS EIGENVALUES (AND EIGEN ESOR0040
C                                VECTORS) OBTAINED FROM SUBROUTINE JACOBI. ESOR0050
C                                H    = ORIGINAL MATRIX(ND,ND).         ESOR0060
C                                U    = EIGENVECTORMATRIX(ND,ND).       ESOR0070
C                                ND   = MAX. DIMENSION OF MATRICES.     ESOR0080
C                                N    = ACTUAL DIMENSION OF MATRICES.   ESOR0090
C                                IVEC=1 WITH EIGENVECTORS.              ESOR0100
C                                     =0   NO EIGENVECTORS.             ESOR0110
C                                W    = WORKINGSPACE(ND).               ESOR0120
C                                IQ   = WORKINGSPACE(ND).               ESOR0130
C-----------------------------------------------------------------------ESOR0140
      REAL H(ND,ND),U(ND,ND),W(ND),DUMMY                                ESOR0150
      INTEGER N,IQ(ND),FDUMMY,I,J,K,IVEC                                ESOR0160
      LOGICAL LOGIC                                                     ESOR0170
C                                                                       ESOR0180
      DO 10 I=1,N                                                       ESOR0190
         W(I)=H(I,I)                                                    ESOR0200
   10    IQ(I)=I                                                        ESOR0210
      J=N                                                               ESOR0220
   20 LOGIC=.FALSE.                                                     ESOR0230
      K=J                                                               ESOR0240
      DO 30 I=2,K                                                       ESOR0250
         IF(W(I-1).GE.W(I)) GO TO 30                                    ESOR0260
         LOGIC=.TRUE.                                                   ESOR0270
```

```
      DUMMY=W(I-1)                                                      ESOR0280
      W(I-1)=W(I)                                                       ESOR0290
      W(I)=DUMMY                                                        ESOR0300
      FDUMMY=IQ(I-1)                                                    ESOR0310
      IQ(I-1)=IQ(I)                                                     ESOR0320
      IQ(I)=FDUMMY                                                      ESOR0330
      J=I-1                                                             ESOR0340
   30 CONTINUE                                                          ESOR0350
      IF (LOGIC) GO TO 20                                               ESOR0360
      IF (IVEC.EQ.0) GO TO 60                                           ESOR0370
      DO 40 I=1,N                                                       ESOR0380
         K=IQ(I)                                                        ESOR0390
         DO 40 J=1,N                                                    ESOR0400
   40       H(J,I)=U(J,K)                                               ESOR0410
      DO 50 I=1,N                                                       ESOR0420
         DO 50 J=1,N                                                    ESOR0430
            U(I,J)=H(I,J)                                               ESOR0440
   50       H(I,J)=0.0                                                  ESOR0450
   60 DO 70 I=1,N                                                       ESOR0460
   70    H(I,I)=W(I)                                                    ESOR0470
      RETURN                                                            ESOR0480
      END                                                               ESOR0490
      BLOCK DATA                                                        BLDA0010
C-----------------------------------------------------------------------BLDA0020
C                                IN THE BLOCK DATA ARE STORED SUBSEQUNTILY*BLDA0030
C                                -THE ABSCISSAE FOR THE LEGENDRE-GAUSS  BLDA0040
C                                 QUADRATURE,STARTING IN A WITH THE 2-ND BLDA0050
C                                 ORDER AND ENDING IN N WITH THE 15-TH  BLDA0060
C                                 ORDER.                                BLDA0070
C                                -THE ABSCISSAE FOR THE JACOBI-GAUSS    BLDA0080
C                                 QUADRATURE OF THE 8-TH ORDER IN O.    BLDA0090
C                                -THE WEIGHTS FOR THE LEGENDRE-GAUSS    BLDA0100
C                                 QUADRATURE,STARTING IN P WITH THE 2-ND BLDA0110
C                                 ORDER AND ENDING IN CC WITH THE 15-TH BLDA0120
C                                 ORDER.                                BLDA0130
C                                -THE WEIGHTS FOR THE JACOBI-GAUSS QUADRA- BLDA0140
C                                 TURE OF THE 8-TH ORDER IN DD.         BLDA0150
C                                -THE FIRST 149 ZEROS OF J0 IN EE AND FF BLDA0160
C                                -THE FIRST 149 ZEROS OF J1 IN GG AND HH BLDA0170
C-----------------------------------------------------------------------BLDA0180
      REAL I,J,K,L,M,N                                                  BLDA0190
      DIMENSION A(2),B(3),C(4),D(5),E(6),F(7),G(8),H(9),I(10),J(11),K(12BLDA0200
     1),L(13),M(14),N(15),O( 8),P(2),Q(3),R(4),S(5),T(6),U(7),V(8),W(9),BLDA0210
     2X(10),Y(11),Z(12),AA(13),BB(14),CC(15),DD( 8),EE(119),FF(30),GG(11BLDA0220
     39),HH(30)                                                         BLDA0230
      COMMON/GAUSS/AGAUSS(16,16),HGAUSS(16,16)                          BLDA0240
      COMMON/GEDATA/BZEROS(149,2),ZEROS(298)                            BLDA0250
      EQUIVALENCE          (AGAUSS(1, 2), A(1)),(AGAUSS(1, 3), B(1)),   BLDA0260
     1(AGAUSS(1, 4), C(1)),(AGAUSS(1, 5), D(1)),(AGAUSS(1, 6), E(1)),   BLDA0270
     2(AGAUSS(1, 7), F(1)),(AGAUSS(1, 8), G(1)),(AGAUSS(1, 9), H(1)),   BLDA0280
     3(AGAUSS(1,10), I(1)),(AGAUSS(1,11), J(1)),(AGAUSS(1,12), K(1)),   BLDA0290
     4(AGAUSS(1,13), L(1)),(AGAUSS(1,14), M(1)),(AGAUSS(1,15), N(1)),   BLDA0300
     5(AGAUSS(1,16), O(1)),(HGAUSS(1, 2), P(1)),(HGAUSS(1, 3), Q(1)),   BLDA0310
     6(HGAUSS(1, 4), R(1)),(HGAUSS(1, 5), S(1)),(HGAUSS(1, 6), T(1)),   BLDA0320
     7(HGAUSS(1, 7), U(1)),(HGAUSS(1, 8), V(1)),(HGAUSS(1, 9), W(1)),   BLDA0330
     8(HGAUSS(1,10), X(1)),(HGAUSS(1,11), Y(1)),(HGAUSS(1,12), Z(1)),   BLDA0340
     9(HGAUSS(1,13),AA(1)),(HGAUSS(1,14),BB(1)),(HGAUSS(1,15),CC(1)),   BLDA0350
     T(HGAUSS(1,16),DD(1)),(BZEROS(1, 1),EE(1)),(BZEROS(120,1),FF(1)),  BLDA0360
     1(BZEROS(1, 2),GG(1)),(BZEROS(120,2),HH(1))                        BLDA0370
      DATA A,B,C,D,E,F,G,H,I,J,K,L,M                                    BLDA0380
     N/-.4472136, 0.4472136,-0.6546537, 0.0000000, 0.6546537,-0.7650554,BLDA0390
```

```
1-0.2852315, 0.2852315, 0.7650553,-0.8302239,-0.4688488, 0.0000000,BLDA0400
2 0.4688488, 0.8302239,-0.8717402,-0.5917003,-0.2092993, 0.2092991,BLDA0410
3 0.5917001, 0.8717400,-0.8997580,-0.6771863,-0.3631175, 0.0000000,BLDA0420
4 0.3631175, 0.6771863, 0.8997580,-0.9195339,-0.7387739,-0.4779250,BLDA0430
5-0.1652790, 0.1652789, 0.4779249, 0.7387738, 0.9195338,-0.9340014,BLDA0440
6-0.7844836,-0.5652354,-0.2957582, 0.0000000, 0.2957581, 0.5652353,BLDA0450
7 0.7844834, 0.9340014,-0.9448975,-0.8192815,-0.6328754,-0.3995310,BLDA0460
8-0.1365529, 0.1365529, 0.3995309, 0.6328753, 0.8192813, 0.9448975,BLDA0470
9-0.9533069,-0.8463538,-0.6861843,-0.4829108,-0.2492869, 0.0000000,BLDA0480
T 0.2492868, 0.4829106, 0.6861842, 0.8463537, 0.9533068,-0.9599299,BLDA0490
1-0.8678104,-0.7288621,-0.5506417,-0.3427235,-0.1163319, 0.1163318,BLDA0500
2 0.3427235, 0.5506415, 0.7288620, 0.8678104, 0.9599298,-0.9652544,BLDA0510
3-0.8850636,-0.7635341,-0.6062477,-0.4206389,-0.2153539, 0.0000000,BLDA0520
4 0.2153538, 0.4206389, 0.6062477, 0.7635341, 0.8850635, 0.9652544,BLDA0530
5-0.9695861,-0.8991729,-0.7920153,-0.6523931,-0.4860575,-0.2998304,BLDA0540
6-0.1013263, 0.1013262, 0.2998304, 0.4860575, 0.6523930, 0.7920151,BLDA0550
7 0.8991728, 0.9695860/                                            BLDA0560
 DATA N,O                                                          BLDA0570
N/-.9731405,-0.9108602,-0.8157166,-0.6910172,-0.5413883,-0.3721744,BLDA0580
1-0.1895120, 0.0000000, 0.1895119, 0.3721744, 0.5413882, 0.6910170,BLDA0590
2 0.8157164, 0.9108602, 0.9731404,-0.9602899,-0.7966665,-0.5255324,BLDA0600
3-0.1834346, 0.1834346, 0.5255324, 0.7966665, 0.9602899/           BLDA0610
 DATA P,Q,R,S,T,U,V,W,X,Y,Z,AA,BB                                  BLDA0620
N/0.8333334, 0.8333331, 0.5444443, 0.7111111, 0.5444444, 0.3784749,BLDA0630
1 0.5548583, 0.5548581, 0.3784750, 0.2768261, 0.4317453, 0.4876190,BLDA0640
2 0.4317455, 0.2768261, 0.2107044, 0.3411230, 0.4124591, 0.4124591,BLDA0650
3 0.3411230, 0.2107046, 0.1654953, 0.2745391, 0.3464290, 0.3715193,BLDA0660
4 0.3464290, 0.2745388, 0.1654955, 0.1333061, 0.2248897, 0.2920431,BLDA0670
5 0.3275404, 0.3275403, 0.2920429, 0.2248897, 0.1333061, 0.1096126,BLDA0680
6 0.1871701, 0.2480485, 0.2868792, 0.3002176, 0.2868798, 0.2480485,BLDA0690
7 0.1871700, 0.1096126, 0.0916847, 0.1579750, 0.2125089, 0.2512758,BLDA0700
8 0.2714060, 0.2714060, 0.2512759, 0.2125089, 0.1579748, 0.0916846,BLDA0710
9 0.0778019, 0.1349820, 0.1836473, 0.2207679, 0.2440163, 0.2519308,BLDA0720
T 0.2440165, 0.2207679, 0.1836473, 0.1349820, 0.0778019, 0.0668373,BLDA0730
1 0.1165870, 0.1600221, 0.1948268, 0.2191266, 0.2316136, 0.2316138,BLDA0740
2 0.2191266, 0.1948268, 0.1600223, 0.1165869, 0.0668375, 0.0580301,BLDA0750
3 0.1016605, 0.1405119, 0.1727902, 0.1969877, 0.2119743, 0.2170480,BLDA0760
4 0.2119743, 0.1969876, 0.1727903, 0.1405120, 0.1016601, 0.0580301,BLDA0770
5 0.0508505, 0.0893939, 0.1242559, 0.1540275, 0.1774924, 0.1936907,BLDA0780
6 0.2019594, 0.2019594, 0.1936906, 0.1774925, 0.1540275, 0.1242554,BLDA0790
7 0.0893940, 0.0508506/                                            BLDA0800
 DATA CC,DD                                                        BLDA0810
N/0.0449221, 0.0791985, 0.1105931, 0.1379879, 0.1603954, 0.1770052,BLDA0820
1 0.1872171, 0.1906618, 0.1872172, 0.1770049, 0.1603951, 0.1379883,BLDA0830
2 0.1105928, 0.0791985, 0.0449221, 0.1012285, 0.2223810, 0.3137067,BLDA0840
3 0.3626838, 0.3626838, 0.3137067, 0.2223810, 0.1012285/           BLDA0850
 DATA EE    /2.404826, 5.520078, 8.653728, 11.79153, 14.93092,      BLDA0860
1 18.07106, 21.21164, 24.35247, 27.49348,  30.63461, 33.77582,      BLDA0870
2 36.91710, 40.05843, 43.19979, 46.34119,  49.48261, 52.62405,      BLDA0880
3 55.76551, 58.90698, 62.04847, 65.18996,  68.33147, 71.47298,      BLDA0890
4 74.61450, 77.75603, 80.89756, 84.03909,  87.18063, 90.32217,      BLDA0900
5 93.46372, 96.60527, 99.74682,102.8883,  106.0299, 109.1715,       BLDA0910
6112.3131, 115.4546, 118.5962, 121.7377,  124.8793, 128.0209,       BLDA0920
7131.1624, 134.3040, 137.4456, 140.5872,  143.7287, 146.8703,       BLDA0930
8150.0119, 153.1535, 156.2950, 159.4366,  162.5782, 165.7198,       BLDA0940
9168.8613, 172.0029, 175.1445, 178.2861,  181.4277, 184.5692,       BLDA0950
T187.7108, 190.8524, 193.9940, 197.1356,  200.2772, 203.4187,       BLDA0960
1206.5603, 209.7019, 212.8435, 215.9850,  219.1267, 222.2682,       BLDA0970
2225.4098, 228.5514, 231.6930, 234.8346,  237.9762, 241.1178,       BLDA0980
3244.2593, 247.4009, 250.5425, 253.6841,  256.8257, 259.9673,       BLDA0990
4263.1089, 266.2504, 269.3920, 272.5336,  275.6752, 278.8168,       BLDA1000
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5281.9584, 285.1000, 288.2416, 291.3831,  294.5247, 297.6663,      BLDA1010
6300.8079, 303.9495, 307.0911, 310.2327,  313.3743, 316.5159,      BLDA1020
7319.6574, 322.7990, 325.9406, 329.0822,  332.2238, 335.3654,      BLDA1030
8338.5070, 341.6486, 344.7902, 347.9317,  351.0733, 354.2149,      BLDA1040
9357.3565, 360.4981, 363.6397, 366.7813,  369.9229, 373.0645/      BLDA1050
 DATA FF / 376.2061, 379.3476, 382.4892,  385.6308, 388.7724,      BLDA1060
1391.9140, 395.0556, 398.1972, 401.3388,  404.4804, 407.6220,      BLDA1070
2410.7635, 413.9051, 417.0467, 420.1883,  423.3299, 426.4715,      BLDA1080
3429.6131, 432.7547, 435.8963, 439.0379,  442.1794, 445.3210,      BLDA1090
4448.4626, 451.6042, 454.7458, 457.8874,  461.0290, 464.1706,      BLDA1100
5467.3122/                                                         BLDA1110
 DATA GG    /3.831706, 7.015587, 10.17347, 13.32369, 16.47063,     BLDA1120
1 19.61586, 22.76008, 25.90367,  29.04683, 32.18968, 35.33231,     BLDA1130
2 38.47477, 41.61709, 44.75932,  47.90146, 51.04354, 54.18555,     BLDA1140
3 57.32753, 60.46946, 63.61136,  66.75323, 69.89507, 73.03690,     BLDA1150
4 76.17870, 79.32049, 82.46226,  85.60402, 88.74577, 91.88750,     BLDA1160
5 95.02923, 98.17095,101.3127,  104.4544, 107.5961, 110.7376,      BLDA1170
6113.8794, 117.0211, 120.1628,  123.3045, 126.4461, 129.5878,      BLDA1180
7132.7295, 135.8711, 139.0128,  142.1544, 145.2961, 148.4377,      BLDA1190
8151.5794, 154.7210, 157.8626,  161.0043, 164.1459, 167.2876,      BLDA1200
9170.4292, 173.5708, 176.7125,  179.8541, 182.9957, 186.1374,      BLDA1210
T189.2790, 192.4206, 195.5622,  198.7038, 201.8455, 204.9871,      BLDA1220
1208.1287, 211.2703, 214.4120,  217.5536, 220.6952, 223.8368,      BLDA1230
2226.9784, 230.1200, 233.2616,  236.4033, 239.5449, 242.6865,      BLDA1240
3245.8281, 248.9697, 252.1113,  255.2529, 258.3945, 261.5362,      BLDA1250
4264.6778, 267.8194, 270.9610,  274.1026, 277.2442, 280.3859,      BLDA1260
5283.5274, 286.6690, 289.8106,  292.9522, 296.0938, 299.2354,      BLDA1270
6302.3771, 305.5187, 308.6603,  311.8019, 314.9435, 318.0851,      BLDA1280
7321.2267, 324.3683, 327.5099,  330.6515, 333.7931, 336.9347,      BLDA1290
8340.0763, 343.2179, 346.3595,  349.5011, 352.6427, 355.7843,      BLDA1300
9358.9259, 362.0675, 365.2091,  368.3507, 371.4923, 374.6339/      BLDA1310
 DATA HH / 377.7755, 380.9171,  384.0587, 387.2003, 390.3419,      BLDA1320
1393.4835, 396.6251, 399.7667,  402.9083, 406.0499, 409.1919,      BLDA1330
2412.3331, 415.4747, 418.6163,  421.7579, 424.8995, 428.0411,      BLDA1340
3431.1827, 434.3243, 437.4659,  440.6075, 443.7491, 446.8907,      BLDA1350
4450.0323, 453.1739, 456.3155,  459.4570, 462.5987, 465.7403,      BLDA1360
5468.8819/                                                         BLDA1370
 END                                                               BLDA1380
```

INPUT GUIDE

The input guide is listed in Tables F1-F8.

Tables F1, F2, and F3

Table 1: Title

TEXT	20A4

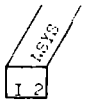


Table 2:

Columns 1-2 NSYS = number of problems

Table 3:

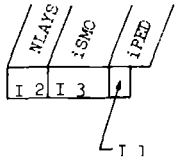
Columns 1-2 NLAYS = number of layers in pavement system

Columns 3-5 iSMO = 0 request for rough computational procedure

iSMO = 1 request for smooth computational procedure (see Note 1)

Column 6 iRED = 0 AK(i) is input in Table 4 or

iRED = 1 ALK(i) is input in Table 4



NOTE (1): The smooth calculation procedure is more stable but less efficient than the rough procedure and is used for systems with frictionless slip between the layers or for cases where numerical instabilities are expected.

Table F4

Table 4: Layer Information: One card for each layer (see Note 2)

Columns 1-10 E(i) = moduli of layer i

Columns 11-20 NU(i) = Poisson's ratio

Columns 21-30 Thick(i) = thickness of layer i

Columns 31-40 AK(i) = interface compliance

or ALK(i) = reduced interface compliance (see Note 3)

Note (2): Columns 21-40 left blank for last layer

(3): AK(i) values are generally very small thus it may be more desirable to use ALK(i) where $ALK(i) = \frac{E_i}{1 + V_i} \cdot AK(i)$

for complete adhesion between layers i and i + 1 set AK(i) = ALK(i) = 0

for almost frictionless slip between layers set $\frac{E_i}{1 + V_i} AK(i) = ALK(i) \geq 1000$

Cards for layers i=1 thru NLAYS - 1

E(i)	NU(i)	THICK(i)	AK(i) or ALK(i)
F 1 0 . 0	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0

Card for layer i = NLAYS

E(i)	NU(i)
F 1 0 . 0	F 1 0 . 0

Tables F5 and F6

Table 5:

Columns 1-2 NLOAD = number of loaded areas

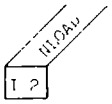


Table 6: Load information: One card for each load

Columns 1-10 LDSTRS(i) = vertical load in units of load for loaded area i

Columns 11-20 RADIUS(i) = radius of loaded area i

Columns 21-30 X(i) = abscissa of center of loaded area

Columns 31-40 Y(i) = ordinate of center of loaded area

Columns 41-50 HOSTR(i) = horizontal load in units of load for loaded area i

Columns 51-60 PSI(i) = angle of HOSTR(i) with respect to positive X-axis in degrees

LDSTRS	RADIUS	X	Y	HOSTR	PSI
F 1 0 . 0	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0

Tables F7 and F8

Table 7:

Columns 1-2 NPOS = the number of position for which stress, strains, and displacements are to be computed

NPOS
I 2

Table 8: Position data; one card for each position

Columns 1-2 LAYER(i) = layer number for position i

Columns 11-20 AX(i) = abscissa of position

Columns 21-30 AY(i) = ordinate of position

Columns 31-40 Depth(i) = depth from pavement surface to position

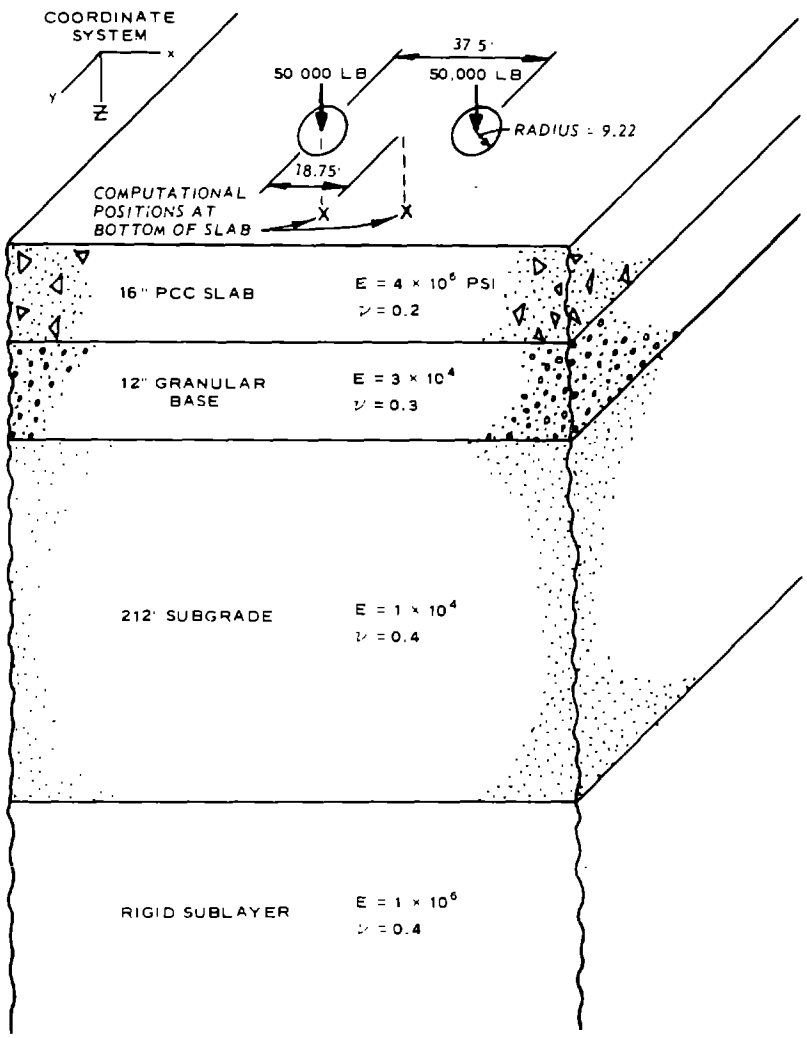
Columns 41-50 ETA(i) = angle from which position is observed with respect to the direction of the tangential loading

LAYER	AX	AY	DEPTH	ETA
I 2	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0	F 1 0 . 0

If another problem is needed return to Table 3 and repeat input thru Table 8.

EXAMPLE PROBLEM

The example problem is for the computation of the tensile stress at two locations at the bottom of a PCC slab which is subjected to the loading of a dual-wheel aircraft gear. Figure F1 shows the pavement section, the characteristics of the applied load, and the locations at which the stress is computed. The coded data for the example problem are given in Table F9, and the output in Table F10.



NOTE COORDINATE FOR COMPUTATIONAL POSITIONS
POSITION NO. 1 x = 0.0, y = 0.0, z = 12
POSITION NO. 2 x = 18.75, y = 0.0, z = 12
COORDINATES FOR LOADS
LOAD NO. 1 x = 0.0, y = 0.0
LOAD NO. 2 x = 37.5, y = 0.0

Figure F1. Diagram for example problem

Table F9

Coded Data for BISAR Program - Example Problem

GENERAL PURPOSE DATA FORM

Coded Data For BISAR Program — Example Problem

EAA R. Austin W. Barker 8/1/78

```
DUAL WHEEL PROBLEM
1
4 1.1
40.0.0.0.0.0.. 0..2 1.6 1.0.0.0.
3.0.0.0.0.. 0..3 1.2 0.
1.0.0.0.0.. 0..4 2.1.2 0.
1.0.0.0.0.0.0.. 0..4
2
50.0.0.0.. 9..2.2 0. 0. 0. 0.
50.0.0.0.. 9..2.2 3.7..5 0. 0. 0.
2
1 0. 0. 1.6 0.
1 1.8..7.5 0. 1.6 0.
```

NES FORM NO 1733
SEPTEMBER 1962

Table F10

Output for Example Problem

```
BBBBBBBBBBBB     III     SSSSSSSSSSS      AAAAAAAAA     RRRRRRRRRRR
BBBBBBBBBBBB     III    SSSSSSSSSSSS     AAAAAAAAAAA    RRRRRRRRRRRR
BB        BBB    III   SSS              AAA       AAA   RR        RRR
BB         BB    III   SS               AA         AA   RR         RR
BB       BBB     III   SSS              AA         AA   RR        RR
BBBBBBBBBBBB     III    SSSSSSSSSSS     AAAAAAAAAAAAA   RRRRRRRRRRRR
BBBBBBBBBBBB     III     SSSSSSSSSSS    AAAAAAAAAAAAA   RRRRRRRRRRR
BB       BBB     III              SSS   AA         AA   RR    RR
BB        BB     III               SS   AA         AA   RR      RR
BB       BBB     III              SSS   AA         AA   RR       RR
BBBBBBBBBBBB     III   SSSSSSSSSSSSS    AA         AA   RR        RR
BBBBBBBBBBBB     III   SSSSSSSSSSSS     AA         AA   RR         RR
```

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DUAL WHEEL PROBLEM

SYSTEM NUMBER 1

LAYER NUMBER	CALCULATION METHOD	YOUNG'S MODULUS	POISSON'S RATIO	THICKNESS	REDUCED SPRINGCOMPL
1	SMOOTH	0.4000E 07	0.2000E 00	0.1600E 02	0.1000E 04
2	SMOOTH	0.3000E 05	0.3000E 00	0.1200E 02	0.
3	SMOOTH	0.1000E 05	0.4000E 00	0.2120E 03	0.
4		0.1000E 07	0.4000E 00		

LOAD NUMBER	NORMAL STRESS	SHEAR STRESS	RADIUS OF LOADED AREA	LOAD - POSITION X	Y	SHEAR DIRECTION
1	0.1872E 03	0.	0.9220E 01	0.	0.	0.
2	0.1872E 03	0.	0.9220E 01	0.3750E 02	0.	0.

POSITION NUMBER 1

LAYER NUMBER 1

COORDINATES

X	Y	Z
0.	0.	0.1600E 02

DISTANCE TO LOAD-AXIS(1) 0.

THETA 0.

DISPLACEMENTS

RADIAL	TANGENTIAL	VERTICAL
0.	0.	0.1738E-01

STRESSES

RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
0.2675E 03	0.2675E 03	-0.3536E 01	0.	0.	0.

STRAINS

RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
0.5368E-04	0.5368E-04	-0.2763E-04	0.	0.	0.

DISTANCE TO LOAD-AXIS(2) 0.3750E 02

THETA 0.3142E 01

DISPLACEMENTS

RADIAL	TANGENTIAL	VERTICAL
0.8756E-03	0.	0.1407E-01

STRESSES

RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
0.3544E 02	0.1002E 03	-0.1551E 01	0.	-0.6448E 00	0.

STRAINS

RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
0.3930E-05	0.2335E-04	-0.7169E-05	0.	-0.1934E-04	0.

	XX	YY	ZZ	XZ	XZ	XY	UX	UY	UZ
TOTAL STRESS	0.303E 03	0.368E 03	-0.509E 01	0.	0.645E 00	0.			
TOTAL STRAIN	0.576E-04	0.770E-04	-0.348E-04	0.	0.193E-06	0.			
TOTAL DISPLACEMENT							-0.876E-03	0.	0.314E-01

PRINCIPAL VALUES AND DIRECTIONS OF TOTAL STRESSES AND STRAINS

	NORMAL STRESS	NORMAL STRAIN	SHEAR STRESS	SHEAR STRAIN	X COMPONENT	Y COMPONENT	Z COMPONENT
MAXIMUM	0.368E 03	0.770E-04			0.	1.000	0.
MINIMAX	0.303E 03	0.576E-04			1.000	0.	0.002
MINIMUM	-0.509E 01	-0.348E-04			-0.002	0.	1.000
MAXIMUM			0.188E 03	0.559E-04	0.001	0.707	-0.707
	0.181E 03				-0.001	0.707	0.707
MINIMAX			0.158E 03	0.462E-04	0.709	0.	-0.706
	0.149E 03				0.706	0.	0.709
MINIMUM			0.324E 02	0.971E-05	-0.707	0.707	-0.001
	0.335E 03				0.707	0.707	0.001

STRAIN ENERGY 0.2297E-01

STRAIN ENERGY OF DISTORTION 0.1190E-01

POSITION NUMBER 2

LAYER NUMBER 1

COORDINATES

X	Y	Z
0.1875E 02	0.	-0.1600E 02

DISTANCE TO LOAD-AXIS(1) 0.1875E 02 — THETA 0.

	RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
DISPLACEMENTS	0.6985E-03	0.	0.1613E-01			
STRESSES	0.1043E 03	0.1694E 03	-0.2292E 01	0.	-0.7379E 00	0.
STRAINS	0.1771E-04	0.3725E-04	-0.1426E-04	0.	-0.2214E-06	0.

DISTANCE TO LOAD-AXIS(2) 0.1875E 02 — THETA 0.31426 01

	RADIAL	TANGENTIAL	VERTICAL	RAD./TANG.	RAD./VERT.	TANG./VERT.
DISPLACEMENTS	0.6985E-03	0.	0.1613E-01			
STRESSES	0.1043E 03	0.1694E 03	-0.2292E 01	0.	-0.7379E 00	0.
STRAINS	0.1771E-04	0.3725E-04	-0.1426E-04	0.	-0.2214E-06	0.

	XX	YY	ZZ	YZ	XZ	XY	UX	UY	UZ
TOTAL STRESS	0.209E 03	0.339E 03	-0.458E 01	0.	0.	0.			
TOTAL STRAIN	0.354E-04	0.745E-04	-0.285E-04	0.	0.	0.			
TOTAL DISPLACEMENT							0.	0.	0.323E-01

PRINCIPAL VALUES AND DIRECTIONS OF TOTAL STRESSES AND STRAINS

	NORMAL STRESS	NORMAL STRAIN	SHEAR STRESS	SHEAR STRAIN	X COMPONENT	Y COMPONENT	Z COMPONENT
MAXIMUM	0.339E 03	0.745E-04			0.	1.000	0.
MINIMAX	0.209E 03	0.354E-04			1.000	0.	0.
MINIMUM	-0.458E 01	-0.285E-04			0.	0.	1.000
MAXIMUM			0.172E 03	0.515E-04	0.	0.707	-0.707
	0.167E 03				0.	0.707	0.707
MINIMAX			0.107E 03	0.320E-04	0.707	0.	-0.707
	0.102E 03				0.707	0.	0.707
MINIMUM			0.651E 02	0.195E-04	-0.707	0.707	0.
	0.274E 03				0.707	0.707	0.

STRAIN ENERGY 0.1638E-01

STRAIN ENERGY OF DISTORTION 0.9015E-02

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16. Abstract

The development and formulation of a design procedure for rigid airport pavements are presented. The design criteria used in the procedure are based on the tensile stress in the portland cement concrete (PCC) slab as computed by layered elastic theory and the strength of the PCC slab as measured in the flexural beam test. The criteria were developed by the analysis of some 60 test sections. Procedures are given for the characterization of the pavement materials both by laboratory testing and by typical values and/or correlation studies. The thickness requirements as determined by the new criteria are compared with the thickness as determined by present Corps of Engineers-Federal Aviation Administration design procedures.

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